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THE SUPPORTING POWER OF PILES.*

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With Discussion by Messrs. E. Sherman Gould, Horace J. Howe, Joseph P. Carlin and Ernest P. Goodrich.

The use of piles in the construction of foundations dates from primitive times. The villages of the prehistoric tribes dwelling on the lakes in Switzerland were almost entirely on piles. The very existence of Venice depends on piles, and modern engineering makes use of millions of piles each year.

Formerly, piles were simply sticks of wood stuck into the mud, but modern engineering makes use of iron and steel piles, and of other formations such as the so-called sand and concrete piles. These are used for various purposes. They often stand with one end upon a firm footing and the other supporting the mass to be carried. In this case the pile acts simply as a column with more or less lateral support.

Sometimes, piles are used simply to make more compact the soil into which they are driven, through the natural compression suffered by the earth which is displaced by the penetration of the pile. The bearing power of the soil is thus increased, about in proportion to the

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ratio of the sectional area of the piles driven, over a given area, to the latter. Usually, however, the piles act directly against the forces to be sustained, and are held in place simply by the resistance of the soil into which they penetrate. This resistance, under increase of the external forces, may prove too small to equilibrate the latter, and movement of the pile thereby results. Piles of the latter class are used sometimes in close clusters, sometimes in close rows, but usually singly. They may be square, hexagonal, round, or of other sections; may be uniform in size, or tapering; pointed, or blunt, etc. They may be put in place by means of water jets, or by continuous pressure, or by the rapid blows of a steam or gunpowder driver, or the slower, more measured blows of the ordinary gravity hammer.

This discussion will be confined solely to the use of the common, tapering, round, wooden pile, driven by the ordinary gravity hammer, which is raised by a rope on a steam drum, falls by its own weight upon the head of the pile and thereby forces it to its destined place, where it is held simply by the contact of the surrounding soil. Further, only the extreme sustaining power of the quiescent loads of piles immediately after being driven as above described, and in such positions as to allow each pile to act by itself, will be discussed. In these times, the determination of this question is very important, and, though considerable has been done along this line, the subject still remains in an undeveloped state as compared with most similar modern engineering questions, and of necessity, perhaps, must ever remain so.

An instructive experiment, throwing some light on the nature of the support given to a pile by the earth into which it is driven, is to take a box with a glass front, fill it with lightly compacted sand, and push down between the glass and the earth half-round sticks, similar to piles in shape. This experiment discloses the fact that a compact cone of earth is formed under the foot of a blunt stick, and remains there, being pushed forward through the ground as the stick descends. This cone acts exactly like the sharp end of a pointed stick or pile. It will always form under any load which soil is required to carry, and, consequently, the bearing power of the latter is one of the elements helping in the support of a loaded pile.

In most soils this bearing power is known to increase with increase in depth below the surface, and it would, therefore, be expected that the bearing power of a pile would increase as it is driven deeper and deeper into the ground. Therefore, if a curve were to be constructed, showing the supporting power of a pile for different penetrations, it should, theoretically, start from a point, the co-ordinates of which are penetration = zero, supporting power equals supporting power of soil, and be a straight line, inclined at some angle with the axes of reference, depending upon the rate of increase in the bearing power of the soil.

The penetration of the stick into the sand discloses, around the pointed end, flow lines along which the earth moves as it is pushed aside and compressed by the penetration of the stick. The extent of the region throughout which movement occurs in this way depends upon the compressibility of the soil at that point. Theoretically, this compressibility should decrease with increase of depth beneath the surface, but the actual variation is so slight as in nowise to affect the supporting power of piles. The displaced soil in contact with the pile is pressed against the latter by its own elasticity and by the influence of the stresses in the surrounding earth. Theoretically, the resistance to motion of the pile from this source should increase directly as the depth below the surface, so that the curve of supporting power already described must be compounded with a second straight line, starting from the origin, and slanting at a certain angle with the axes of reference, depending upon the amount of friction observed. The actual amount of this frictional resistance varies greatly with many circumstances.

The fact that piles enter loose ground and mud with equal penetrations for equal blows, however far driven, would tend to show that the frictional resistance was so small as not to be observable, and that the main resistance was in this case the supporting power of the ground beneath the pile. On the other hand, where the pile has been driven between stones or logs into hard or compact soil, the frictional resistance offered must often be far greater than all other means of support. The above-mentioned small frictional resistance, however, may be greatly increased after lapse of time by the compacting of the earth around the pile from the driving of others near it, or through the natural settlement back against the pile of the soil forced away while it was being driven.

When a pile is supported entirely by the frictional resistance, the actual region supporting the load is some deep ground level at which the frictional resistance holding the pile has been transferred through the earth in the shape of a conoid of pressure, the base of which gives a total bearing value equal to the load and a unit bearing value which the earth at that lower level will support. Each kind and degree of compactness of earth will give a different angle for the slope of the conoidal surface. When the frictional resistance is relatively small, more of the pile must be in the ground, or the pile will settle through the immediately surrounding earth. Under excessive load, if the bearing power of the earth is small and the frictional resistance rather high, the pile will carry down the earth surrounding and in contact with it.

As the exact nature of the soil is seldom known, any criterion based upon the same would be valueless in determining the supporting power of a pile. The engineer can be guided by any information he may have on this point only in the spacing of the piles one from another. When the piles are held by the real bearing power of the soil they may be driven solidly if thought best, and in any case they will act simply as columns. When supported by frictional resistance, they must be driven so far apart, or to such a depth, that the increased area of bearing developed by the conoid of pressure having the required altitude of frictional resistance meets a level which will afford the required support before intersecting the conoid of a neighboring pile.

No criterion for the supporting power can be determined with absolute certainty, because one cannot say that after whatever test may be made, a pile will act under the next identically similar test in exactly the same way. The probabilities are very great, however, that if similar conditions are observed as far as possible, a pile under two similar tests will act in nearly the same manner, and with perhaps a little less probability that two neighboring piles will act alike when subjected to similar tests.

If a pile is found to begin to sink under a given quiescent load applied several days after it has been put in place, it will probably begin to sink a second time at nearly the same load if similarly applied. The probabilities are nearly as great that a neighboring pile, showing the same phenomena while being driven, will carry the same load. This probability must, therefore, be the basis for any practical determination of the question. Of course, the most satisfactory way is to actually test as many as possible of the piles under question with quiescent loads.

It is a great pity that more numerous determinations of this character have not been made in the past, or, if made, that the records of the same have not become available to engineers. Almost all the information so far published is to be found in "Piles and Pile Driving," edited by the late A. M. Wellington, M. Am. Soc. C. E., who quotes extensively from a compilation* by John C. Trautwine, Jr., Assoc. Am. Soc. C. E., and to which attention is herewith especially directed. Another valuable paper† is entitled "Some Instances of Piles and Pile-driving, New and Old," by Horace J. Howe, M. Am. Soc. C. E.

As no criterion for supporting power can be obtained from the soil, the phenomena observable during the driving of a pile must be examined.

A study of the various phenomena involved in the blow of a falling body striking a pile and forcing it into the ground is intricate. The relation of the phenomena to the ultimate supporting power of the pile is at best quite uncertain. It is the relation of a case of impact to one of quiescent pressure.

A body free to fall under the influence of the attraction of the earth does so with an accelerated motion, thereby increasing in kinetic energy. If this body strikes another, both are compressed to amounts depending upon their elasticity, and the second is set in motion when the amount of compression has reached such a point that its force overcomes whatever force of hindrance may exist. The latter force tends to bring the second body to rest, or both bodies, provided their relative elasticities are such that they have not separated. This force may be constant or variable, and probably at the last instant of its action its value would equal in amount the ultimate resistance offered by this second body to being moved by a slowly and steadily increasing force.

The following mathematical treatment is believed to cover as thoroughly as possible the various phases of the problem.

Let $V =$ velocity of hammer at pile head (actual);

$g =$ acceleration due to gravity;

$h =$ height of fall of hammer.

Then $V = (2\,g\,h)^{\frac{1}{2}}$, theoretically.

* *Transactions*, Am. Soc. C. E., Vol. xxvii, pp. 148–160.

† *Journal of the Association of Engineering Societies*, Vol. xx, p. 257.

Let $V = (b\,h)^m + x$, practically, as found by experiment;

w = velocity of hammer and pile at instant when both move together.

Then $V - w$ = velocity lost by hammer.

Let M_h = mass of hammer.

Then $M_h\,g$ = weight of hammer.

Let W_h = weight of hammer.

Then $\frac{W_h}{g} = M_h$.

Let M_p = mass of pile;

W_p = weight of pile;

M_g = mass of earth moved in connection with pile;

W_g = weight of earth moved in connection with pile.

Then $\frac{W_p + W_g}{g} = M_p + M_g$;

$\frac{W_h}{g}\,V$ = momentum of hammer at pile head;

$\frac{W_h}{g}\,(V - w)$ = momentum lost by hammer;

$\frac{W_p + W_g}{g}\,w$ = momentum gained by pile and earth moved by it.

But $\frac{W_h}{g}\,(V - w) = \frac{W_p + W_g}{g}\,w$;

$$w = \frac{W_h}{W_h + W_p + W_g}\,V.$$

Let $\frac{W_h}{W_h + W_p + W_g} = R_w$, be ratio of weight of hammer to that of pile and hammer combined.

Then $w = R_w\,[(b\,h)^m + x]$.

Let u = varying velocity of pile after instant pile and hammer move together.

Let $u = (K\,w)^r + y$.

Then $u_o = \{K\,R_w\,[(b\,h)^m + x]\}^r + y$ is initial velocity.

Let t = time occupied in stopping motion of pile;

p = penetration of pile;

$p = C\,t - z$, be law of variation of penetration with time.

Then $\frac{d\,p}{d\,t} = u$.

$u = n\,C\,t^{n-1}$.

$\frac{d\,u}{d\,t} = n\,(n - 1)\,C\,t^{n-2}$ is acceleration at any instant.

Let f = force bringing pile to rest.

Then $f = (M_h + M_p + M_g) \frac{d\,u}{d\,t}$.

M_h will equal zero if hammer and pile separate immediately after instant both move as one.

$$f = (M_h + M_p + M_g)\, n\,(n-1)\, C t^{n-2},$$

$$\frac{f}{u} = \frac{(M_h + M_p + M_g)\, n\,(n-1)\, C t^{n-2}}{n\, C t^{n-1}}.$$

$$\frac{f}{u} = (M_h + M_p + M_g) \frac{n-1}{t}.$$

$$t = (M_h + M_p + M_g) \frac{(n-1)\, u}{f}.$$

$$\frac{u}{p+z} = \frac{n\, C t^{n-1}}{C t^n}.$$

$$\frac{u}{p+z} = \frac{n}{t}.$$

$$t = \frac{n\,(p+z)}{u}.$$

$$(M_h + M_p + M_g) \frac{(n-1)\, u}{f} = \frac{n\,(p+z)}{u}.$$

$$f = (M_h + M_p + M_g) \frac{(n-1)}{n} \frac{u^2}{p+z}.$$

Let f_o = initial force.

Then $f_o = (M_h + M_p + M_g) \frac{n-1}{n} \frac{u_o^2}{p+z}$.

$$f_o = (M_h + M_p + M_g) \frac{n-1}{n} \times \frac{\{K R [(b\, h)^m + x]\}^r + y}{p+z}$$

Let z = penetration lost through crushing of head, heating of head, compressing pile and hammer;

v = work done in crushing head, heating head, etc.;

q = quantity of work done in compressing pile and hammer.

When the hammer strikes the pile, the pressure between the pile and the hammer, which tends to again separate the same, will increase from zero up to the value of f_o, at which instant the pile as a whole will begin to move.

Let $a f_o$ be average of force up to instant pile and hammer move together;

e' = coefficient of elasticity of material of hammer;

L' = average length of hammer.

Then $\frac{L'}{e'}$ = total compression per unit of compressive force for each unit area.

Let s' = average sectional area of hammer;

p' = average force throughout length of hammer tending to compress same at each instant.

Then $\frac{p'}{s'}$ = force per unit area of hammer;

$\frac{p'\,L'}{s'\,e'}$ = total compression suffered by hammer;

$a f_o \frac{p'\,L'}{s'\,e'}$ = work done in compressing hammer.

Let $p' = c' f_o$;

Then $a\,c' f_o^2 \frac{L'}{s'\,e'}$ = work done in compressing hammer.

Let e = coefficient of elasticity of material of pile;

L = average length of pile;

s = average sectional area of pile;

c = fraction denoting portion of f equaling average pressure throughout length of pile tending to compress same.

Then $a\,c\,f_o^2 \frac{L}{s\,e}$ = energy consumed in compressing pile;

$$q = a f_o^2 \left(\frac{c\,L}{s\,e} + \frac{c'\,L'}{s'\,e'}\right);$$

$$z = \frac{q + v}{f_o};$$

$$z = a f_o \left(\frac{c\,L}{s\,e} + \frac{c'\,L'}{s'\,e'}\right) + \frac{v}{f_o};$$

$$f_o = (M_h + M_p + M_g)\,\frac{n - l}{n} \times \frac{[\{K R_w [(b\,h)^m + x]\}^r + y]^2}{p + a f_o \left(\frac{c\,L}{s\,e} + \frac{c'\,L'}{s'\,e'}\right) + \frac{r}{f_o}}$$

Let F = final force;

$$F = j f_o;$$

$$F = \frac{-j\,p}{2a\left(\frac{c\,L}{s\,e} + \frac{c'\,L'}{s'\,e'}\right)} + \frac{j}{2a\left(\frac{c\,L}{s\,e} + \frac{c'\,L'}{s'\,e'}\right)} \times$$

$$\sqrt{p^2 + 4a\left(\frac{c\,L}{s\,e} + \frac{c'\,L'}{s'\,e'}\right)\left(\frac{W_h + W_p + W_g}{g}\right)\frac{n-1}{n} \times \left[\{K R_w [(b\,h)^m + x]\}^r + y\right]^2 - 4 a v \left(\frac{c\,L}{s\,e} + \frac{c'\,L'}{s'\,e'}\right)}.$$

This expression is exceedingly long and unwieldy, but, before discussing how it may be shortened, a glance at its relation to other proposed formulas will be of interest. Rudolph Hering, M. Am. Soc. C. E., in a monograph entitled "Bearing Piles,"* has collected and collated fourteen different formulas. Those there given, together with some others, are here reproduced for reference, together with their derivation from the writer's formula by the substitution of certain values for the unknowns.

Trautwine: $60\ W_h\ \sqrt[3]{h}$ (if p is very small).

$$\frac{5 W\ \sqrt[3]{h}}{p+1}.$$

McAlpine: $80\ \{W_h + (0.228 \sqrt{h} - 1)\ 2\ 240\}.$

Rankine: $2\left\{\frac{-S e p}{L} + \sqrt{\frac{S e W_h h}{L} + \frac{s^2 e^2 p^2}{L^2}}\right\}.$

Weisbach:

$$\frac{s' e' s e}{c L s' e' + c' L' s e}\left(\sqrt{p^2 + 2\frac{c L s' e' + c' L' s e}{s' e' s e} W_h h} - p\right).$$

Redtenbacher: $-\frac{e p}{L} + \sqrt{\frac{2 e W_h^2 h}{s L (W_h + W_p)} + \left(\frac{e p}{L}\right)^2}.$

Brix and Becker: $\frac{W_h^2 h W_p}{p (W_h + W_p)^2}.$

Weisbach: $\frac{W_h^2 h}{p (W_h + W_p)} + (W_h + W_p).$

Weisbach: Mason: $\frac{W_h^2 h}{p (W_h + W_p)}.$

Weisbach: Sanders: Molesworth: $\frac{W_h h}{p}$. (Sanders gives $3 p$.)

Nystrom: $\frac{W_h^3 h}{p (W_h + W_p)^2}.$

Baker:

$$\sqrt{W_h h \frac{12 s' e' s e}{3 L' s e + 4 L s' e'} + \frac{36 p^2 s'^2 e'^2 s' e'}{(3 L^1 s e + 4 L s' e)^2}} - \frac{6 p s' e' s e}{3 L' s e + 4 L s' e'}.$$

* Eng. News Pub. Co., New York, 1878.

Wellington: (*Engineering News* formula.) $\frac{W_h h}{p+1}$.

Crowell: $\frac{W_h h}{p + 0.1 + \text{etc.}}$.

Hurtzig: $\sqrt{500\ W_h h + (250\ p)^2} - 250\ p$.

A glance at these formulas reveals the following facts:

Most authors neglect friction in the fall of the hammer, so that in the writer's formula $b = 2g$, $m = \frac{1}{2}$, $x = 0$.

Most authors consider that the pile and the hammer do not separate; then $u = w$, and $r = 1$, $K = 1$, $y = 0$.

All neglect energy lost by heating, etc.; then $v = 0$.

They consider, without so stating, that initial and final forces are equal; then $j = 1$.

They also consider, without so stating, that the penetration varies as the square of the time; then $n = 2$.

They also consider $W_g = 0$.

With these assumptions:

$$F = \frac{-p}{2a\left(\frac{cl}{se} + \frac{c'\,l'}{s'\,e'}\right)} + \frac{1}{2\,a\left(\frac{cl}{se} + \frac{c'\,L'}{s'\,e'}\right)}\sqrt{p^2 + 4\,a\left(\frac{cL}{se} + \frac{c'\,L'}{s'\,e'}\right) W_h h R_w}.$$

Weisbach assumes, besides the foregoing, that the pressure is uniform throughout the pile and the hammer; then $c' = c = 1$. He also assumes that the pressure increases uniformly from zero up to f, then $a = \frac{1}{2}$. His formula, therefore, is

$$F = \frac{s'\,e'\,s\,e}{c\,L\,s'\,e' + c'\,L'\,s\,e}\left(\sqrt{p^2 + 2\,\frac{c\,L\,s'\,e' + c'\,L'\,s\,e}{s'\,e' + s\,e}\,W_h h} - p\right).$$

Rankine neglects compression in the hammer, that is, $e' = \infty$. He also assumes that the pressure throughout the pile varies uniformly; that is, $c = \frac{1}{2}$. He also assumes $a = \frac{1}{2}$.

His formula, therefore, is

$$F = \sqrt{\frac{4\,s\,e\,W_h h}{L} + \frac{4\,s^2\,e^2\,p^2}{L^2}} - \frac{2\,s\,e\,p}{L}.$$

Baker assumes that the pressure in the hammer varies uniformly; that is, $c' = \frac{1}{2}$; and he takes $a = \frac{1}{2}$. He also assumes c to be $\frac{2}{3}$, stating that the average pressure is probably below the center of the pile.*

His formula then becomes,

$$F = \sqrt{W_h\, h \frac{12\, s'\, e'\, s\, e}{3\, L' s\, e + 4\, L\, s' e'} + \frac{36\, p^2\, s'^2 e'^2 s' e'}{(3\, L'\, s\, e + 4\, L s'\, e')^2}} - \frac{6\, p\, s' e' s\, e}{3\, L' s e + 4\, L\, s' e'}.$$

This he reduces to

$F = 100 \left(\sqrt{W_h h + (50\, p)^2} - 50\ p\right)$ by assuming 5 000 for the quantity $\frac{6\, s' e' s\, e}{3\, L'\, s\, e + 4\, L\, s'\, e'}$.

Hurtzig's formula is the same as the latter, except for the numerical quantities.

Redtenbacher assumes the first mentioned values for all constants except K, and also that $e' = \infty$, $\frac{2\, a\, c}{s} = 1$. He makes $K = \frac{\sqrt{2}}{\sqrt{s}}$, and finds that

$$F = \frac{-p\, e}{L} + \sqrt{\left(\frac{p\, e}{L}\right)^2 + \frac{2\, e\ W_h^2\, h}{s\, L\, (W_h + W_p)}}.$$

Many authors neglect the compression of the pile and hammer entirely; then $e = e' = \infty$. Letting, also, $W_g = 0$, $v = 0$, $j = 1$, $n = 2$, $a = \frac{1}{2}$, as before, and as most writers do, we have

$$F = \frac{W_h + W_p}{2\, p\, g} \left[\left\{ K\, R_w \left[(b\, h)^m + x\right] \right\}^r + y \right]^2.$$

Nystrom's work seems to take $x = 0$, $y = 0$, $K = 1$, $r = \frac{3}{2}$, $m = \frac{1}{3}$, $b = 2\, g$; that is, in fact, that the friction is such as to make the velocity of fall vary as the cube root of the height and u vary as $W^{\frac{3}{2}}$. His formula is, then,

$$F = \frac{W_h^3\, h}{(W_h + W_p)^2\ p}.$$

Again, if $u = \sqrt{R_w\, w}$, and $V = 2\, g \left(h + \frac{p}{R^2_w}\right)$, then $y = 0$, $r = \frac{1}{2}$, $K = R_w$, $m = 1$, $b = 2\, g$, $x = \frac{2\, p\, g}{R_w^{\ 2}}$, $F = \frac{W_h^2\, h}{p\ (W_h + W_p)} + W_h + W_p$, which is one of Weisbach's forms. He reaches it through an entirely different course of reasoning, however.

* "Treatise on Masonry Construction," page 237.

Brix and Becker assume $x = 0$, and $K = \frac{R_{x} W_p}{W_h + W_p}$, instead of as in the last case, and find $F = \frac{W_h^2 W_p h}{(W_h + W_p)^2 p}$.

Mason's work assumes no friction; that is, $b = 2g$, $m = \frac{1}{2}$, also, $r = 1$, $K = 1$, $x = 0$, $y = 0$. This gives

$$F = \frac{W_h^2 h}{(W_h + W_p) p},$$

which is also one of the forms given by Weisbach.

Many authors neglect the weight of the pile entirely. If, in the last case, $W_p = 0$, $F = \frac{W_h h}{p}$, which is nearly Sander's formula, and Weisbach's final form.

Another group of formulas, having a constant besides p in the denominator, may be obtained as follows:

As before, let $v = 0$, $j = 1$, $n = 2$, $a = \frac{1}{2}$, $x = 0$, $y = 0$, $K = 1$, $r = 1$, $b = 2g$, $W_g = 0$. And, also, if the pile and hammer are such that $\frac{c L}{s e} + \frac{c' L'}{s' e'} = \frac{2 N}{f_o}$, or $N = \frac{g}{f_o}$, then the value of F is

$$\frac{W_h + W_p}{2 g} \left\{ \frac{W_h}{W_h + W_p} (2 g h)^m \right\}^2 \frac{1}{p + N}.$$

Of course, N may be assumed as a quantity of any degree of complexity

If, now, $m = \frac{1}{2}$ and $W_p = 0$, $F = \frac{W_h h}{p + N}$, which is practically Crowell's form.

If the velocity varies as the sixth root of the fall, $m = \frac{1}{6}$, and if the weight of the pile be neglected, $W_p = 0$, and if $N = 1$,

$F = \frac{\text{constant } W_h \sqrt[3]{h}}{p + 1}$, which is nearly Trautwine's formula.

If friction be neglected, $m = \frac{1}{2}$, then if $W_p = 0$, and $N = 1$, $F = \frac{W_h h}{p + 1}$, which is really Wellington's or the *Engineering News* formula, except for the factor of safety.

Having thus obtained a general formula, which, in one or another of its modified forms, is the one used by different authors in determining the supporting power of a pile, it remains to be seen what effects the various assumptions made by them actually have on the results obtained.

The wide variation observable proves conclusively that some of the assumptions made are seriously in error. Actual experimental determination of the various quantities entering the theoretical formula is the only way to find the true value in a given case. The writer's formula, being entirely theoretical, involves many quantities, the values of which have not hitherto been ascertained. The first efforts made by the writer along this line were to find the law of variation of force, the time interval occupied by the blow of the hammer, and the movement of the pile. In other words, the values of n and t were to be ascertained.

In the report of Brevet Lieut.-Col. James L. Mason, Corps of Engineers, U. S. A., concerning the foundation of Fort Montgomery, the following statements appear:

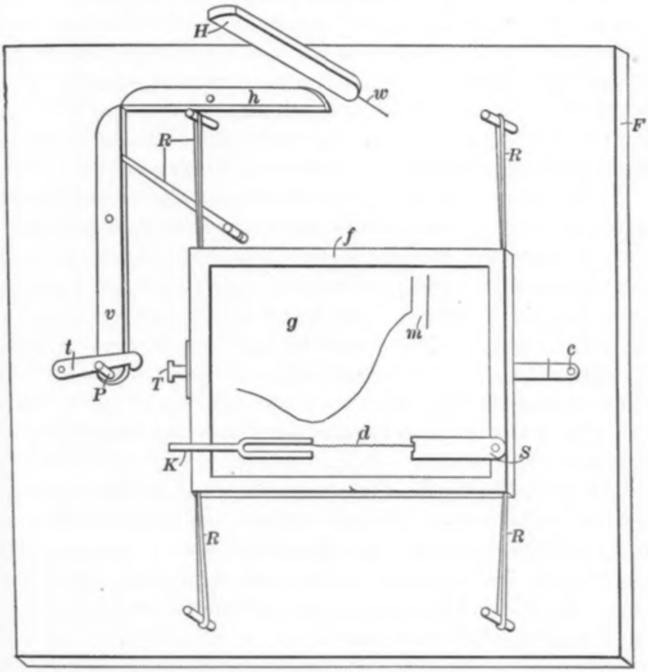
"The calculation (of the supporting power of a pile) is very easy, by the supposition that the retarding force is constant during the very short period that it takes to destroy the motion of the pile. This time was not measured. To measure it might be a difficult mechanical problem. Now, the variation of the force may be according to a simple or complex law. Its variations may be steadily in one direction, that is, constantly (not uniformly) to increase or constantly to diminish the intensity that the force had at the beginning of the motion, or they may be the reverse, tending during one portion of the time to increase, and at another to diminish the value of the force at the commencement of the motion."

In connection with the construction of the sub-foundation of the Sherman Statue, situated just south of the Treasury Building in Washington, D. C., a machine was contrived which should show besides the exact amount of the vertical motion of the pile, the time occupied by this motion, the velocity of the hammer as it struck the pile, and of the pile at each instant of its movement, and also the amount of compression suffered by the head of the pile from the blow of the hammer.

This apparatus (Fig. 1) consisted of a frame which could be held near the head of the pile. It was at first held in position by a vertical leg resting on the ground and two braces at right angles fastened to the frame and resting on the ground. Later, it was held by suitable arms resting against the guides in which the hammer of the pile-driver moved.

The second method was more readily managed, but the apparatus

RECORDING DEVICE FOR PILE DRIVER.



F. Frame of Apparatus.	*R.R.* Rubber Bands.
f. Frame of Smoked glass.	*K.* Tuning fork.
g. Smoked glass.	*S.* Strip to release fork.
c. Catch.	*P.* Peg to raise trigger.
t. Trigger.	*H.* Stick in hammer.
T. T-shaped dog.	*w.* Wire to mark on glass.
v. Vertical lever.	*m.* Marks of wire.
h. Horizontal lever.	*d.* Sinusoid,

Fig. 1.

suffered somewhat from the jar of the frame-work of the derrick. Moving in front of this frame was a piece of smoked glass, about 6 by 8 ins. on a side, rigidly held in a light wood frame. This latter was supported by four rubber bands fastened to the four corners of the frame surrounding the glass, and stretched vertically above and below those corners to four pegs set in the frame of the apparatus. These rubber bands could be so adjusted on the several pegs that the glass might move horizontally, parallel with the frame, and when drawn aside and released would vibrate from side to side horizontally, moving in a vertical plane parallel with the frame of the apparatus.

At one side was a catch to stop and hold the glass at the end of a single vibration. On the opposite end of the frame holding the glass was a T-shaped metal dog serving to engage the trigger which, when tripped, released the frame, allowing it to vibrate after it had been drawn to that side and set. The trigger was simply a notched piece of metal pivoted on a level with the dog. This trigger was released by a peg in the end of a vertical lever, so pivoted that, as the peg moved around the fulcrum, it would raise the trigger. The upper end of the lever was held against the end of a second horizontal lever by a strong rubber band.

This second lever was pivoted near its center, so that, when its free end was lowered, its outer end released the vertical lever which in turn released the trigger, thus allowing the glass to make a single half vibration, and be caught by the catch on the other side of the frame. The horizontal lever was struck, as the hammer fell, by a stick projecting from a hole in the latter.

Projecting from the end of the stick was a wire which made a mark on the smoked glass as the hammer fell and as the glass moved. A tuning-fork was placed horizontally in front of the glass, so that a wire fastened to one of the prongs would scratch a path on the glass as the latter moved. When the glass had been set, and was held by the trigger, the prongs of the tuning-fork were pressed together and held in that position by a strip of metal with a square notch in its end, into which the prongs fitted. The other end of this strip was fastened to the frame holding the glass, so that when the latter began its half vibration the strip was pulled off, allowing the fork to vibrate and thus trace a sinusoid upon the smoked glass. The horizontal motion of the glass, theoretically, would be accelerated first positively and then negatively, but the sinusoid traced by the fork showed an almost absolutely uniform motion. From a test, the fork was afterward found to vibrate 500 times a second.

In each observation taken with the machine, the hammer was allowed to remain upon the head of the pile after a blow, and the apparatus was then so adjusted that the wire in the end of the stick attached to the hammer was about 2 ins. below the top of the smoked glass. The hammer was then raised to the desired height, the trigger set, and the hammer allowed to fall. The slant of the line upon the glass, from its top down to the point where the hammer struck the pile, would measure the velocity with which the hammer struck.

In the observations here described, this could not be relied upon to give true results because of the large amount (about 2 ins.) of lateral motion possible by the hammer in the guides. At the instant of striking the pile, the slant of the line, theoretically, should change, making a greater angle with the vertical, thus showing the reduced velocity of the system composed of the pile and hammer. As to the remainder of the motion, nothing could be predicted, as no one knew the law of force according to which the pile moved. However, theory predicted that there would be a slight rise in the curve after becoming horizontal, due to the reaction against the compression of the head of the pile, due to the impact of the hammer.

A study of the forms of the curves obtained is very instructive, and brings to light many facts as to the action of piles under the effect of the blow of a pile-driver hammer. What may be called a typical diagram is shown in Fig. 3. In it, the line, $A\ B$, is that made by the hammer in the last 2 ins. of its fall; and its slant, tan. $A\ B'\ D'$, should give the velocity of the hammer when it struck the pile. This point of striking is at B. The curve, $B\ C\ D$, shows the velocity of the pile at each instant of its descent. The vertical distance, $B\ D'$, shows the penetration of the pile, $B\ E'$, and the compression, $E'\ D'$, suffered by the head of the pile under that blow.

The time occupied is measured by the horizontal distance, $D'\ D$, measured in terms of the sinusoid above. At the point, D, the pile ceased to descend, and the vertical distance, $E\ D''$, shows the compression which the head of the pile underwent, as measured by the reaction of the pile head. This reaction occupied an interval of time measured by the horizontal distance, $D\ D''$. At the point, E, the hammer left the pile in the rebound, and the line, EF, was drawn in the irregular motion of the rebounding hammer. According to this diagram, the velocity of the hammer was about 57 ft. a second, corresponding to a fall of 59 ft., whereas, in reality, the fall was 20 ft. The pile sunk 2.56 ins., and occupied about $\frac{235}{5000}$, or 0.047, second in doing so. The compression of the head was about 0.05 in. and the reaction took place in about $\frac{30}{5000}$, or 0.006, second.

Fig. 4 is a diagram taken on a pile with a very much broomed head. The amount of compression suffered by the pile is here very large, and, consequently, the point at which the hammer struck the pile is nearly lost because the motion was first taken up by the head; and the pile, as a whole, partook of the motion only gradually. Figs. 3 and 4 were taken from piles when their points were penetrating a stratum of soft, dry, loamy clay; and the diagrams are very smooth and regular. From these two figures, it would appear that the law of variation of velocity is such that the penetration, measured from the deepest point, varies as the square of the time measured from the final instant. That is,

$$p \propto t^2, \quad \text{or} \quad p = c\,t^2.$$

If c be chosen as 0.3, on the scale of the diagram,

$$p = 0.3\,t^2,$$

and the dotted curve of Fig. 3, which shows the locus of this equation, coincides almost exactly with the diagram. Necessarily, the value of this constant will change for different kinds and degrees of compactness, density, etc., of the soil being penetrated, and with the size and shape of the pile being driven.

Figs. 5, 6 and 7 are diagrams taken when the point of the pile was penetrating more heterogeneous material, clay mixed with gravel. Fig. 7 is an interesting study of the action of a pile penetrating such a stratum. For the first two or three thousandths of a second, the pile did not move perceptibly, but almost immediately increased, with a rapid acceleration, until it had sunk about 0.85 in., when it encountered an increased resistance which slackened its speed to some extent. It gained slightly in velocity as it penetrated that obstacle, but rapidly fell off after a few thousandths more, and came to rest in obedience to the law above enunciated, after sinking 2.20 ins. in all, and occupying about 0.034 second in the process.

Occasionally, a pile would appear to sink with a uniform velocity, as shown in Fig. 8. This velocity sometimes appeared to be exactly that of the falling hammer, and the diagram would take the form shown in Fig. 9. It would seem as if the pile were penetrating a soft stratum and suddenly struck some obstacle, for it was several times observed that the diagram of the next blow was similar to Fig. 7.

Fig. 10 is a diagram of a type occasionally encountered. It seems to be simply a modification of Figs. 8 and 9. From the position of the apparatus, it was evident that the hammer struck the pile at a point near the star in the figure. The pile seemed to respond instantly to the hammer, but to fall off to some extent in velocity, down to the point when it encountered the hard stratum. The foregoing diagrams were all taken from among the last three or four blows which each pile received, and in every case it seemed to be sinking about the same distance under each of any number of similar blows, however deep it was driven.

At a later date, a heavy cylinder was mounted upon pivots and made to rotate rapidly by the act of pulling a string wound around one end, as a top is spun (Fig. 11). This rapidly rotating cylinder was held so that a wire, firmly attached to the pile to be driven, would make a mark on the smoked paper on the surface of the cylinder. Some of the results obtained are shown in Figs. 12, 13 and 14. No relation seems apparent between the curves obtained with the wire on the hammer and with it on the pile.

Having thus obtained numerous graphical records of the pile's movement, a very careful study was required to determine the exact law of variation. The writer was fortunate enough to have access to a Coradi's graphical integrating machine. With the help of this instrument it was possible to draw the curves which are the first, second, etc., differentials of the curves first obtained. These are shown in Figs. 15 and 16. While the original curves are necessarily irregular, and doubtless the exact law of variation in no two cases is exactly identical, and in no case is uniform throughout the whole extent of motion; still, it would seem, from a study of the curves, as if the variation of the penetration was as the square of the time in the average case, and that it was best to make that important primary assumption in the building of a formula. The curves also show that in the majority of cases the final intensity of force was the same as the initial intensity. This, in the writer's general formula, makes $n = 2$ and $j = 1$.

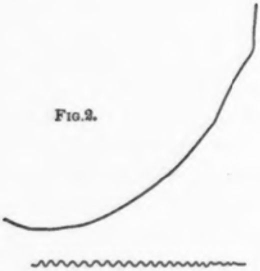


Fig. 2.

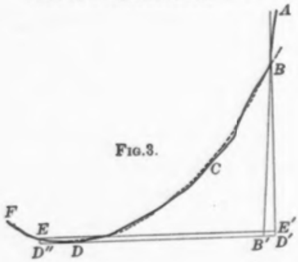


Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

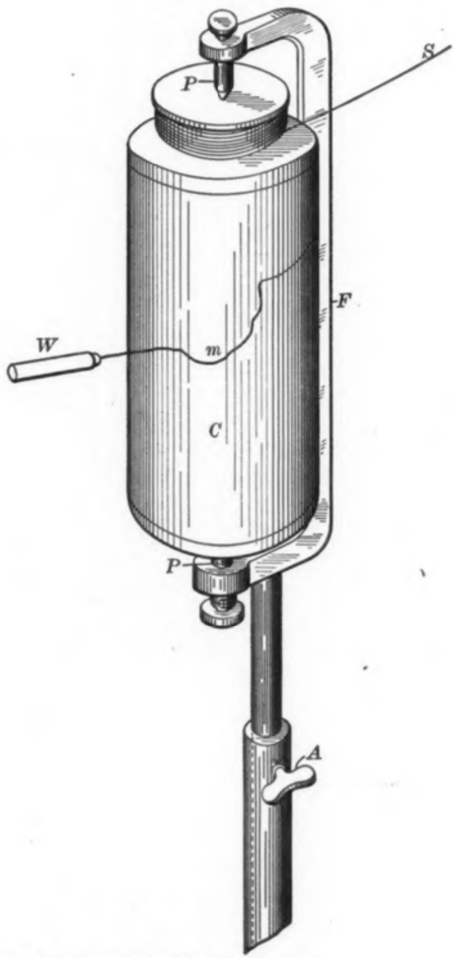
Fig. 7.

Fig. 8.

Fig. 9.

Fig. 10.

RECORDING DEVICE FOR PILE DRIVER.



C. Cylinder covered with smoked paper.
F. Frame. W. Wire attached to pile.
P. Pivots. m. Mark on paper.
S. String. A. Adjusting screw for support.

Fig. 11.

FIG. 12.

FIG. 13.

FIG. 14.

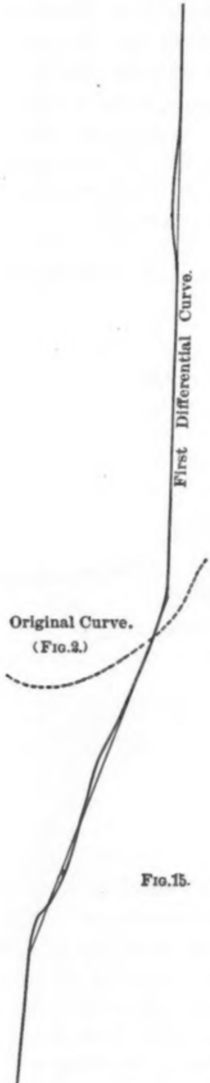

First Differential Curve.
Original Curve.
(Fig.2.)

Fig.15.

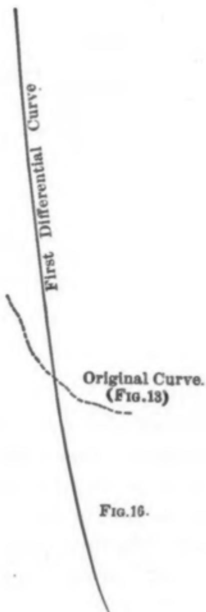
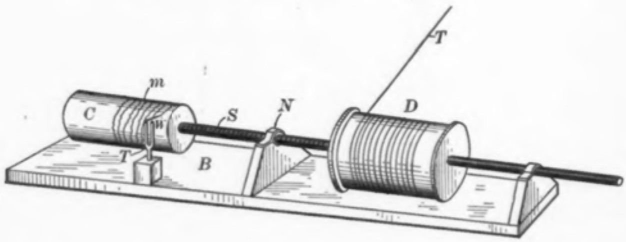

First Differential Curve
Original Curve.
(Fig.13)

Fig.16.

As far as can be seen from the diagrams, the value of a, also, is equal to $\frac{1}{2}$. The diagrams also prove beyond a doubt that the hammer remains in contact with the pile until its motion has entirely ceased. The initial velocity, u_0, of the writer's formula, which is that of the pile itself as distinguished from that of the hammer, or hammer and pile combined, is then equal to w, which is that of the hammer and pile at the instant they move together after all compression has taken place. Therefore, in the writer's formula the quantities r, K and y have the values 1, 1 and 0, respectively. Also, M_h is not equal to zero in the latter portion of this investigation, but must be included in the formula.

RECORDING DEVICE FOR FALL OF HAMMER.



C. Cylinder, covered with smoked paper.
T. String attached to hammer.
D. Drum acting as reel.
B. Base.
S. Threaded axle.
N. Fixed nut.
T. Tuning fork.
m. Mark.

W. Wire on tuning fork.

FIG. 17.

Few authors seem to think that the actual velocity with which the hammer falls differs materially from its theoretical value. To determine this point, two drums were mounted upon the same axle, which could be given a lateral motion by having a threaded portion pass through a fixed nut (Fig. 17). A tuning fork, with a wire soldered to one prong, was made to mark on the smoked surface of one drum as the whole was made to revolve by the unwinding of the string from the other drum. The free end of the string was fastened to a pile-driver

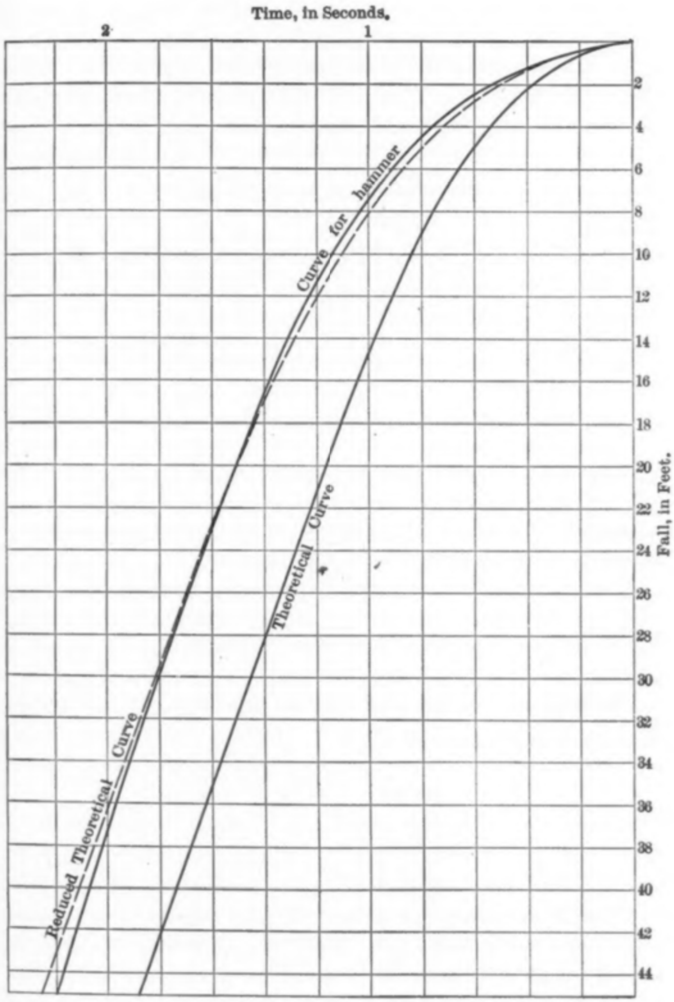

Time, in Seconds.
2
1
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
42
44
Fall, in Feet.
Curve for hammer
Theoretical Curve
Reduced Theoretical Curve

FIG. 18.

hammer which was allowed to fall in the usual way while the observations were taken. What is considered a typical curve of velocity is shown in Fig. 18, together with the theoretical one. It will be seen from the figure that, after the hammer has fallen about 10 ft. the curves are very nearly parallel. If $2g$ were made $1.15g$, the dotted curve is the one obtained, which is seen to agree very closely with the actual curve obtained experimentally. It may be assumed, then, that the quantities b, m and x of the writer's formula should be $1.15g$, $\frac{1}{2}$ and zero, respectively.

In the discussion of Mr. Crowell's paper by G. B. Nicholson,* M. Am. Soc. C. E., the following examples of the friction suffered by a hammer falling with the usual rope attached are given:

Pile No.	Penetration in Feet: Free fall.	Penetration in Feet: Rope attached.
1	0.7	0.5
2	0.9	0.7
3	0.4	0.32

Upon the assumption that the penetrations are proportional to the velocity of the hammer, the values of b, in the writer's formula, can be computed. These are 1.02, 1.21 and 1.28, respectively, and they average 1.15, which is the value already assumed.

The only other quantity concerning which observations could be made is v. Its value, however, is so very uncertain, in any given case, and so subject to variation, that its consideration will be deferred a little.

The writer's formula, with the above-mentioned substitutions made, is as follows, letting also $v' = \frac{2v}{W_h\,1.15\,h}$:

$$F = \frac{-p}{\left(\frac{cL}{s\,e} + \frac{c'\,L'}{s'\,e'}\right)} + \frac{1}{\frac{cL}{s\,e} + \frac{c'\,L'}{s'\,e'}} \times \sqrt{p^2 + \left(\frac{cL}{s\,e} + \frac{c'\,L'}{s'\,e'}\right)(R_w - v')\,W_h \times 1.15\,h}.$$

Practical considerations make it virtually impossible to measure accurately a penetration of less than $\frac{1}{8}$ in., and it is believed that in making actual determinations it should never be undertaken. It is equally troublesome to obtain any result which can be guaranteed as being within $\frac{1}{8}$ in. of being exact. With a total penetration as large as

* *Transactions*, Am. Soc. C. E., Vol. xxvii, page 172.

4 ins. (which is seldom observed), a variation of $\frac{1}{8}$ in. would make this penetration liable to 3% error. Assuming the following approximate values for the other quantities in the formula: $c = \frac{1}{2}$, $c' = \frac{1}{2}$, $e =$ 1 600 000, $e' = 17\ 000\ 000$, $L = 700$, $L' = 40$, $s = 75$, $s' = 288$, $W_p = 2\ 000$, $W_h = 3\ 000$, $h = 180$, $W_g = 1\ 000$, $v' = 2\%$, we obtain for F, in terms of p,

$$F = 479\ 065\ (\sqrt{p^2 + 0.621} - p).$$

Differentiating this expression with respect to p, we observe that the value F changes 8 623 times as fast as p when the latter is 4, and 102 040 times as fast when $p = 1$; and, consequently, an error of 3% in the observed value of p will involve the value of F in at least 3.1% of error in the first case and 23% in the last case. Consequently, any quantities or sets of quantities in the formula which will not change F by 3% when neglected, can, to good advantage, be dropped from further consideration. Further, the liability to error is so enormous with small penetrations that no penetration should be trusted much less than 1 in., and no formula can be guaranteed within a reasonable percentage of error for less penetrations.

The variation caused by the omission of the factors which increase the compression of the pile and hammer are the first which suggest themselves for investigation.

Let $A = \frac{cL}{se} + \frac{c'\,L'}{s'\,e'}$, which can have a value between 0.0000058018 for long piles of soft wood driven by a large, soft, iron hammer, and 0.00000140302 for short, hardwood piles, and a light, tough, iron hammer. Assuming $W_p = 2\ 000$, $W_h = 3\ 000$, $h = 180$, $W_g = 1\ 000$, $v' = 2\%$, $p = 1$, we obtain for the relation between F and A.

$$A\,F^2 + 2F = 298\ 080.$$

If $A =$ zero, $F = 149\ 040$; and if A is equal to its maximum value given above, $F =$ about 112 034. This is an extreme variation, in extraordinary cases, of 33 per cent. With a value of 0.000002083, which is upon the assumptions of the last paragraph, $F = 131\ 000$, and there is a variation of 17 per cent. Extreme variation in the hammer is found to produce only $\frac{7}{100}$ of 1% variation, so that the quantities c', L', s' and e' may be neglected, simply assuming that the value of $\frac{c'\,L'}{s'\,e'}$ is zero.

In place of $\frac{cL}{s\,e}$, use C, and the value of F now is:

$$F = \frac{-p}{C} + \frac{1}{C} \sqrt{p^2 + 1.15\, C\, W_h\, h\, (R_w - v')}.$$

The matter of a proper value to be assigned to the v' of this formula is one largely of conjecture. Many authors say that their formulas are to be applied only to a pile having a head firm and free from all brooming. A strict conformity to this dictum would require that practically every pile to be tested should have its head sawed or adzed off just before the test blow is struck. This is almost always impracticable, and thus some value should be assigned to this quantity. From the record of the pile driven by a Nasmyth pile-driver,* by D. J. Whittemore, Past-President, Am. Soc. C. E., it would appear that about 52%, only, of the available energy was actually consumed, on the average, in driving the pile. In driving the pile from the twelfth to the twenty-second foot of penetration, 4 682 blows were struck, or an average of 468 per foot. Immediately after adzing off the head, each of two different times, only 275 and 213 blows, respectively, were required to drive the pile the next foot. Averaging these two would give only 244, which would have been required under first-class conditions. This affords the means for arriving at the above result. The loss in this case is considered excessive.

Professor Franz Kreuter, of Munich, in 1896 presented a paper† in which he showed how the total lost energy could be found by two sets of observations on a pile. Upon the assumption that the loss of energy is the same for falls of hammer not very widely varying, or is proportional to the same, and also that the supporting power is not dependent upon the fall, the value of v', in the writer's formula, can be computed from the two following equations:

$$F = \frac{-p}{C} + \frac{1}{C} \sqrt{p^2 + 1.15\, C\, W_h\, h\, (R_w - v')}, \text{ and}$$

$$F = \frac{-p'}{C} + \frac{1}{C} \sqrt{p'^2 + 1.15\, C\, W_h\, h'\, (R_w - v')}.$$

Observations were made for quite a number of piles, and the corresponding computations of v' made. They were found to vary largely, but v' did not usually exceed 5%, and remained near 2% in most cases where the piles were sound and well driven.

Substituting, as before, values in the last found formula, and letting v' = zero, F then equals 134 400. If v' = 5%, F equals

* *Transactions*, Am. Soc. C. E., Vol. xii, p. 441.

† *Minutes of Proceedings*, Inst. C. E., Vol. cxxiv, Pt. ii.

124 800—a considerable variation. Without making just such observations as the foregoing, and reducing them, it is absolutely impossible to judge of the size of v'. It is needless to say that such computations are exceedingly irksome, and, according to modern practice, carrying them out would be deemed a needless refinement. When, in

DIAGRAM FOR DETERMINATION OF ENERGY LOST.

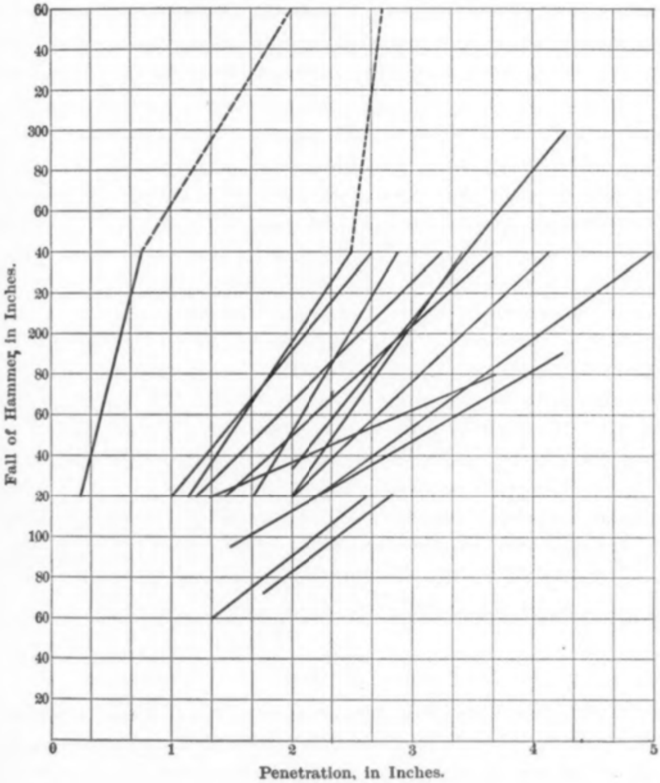


FIG. 19.

the above example, $v' = 2\%$, $F = 130\,560$. A variation of v', 1% either way would involve F in a change of 1.4 per cent. To eliminate the quantity C from the formula, another value of v' might be obtained from two equations in which C had been omitted, so that the value of v' thus obtained should include losses due to compression of the pile as well as heating and crushing of its head. The same observations used for v' above were used, but results were obtained graphically instead of analytically, as follows: Heights of fall were plotted as ordinates and penetrations as abscissas, and the line connecting the two points thus determined for each pile was extended to intersect the ordinate axis. This point would show what approximate fall was required to overcome all losses, and its ratio to the average fall in each case would give the value of v' required. Of course, these varied greatly, but averaged less than 10%, even with

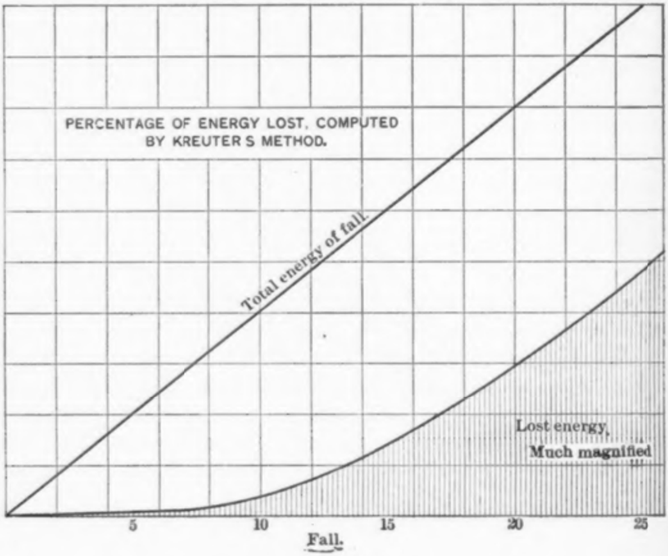


FIG. 20.

some very badly broomed piles. A few plotted observations are shown in Fig. 19. In the cases of the piles driven by the Nasmyth pile-driver and the one shown in Fig. 3, the losses are 48 and 2%, respectively. A very interesting result, shown in Fig. 20, was obtained by computing approximately the percentage of lost energy due to all causes at the different observed falls. It there appears that the loss of energy increases with the fall. The writer's observations tend to show that the quantities involving the compression in the pile can be neglected, and their effect compensated for when the piles are sound and well driven, if we make $v' = 2\%$ in the formula. On the other hand, the formula is liable to a 20% error, with poorly driven piles and falls materially less than 15 ft., at which point the value of b is nearly 1.15 g.

Making these substitutions, gives for F

$$F = \frac{0.575\ W_h\ h\ (R_w - 0.02)}{p}.$$

The only remaining unknown quantity is R_w. From McAlpine's experiments at the Brooklyn Navy Yard, he concludes* "that, by adding to the weight of the ram, the sustaining power of the pile was increased 0.7 to 0.9 of the amount due to the ratio of the augmented weight of the ram." No experiments were made by the writer concerning this point, as R_w also involves the absolutely unknowable quantity W_g. The experiment with the box of sand, and estimates made of the dirt found clinging to piles withdrawn from the earth, convince the writer that with 70-ft. piles, weighing about 2 000 lbs., W_g should be not less than 1 000 lbs. The assumptions which we are finally forced to make if we desire to reduce the formula still further, involve us in variations which may in special cases amount to 33 per cent. Should this happen in combination with other cumulative errors, the final value obtained may be in error by 50 per cent. On the other hand, if a sound, well-driven pile, weighing somewhat less than the hammer, be tested by a fall of a hammer of about 15 ft., and shows a penetration of about 1 in., the writer feels confident that the final formula will give its supporting power immediately after driving, within a probable error of considerably less than 10 per cent.

If we let $R_w = \frac{1}{2}$, F then is $F = 0.276 \frac{W_h\ h}{p}$, where h and p are in inches and W_h in pounds.

If it is desired to have h given in feet, the formula becomes $F = \frac{10\ W_h\ H}{3 p}$, or ten times hammer, times height, divided by three times penetration.

It yet remains to show the relation of this formula to those of other authors. In making this comparison it is to be remembered, however, that this formula was not built to give accurate results under any and all conditions.

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* *Minutes of Proceedings*, Inst. C. E., vol. xxvii, p. 8.

TABLE No. 1.—Tests of Supporting Power of Piles by Loading.

Locality.	Weight of hammer, in pounds.	Hammer fall, in feet.	Penetration of pile, in inches.	Supporting Power, in Pounds.	
				Estimated or observed.	Computed.
Philadelphia	1 600	36	18	14 560	10 666
Mississippi	1 600	25	3	22 400*	44 444
Perth Amboy	1 700	25	2	> 44 800	71 000
Proctorsville	910	5	0.35	62 500	42 000
Brooklyn	2 240	30	0.2	>224 000	1 224 000
Lake Ponchartrain	2 500	30	12	> 22 400	21 000
Aquia Creek	2 000	2	6.50	2 000	2 052
Dordrecht	2 205	25	0.375	13 440(?)	49 020
Buffalo	1 900	29	1.5	75 000	122 000
Fort Delaware	1 900	6	1	15 000	38 000
Boston†	1 710	10	0.7	76 000	81 000

Fig. 21 shows the relationship which exists with other formulas for 3 000, 2 000 and 1 000-lb. hammers and a 15-ft. fall. Table No. 1 shows the actual supporting power of piles (selected from Tables A and B,‡ referred to in connection with Mr. Trautwine's compilation), and others, such as can be used with this formula, in relation to the results indicated by it. Comparison will show that this formula is far closer than any other except Trautwine's, and in the cases selected, and given in Table No. 1, it is nearer than his in seven out of eleven.

It is recommended that in making tests for the supporting power of piles, a standard fall of hammer be adopted and specified for making all determinations. It is recommended that 15 ft. be adopted, and this is done for the following reasons:

(*a*) This height of fall produces a good, observable penetration with any but very light hammers, or for piles in extremely compact soils.

(*b*) The penetration is not excessive for any but very heavy hammers or for piles in very light soils.

(*c*) All frames are large enough to afford this fall.

(*d*) The lost energy is comparatively small.

(*e*) Nearly all formulas give nearly the same values through this region of variation.

(*f*) The writer's formula is especially built for this fall.

Finally, a specification similar to the following, in its main features, is especially recommended:

* From locomotives in use.

† *Journal of the Association of Engineering Societies*, vol. xx. p. 260.

‡ *Transactions*, Am. Soc. C. E., Vol. xxvii, pp. 148–151.

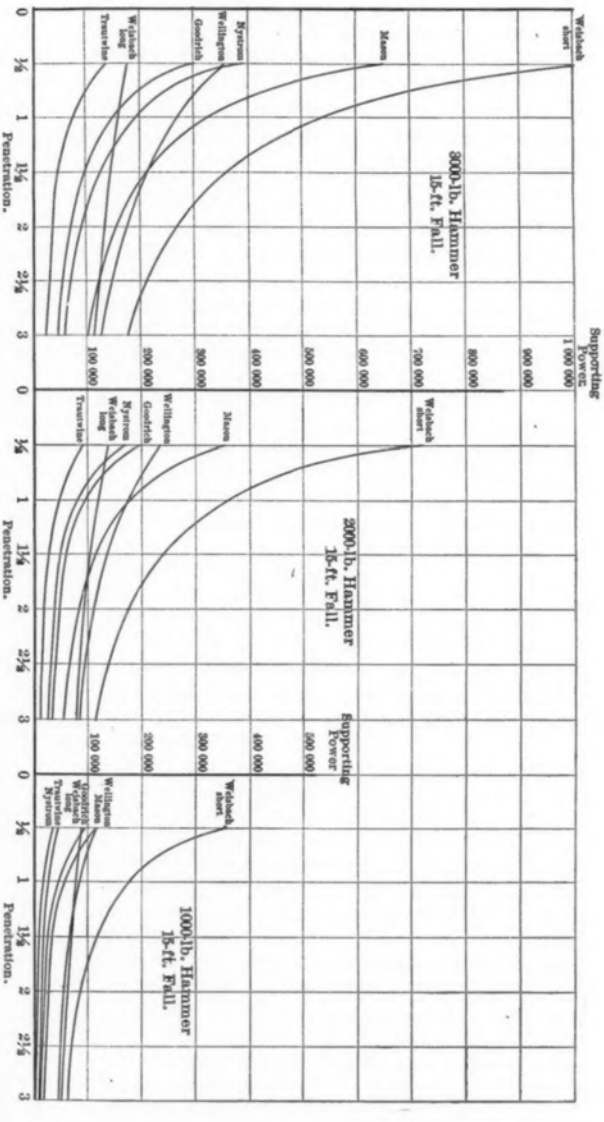
3000-lb. Hammer
15-ft. Fall.
2000-lb. Hammer
15-ft. Fall.
1000-lb. Hammer
15-ft. Fall.
Supporting Power.
Penetration.
Weisbach short
Mason
Nystrom
Wellington
Goodrich
Weisbach long
Trautwine

Fig. 21.

"Piles shall be driven to such depths that the last blow of a 3 000-lb. hammer freely falling 15 ft. shall not produce a penetration greater than 1 in., or an equivalent penetration directly proportional to the weight of the hammer."

It is believed that such piles will support an ultimate load within 10% of 75 tons; and that designers can more easily determine the necessary pile spacing and the most desirable factor of safety to be used in individual cases, and make the pile-drivers follow a standard specification, than otherwise.

In securing data and checking results, the writer has been assisted materially by P. J. Cleaver, Jun. Am. Soc. C. E., who has acted as Assistant Engineer at the New York Navy Yard for several years.

DISCUSSION.

E. Sherman Gould, M. Am. Soc. C. E. (by letter).—This paper is valuable in that it exhausts completely the mathematical side of the question of pile-driving, which comes up at intervals with greater or less profit to the profession. No such thorough analysis as is given by the author can ever be deemed useless, because it either establishes a rational formula or demonstrates that none such can exist. It appears to the writer that this paper proves the latter proposition. Mr. Gould.

As a practical art, pile-driving depends wholly and exclusively upon practical experience. Mathematics has nothing whatever to do with it. At most, practice may utilize, under great reserve, some approved empirical formula, which is indeed only the embodiment and concise expression of practical experience. This is abundantly proved by the fact that the practice of pile-driving has been carried to a high degree of perfection, while the science is not yet established.

In the every-day practice of ordinary pile-driving on land and water, it is known that yellow pine piles, 10 to 15 ins. in diameter at the butt, driven to a practical refusal, or until they "fetch up," with a hammer weighing from 2 000 to 4 000 lbs., with a fall of from 5 to 20 ft., all according to circumstances, give satisfactory results; the only remaining question being how many shall be driven in the given area. In all ordinary work, considerations of continuity of bearing result in spacing the piles so close together that their bearing capacity, as determined by any known formula, vastly exceeds the weight to be placed upon them. When any doubt exists as to the probable length of piles required, recourse is had to an actual test. If piles of great length are to be driven in uncertain ground, to support an important structure, the best available expert advice should be invoked, rather than the best mathematical talent.

The only obstacle to reducing any engineering proposition to a rational formula is uncertainty as to data. If all the data are known, to a certainty, the problem falls inevitably within the iron grasp of analysis, from which it cannot escape, and to which it must yield its secret. In pile-driving, data are conspicuously lacking. Of course the weight of the hammer, the height of fall, etc., are known, but all these factors are affected by unknowable and varying coefficients. What are the conditions under which a 4 000-lb. hammer, falling 20 ft., strikes the head of a pile, 15 ins. at the butt? In the first place, the force of the blow is entirely under the control of the man who has one hand on the throttle, and the other on the drum-brake. How lightly he can tap the pile! He can almost crack a hickory nut on it without injuring the kernel. But, admitting that he acts in good faith, and "lets go altogether," the falling weight must overhaul the rope and revolve the drum in its descent, and also overcome the friction of the

Mr. Gould. leaders. Several men will be keeping the pile in position by jamming handspikes between it and the leaders, and by hauling it in by the headlines on the winch. There is also the brooming of the head of the pile, above the ring, upon which the hammer cushions itself to a greater or less extent. All these resistances are to be added to that offered to the pile by the substance through which it is being driven, and these circumstances—and many more—destroy all hope of a rational formula.

In the writer's opinion, no formula can be applied, and no intelligent guess made as to the bearing capacity of a pile, unless it is driven to a practical refusal of, say, 1 inch. The best way is to drive the pile down rapidly until it either refuses a high fall, or has gone down nearly to grade, and, in the latter case, to reduce the fall till refusal is exhibited, and then, if desired, apply the formula. He believes, also, that refusal can be estimated by eye without actual measurement; it is very noticeable when penetration becomes labored, and the pile is beginning to fetch up. All attempts at refinement of measurements of the fall and penetration have the fatal defect of retarding the rapidity with which the pile is sent home. When piles are driven down to a hard substratum there is, of course, no difficulty in telling when they are home, and all hammering should then be stopped.

Another element which makes for safety, but which baffles calculation, is the clinging action of the material through which the pile is driven, and which action is set up immediately after it has been allowed to come to rest. It is often impossible to draw a defective pile even a very short time after it has been driven, unless a few blows be given by the hammer to start it, when it may come up very easily. A pile which has gone down readily to-day may utterly refuse all further penetration under the same hammer and fall to-morrow; for this reason, driving should be continuous, till the pile is home.

The writer observes with some surprise that the author makes no mention of the paper* presented by Charles H. Haswell, M. Am. Soc. C. E., which gave rise to an instructive discussion. In this paper Mr. Haswell gave a formula, which, admitting a set of ½ in., and a factor of safety of 6, reduces to

$$L = 5\ W \sqrt{h} \qquad (1)$$

in which $L =$ safe load, and $W =$ weight of hammer, both in the same unit, and $h =$ fall, in feet. Wellington's formula, given in the same paper, admitting a set of 1 in., reduces to

$$L = W h \qquad (2)$$

the nomenclature remaining the same. If a set of ½ in. be admitted, the same as in Equation (1), then Equation (2) reduces to

$$L = 1.33\ W h \qquad (3)$$

* *Transactions*, Am. Soc. C. E., vol. xiii., p. 267.

For falls of between 10 and 20 ft., Equations (1) and (3) give nearly equal values of L. Either is serviceable under average conditions. Mr. Gould.

In thus exalting the part which the trained judgment plays in the art of pile-driving, the writer does not wish to detract in the least from the high degree of analytical ability displayed by the author.

HORACE J. HOWE, M. Am. Soc. C. E.—Mr. Goodrich has referred to the speaker's paper on piles and pile-driving,* which went into the subject from a historical and experimental standpoint. Mr. Howe.

Since that time few detailed tests have come to the speaker's notice, the most interesting being those at the Annapolis Naval Academy during the construction of the sea-wall, a year or more ago, and partially described by J. P. Carlin,† Jun. Am. Soc. C. E., of the contracting firm which did this work. He states that five piles were loaded singly, to the ultimate, and that the results were compared with the Wellington (*Engineering News*) formula, as shown in Table No. 2.

TABLE No. 2.

Pile.	Safe load, as per formula.	Actual ultimate load.	Remarks.
1	38 000	75 000	Point in mud and sand.
2	40 000	85 090	" " "
3	32 000	34 000	Point in mud.
4	33 800	38 000	Pile in sand.
5	50 000	110 000	" "

The third test pile was entirely in mud, and the factor of safety was assumed as one, in designing the sea-wall at that point. It seems, however, that both the formula and the load actually supported gave figures which were too small, and that the wall settled for 100 ft. in length.

Mr. Carlin can doubtless supply further details.

In the paper above referred to, after a somewhat exhaustive review, the speaker called attention to the fact that single test piles, whether separate or taken from a cluster, are inadequate; and that a test is accurately adequate only when it fulfills all of the subsequent designed conditions of loading, and covers a sufficient area and lasts for a sufficient period of time. The failure of this wall is testimony as to the soundness of these conclusions.

Half a century ago, at Fort Delaware, Major Sanders made two sets of experiments on clusters of four piles each, and extended his observations on the same for some years; afterward evolving his well-known formula. Recent reports (1897) as to the masonry indicate no settlement, and the conclusion is that, under exactly those conditions, Sanders' formula is to be considered applicable.

* *Journal of the Association of Engineering Societies*, April, 1898.

† *The Engineering Record*, May 11th, 1901.

Mr. Howe. Looked at from the standpoint of his contribution to the subject, it may be that a formula is not such a forlorn hope, after all, as some have been in the habit of thinking; and that in expert hands it may attain positive value.

It is hoped that the profession may hear further from Mr. Goodrich along these lines, and that he will extend his clever observations and give us and himself the satisfaction of checking, or, if necessary, revising his mathematics.

Mr. Carlin. JOSEPH P. CARLIN, Jun. Am. Soc. C. E. (by letter).—In an article* by the writer a description is given of some tests of piles, in connection with the construction of a sea-wall at the Annapolis Naval Academy, of which the following is an abstract:

Tests Nos. 1 and 2 were on the same pile, although the second test was on the pile after it had been driven 6 ft. further.

The original borings had indicated hard bottom at a uniform depth of 40 ft. below the river bed, excepting at one point, where they showed a depth of mud of 70 ft., with 7 ft. more of mud and sand before hard bottom was reached. This point, therefore, was selected for the site of the tests.

The usual data were taken during the driving. Then the head of the pile was squared and dowelled, and a timber frame, slung by means of wire-rope lashings, was securely fastened about the 3-in. steel dowel. To prevent lateral swaying, four guys were run out from the head, and their ends made fast to powerful kedge anchors. These guys were very nearly horizontal. The pile was then loaded with anchor chains and shot, each shot having been weighed separately and tagged. The frame weighed more than 9 tons.

The sea-wall, along the site of the tests, has been finished. For about 200 ft. of wall the piles were in sand, and there has been no settlement. The next 100 ft., however, was in mud (Test No. 3), and there has been a settlement of 10 ins., which was subsequently arrested by driving additional re-enforcement piles of greater length, and blocking up from them under the wall.

In conclusion, the writer believes that, while the first two tests indicated fair results, the third demonstrated that, considered independently, the Wellington formula, or any other, is practically useless. The fourth and fifth tests give a striking lack of uniformity in the results.

It seems to be very necessary that the conditions surrounding the pile be the same as will be the case, ultimately, in the permanent structure; and that this test pile be observed during a period as long as the opportunity will permit, certainly not less than two or three months. If it be not possible to apply this time test, a carrying-capacity experiment would be no more satisfactory than the resort to the ordinary penetration records, together with an intelligent investi-

* *The Engineering Record*, May 11th, 1901.

gation of borings, both governed by experience with variable conditions. Mr. Carlin.

ERNEST P. GOODRICH, Jun. Am. Soc. C. E. (by letter).—No pile formula can give more than an approximation to the supporting power of the special pile observed, and only at the time of driving; but, with an intimate knowledge of the soil conditions, a good pile formula becomes of value, and considerable money often can be saved, at the time of driving, through its proper application. This is where the science of pile driving can influence the art. Mr. Goodrich.

TABLE No. 3.—VARIATIONS IN SUPPORTING POWER, FROM VARYING ASSUMPTIONS.

v'	R_W	Formula.	F for p equals 1.	F for p equals $\frac{1}{4}$.	F for p equals 4.	Percentage of Error.
0	....	$0.287 \frac{W h}{p}$	156 000	....	....	4.3
2	....	$0.276 \frac{W h}{p}$	149 040	....	....	0.0
5	....	$0.256 \frac{W h}{p}$	138 000	....	....	6.0
10	....	$0.230 \frac{W h}{p}$	124 000	....	....	11
20	....	$0.173 \frac{W h}{p}$	93 500	....	....	37
2	$\frac{1}{2}$	$0.276 \frac{W h}{p}$	149 040	....	....	0.0
2	$\frac{3}{8}$	$0.204 \frac{W h}{p}$	110 160	....	....	26
2	$\frac{1}{4}$	$0.132 \frac{W h}{p}$	71 280	....	....	52
2	....	$3.312 \frac{W H}{p}$	149 040	....	....	0.0
2	....	$3.312 \frac{W H}{p}$	....	596 160	....	0.0
2	....	$3.312 \frac{W H}{p}$	....	....	37 260	0.0
2	....	$10 \frac{W H}{3 p}$	150 000	....	....	0.6
2	....	$10 \frac{WH}{3 p}$	....	600 000	....	0.6
2	....	$10 \frac{W H}{3 p}$	....	....	37 500	0.6
0	....	$\frac{10 W H}{3 p + \frac{1}{10}}$	145 100	....	....	2
0	....	$\frac{10 W H}{3 p + \frac{1}{10}}$	....	529 411	....	11
0	....	$\frac{10 W H}{3 p + \frac{1}{10}}$	....	....	37 190	0.1
0	....	$\frac{3.444 W H}{p + 0.04}$	148 058	....	....	0.6
0	....	$\frac{3.444 W H}{p + 0.04}$	....	534 414	....	10
0	....	$\frac{3.444 W H}{p + 0.04}$	....	....	38 360	3.0

Mr. Goodrich. Of course, it is eminently better to test piles under the actual conditions to be encountered; but this is almost invariably impossible, the few actual tests of even single piles showing this conclusively.

After the accumulation of whatever evidence and experience has come down to us, we seem to be justified in assuming that Sanders' formula, as it is usually known, when applied to penetrations of from ½ to 1 in., comes sufficiently near the truth, with perhaps the need of a slight change in his constant.

Two other formulas, somewhat different in character in that they involve falls of at least two different heights, may be added to the list for reference:

Haagsma: $\frac{h - h'}{p - p'} \times \frac{W_h}{(W_h + W_p)}$

Morrison: / Kreuter: $\left.\right\} \frac{h - h'}{p - p'} \times W_h.$

Haswell: constant $\times\ W_h \sqrt{h}$

McAlpine: $30\ [\,W_h + (0.228 \sqrt{h} - 1)\ 2240].$

Haswell's formula is based on a penetration of ½ in. only, and hence is hardly comparable with the others in that it does not involve a variable p. In this respect, it is like that of McAlpine, who also has $\sqrt{h}$. Their observations and conclusions, which make the supporting power vary with the square root of the fall, are at variance with the work of the writer and of all other observers.

To show the possibility of wide variation, even with a most carefully prepared formula, for small variation in conditions, Table No. 3 has been prepared. It is believed to be self-explanatory. In computing F, W_h has been taken as 3 000, h as 180, H as 15, p various, v' various and R_W various.

The Annapolis tests, which had entirely escaped the attention of the writer, are of great interest. The actual load and the load computed by the writer's final formula are shown in Table No. 4.

TABLE No. 4.

Number.	Length.	Point.	Butt.	Hammer.	Fall.	Penetration.	Actual load.	Formula: $\frac{10\ W\ H}{3\ p}$	Nature of Soil.
1...	91	7	22	2 300	22	1¾	75 000	96 500	Water, 12 ft.; mud, 60 ft.; sand, 6 ft.
2...	91	7	22	2 300	22	1½	85 090	112 600	Water, 12 ft.; mud, 60 ft.; sand 12 ft.
3...	73	9	18	2 300	33½	3¾	34 000	67 000	Water, 12 ft.; mud, 61 ft.
4...	30	12	8	2 300	22	2	38 000	84 500	Sand.
5...	32	13	9	2 300	22	1	110 000	168 666	Sand.

When the actual values of R_w and properly assumed values of v' are used, much closer computed results are found, but, under ordinary circumstances, such refinements are not practicable. Mr. Goodrich.

The peculiar, and apparently erratic, variation in the results can be readily and satisfactorily explained by the soil conditions, but strongly go to prove that a pile formula alone, without other knowledge, is indeed a poor crutch.

The matter of a proper factor of safety is believed to be one which must be settled specially by each engineer, for each piece of work in hand, from known soil conditions and the uses to which the foundation will be put.

It is well to note that a pile will fail by crushing with a load of approximately 6 000 lbs. multiplied by the square of the diameter, in inches; and loads greater than given by this, where found by pile formulas, should be discarded.

A series of experiments somewhat similar to those made by the writer and illustrated in Figs. 2 to 14, was carried out by Mr. J. M. Heppel, in England, some years previous to 1867, but his description* is too meager to afford much information.

The writer begs to acknowledge his appreciation of the kind remarks made by those discussing his work.

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* *Minutes of Proceedings*, Inst. C. E., Vol. xxvii, p. 42.