Some aspects on sheet pile wall analysis, soil structure interaction

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1 Summary

This paper raises some questions on how to perform sheet pile wall analysis with focus on the soil structure interaction in urban areas. The main challenge during the physical excavation works is to limit the deformations involved in order to minimise damage on adjacent structures. The deformations depend largely on the excavation and strutting procedures, but also on the properties of the structural elements like the soil, the sheet pile and strutting members. The detailed design procedure involves numerical analyses, national regulations and common practice considerations.

2 Introduction to the problem

Several parameters enter the analysis of a sheet pile wall. Discussion will be made on the issue of determining the earth pressure itself as function of boundary conditions and shear mobilisation.

The interaction between the steel designer and the geotechnical engineer will be of particular interest with respect to the method of partial coefficients as introduced by the Eurocode 7 and the fact that Eurocode 3 allows the development of plastic hinges in sheet pile design.

An example will be hand-calculated and also calculated by means of a FEM-analysis, as well as through a beam/ spring system (Winkler model).

3 Summary of different methods for retaining wall analysis

3.1 Limit equilibrium analysis

Limit equilibrium analyses are normally performed by the simple earth pressure formula, Eq 1, which relates effective active and passive earth pressures to the effective overburden by means of earth pressure coefficients as shown in Figure 1:

$$p'_{\substack{A\\P}} + a = K_{\substack{A\\P}}(p'_v + a) \qquad \text{Eq 1}$$

The author likes to draw the attention to the importance of judging the mobilised roughness ratio r or the wall friction angle $\tan\delta = r\tan\rho$ as well as the shear mobilisation f or the mobilised friction angle $\tan\rho = f\tan\phi$. One often sees the unnecessary restriction of $r=0$ and $f=1$ used, limiting the versatility of the simple analysing methods.

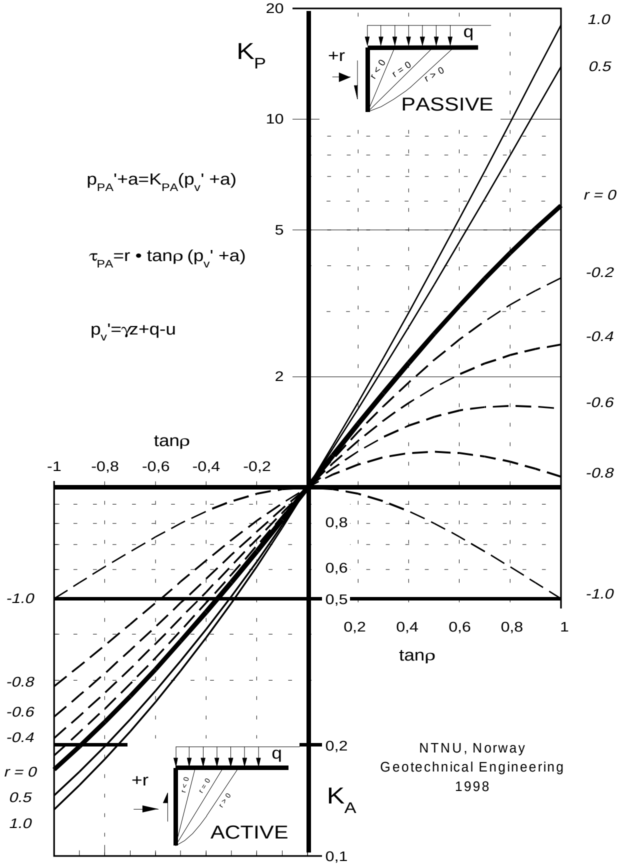


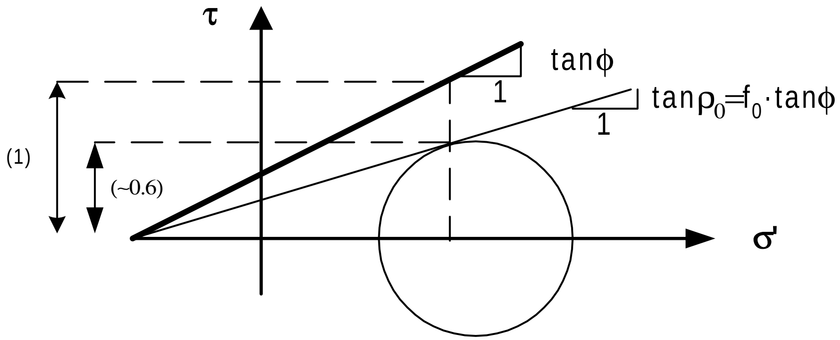
Figure 1. Earth pressure coefficients

3.1.1 The mobilised roughness ratio r

The mobilised roughness ratio *r* is clearly dependant on the vertical equilibrium of the wall element and is influenced by the direction of the anchors or struts supporting the wall as well as the relative vertical displacements between the wall and the adjacent soil masses. The absolute value of *r* is limited by the ultimate friction $\tan\delta_{ult} = |r|_{ult} \tan\rho$ available for the wall-soil interface. The direction of the interface shear stresses is of great importance; thus the *r* is given a sign convention. Special care should be taken to judge whether negative values of *r* can develop since this implies increased active and decreased passive earth pressures as compared to the case of r=0, in both cases requiring stronger structural elements and deeper sheet piles.

3.1.2 The degree of shear mobilisation f

The mobilised shear tanρ=f tanϕ clearly depends on the initial stress situation and the overall strain level caused by the displacements of the sheet pile wall. The partial material factor γ_m specified in for instance EUROCODE 7 relates to maximum allowed value of f=f_{max} through the formula, Eq 2:



$$f_{\max} = \frac{1}{\gamma_m} \qquad \text{Eq 2}$$

Figure 2. Shear mobilisation f (f=f_0 at rest)

In a normally consolidated sediment the degree of shear mobilisation f=f_0 may typically be in the order of 0.6. This is supported by the often cited empirical Jaky finding (middle part of Eq 3) as indicated in Figure 3.

$$K_0 = 1 - \sin\varphi = \frac{1-\sin\rho}{1+\sin\rho} \qquad \text{Eq 3}$$

When the excavation starts, the degree of shear mobilisation increases towards f=1 behind the sheet pile decreasing the earth pressure coefficient K_A(r=r, tanρ=tanϕ). How far towards "failure" or yield the soil is brought, clearly depends on the deformation level. Thus, if the designer chooses to utilise the yield value f=1 for his design this implies that the deformations of the sheet pile walls are correspondingly large, although the deformation does not enter the calculation as such. Also note that if the sheet pile wall is loaded in an unfavourable way so that negative roughness ratio *r* develops; the earth pressure may increase, even when the deformation level and consequently the degree of shear mobilisation *f* increases.

By utilising the *f* and *r* parameters a given average stress situation along the sheet pile wall may be mapped into some average values of *r* and thus *f*, giving a gross feeling for safety levels and deformation magnitudes.

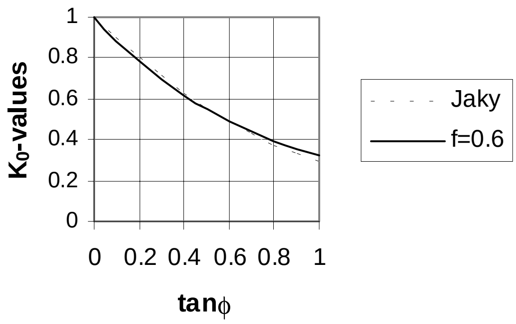


Figure 3. Earth pressure coefficient K_0

3.2 Beam-spring approaches.

Beam-spring calculation methods cover a variety of models, ranging from simple uncoupled springs with constant stiffness to semi-empirically based spring stiffness with maximum capacities superimposed.

For example, one may relate the ultimate earth pressures to the *r* and *f* parameters varying along the wall, thus obtaining some kind of influence between shear and normal stresses acting in the wall-soil interface based on down-scaled "plastic" zones behind the wall. By connecting the spring stiffness to the *r* and *f* parameters one may even obtain some logical (but crude!) description of the stress-deformation characteristics for the springs.

However, no matter how elaborate the beam-spring approaches are evolved, such approaches can never model far field behaviour and thus these models will not have the ability of describing the *wall-soil-adjacent_structure* behaviour often needed in urban areas.

3.3 Full numerical analysis.

Full numerical analyses model the wall, soil and adjacent structures at the same time. Current limitations in numerical analyses stemming from computational limits reduces every day, leading steadily more complex problems to be analysed,

even on a routine level. The computer codes are improved dramatically with respect to user friendly input of data and check on input parameters, removing some of the former requirements on deep knowledge in how the computer codes and mathematical formulations actually functions. These facts clearly open for wider use of heavy numerical simulations of the whole execution of the construction works.
The user friendliness also opens for the temptation of doing the analysis without having a sufficient experience in judging the assumptions involved in simulating the real problem. This holds for defining construction stages and sequences as well as for specifying boundaries and material parameters relevant for the construction stage at hand. It should be strongly advised to do simple hand calculations together with the elaborate numerical analysis. In this way the designer is forced to identify and rationalise (at least for himself) the discrepancies between the "green field" behaviour and the elaborate FEM-analysis.

4 Excavation problems are difficult.

The modern society demands the utilisation of underground space in densely populated areas. Excavations greatly influence nearby and even more distant structures. Thus a proper design of the support structures and excavation procedures requires quite versatile analysing tools capable of incorporating a number of influencing factors in order to determine optimum bracing and excavation procedures.
Conventional earth pressure calculation procedures do not take deformations directly into account; thus such analyses provide little or no information on the deformations to be expected. However, the deformation level is most important for a proper judgement of the damage one may expect for the existing structures during the actual carrying out the excavation and construction works.
This lack of analysing tools has often been remedied by inspecting empirical experience. These locally acquired databases of measured displacement levels do incorporate local soil conditions as well as local construction practice and are locally very valuable. They also shortcut the difficulties in judging the effects of sample disturbances during sampling and laboratory testing as well as erroneous or imperfect analysing methods. The empirical databases have clear limitations when the structure, ground conditions or the excavation/ construction methods exceeds former projects forming the bases for the experience.
More refined analysing tools are clearly needed.

5 Different methods for full numerical analysis.

In order to model an excavation in a way that accommodate for complex loadings, geometry and material behaviour some kind of discretisation has to be done. Several numerical techniques are in use.

5.1 Numerical techniques

The Finite Difference Method **(FDM)** is widely used for problems with fairly well defined field functions and reasonable simple geometry of the boundaries. Pure elasticity and steady state ground water seepage are frequently modelled by FDM.
The Finite Element Method **(FEM)** is the method that has gained most popularity in structural (and geotechnical) engineering. Real problems with soil layers, soil-structure boundaries and different material behaviour in different zones may be modelled quite conveniently.
The Boundary Element Method **(BEM)** may also be applied. Boundary elements are introduced at the boundary to simulate the far field interaction with the near field boundary. Typically the theories of elastic half space behaviour are utilised.

5.2 Material behaviour

The material behaviour is described by the constitutive equations. Several reference platforms are introduced.

5.2.1 Elastic material models

Stress and strain increment directions coincide. Material parameters may vary with the stress or strain level and direction of loading (loading, unloading), bilinear and multi-linear models. Plasticity phenomena (where the orientation of the principle directions of the stress and strain increments do not necessarily coincide) can not be simulated, no matter how complicated the material behaviour is described.

5.2.2 Elasto-plastic material models

A yield surface is described in the stress space. Stress states within the yield surface are described by linear to multi-linear elastic material behaviour. When the stress states reaches the yield surface, the material behaviour is described by the theory of plasticity where the plastic strains are described by a flow rule. During the numerical iteration process stress states falling temporarily outside the yield surface are modified by some iterative technique so that they finally end up at the surface. A number of iteration strategies are available. Unfortunately different programming codes of essentially the same material model may lead to different answers.

5.2.3 Strain hardening and strain-softening material models

In real soil plastic strain occurs even before reaching "failure". Strain hardening and softening models introduce plastic strains before reaching the ultimate yield surface. The plastic strains modify the yield surface during hardening or softening.

5.2.4 Stress history

Real soil behaviour strongly depends on previous and current stress paths, *stress history*. This may be described by introducing several moving yield surfaces or "history surfaces".

6 A set of EU reference models is needed collating EU experience.

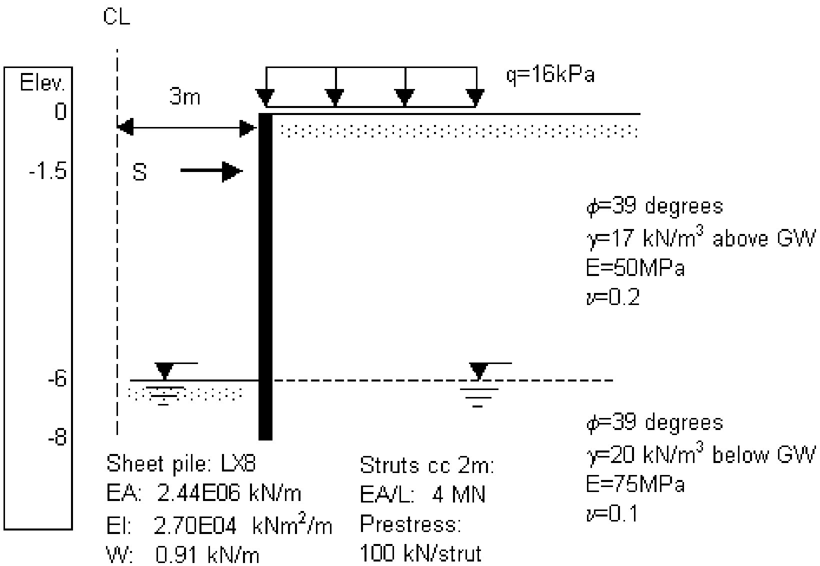
Experience shows that program codes intended to be identical may lead to different answers of the same problem. It therefor seems wise to establish a *reference data bank* with input and output data obtained by a limited number of *"reference programs"* which are generally accepted in the different countries. Such a data bank will serve the purpose for quality assurance of new program codes, for comparisons of different soil models and for stimulating a variety of different problems to be solvable by the codes.
An activity as proposed will require significant efforts by the members of the Cost 7-action. Nevertheless it seems worth striving for!

7 Example analysis

In this section an idealised excavation is analysed in different ways to form a basis for some discussions of features demonstrated by the different methods. The analyses made are:

- "Classic" hand calculus based on the coefficients in Figure 1
- A beam-spring analysis performed through program SPUNT A3
- A FEM analysis based on EP soil model performed through program PLAXIS
- A FEM analysis based on HS soil model performed through program PLAXIS

EP is a pure linear elastic - perfect plastic model, HS (hardening soil) is based on Mohr-Coulomb strength with cap and a flow rule.



7.1 Basic data

Figure 4. Basic data for the excavation example

The example chosen is a strutted excavation *6m deep* and *6m wide* with basic data as given in Figure 4. The soil consists of fairly normally consolidated sand with the ground water level at the intended excavation level 6m below terrain. The initial stress state is assumed to be characterised by $K_0=0.5$. Cohesion is set to 1 kPa

The excavation stages are intended to be as follows:

- Excavation to 2m depth
- Placing of the struts at level -1.5, pre-stressing to 100 kN/strut (cc 2m)
- Excavation to elevation -6
- Loading the terrain with 16 kPa

The results from the analyses are given for the final stage in diagrams due to the limited space available.

7.2 Classic earth pressure calculations

Only the final stage of excavation is analysed. "Classic" earth pressure distribution with r=+0.5 is assumed, the strut load and average degree of shear mobilisation are determined. Moment and horizontal force equilibrium is obtained for γ_m=1.25, i.e. f= 0.8 implying K_A=0.26 and K_P=5.4 and for a strut load of S=77kN/m. Vertical force equilibrium requires a mobilised tip resistance of 24kN/m. The results from these calculations are given in the diagrams for reference.
The average horizontal displacement may be estimated at 3m depth from the formula given in Eq 4:

$$\bar{\delta}_h = \frac{\overline{\Delta\sigma}_h}{5M_0} H = \frac{(K_0 - K_A)(\gamma \cdot \bar{z} + q + a)}{5m_0 p_a \sqrt{\frac{\bar{\sigma}}{p_a}}} H = 0.002m \qquad \text{Eq 4}$$

Here M_0 is the oedometric virgin modulus, Eq 5, assigned the values $m_0=300$ and $n=0.5$. Since the displacements are grossly caused by swelling, a factor of 5 is introduced as an empirical relationship between virgin and unloading stiffness. This average horizontal displacement is plotted in Figure 15, for comparison.

7.3 Beam-spring analysis (Spunt A3)

In this program the ultimate earth pressures are limited by earth pressures determined by "classic" earth pressure theory, including the roughness ratio *r* (see section 3.1.1) and the degree of shear mobilisation *f* (see section 3.1.2). The soil stiffness upon loading is characterised based on the constrained modulus, Eq 5:

$$\frac{\partial\sigma'}{\partial\varepsilon} = M_0 = m_0 \cdot p_a (\frac{\sigma'}{p_a})^{1-n} \qquad \text{Eq 5}$$

as described by Janbu for constrained (oedometric) conditions and pragmatically modified to:

$$\frac{\partial\sigma'}{\partial\varepsilon} = M = \frac{1-f}{1-f_0} m_0 p_a (\frac{\sigma'}{p_a})^{1-n} \qquad \text{Eq 6}$$

in order to describe the loss of shear stiffness versus failure (*M=0* for *f=1*),Eq 6. The values assigned are again $m_0=300$ and $n=0.5$. These parameters correspond fairly well with the E modulus given. The rest of the input parameters for Spunt A3 are defined in Figure 4.

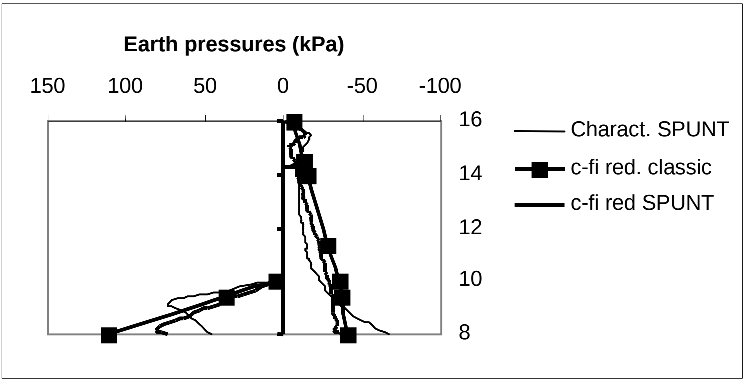


Figure 5. Earth pressures obtained by SPUNT

Results are shown in Figure 5 to Figure 7. In these diagrams the situation "Charact." is the "real" situation, provided the chosen parameters are correct (characteristic values), and the "c-fi reduction" is the case where the soil strength is

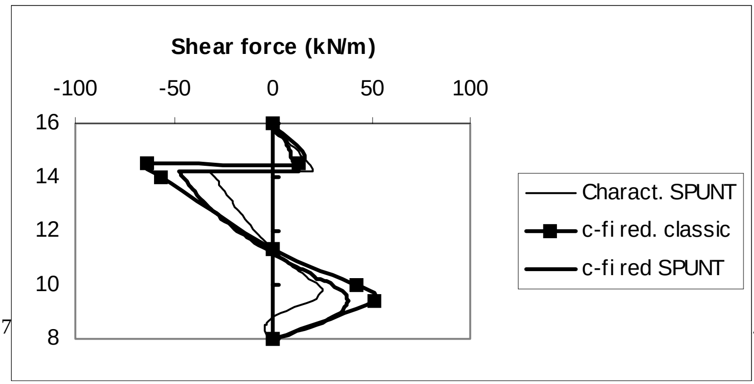


Figure 6. Shear forces obtained by SPUNT

formally reduced with a factor of γ_m=1.4 just large enough to reach an ultimate limit state (ULS). If the actual strength is less than the factored value failure will occur. In this way the numerical procedure may be used to check the material coefficient required in for instance Eurocode 7.
Note the great difference in the earth pressure *distribution* between the two states, leading to a dramatic increase of the bending moment. Also note that there is a fair similarity between the "classic" and the "ci-fi" situations since they both reflect reduced soil strength. SPUNT leads to a more favourable situation due to the higher pressures above the strut level. These pressures exert a stabilising moment on the sheet pile. Thus a higher material factor γ_m is achieved by SPUNT than by the classic assumptions, 1.4 versus 1.24.

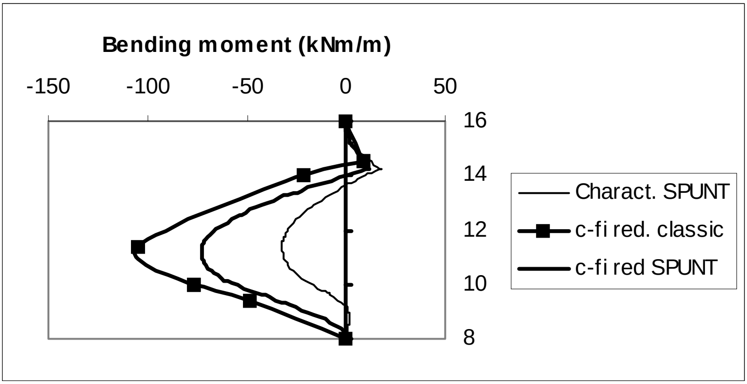


Figure 7. Bending moments obtained by SPUNT

7.4 Full numerical analysis by the FEM, Elasto-Plastic soil model.

The program used for the analysis was the PLAXIS FEM code for Windows 95.

The FEM model extends *20m horizontally* from the centreline of the excavation. The bottom boundary is set at *elevation 0* whereas the original terrain lies at *elevation +16*. Two soil layers are defined, dry sand above *elevation +8* and saturated sand (or wet sand) below *elevation +8*. The sheet pile wall is defined through a vertical beam, *8m long*. The strut is defined as a fixed end anchor at *elevation +14.5*. Two surface loads are defined, A-A extending *3m* and B-B extending the remaining *14m* over the surface. Due to the passive earth pressure resistance above the strut level the load A-A is set to zero.

The sheet pile is "surrounded" by interface elements describing an ultimate roughness ratio $|r_{ult}|=0.5$ at the *soil-sheet_pile* interface. In order to partly dampen the singularity problems at skirt tip the interface elements are extended down to

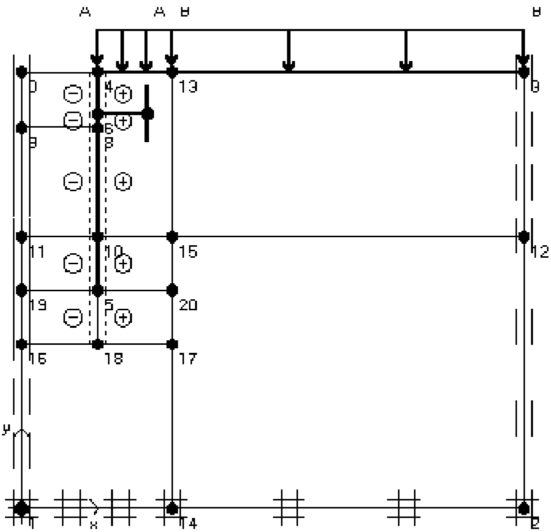


Figure 8. The FEM model

elevation 6, 2m below the skirt tip. The pore pressures are defined to vary hydrostatic from *elevation +10*. The modelling is shown in Figure 8.

The **elasto-plastic model (EP)** was run with *coarse mesh, 15 nodes triangular elements* leading to 97 elements, 883 nodes and 1164 stress points. The c-fi reduction phase indicates a material factor of γ=1.5 or an average degree of shear mobilisation f=0.67.

The **hard soil model (HS)** was run with *coarse mesh* **(HSC)**, *6 nodes triangular elements* leading to 99 elements, 252 nodes and 297 stress points, and with *fine mesh* **(HSF)**, *6 nodes triangular elements* leading to 973 elements 2070 nodes and 2919 stress points. Both runs simulated the excavation in four stages, (1) excavating to 2m depth, (2) strutting to 100 kN per strut (cc 2m), (3) excavating to 6m depth and (4) applying surface load of 16 kPa. Finally the soil strength (c and ϕ) was reduced until an *ultimate limit state ULS* was reached, revealing a material factor γ_m of 1.9 and 1.6 for the coarse and fine mesh options respectively. Figure 9 shows the pattern of plastified points at this stage. The results from the PLAXIS runs are shown in Figure 11 to Figure 14. Note the diminutive effect of the *surface load*.

Also note the great effect on the bending moment (300% increase) of introducing the *reduced soil strength* (ci-fi red.). A relatively small safety margin on the soil strength (1.5) leads to a great factor (4.0) in the loading effects (bending moment) for the sheet pile.

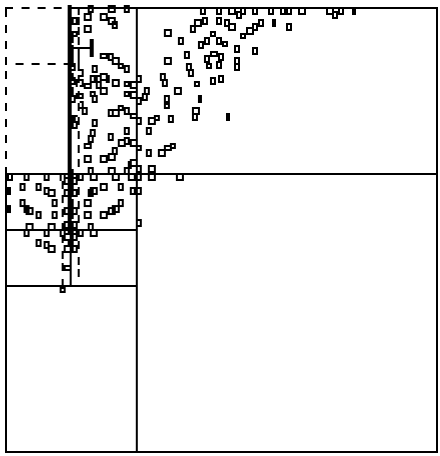


Figure 9. Pattern of plastified stress points at the end of c-fi reduction

This is a strong argument for the principle of partial safety factors. A lumped factor of safety does not distinguish between soil and steel and may therefore mislead the judgement of the real safety level grossly. Neither can a general load factor γ_f normally in the order of 1.6 compass an uncertainty margin of 1.5 on the soil strength parameters.

7.4.1 Ground deformations

The FEM analysis offers predictions of the surface deformations due to the excavation. This is a significant benefit compared to the previous analysis, and a must for interaction analysis. However, the deformations obtained by the EP-soil model may not be realistic. Since the soil is given the same stiffness for loading and for unloading the deformation pattern will be mainly governed by the decrease in the general stress level caused by the removal of the excavated masses, see

Figure 10. The figure shows the deformation pattern of the soil surface after excavation, but before applying the surface load. However, the information may not be realistic for simple soil models as demonstrated here.

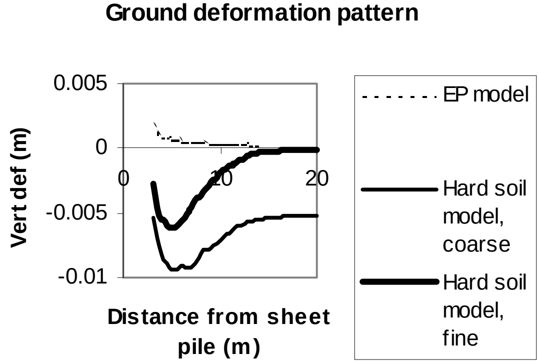


Figure 10. Ground deformations, fully excavated, no surface load

7.5 Discussion of the result

7.5.1 Main result

The main results affecting the design of the sheet pile wall are *the safety level* required (determining the necessary embedment), *the strut load* and *the maximum bending moment* (determining the dimensions of the structural components). In order to form a basis for better understanding of the results; earth pressure diagrams are shown in Figure 11 and Figure 12. For judging the sheet pile wall

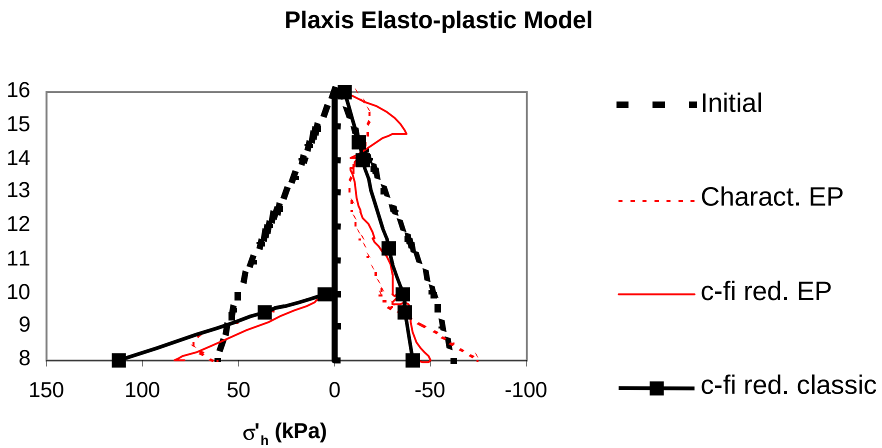


Figure 11. Earth pressures obtained by classic theory and FEM, Elasto-Plastic Model

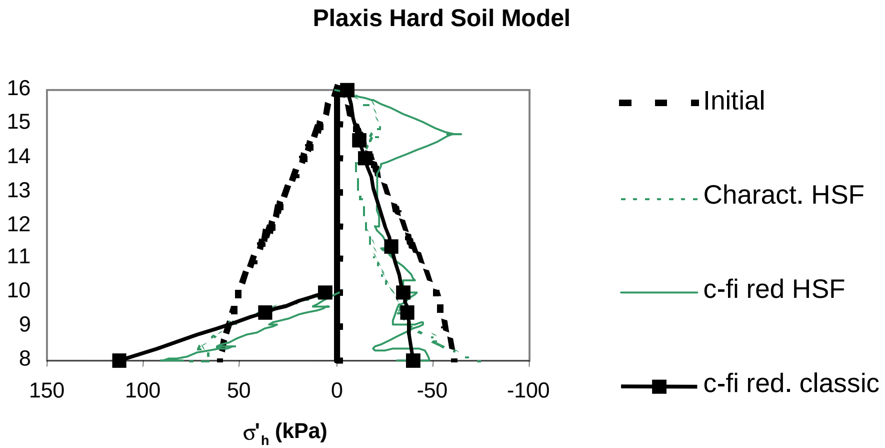


Figure 12. Earth pressures obtained by classic theory and FEM, Hardening Soil Model

itself shear force diagrams for shear force and bending moment are shown in Figure 13 and Figure 14, respectively. The diagrams yield the characteristic ultimate situation of 6m excavation depth and with the

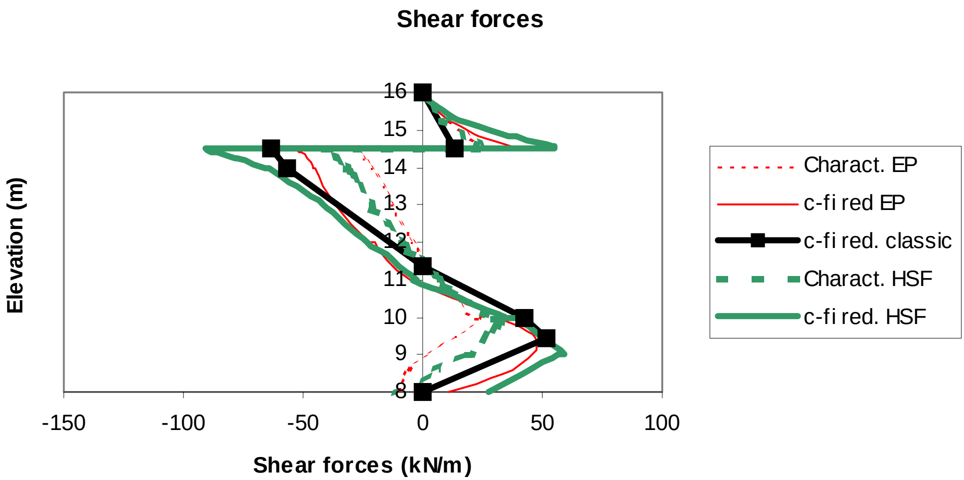


Figure 13. Shear forces obtained by classic theory and FEM.

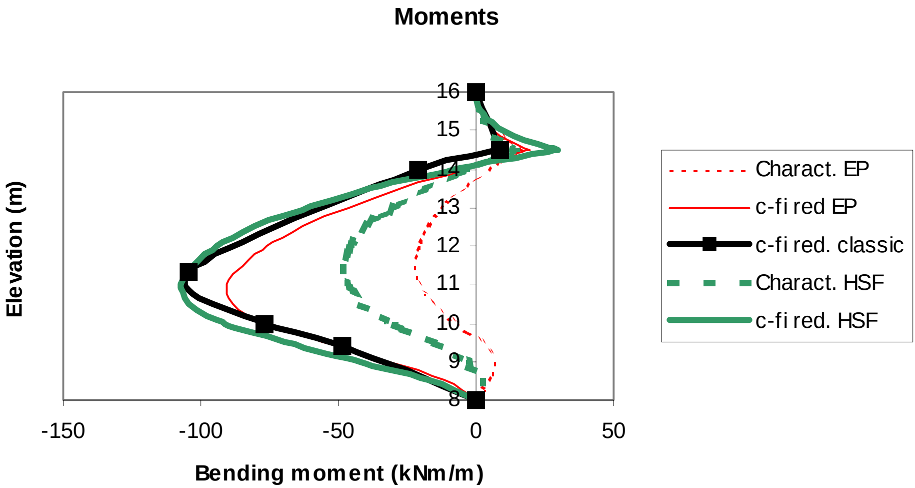


Figure 14. Bending moments obtained by classic theory and FEM

surface load of 16kPa. The "c-fi reduction" values are also given as well as the "classic" approach, for comparison.

The significant findings are summed up in Table 1. All values are given without signs, irrespectively of any sign conventions. The fine mesh values are used for the Hard Soil model.

Table 1. Significant findings for design

	Class.	Spunt A3 Charact	Spunt A3 c-fi-red	FEM E-P Charact	FEM E-P c-fi red	FEM H-S Charact	FEM H-S c-fi red
γ_m	1.24		1.4		1.5		1.6
Strut load	77	53	60	52	87	64	143
Q_{max}	63	32	48	29	53	41	89
Q_{tip}	0	0	0	10	11	11	27
M_{max}	104	32	73	22	90	48	107

Note that the safety margin γ_m differs quite substantially between the "classical" approach and the "numerical" approaches SPUNT and PLAXIS. Two effects are clearly visible. One effect is the earth pressures above the strut level. In the "classical" approach these stresses are limited to be at the active state level, whereas the numerical approaches end up with substantially higher stresses forming a stabilising moment on the wall.

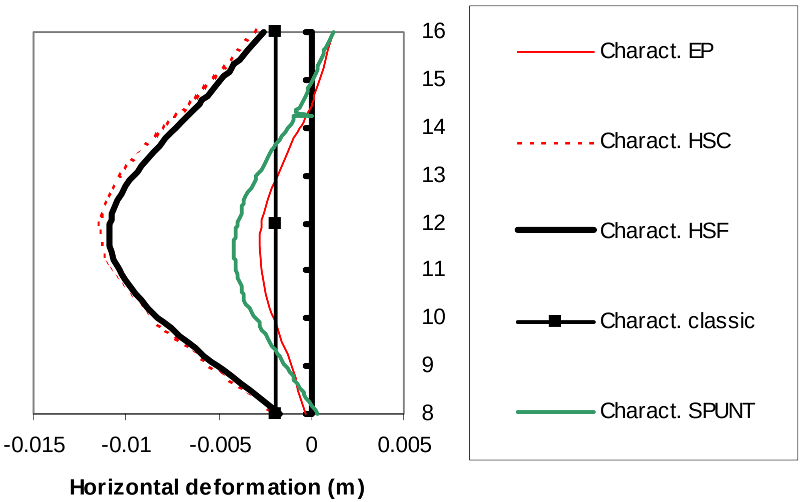


Figure 15. Horizontal displacements of the sheet pile wall, 6m excavation.

Also note the great difference in maximum bending moments, mostly depending on the assumed *mobilisation* of the soil strength, formalised through the factor of safety (or degree of shear mobilisation). One may observe that the hand calculus and SPUNT agrees fairly well, but fail in predicting the horizontal displacements since the far field displacements are not included.

7.5.2 Level of safety

The *material factors* (γ_m) used in a partial factor design method are aiming at covering reasonable uncertainty margins on the assessment of the material parameters. The use of *load factors* (γ_f) are aiming at covering reasonable margins on the assessment of loads or loading effects (like bending moment and strut load). In a design process the designer must ensure that proper values are introduced according to national regulations.

The bending moment *capacity* of the sheet pile wall may be assessed within rather narrow margins. Thus a rather narrow margin is required for the material factor to be applied on the sheet pile bending moment capacity.
The soil strength parameters may be more variable. The "boxed value" of γ_m=1.25 proposed in EUROCODE 7 is often regarded as a sensible value for the *material factor* γ_m applied to the strength parameters c and tanϕ. It is interesting to note that this leads to a corresponding 300% increase in the load effect in the EP example. Thus if this "uncertainty" in the assessment of the real soil strength (and thus in assessment of the earth pressure) should be formalised by using a *load factor* γ_f applied to some "characteristic earth pressure" instead, this factor must be given the value of γ_m=4 in this case! Luckily the HSF model rather indicates a 100% increase to be more realistic, but even γ_m=2 may be a rather high value to apply as a load factor for a general practising structural engineer.
My suggestion is that one should stick to a partial factor strategy covering the uncertainty margin on the soil parameters rather than factoring some "characteristic" value of the earth pressure by a load factor as frequently proposed.

7.5.3 Soil shear mobilisation f

Initially K_0=0.5 was assumed. This implies that tanρ_0=0.35 and f_0=0.43 characterise the stress state before excavation.
After completing our simulations of the excavation and strutting operations we obtain average shear level parameters as given in Table 2.
One may observe that the "Classic" analysis is most pessimistic closely followed by the Beam-spring analysis (SPUNT A3). The FEM analyses are more optimistic. As stated before, one contribution to the discrepancy is a false shear force acting at the skirt tip in the PLAXIS analyses. Thus PLAXIS overestimates the soil capacity. Another contribution is caused by a passive stress state in the upper soil layer, neglected the "Classic" analysis. The Danish codes have included this passive stress field ever since Brinch Hansen introduced them in the 1960-ties. In Norway we have been somewhat reluctant in utilising this margin. Arguments for this may be long term creep, frost actions (thawing) and possible local excavations. The sheet pile length is sufficient according to current Norwegian Application Rules and EUROCODE 7.

Table 2. Average shear level obtained in the analyses

Analysis	Classic	SPUNT A3	PLAXIS EP	PLAXIS HSC	PLAXIS HSF
γ_m	1.24	1.4	1.54	1.92	1.64
f	0.81	0.71	0.65	0.52	0.61
tanρ	0.66	0.57	0.34	0.42	0.49

7.5.4 Strut load

The strut loads obtained are given in Table 3. One may observe that the "classic" approach leads to very optimistic strut load and SPUNT even more optimistic. The high load levels in the FEM analyses goes together with the passive earth pressures in the upper soil layers. If these stress levels are mobilised the "classic" analysis grossly underestimates the load level. It may be relevant to remember the old advice on being generous with the struts and rather save on the sheet piles.

Table 3. Strut loads obtained in the analyses

Analysis	Classic	SPUNT A3	PLAXIS EP	PLAXIS HSC	PLAXIS HSF
Charact.		52	53	65	64
Design (?)	77	59	83	132	143

8 Conclusions

The demands of accuracy in designing excavation projects in town areas clearly asks for bringing comprehensive analytical models like the Finite Element Method into practice. As demonstrated in the above simple example this requires some discussion and maybe even formal training in bringing together the practical experience, simple estimates and full numerical analysis. It seems clear that simple elasto-plastic soil models may be rather misleading, thus more sophisticated (but numerically sturdy) soil models must be used in common practice so that the designers develop a feeling for potentials and pitfalls.

Co-operation areas like the COST C7 action may form a forum for developing a base of calibrated calculation examples that can serve as a training ground and a reference for students and practising engineers.

The activity may also help to a harmonising of how one may utilise the FEM technique in the framework of the design principles of partial factors as laid down in the newly issued Eurocode 7, (ENV 1997-7). This seems to be a fairly open question, and any harmonising and clarifying activity should be welcomed.