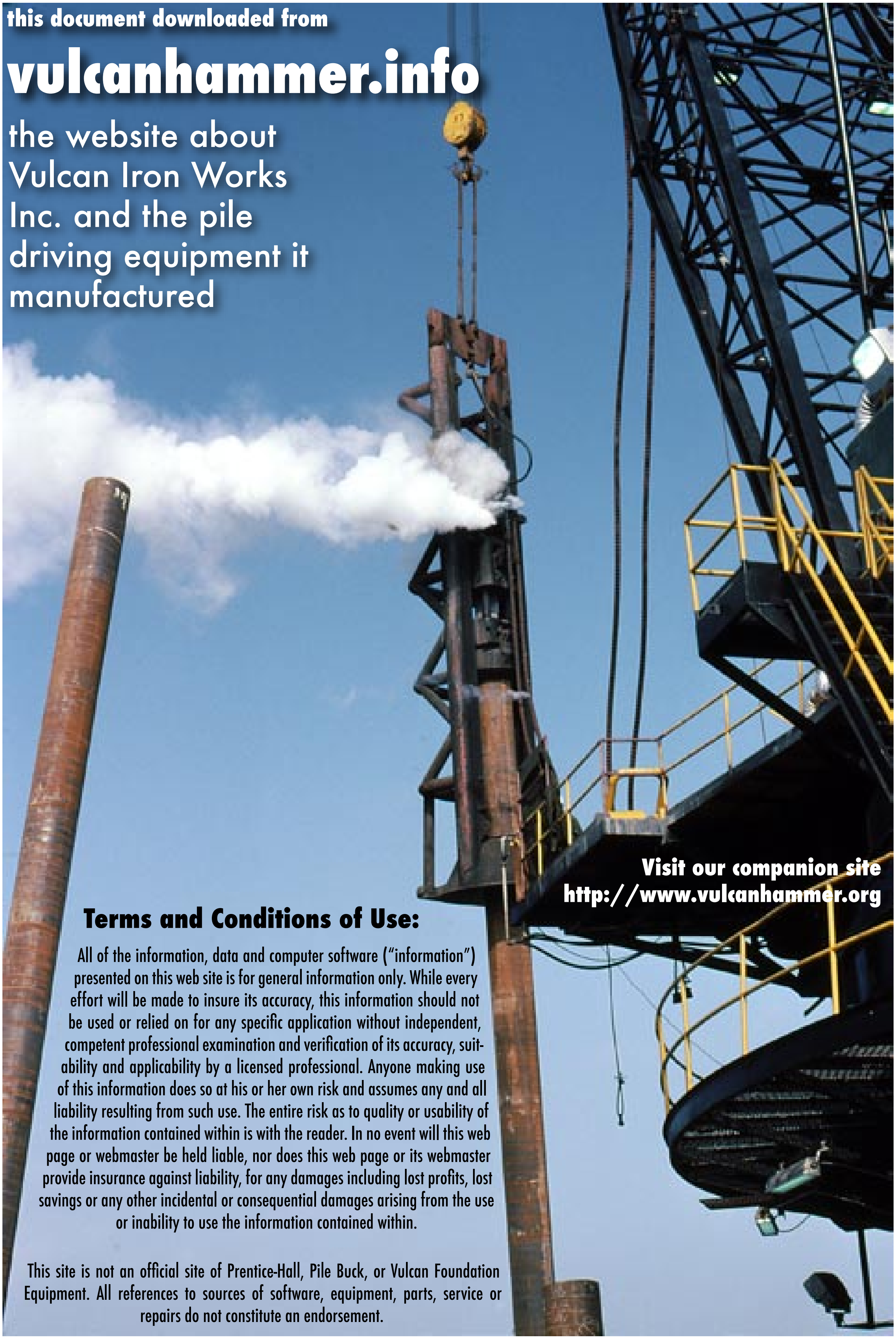

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hat Happens When Hammer Hits Pile

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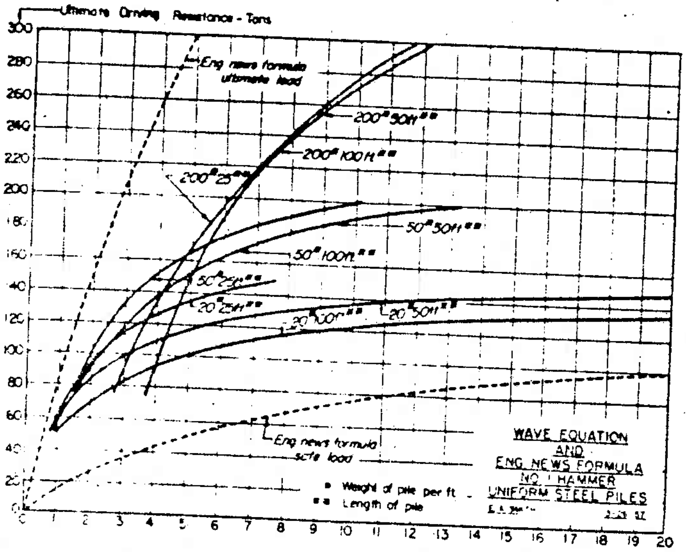
Now it can be told:

Up to now no one had been known to solve manually the complicated wave equation as it applies to action of a pile under the blow of a hammer. Today it takes but seconds to solve "mechanically" for driving resistance information such as plotted here.



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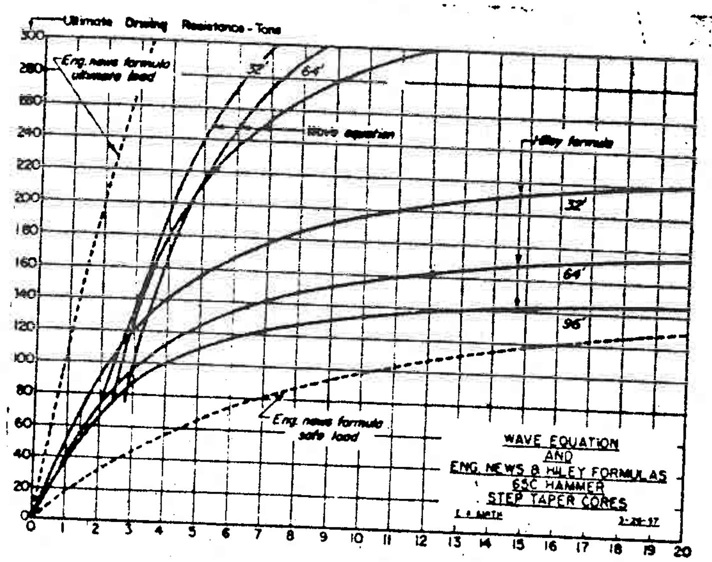


2

Electronic digital computers are solving the wave equation to answer the question . . .

What Happens When Hammer Hits Pile

Edward A. Smith
Chief Mechanical Engineer
Raymond Concrete Pile Company

It is now possible, thanks to electronic digital computers, to calculate in a practical and economical manner what happens at the instant a hammer hits a pile. As a result, it is possible to tell with far more accuracy than ever before that a certain type pile, driven with a particular hammer to a determinate number of blows per inch, is being driven against a definite number of tons ultimate driving resistance.

This knowledge is of course important because, in a majority of cases, the ultimate driving resistance may be used with a factor of safety of 3 or 4 to determine the safe bearing capacity of a pile. Exceptions occur with soils that either "set up" or "relax" after driving, so a knowledge of ground characteristics is essential, but in the main, the ability to compute the driving resistance accurately will greatly improve the engineer's position in determining the safe bearing value of a pile. All this is now possible because electronic digital computers are available to solve by numerical methods the difficult math involved in applying the "wave equation" to pile driving problems.

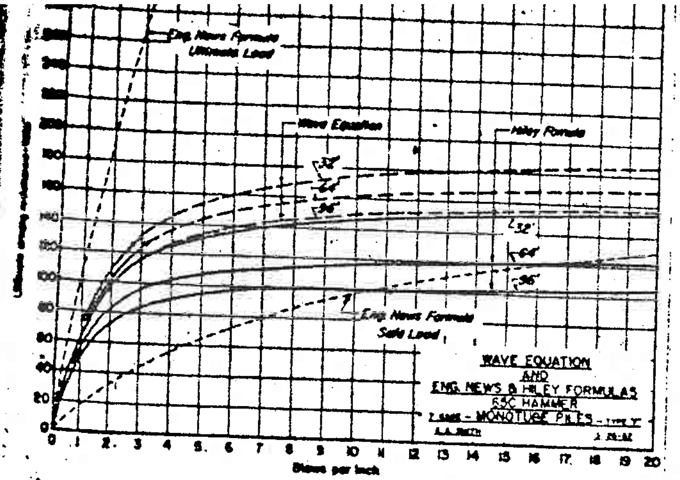
• **The Wave Equation**—The wave equation itself is not new—it has been known to mathematicians for many years. In general, it describes how waves progress from one point to another; specifically, it may be used to illustrate the wave action produced in a long object by a force suddenly applied at one end. In his book "Theory of Elasticity" (McGraw-Hill, 1934) Timoshenko showed how the wave equation might be used to calculate longitudinal wave action in a uniform steel rod.

Furthermore, the idea of applying the wave equation to pile driving is not new. In 1940 and in 1941, A. E. Cummings referred to the subject, and credited D. V. Isaacs of Australia with being, in 1931, the first to suggest its application to pile driving. Cummings also stated that the British Building Research Board in 1938 demonstrated that the behavior of full sized piles under actual field conditions can be predicted by means of the wave theory.

Yet, the wave theory has not, until very recently, been applied to pile driving. The reason for this is simple, and it was given by Cummings—the calculations involved were too difficult. As given by Timoshenko, the wave equation:

$$\frac{\delta^2 u}{\delta t^2} = C^2 \frac{\delta^2 u}{\delta x^2}$$

is a second order partial differential equation. For very simple cases, as



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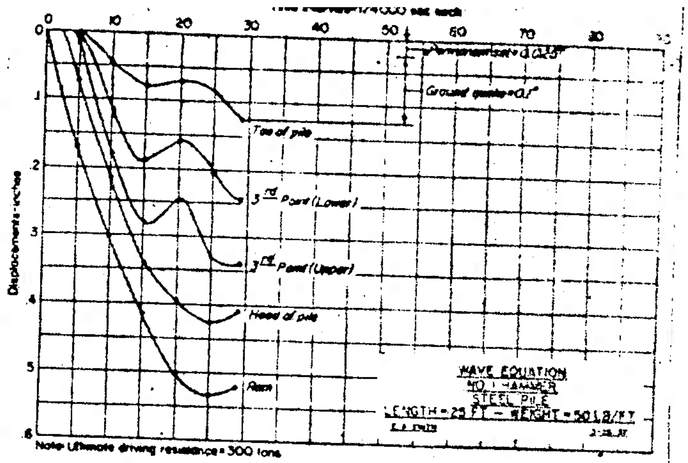

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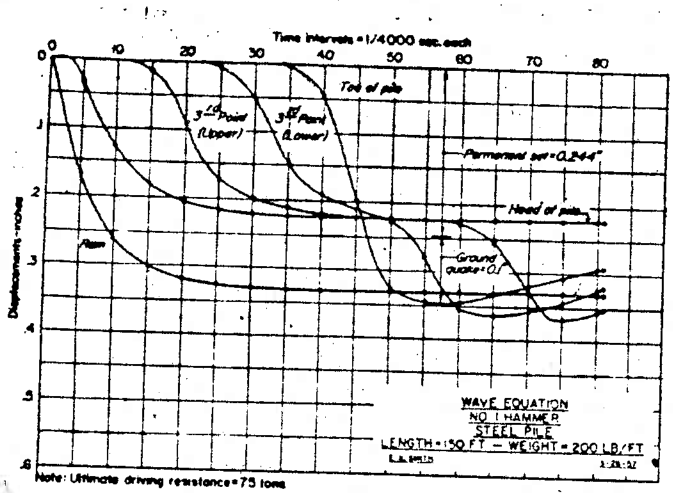

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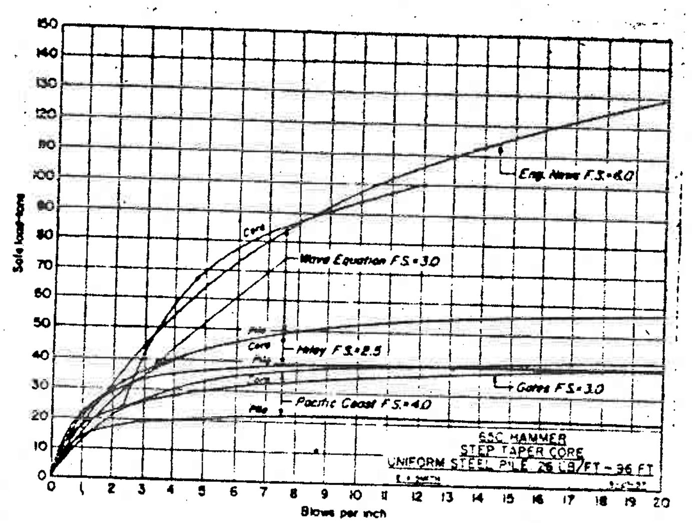

6

when a known force is suddenly applied at one end of a uniform steel rod, the equation can be solved by ordinary calculus. But when the equation is complicated by considerations of the actions of the ram, the cap block, the pile, and the ground, the problem becomes so difficult that no one has been known to solve it.

• Computers to rescue—However, the picture has been changed in the last few years by the development of electronic digital computers such as the Sperry Rand "Univac" and the IBM "704." Not only can these machines solve the wave equation as applied to pile driving—but they can do it in a matter of seconds.

The Raymond Concrete Pile Co., working in conjunction with IBM, has made over 250 such computations involving various combinations of pile and hammer types. The mathematical methods used have been checked in many ways and proved reliable. From the results of these calculations, the writer will draw some conclusions, subject to the following qualifications:

1. The importance of variations in ground characteristics as explained in a previous paragraph must be considered as a limiting factor in arriving at definitive conclusions.

2. In applying the wave theory to pile driving, it is possible, with the computers, to include side friction as part of the calculation and in the immediate future this will be done, but thus far the calculations have been made on the basis of an end bearing pile with no side friction.

3. It is not the intent of the writer to offer a new and different pile formula. The problem is far too complicated to be expressed accurately by a single reasonably simple formula. Possibly a formula could be devised with certain constants that would be changed to suit different types of driving conditions, but, for now at least, the results of the calculations are presented only in graph form.

• Conclusions—Graph No. 1 shows ultimate driving resistances for various lengths and weights of straight-sided steel piles driven with a No. 1 Vulcan hammer. For comparative purposes the ENGINEERING NEWS formula is also plotted both in its ordinary safe-load form and also in its ultimate resistance form based on the fact that this formula includes a factor of safety of 6. It will be noted that the wave equation curves show considerable variation from one another and indicate that use of the ENGINEERING NEWS formula results in a factor of safety that varies from a little more than one to as much as four.

Graph No. 2 shows a comparison between the ENGINEERING NEWS formula, the Hiley formula and the wave equation for step-taper piles. It will be noted that in general the wave equation indicates that step-taper piles can take quite high loads, much more than allowed by the Hiley formula. It must be remembered that the Hiley formula can be made to give widely varying results depending upon what constants are used with it. In preparation of the graph, therefore, the constants used corresponded to those used with the Wave Equation.

Graph No. 3 shows a comparison between the ENGINEERING NEWS for-

[illegible] Hiley formula and the wave [illegible] for No. 7 gage Monotube [illegible]. In general the wave equation [illegible] the Hiley formula [illegible] light piles [illegible].

[illegible] No. 4 and No. [illegible] show [illegible] why the wave equation [illegible] Hiley formula [illegible] well [illegible] but disagrees with [illegible]. Graph No. 4 shows [illegible] of maximum penetration [illegible] light pile is entirely in compression. This is the condition [illegible]. In Graph No. 5, however, it is [illegible] in the fact that the waves [illegible] other, that [illegible] pile section, it is possible for [illegible] the pile to be in compression [illegible] moment of maximum penetration [illegible] while the rest of the pile is actually [illegible] tension. This is very different from [illegible] assumption.

[illegible] No. [illegible] shows a comparison on a safe load basis between the wave equation with a factor of safety of 3 and certain formulas employing the recommended or usual factors of safety.

From all these data it must be concluded that the Hiley formula tends to penalize the heavy or long pile and to favor the light and short pile. This comes about because the Hiley formula is shown by the Wave Equation calculations to contain incorrect assumptions as follows:

1. It assumes that the compression of the capblock, the compression of the full length of the pile, and the elastic compression of the ground all occur at one and the same instant of time. This is reasonably accurate for a light pile, but inaccurate for a very heavy pile.

2. It assumes that the percentage of energy transmitted from the hammer to the pile is dependent on the relative weights of the ram and the entire pile. This assumption is reasonably accurate for a short pile, but inaccurate for a long pile.

Chellis, in his book "Pile Foundations" (McGraw-Hill, 1951) lists 38 pile formulas that have been proposed and used. No one of them can be accurate for all types and lengths of piles because they all fail to consider the effects of wave action. The newly acquired ability to solve the wave equation as applied to pile driving offers a means of obtaining a truly mathematical solution.

In due course it is expected that additional calculations will be made covering a variety of hammer and pile types, and that the results will be published in the form of tables, resembling logarithmic and trigonometric tables. Meanwhile, it is hoped that the graphs here presented will prove both informative and provacative.
