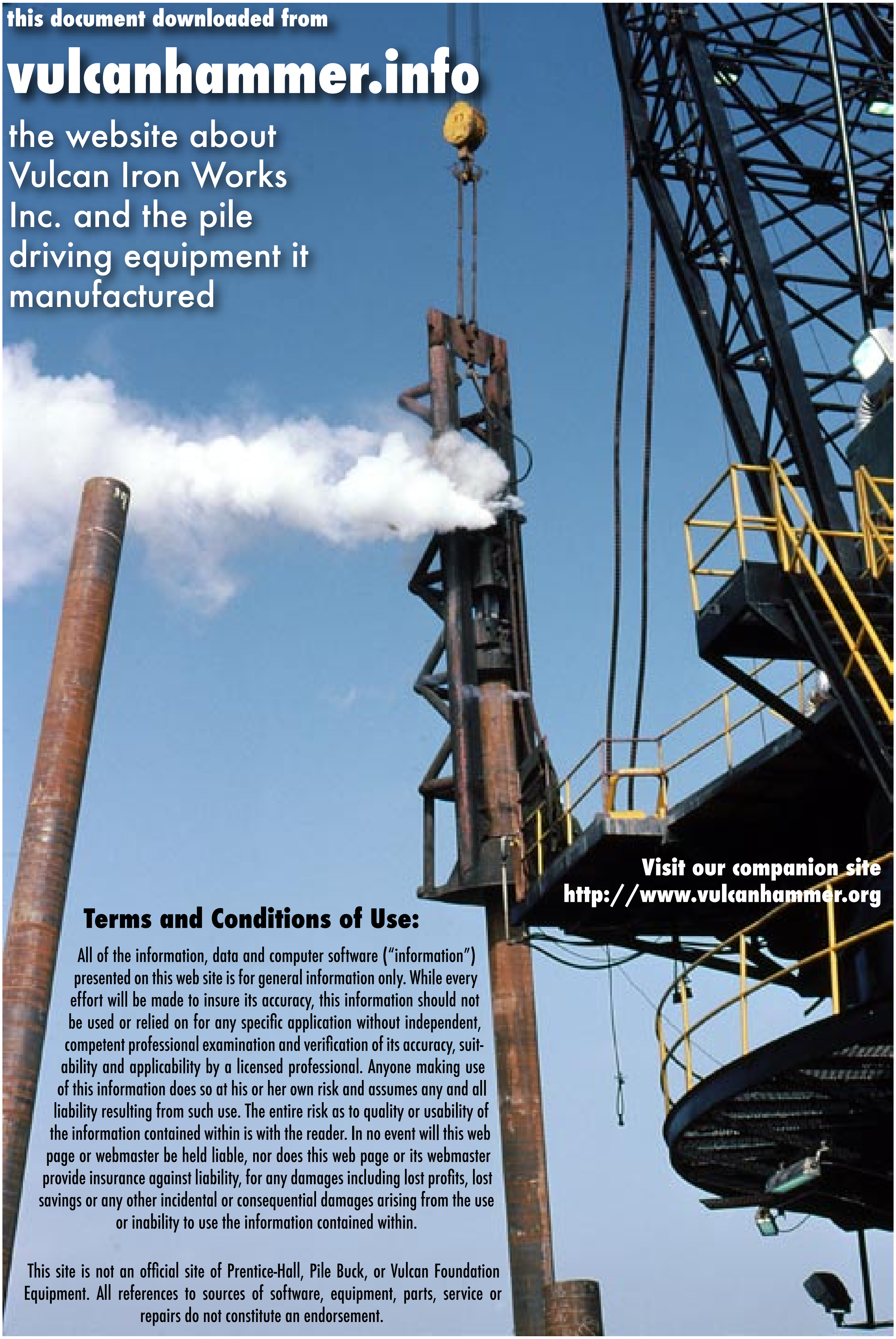

# Analysis of Pile Drivi

*Early this year, Dr. Tonis Raamot was appointed Chief Civil Engineer of the Raymond Concrete Pile Division. Since 1964 he had served as consultant to Raymond International on soils and foundations as well as attending to his duties as a professor of civil engineering at Newark College of Engineering. Dr. Raamot, a graduate of Columbia University (BA, BS and MS) and the University of Illinois (Ph.D.), served with the Civil Engineering Corps of the U. S. Navy for three years. With considerable experience in the design and construction of pile foundations, Dr. Raamot has devoted much time to analyzing pile driving mechanics.*

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Editor's Note:

Readers of Foundation Facts were introduced to the Wave Equation as it is applied to pile driving in an article by C. R. Graff which appeared in the Fall 1965 issue. Dr. Raamot presents in greater detail the fundamentals for applying the Wave Equation and examples of data obtained from actual pile driving problems.

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While a great number of judgement factors enter into pile driving, it is now possible to analyze the most important ingredient of that so-called brutal procedure—the dynamics of energy transmission along the pile. A mathematical solution to this problem was obtained about twenty years ago by E. A. Smith, retired Chief Mechanical Engineer of Raymond. Because of the tediousness of the great number of hand calculations required to arrive at an answer, Smith's method found little use, and therefore attracted little attention among engineers or contractors. With the availability of high-speed computers, however, it has become practical to make the necessary calculations to predetermine driving criteria for any pile-hammer-soil combination.

Smith's method is actually the numerical equivalent of the wave equation, a differential equation that describes the mechanics of force transmission along an elastic rod that has been subjected to impact by a

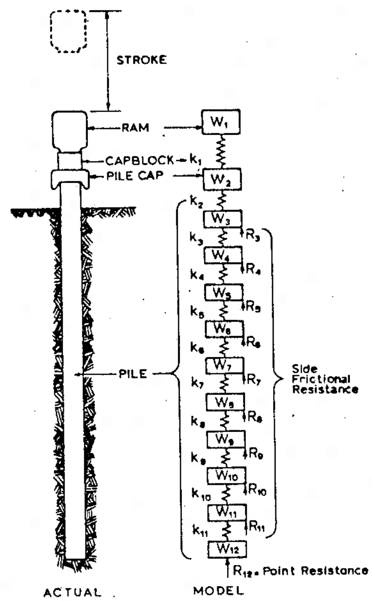


Fig. 1. Mathematical mo..el of pile

mass having a specified initial velocity. Figure 1 shows the appropriate mathematical model. The ram is represented by a weight, the capblock by a spring and the pile by a series of weight-spring segments. Spring constants for various types of capblocks are obtained from test results. Those of the pile segments can be calculated by the formula

$$k = \frac{AE}{L}$$

where k is the spring constant (lbs/in), A is the cross sectional area of the pile ($in^2$), E is the modulus of elasticity of the pile material ($lbs/in^2$) and L is the length of the pile segment (in).

The soil resistance R consists of a combination of springs to simulate the ground quake, and dashpots to account for the dynamic response of the soil. The ultimate load in the springs equals the ultimate capacity of the soil material.

The numerical method used to describe the behavior of this mathematical model is very simple and does not really require the use of a computer if great accuracy is not desired. Figure 2 illustrates the force build-up in the pile during the very early stages of impact. Simply described, at the instant of impact, the ram has an initial velocity and acceleration that can be converted into distance for any desired time increment by the simple equation $y = v\,(\triangle\, t)$. The distance (y) that the ram has traveled after a very short time interval after impact is equal to the compression of the capblock. The force in the capblock can then be simply calculated by the equation $P = ky$ where k is the spring constant of the capblock. The force built up in the capblock in turn gives rise to acceleration (a) of the pile cap in accordance with the well known formula $F = ma$, where m represents the mass of the pile cap. This acceleration results in velocity and movement of the pile cap, which then begins to compress the first segment of the pile. Again, the force in the first segment of the pile is directly proportional to the spring constant of the pile segment and the distance the pile cap has traveled. In the meantime, the ram has travelled an addi-

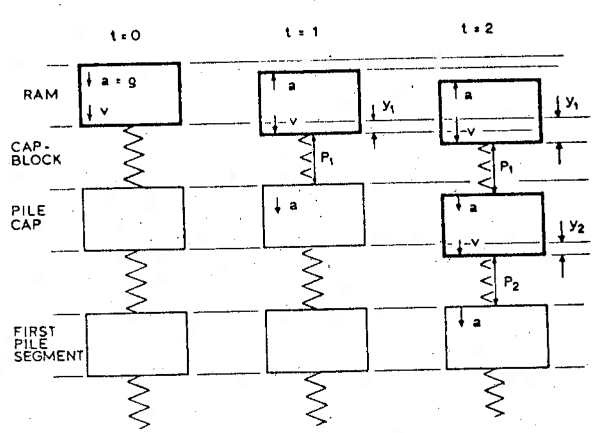


Fig. 2. Force build-up in pile

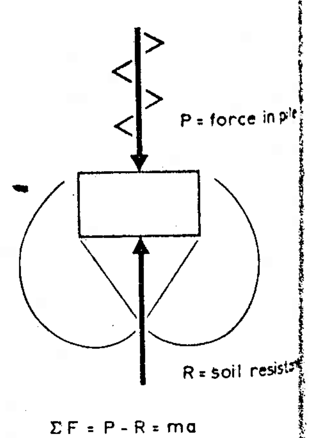


Fig. 3. Forces acting at pile tip

# ...by the Wave Equation

*by Tonis Raamot*

...nal distance and the difference between that distance and the travel of the pile cap then represents the compression in the capblock at that particular time, and a new force in the capblock can be calculated. This procedure can be repeated for many time intervals, and the forces in the capblock and in the pile can be traced for any desired length of time. With very little effort, an engineer can, by a hand calculation, solve an actual case in order to understand the application of the wave equation to the dynamics of pile driving.

Figure 3 represents the mechanics of the last segment of the pile, or the pile tip. In this figure, R is the ultimate soil resistance and P is the force in the pile at any given time. For values of P less than R, only enough soil resistance is mobilized to counteract the force in the pile. However, at a time when P exceeds R, the net force must again be equal to mass times acceleration and downward movement of the pile point occurs, resulting in a set of the pile. The set then is determined by the magnitude and the duration of the force in the pile.

Figure 4 illustrates the transmission of the force wave in a Step-Taper mandrel driven pile. The force in the pile is shown at different times after the initial impact of the hammer. The maximum force at the pile head occurs .0028 seconds after impact. Shortly after, the tip of the pile is beginning to feel some force. The force at the pile tip then begins to build up as shown in the lower part of the graph. No movement of the pile can take place until the force in the pile point has exceeded the soil resistance at the point. Hence, for this example, useful work on the pile tip is done only for about .003 seconds beginning .0045 seconds after initial impact. If dynamic effects of soil resistance are neglected, the magnitude and duration of this useful force could be represented by the shaded area in Figure 4. The resulting set per blow for this case is .026 inches, which corresponds to about 38 blows per inch.

Typical shapes of the stress wave are shown in Figure 5. The vertical axis represents the pile length. The magnitude of the force in the pile is plotted horizontally along the various points of the pile. If a pile is very stiff, that is it has a large cross-sectional area or is made of material having a high modulus of elasticity, the shape of the stress curve is similar to the one labeled "hard". This stress

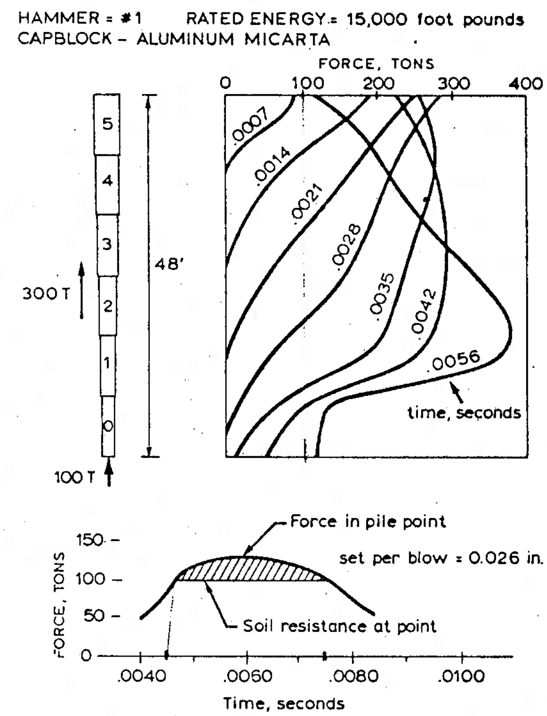


Fig. 4. Illustration of force waves in pile

wave is shorter but it has a higher peak force. A soft or springy pile will tend to cushion the blow of the ram. Therefore, the ram is in longer contact with the pile and the force trans-

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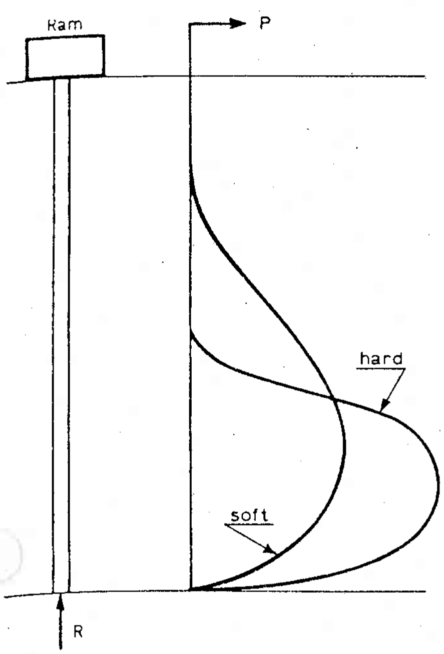


Fig. 5. Effect of pile and capblock stiffness on force wave

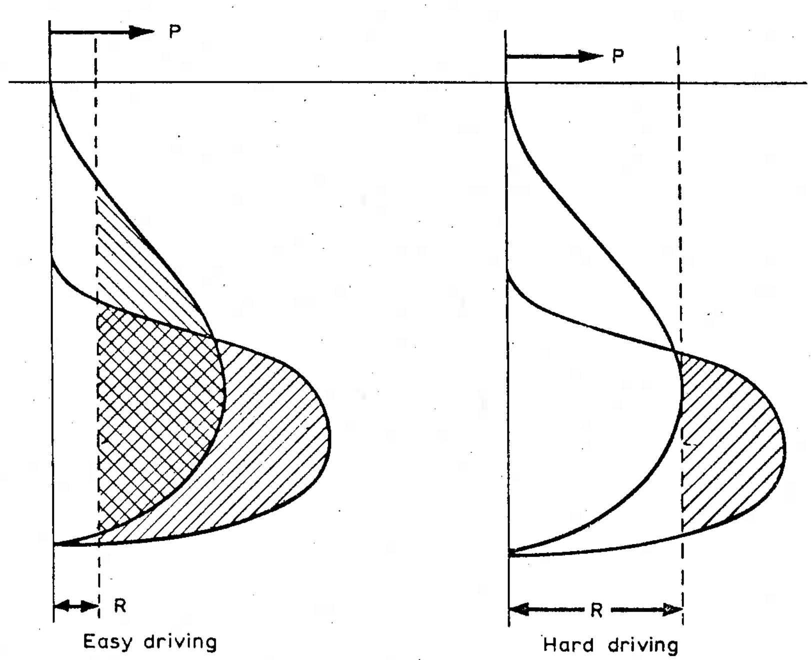


Fig. 6. Illustration of useful driving energy in piles of different stiffness

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# Analysis of Pile Driving by the Wave Equation

mitted by the ram is generally of smaller magnitude than the force generated in the stiff pile. The curve labeled "soft" illustrates the relative shape of the stress wave for this case. The terms hard and soft in Figure 5 can also be applied to capblocks.

Figure 6 shows the effect of the stiffness of the pile-capblock system on pile driving. As previously shown in Figure 3, the pile can advance only when the force in the pile overcomes the soil resistance. In the case of easy driving, the soil resistance R is small and both stress waves are effective in advancing the pile point as indicated by the shaded areas on the graph. As the soil resistance increases, however, the soft pile will meet refusal earlier than the stiff pile. Absolute refusal is attained when the soil resistance becomes equal to the maximum force in the pile. At a soil resistance that prevents a springy pile from penetrating further, there is still enough energy left in the stiff pile to cause further penetration. Hence, a stiff pile driven with a hard capblock can be driven to a much greater capacity than a soft pile driven with a soft capblock.

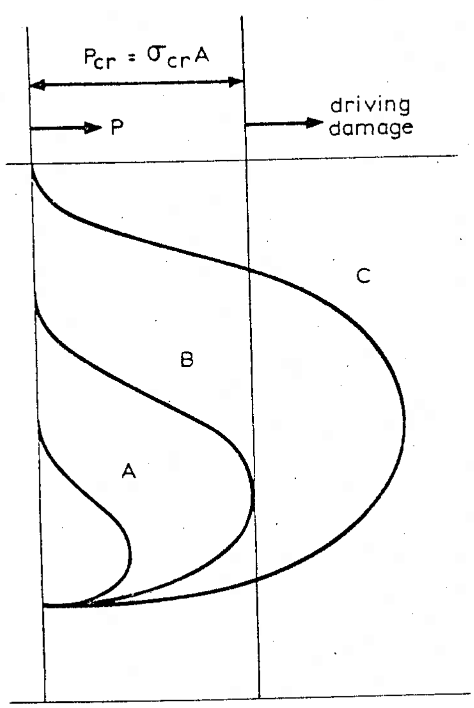


Fig. 7. Effect of hammer size on driving stresses

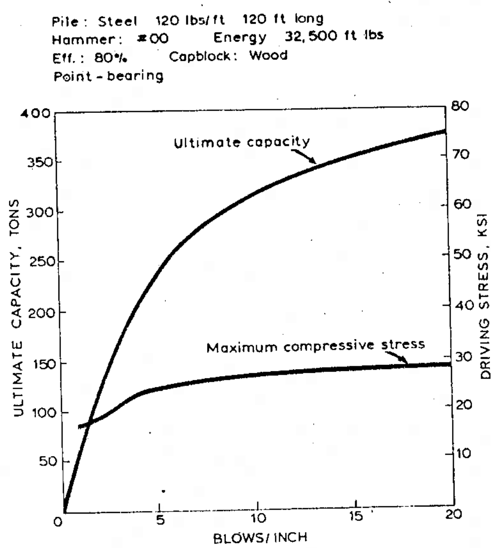


Fig. 8. Typical results of wave equation analysis

The stress level that a pile can sustain without driving damage is constant for any given pile material. This principle is illustrated in Figure 7, where the vertical axis again represents the length of the pile. $P_{cr}$ is the maximum driving force that the pile can sustain without damage. Curve A is a stress wave caused by a light hammer that obviously does not damage the pile. The hammer associated with curve B is on the verge of causing driving damage, while a still heavier hammer, represented by curve C, would cause considerable pile damage during driving.

The results of a wave equation analysis can be represented in graphical form as shown in Figure 8. The shape of this graph is familiar to engineers, with pile capacity plotted against driving resistance in blows per inch. The only difference is that instead of working pile capacity, this curve represents the ultimate capacity of the pile without a factor of safety. An appropriate factor of safety should be chosen by the engineer to determine allowable working loads. In addition, maximum stresses are obtained to determine whether or not the pile will be damaged during driving.

A typical case history illustrating the use of the wave equation results is shown in Figure 9. Pipe Step-Taper piles about 110 feet long were to be driven into limerock to a specified resistance. Three test piles were driven with a 19,500 ft. lb. hammer. These piles were load tested and all three failed to carry a 100-ton test load. It was decided that a heavier hammer was needed to develop additional pile capacity. A 32,500 ft. lb. hammer was sent to the job site. When this hammer was used on the same piles, it was found that the pipe folded up during hard driving. The job was finally done with a 24,375 ft. lb. hammer, which was able to develop the test load and yet not damage the pile. This short case history demonstrates the delicate balance between pile capacity and driving stress.

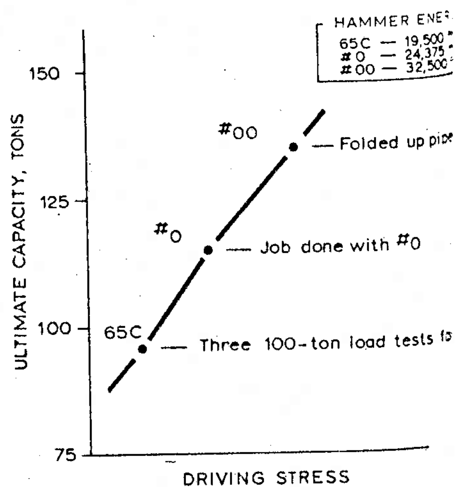


Fig. 9. Study of actual job using wave equation analysis

The wave equation describes only the structural dynamics of pile driving and does not solve any of the associated soil mechanics problems. It will not predict the length of piles, nor will it predict the long-term behavior of the soil surrounding the piles. Hence, it does not replace judgement and experience developed by engineers and contractors in the field of pile driving. It merely assists to systematize existing knowledge by accurately describing the energy transfer from the hammer to the tip of the pile. It thus fulfills the function of empirical dynamic pile driving formulas in a much more accurate and comprehensive manner. □