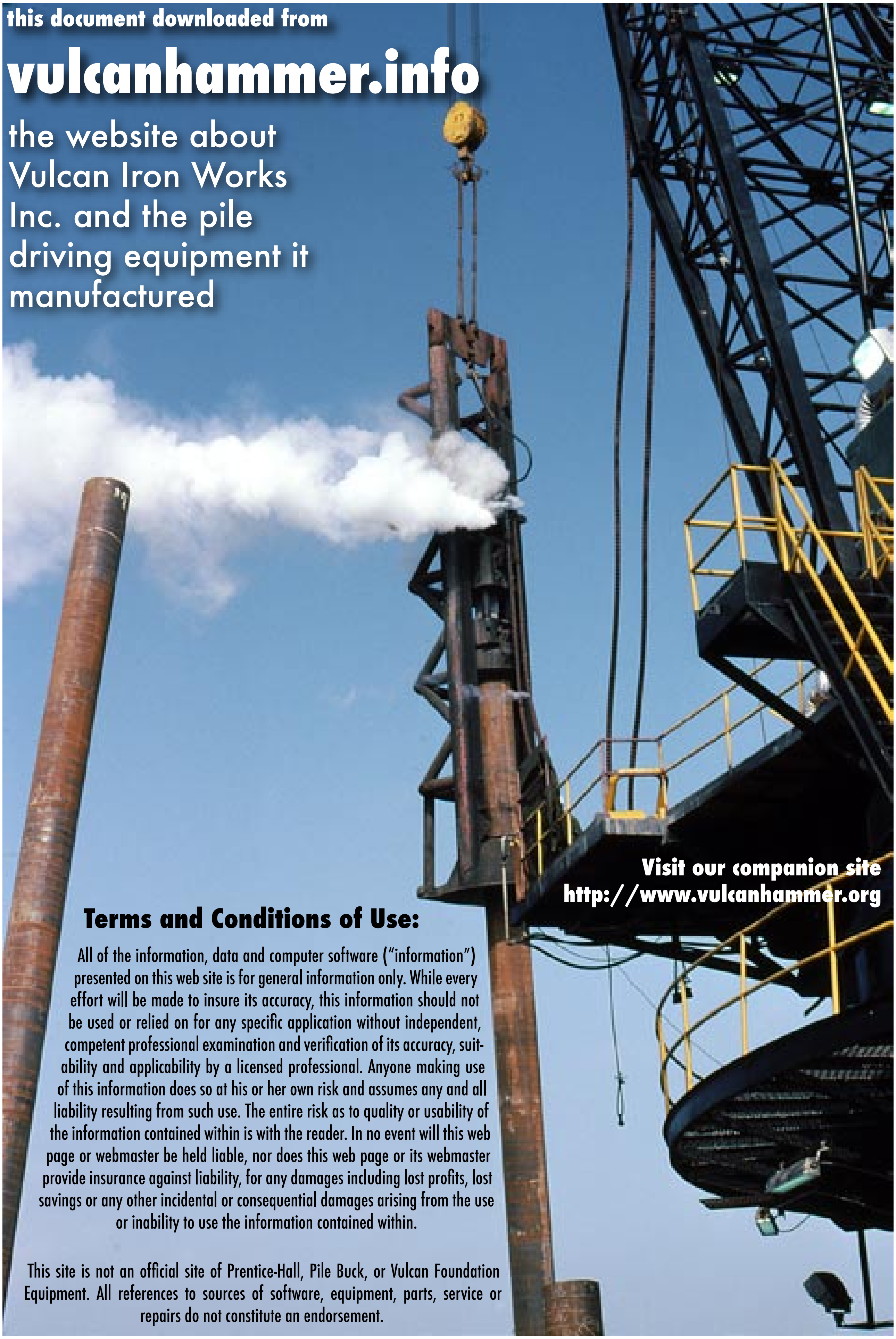

The Wave Equatic

by C. Russ Graff
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For many years engineers have tried to find reliable methods of determining the static bearing capacity of piles from observations of their behavior during driving and from dynamic energy considerations. Eytelwein published his formula in 1820; Retenbacher's so-called "complete formula" was published in Germany in 1858, over 100 years ago. Chellis in his book on pile foundations lists 38 different formulas that have been proposed and used.

During the past 35 years, engineers working in the field of soil mechanics have come to put less and less reliance on the dynamic approach to determining pile capacities. In fact, many practicing engineers have come to depend completely on laboratory determined soil characteristics and have used measured driving resistances only as a means of field control of pile driving. The trend toward discarding the dynamic approach has occurred because experience has shown that the old dynamic pile driving formulas gave very erratic results.

During the past decade a new dynamic approach has been developed. It is now possible with electronic digital computers to perform tedious and complicated calculations in a very short time. These computers have made it possible to solve problems with a large number of variables.

Timoshenko, in his "Theory of Elasticity" published in 1934, showed a second order differential equation for calculating wave action in a uniform steel rod. When an attempt was made to apply this equation to the driving of piles the addition of such necessary factors as compression of the cushion block, action of the hammer and compression of the ground, made the resulting equations too complicated to solve. The use of electronic digital computers has changed this completely. By the wave equation approach it is now possible to calculate in a practical and economical way what happens when a hammer hits a pile. It is now possible by this method to calculate with reasonable accuracy the stresses in the pile and the ultimate resistance at the instant of driving.

There has been a growing tendency in past years to ignore dynamic considerations in evaluating piles. It has always seemed that there must be a relationship between driving resistance and static bearing capacity, although that relationship might be complicated and obscure and might not fit any known formula. It also seemed that to disregard the data obtained from observation of pile behavior during driving was to ignore useful data. Driving resistance criteria must be used in the judgment derived from experience.

One of the advantages of observing and recording pile driving resistances is that it provides a method of discovering local variations in the soils across a particular site. When pile capacities are determined from laboratory tests on soil samples, one of the weak points is that there is seldom really adequate data. The soils on a given site are seldom homogeneous, the soil strata are usually variable in thickness and are frequently discontinuous. Because of economic considerations the engineer very often does not have an adequate number of borings and must rely on interpolation and extrapolation in making his calculations and recommendations. When driving resistance is observed and recorded it can be used to fill in the gaps in the knowledge of the soil conditions on any given site. Each pile is in effect a test pile, and the required lengths can be adjusted as the job proceeds to fit the newly discovered conditions.

However, if dynamic considerations are to be used we need a better understanding of what happens to the pile during driving. The principal reason that piledriving formulas have fallen into disrepute with soils engineers is that they have failed to account for the erratic behavior of different types of piles in different soil conditions. In order to understand why this is so it might be useful to review the basis for dynamic formulas.

Most dynamic formulas start with the simple energy equation $Wh = Rs$. In this equation the left side represents the energy developed by the piledriving ram W falling through a height h, and the right side is the work done against resistance R through a distance s. This basic equation has a fallacy in that it assumes that R is a constant through the distance s. We would expect this would rarely be true. For example, for piles driven in sand, R would probably increase through the distance s, while for piles driven in clay the opposite would probably be true. R, then, must be considered to be an average value. The basic formula must be further modified to take into account the energy losses that occur, such as deformation of the ram, compression of the cap or cushion block, compression of the pile and compression of the soil. When Wellington derived the Engineering News Formula he based it on his experience with the driving of piles and used a work diagram in which he took into consideration the variable nature of the soil resistance, R. When his formula is used for the types of piles and hammers with which he was familiar and for safe loads in the lower ranges, it gives as good results as the so-called complete formulas which attempt to introduce constants for the various energy losses. However, it falls down completely when used outside the range of conditions on which it was based.

Most other dynamic piledriving formulas have used the basic energy equation as a starting point and by theoretical mechanics and mathematics have attempted to take into account all the energy losses. The large number of formulas that have been presented indicates the wide variety of assumptions that have been made as to these energy losses.

Over the years a number of studies have been made comparing load test results with pile capacities determined by dynamic formulas. They all show very erratic results. The reasons for the poor results fall into two categories. (1) The dynamic formulas themselves do not truly represent the behavior of piles under driving and (2) no dynamic formulas can take into consideration soil phenomena such as "set up" in the ground, relaxation of the ground, and consolidation of soils under long-time load. The fundamental errors in pile driving formulas were covered very thoroughly by A. E. Cummings,[1] who concluded that customary methods of evaluating energy losses are erroneous. The three usual energy loss deductions

[1] "Dynamic Pile Driving Formulas" by A. E. Cummings, Journal of The Boston Society of Civil Engineers January 1940.

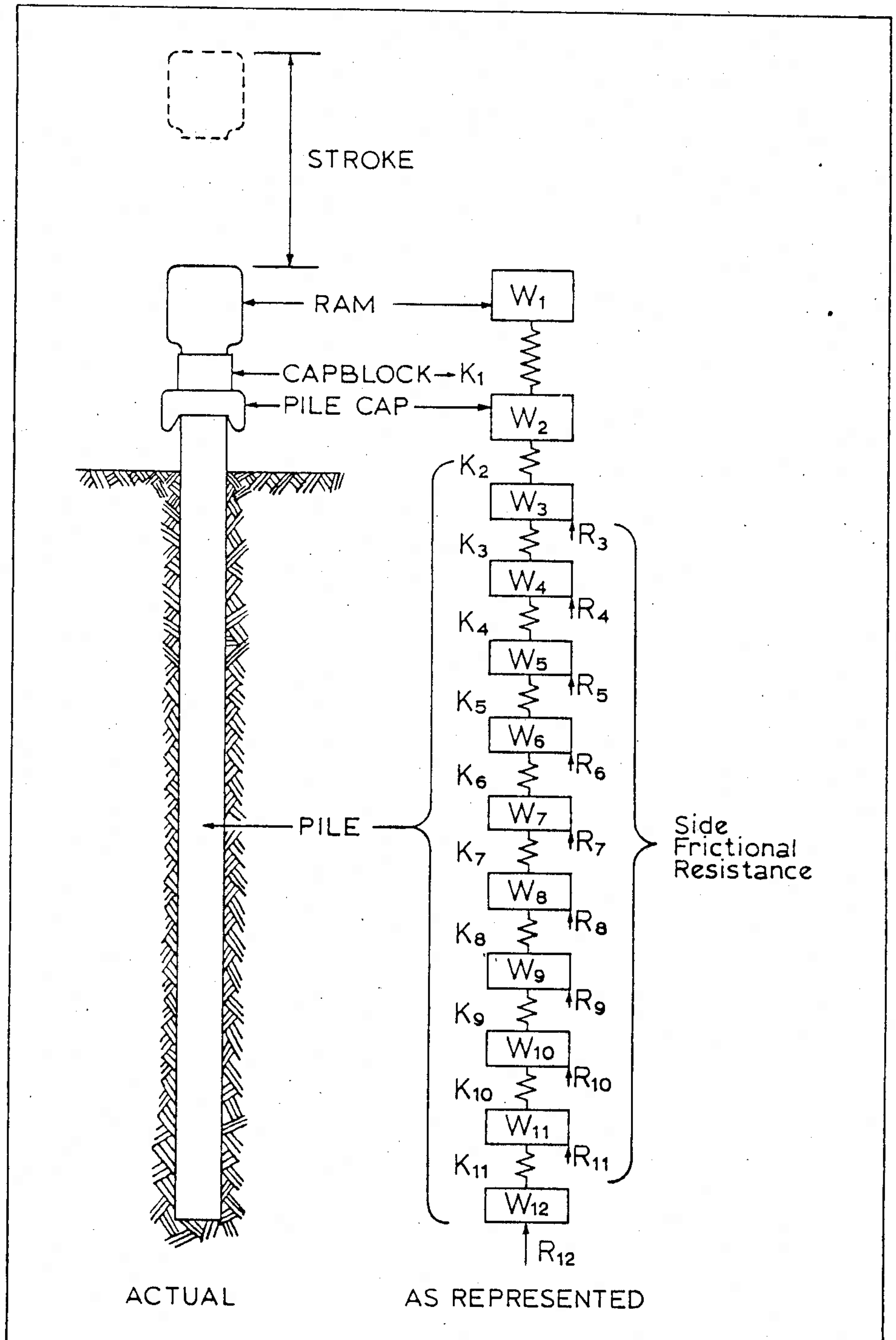
nd Pile Driving

...re: (1) Only the temporary elastic ...mpression, (2). only the Newtonian loss (i.e. the inertia loss), and (3) one and two combined. All these are defective because (a) the temporary elastic compression is taken from static theory, not from dynamic theory, (b) the Newtonian theory of impact does not apply to elongated bodies, and (c) some losses are deducted twice when both (a) and (b) above are combined.

The wave equation approach was suggested by Cummings as the most likely to give good results. However, at the time he wrote his paper a practical method of solving the wave equation was not available.

About forty years ago a young man, E. A. Smith, came to work for the Raymond Concrete Pile Co. fresh out of college. He started to work in the mechanical engineering department, and his first assignment was to try to find out what happened to a pile under driving. He was a skillful mathematician and he spent a number of months on the project, doing a great deal of re-...rch. He finally reported that the problem was too complicated to be solved by mathematical means because of the many variables involved. He was able to cite authorities to support his conclusion. Over the years the problem was tucked away in a corner of his mind to be brought out occasionally, re-examined, and then tucked away again. With the development of electronic computers he finally saw the possibility of a solution to the problem. He worked up a general method to solve impact and longitudinal wave transmission problems in a step by step process and wrote a few preliminary papers followed by a complete paper[2] published in 1962. He has since had the satisfaction of having his method tested and approved by authorities in this field.

It might be interesting to give a brief description of the development of the basic formulas. The pile is represented as a series of concentrated weights separated by weightless springs. Each weight ...acted upon by the two forces of the ...jacent springs and may also be acted upon by any given external force. The time during which the action occurs is divided into small time intervals. The time intervals must be small enough so that it may be assumed, with negligible error, that all velocities, forces and displacements will have fixed values during the interval. The numerical calculation is a step by step process which can be worked out with a slide rule, but if the time interval is very small the number of steps becomes exceedingly large. This is where the electronic digital computer makes the method practical for complicated problems.



For each ideal concentrated weight and its associated spring there are eight constants. (1) The concentrated weight, (2) the spring constant, (3) the external force, (4) the acceleration of gravity, (5) Young's modulus of elasticity, (6) the cross-sectional area, (7) the unit length and (8) the time interval. The calculation uses five variables for each weight for each time interval. These are: (1) The displacement of the weight from its initial

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[2] "Pile-driving Analysis by the Wave Equation", by E. A. Smith, Transactions, ASCE. Volume 127, 1962, Part 1, Page 1145.

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THE WAVE EQUATION AND PILE DRIVING

position, (2) the compression of the spring, (4) the net force acting on the weight and (5) the velocity of the weight.

Each variable has an equation which may be solved to give a numerical value for each time interval. (1) The displacement is figured from the displacement for the preceding interval, plus its velocity in the preceding interval multiplied by the time interval. (2) The compression of this spring is merely the difference between the displacement of adjacent weights. (3) The force exerted by this spring is the compression of the spring times the spring constant. (4) The net force is arrived at by simple addition and subtraction of forces previously calculated or given, and (5) the velocity of the weight for any interval is the velocity of the preceding interval plus an increment of velocity acquired during the time interval. This increment is calculated from Newton's law that change of velocity equals force times the time divided by the mass.

When the calculation is done by hand it requires a vertical column for each variable for each concentrated weight and a horizontal line for each time interval in each column. Each entry is calculated from entries previously made plus given constants. This is what makes it possible to set up the problem on an electronic computer.

It is impossible in the space available to present any more than this very general idea of the wave equation, but it should suffice to stimulate further investigation. With this method it is possible to calculate accurately the stresses in a pile and also the ultimate resistance at the instant of driving. This fills a gap in our understanding of pile behavior.

The wave equation approach is merely another tool for the soils engineer and will not replace present methods of determining static bearing capacity. There is no difficulty in using the method for piles of varying sections and for other than pure end bearing. Constants can be introduced for any assumed condition of frictional resistance.

At the present stage of development of the art and science of soils engineering, there is tendency to design piles purely on the pile-soil relationship and to ignore the fact that piles are installed by dynamic methods. With the trend toward higher and higher loads we are in sore need of some method of calculating driving stresses in piles. This wave equation approach seems to be the most practical and reliable. By this method it will be possible to determine whether a pile of given material, shape and length can be driven without damage to the depth required from computations based on contact area and laboratory determined soil characteristics.

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The Myth of Manhattan

as far north as West 230th Street in the Bronx. When a cut was made across Harlem to widen the River and improve navigation, the old bend in the river dried up and disappeared. This stranded a large piece of land that still belongs to Manhattan, although it now adjoins the Bronx on the opposite side of the river.

Not too many years ago, New Yorkers could sail or row a small boat completely across lower Manhattan. The boater could enter a creek on the East River just above the site of the present Brooklyn Bridge, sail about a half mile to an irregular shaped body of water (called Collect Pond) just north of the present Municipal Building, then continue on another creek to the Hudson River. This last creek was later made into a canal—and still later was filled in to become the present Canal Street. Incidentally, Collect Pond, once renowned for its scenic beauty and a popular spot for boating and ice skating, was the site of the first steamboat trials. Today, in its filled-in stage, the pond is a headache for foundation engineers who must drive extremely long piles to reach firm bearing strata.

In uptown Manhattan, a complete crossing of the island by boat could not be made. From a point that is now 108th Street and the East River, the boater could paddle upstream on what was known as Mill Creek to the northern tip of Central Park and then on to Morningside Park. If the boater could carry his boat over a small portage, he was on his way to the Hudson River.

Today, there is little resemblance to New York City's topography of a hundred years ago. With the exception of Central Park and some small scattered parks throughout the City, there are few of the original green knolls, swamps, ponds and creeks remaining.

EDITOR'S NOTE:

Much of Mr. Nalen's familiarity with the physical evolution of New York City comes from an array of priceless maps, some of pre-Revolution vintage, which he has collected over the years. These geologic and topographic charts and maps accurately spot the old shoreline, swamps, ponds, knolls and creeks that sculptured the city's landscape in bygone days.

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UTAH SAND DRAINS

drive the sand drains. The pipe mandrel used on Interstate 15, for example, had a 14 inch O.D. and a ¾ inch wall thickness. This mandrel is closed at the lower end with a 16-inch diameter plate that is attached to the mandrel by a hinge.

At the upper end of the mandrel, there is a hopper with a flap gate opening where the sand enters the pipe. The mandrel was driven with a steam hammer with an energy rating of 19,500 ft. lbs.

As the mandrel is being driven, a skip loaded with sand is hoisted up to the hopper and the sand spills into the mandrel. The air-tight flap gate on the mandrel is closed when loading is completed and air under pressure is introduced into the mandrel.

The air pressure is gradually increased until it reaches approximately 100 psi at the time driving is completed. Upon reaching required depth, the mandrel is extracted, at which time the bottom plate opens and air pressure forces the sand out of the mandrel as it is lifted leaving a column of sand in place.

The effectiveness of vertical sand drains has been proven many times by a system of settlement platforms installed in fills and checked periodically. Recently, near Brigham City, a compacted test fill 20 feet high was placed without sand drains. After 300 days in place, the settlement measured 0.75 feet. Sand drains were then installed by pre-boring through the compacted fill. Test results show that in another 120 days the fill had settled an additional 2.25 feet.

The design for this section of Interstate 15 was performed by Porter, Urquhart, McCreary and O'Brien, Salt Lake City, Utah.