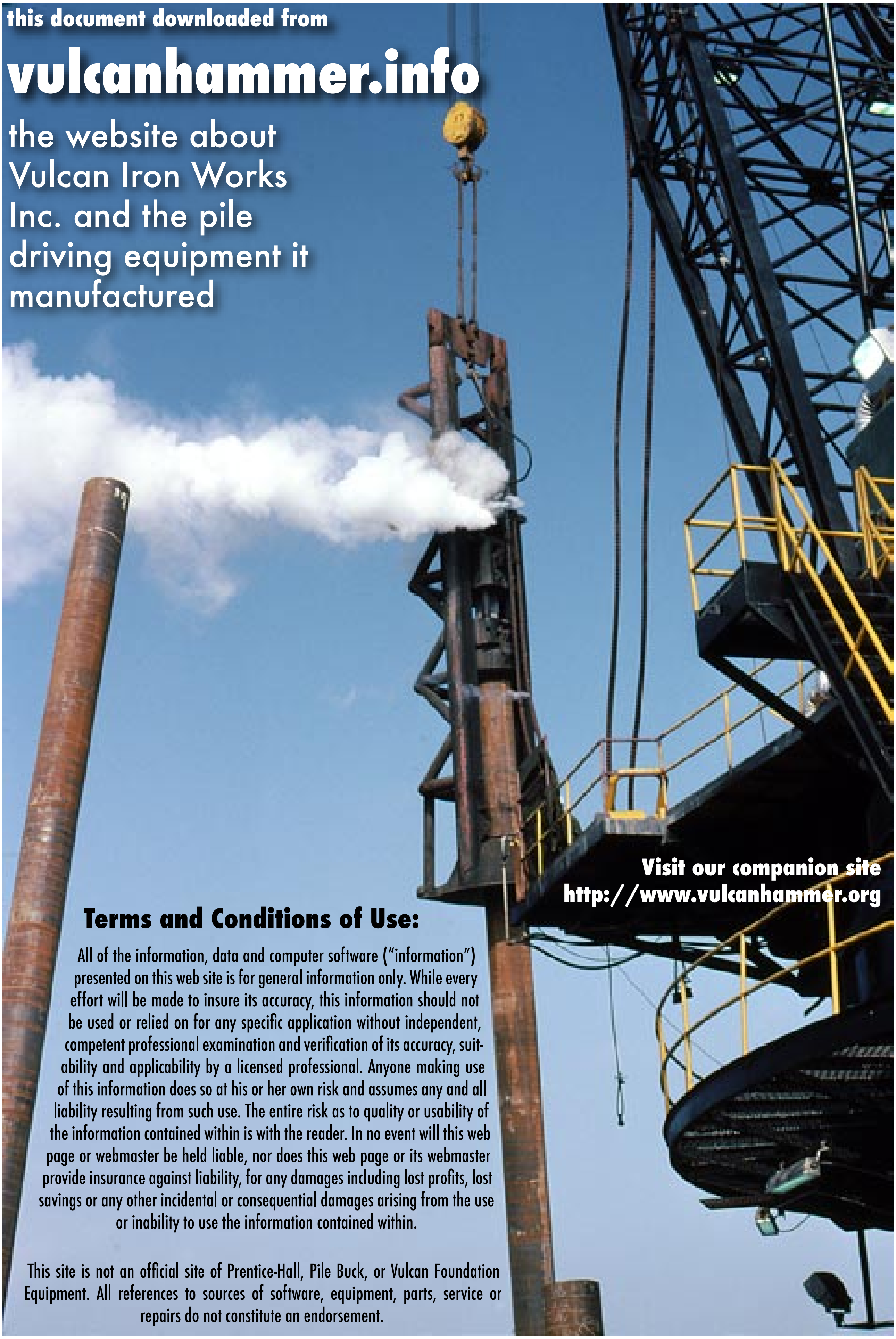

TEXAS
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DEPARTMENT

COOPERATIVE
RESEARCH

A COMPARISON OF DYNAMIC PILE DRIVING FORMULAS WITH THE WAVE EQUATION

in cooperation with the
Department of Transportation
Federal Highway Administration
Bureau of Public Roads

RESEARCH REPORT 33-12
STUDY 2-5-62-33
PILING BEHAVIOR

A COMPARISON OF DYNAMIC PILE DRIVING FORMULAS WITH THE WAVE EQUATION

by

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Research Report 33-12

Piling Behavior
Research Study No. 2-5-62-33

Sponsored by
The Texas Highway Department
in cooperation with the
U. S. Department of Transportation, Federal Highway Administration
Bureau of Public Roads

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TEXAS TRANSPORTATION INSTITUTE
Texas A&M University
College Station, Texas

FOREWORD

The information contained in this report was developed on research study 2-5-62-33 entitled "Piling Behavior" which is a cooperative research study sponsored jointly by the Texas Highway Department and the U. S. Department of Transportation, Federal Highway Administration, Bureau of Public Roads.

The broad objective of the research described in this report is to establish whether the resistance to penetration predicted by any of the commonly used pile driving formulas agrees with that found by the wave equation, and if so, to determine which piles, driving hammers, and soil resistances provide agreement.

The use of the wave equation to determine the range of accuracy for various pile driving formulas would enable them to be used with far greater confidence than is presently the case.

The opinions, findings, and conclusions expressed in this publication are those of the authors and not necessarily those of the Bureau of Public Roads.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Appendix

Introduction

The use of the wave equation to investigate the dynamic behavior of piling during driving has become more and more popular[1, 2] during the past several years. Widespread interest in the method had its beginning in 1960 when E. A. L. Smith[3] used a numerical solution to investigate the effects of such factors as ram weight, ram velocity, cushion and pile properties, and the dynamic behavior of soil during driving. Since then, vast quantities of data have been amassed in experimentation to determine more accurate values for the input variables required,[4, 5, 6, 7, 8] and a multitude of full-scale pile tests have been correlated with the wave equation.[9, 10, 11] These correlation studies have proven that the wave equation is more accurate than other methods and can be used with reasonable confidence. Because of this, the method is becoming widely used and recommended in the literature by foundation experts.[1]

However, as noted by Chellis,[12] a wave equation analysis required the use of a high speed digital computer; before an engineer could utilize the method, he had to develop a relatively complex computer program which was both time consuming and expensive. Therefore, even though the wave equation might be far more accurate, the simplicity of the dynamic pile driving formulas made their use attractive, especially for field use.

For this reason, Chellis and others[13] suggested that the wave equation should be used to determine if there might exist ranges of application through which simplified dynamic formulas were reasonably accurate. He suggested that, "if it can be determined that the Hiley or *Engineering News* Formula results are safe and that ultimate driving resistances are in reasonable agreement with wave equation results in any general range of conditions, then such formulas might permissibly be used within such limits. This might enable such simple formulas to be quickly applied in the office or field, thus avoiding the necessity of access to a computer in order to be sure of obtaining sufficiently reliable and economic results."

This report presents the results of such a study, and demonstrates that it is indeed possible to find ranges of agreement between the wave equation and certain pile driving formulas.

Problems Investigated

The dynamic behavior of a pile during driving is extremely complex, and involves a multitude of variables including the type of hammer, driving accessories, type of cushion, dimensions and properties of the pile, as well as the properties of the supporting soil medium. Since it was obviously impossible to compare the pile driving formulas with the wave equation for every possible combination of variables, the study was limited to the following:

1. *Hammers and pile driving assemblies.* Three pile driving hammers were studied: the Vulcan No. 1, Vulcan 80C, and Delmag D-22. The hammer properties and operating characteristics listed in Table 1 were determined from previous research conducted by the authors and published through the Texas Transportation Institute.[14] Typical hammer assemblies for the Vulcan No. 1 and Vulcan 80C, driving steel, and concrete piles, are illustrated in Figures 1 and 2, respectively. The Delmag D-22 was chosen as a typical diesel hammer, and typical driving assemblies used to drive steel and concrete piles are illustrated in Figures 3 and 4, respectively.

2. *Pile material.* Two pile materials, steel and concrete, were used in this study. A modulus of elasticity of 30 x 10^6 psi was assumed for steel, and 5 x 10^6 psi for concrete.

3. *Pile length.* To determine the limits of accuracy of the pile driving formulas for various pile lengths, piles having lengths of 30, 60, 100, and 140 ft. were analyzed.

4. *Cross-sectional areas of pile.* Three typical cross-sectional areas were used in the study for each type of pile; the steel piles had cross-sectional areas of 10, 20, and 30 sq. in., whereas the concrete piles had cross-sectional areas of 150, 275, and 400 sq. in.

5. *Magnitude of soil resistance.* To maintain a reasonable number of problems for analysis, only two soil resistances for each pile were analyzed. The first resistance represented moderate driving (a low

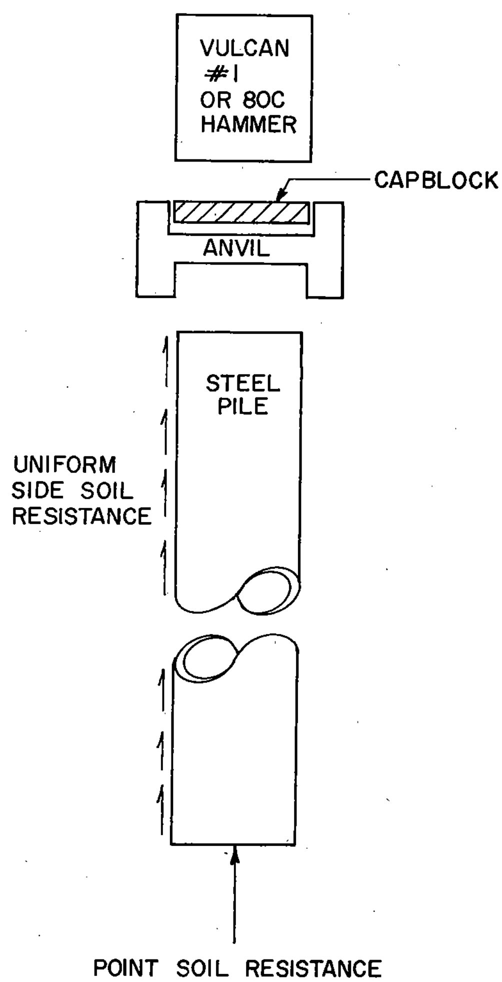

Figure 1. Pile driving assembly of Vulcan No. 1 and Vulcan 80C hammers—steel pile.

TABLE 1. HAMMER PROPERTIES

Hammer	Ram Weight (lbs)	Anvil Weight (lbs)	Helmet Weight (lbs)	Hammer Effi-ciency	Rated Energy Output (ft lbs)	Actual Energy Output (ft lbs)	Capblock Stiffness (kips/in.)	Cushion Stiffness (kips/in.)	Explosive Force (kips)	Coeffi-cient of Resti-tution
Vulcan No. 1	5,000	none	1,000	0.75	15,000	11,250	1,080	2,000	0	0.8
Vulcan 80C	8,000	none	1,000	0.85	24,800	21,500	1,080	2,000	0	0.8
Delmag D-22	4,850	1,576	1,200	1.00	39,700	29,100*	23,800	2,000	158.7	0.8

*Actual energy output of diesel hammer was determined by method presented in Ref. 8, E = Wh (efficiency) where W = ram weight and h = actual ram stroke (6 ft in this case).

number of blows per foot) for the particular hammer, while the second was intended to simulate relatively hard driving. For the Vulcan No. 1, soil resistances of 50 and 200 kips were used. For the Vulcan 80C and Delmag D-22 hammers, soil resistances of 100 and 400 kips were used.

6. *Soil resistance distribution.* For each of the previously mentioned cases, two distributions of soil resistance were studied. These distributions were as follows: (a.) all soil resistance at the point of the pile (no side friction), and (b.) all soil resistance uniformly distributed along the side of the pile (no point resistance).

7. *Other factors.* Other factors known to affect the behavior of piling during driving were determined from experimental information published previously by the authors,[14] and were held constant. These included such factors as hammer efficiency, cushion stiffness, coefficient of restitution, and others. These factors are listed in Table 1.

Scope of the Investigation

Although this study was obviously small with respect to the number of variations possible, the results should give some indication as to the relative accuracy of the commonly used pile-driving formulas and demonstrate at least one method by which considerable useful data can be developed by future studies.

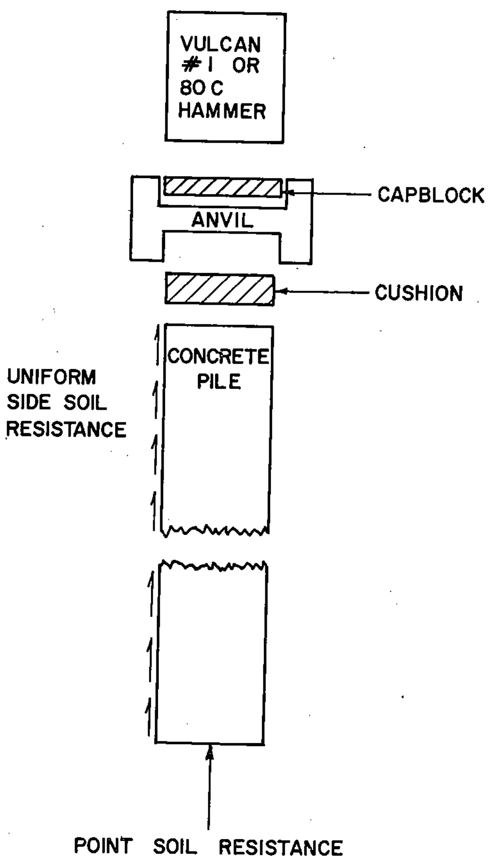

Figure 2. Pile driving assembly of Vulcan No. 1 and Vulcan 80C hammers—concrete pile.

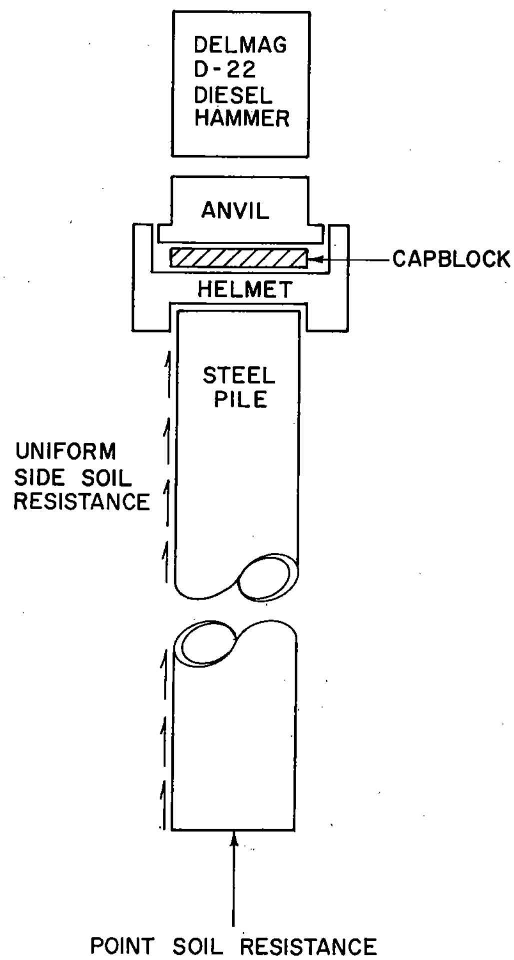

Figure 3. Driving assembly of the Delmag D-22 hammer—steel pile.

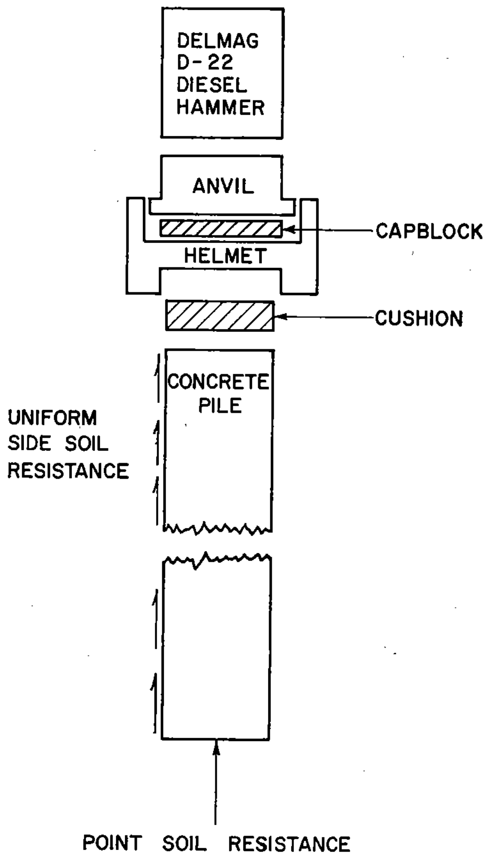

Figure 4. Driving assembly of the Delmag D-22 hammer—concrete pile.

Even though only two or three changes were made in each significant variable, this study required the solution of 288 problems by the wave equation and by each of the 11 pile-driving formulas. It is therefore obvious that a complete study encompassing every type of hammer, pile, and soil would be relatively expensive and difficult.

Tables 2 and 3 summarize the variables used in this study.

TABLE 2. SUMMARY OF HAMMERS AND SOIL PARAMETERS USED IN THE STUDY

Hammer Type	Soil Resistance (kips) RU_w	Soil Distribution	Case
Vulcan #1	50	Point only	A
		Side only	B
	200	Point only	C
		Side only	D
Vulcan 80C	100	Point only	E
		Side only	F
	400	Point only	G
		Side only	H
Delmag D-22	100	Point only	I
		Side only	J
	400	Point only	K
		Side only	L

*Each pile listed in Table 3 was solved.

TABLE 3. SUMMARY OF PILES ANALYZED

Pile Material	Pile Area (in.^{2})	Pile Length (ft.)	Number
Steel	10	30	1
		60	2
		100	3
		140	4
	20	30	5
		60	6
		100	7
		140	8
	30	30	9
		60	10
		100	11
		140	12
Concrete	150	30	13
		60	14
		100	15
		140	16
	275	30	17
		60	18
		100	19
		140	20
	400	30	21
		60	22
		100	23
		140	24

Pile Driving Formulas Used In This Investigation

Because of the vast amount of information previously published concerning the derivation, use, and practical application of both the wave equation[3] and the pile driving formulas[17] used in this investigation, background material will not be presented here. For those interested in the history and prior research which has been done on the wave equation, the references presented at the end of this report should be helpful.

The dynamic pile driving formulas are not presented in their most commonly used form, but rather have been modified to assure that the units are consistent, and to omit the factor of safety normally incorporated in the formula (if one exists). This permits a direct comparison between the ultimate resistance to penetration at the time of driving predicted by the wave equation and that given by the driving formula.

In this study, eleven different dynamic pile-driving formulas were used to predict the capacities of 288 steel and concrete piles of varying lengths and cross sectional areas. These formulas are presented in Table 4.

As was expected in the work, it was sometimes difficult to determine the constants to be used in the formula, since the values recommended in the literature varied widely. It should be noted that the authors did not intend to use the constants they considered the most accurate, but rather those equations and constants most commonly used in the field.

The authors understand that many who read this report may feel that better results might have been obtained had some different form of the equations been used, or perhaps different constants applied. For this reason, the results of the wave equation analysis are tabulated in the Appendix.

This information should enable the reader to compare the results predicted by the use of other constants or pile driving formulas with that given by the wave equation.

TABLE 4. PILE DRIVING FORMULAS USED IN THIS INVESTIGATION

(All factors of safety have been removed)

Engineering News

$$RU_F = \frac{U}{S + C}$$

Michigan Engineering News

$$RU_F = \frac{U}{S + 0.1}\left[\frac{W_r + e^2W_P}{W_r + W_P}\right]$$

Eytelwein

$$RU_F = \frac{U}{S + 0.1\left(\frac{W_P}{W_r}\right)}$$

Navy-McKay

$$RU_F = \frac{U}{S\left[1 + 0.3\left(\frac{W_P}{W_r}\right)\right]}$$

Hiley

$$RU_F = \left[\frac{U(e_f)}{S + 0.5(C_1 + C_2 + C_3)}\right]\left[\frac{W_r + e^2W_P}{W_r + W_P}\right]$$

Terzaghi

$$RU_F = \frac{AE}{L}\left[-S + \sqrt{S^2 + \frac{2U}{AE/L}\left[\frac{W_r + e^2W_P}{W_r + W_P}\right]}\right]$$

Redtenbacher

$$RU_F = \frac{AE}{L}\left[-S + \sqrt{S^2 + \frac{2U}{AE/L}\left[\frac{W_r}{W_r + W_P}\right]}\right]$$

Pacific Coast

$$RU_F = \frac{AE}{2L}\left[-S + \sqrt{S^2 + \frac{4U}{AE/L}\left[\frac{W_r + KW_P}{W_r + W_P}\right]}\right]$$

Canadian Building Code

$$RU_F = \frac{-S + \sqrt{S^2 + 4\,(\text{Can }1)\ (\text{Can }2)}}{2(\text{Can }2)}$$

Rankine

$$RU_F = 2\,\frac{AE}{L}\left[-S + \sqrt{S^2 + \frac{U}{AE/L}}\right]$$

Gates

$$RU_F = 247\ (\sqrt{U})\ \left(\log \frac{10}{S}\right)$$

where:

RU_F = Ultimate load capacity of pile at time of driving predicted by the driving formula (lbs),

RU_W = Ultimate resistance to penetration at the time of driving given by the wave equation.

U = Rated energy output of hammer (in. lbs),

e_f = Mechanical efficiency of hammer,

S = Permanent set of pile per blow (in.),

C = 0.1 for steam and diesel hammers and 1.0 for drop hammers,

e = Coefficient of restitution of cushion,

W_r = Weight of ram (lbs),

W_P = Weight of pile (lbs),

C_1 = 0.1,

$C_2 = \frac{RU_F}{AE}$,

$C_3 = 0.20\,\frac{RU_F}{A}$,

K = 0.25 for steel piles and .10 for concrete piles,

A = Cross-sectional area of pile (in.^{2}),

E = Modulus of elasticity of pile (psi),

L = Pile length (in.),

$$\text{Can }1 = U\left(\frac{W_r + e^2W_P}{W_r + W_P}\right),\ \text{and}$$

$$\text{Can }2 = \frac{1}{2AE/L} + \frac{1}{20{,}000A}$$

Correlation Procedure

During the course of this investigation, a total of 13 different methods were used to predict the resistance to penetration for each of the previously mentioned problems.

In each case, the wave equation was first used to solve the problem to determine the dynamic behavior of the pile for a single blow of the hammer. This solution gave information regarding the permanent set of the pile, temporary compression of the capblock, cushion block, and pile, elastic rebound of the pile, and all other variables required as input by the dynamic pile driving formulas. These values were then substituted into each dynamic pile-driving formula to determine its prediction for resistance to penetration, RU_F, for that case. The resistance to penetration RU_W predicted by the wave equation was then divided by the value predicted by the formula being considered.

For example, assume that a 100 ft. long steel pile with an area of 10 in.^{2} is driven by a Vulcan No. 1 hammer against a soil resistance RU_W of 50 kips, and that this resistance acts at the point of the pile (no side friction). When this problem is analyzed by the wave equation, the resulting permanent set is found to be 1.21 in. (see Table A1). Therefore, according to the *Engineering News* Formula given in Table 4, the ultimate load capacity of the pile at the time of driving will be:

$$RU_F = \frac{(15{,}000)\ \ (12)}{1.21\ +\ 0.1} = 137 \text{ kips}$$

Thus, for this particular case, the ultimate soil resistance to penetration at the time of driving predicted by the *Engineering News* Formula is high. Calculating the ratio of

$$\frac{RU_W}{RU_F} = \frac{50 \text{ kips}}{137 \text{ kips}} = 0.364,$$

this point is plotted in Figure 5 under steel piles with a cross-sectional area of 10 sq. in. and a length of 100 ft.

Similarly, the values predicted by the other 10 formulas were determined, and their ratios RU_W/RU_F plotted.

Thus, by using these graphs, the engineer can readily determine the relative agreement between the pile driving formulas considered and the wave equation.

Discussion of Results

The results of the correlations between the wave equation and the dynamic formulas are both surprising and extremely informative. As noted in Figures 5 through 7 the ratio of the resistance predicted by the *Engineering News* Formula and the wave equation is amazingly constant. This is not to say that the results are accurate, since the curves are consistently grouped around $RU_W/RU_F = 0.5$. This would indicate that at least for these cases the *Engineering News* Formula consistently predicts an ultimate value approximately twice the true resistance to penetration, such that when the recommended safety factor of 6 is applied to the equation, the true factor of safety would only be 3.

Nevertheless, the consistency of this formula is quite surprising, especially considering the amount of research which has recently been published condemning the method.[15, 16] This is not to imply that the *Engineering News* Formula is without proponents. In 1965, the Michigan State Highway Commission[17] completed an exhaustive research program designed to obtain a better understanding of the complex problem of pile driving, and to evaluate a number of pile driving formulas. Their research led them to modify the *Engineering News* Formula as noted in Table 2. This modification has been noted herein as the Michigan *Engineering News* Formula.

It is important to note that the entire Michigan research program dealt with long, slender, and relatively lightweight steel piling. As noted in Figures 8 through 10, it is seen that their proposed pile-driving formula gives results which are as consistent and accurate as the *Engineering News* Formula, for lightweight steel piling at least. However, Figures 8 through 10 also point out the complete inadequacy of the Michigan Formula for predicting capacities for heavy concrete piles. Similar results were found when attempting to predict the bearing capacity for extremely heavy steel piles, and for lightweight shell piles driven by heavy solid steel mandrels. For example, as noted in Figure 8, if a concrete pile having an area of 400 in.^{2} and a length of 100 ft. was to be driven to a side frictional resistance of 50 kips by a Vulcan No. 1 hammer, the Michigan Formula would not indicate satisfactory resistance to penetration until an actual resistance of 115 kips was obtained. Thus, there would be an unseen factor of safety of 2.3 beyond the factor normally used.

This is by no means meant to detract from the usefulness of the Michigan Formula, but rather to emphasize the potential danger of extrapolating such equations which were derived using only a certain type of pile under certain conditions.

As seen in Figures 11 through 16, the Eytelwein and Navy-McKay Formulas are relatively inconsistent.

The results for the Hiley Formula noted in Figures 17 through 19 are extremely interesting. Although the angle and spread of the curves is much greater than that for the *Engineering News* Formula, notice that they in general center about a ratio of $RU_W/RU_F = 1.0$, especially for steel piles with hard driving resistance at point. Predictions for long and/or heavy concrete piles show far less agreement, as would be expected from previous experience. None-the-less, the method's popularity is clearly indicated for steel point bearing piles since it comes closer to predicting the true average pile capacity for a greater variety of steel piles and soil conditions than any of the other formulas analyzed.

Figures 20 through 22 illustrate the results obtained for the Terzaghi Formula. Here again, the results vary, agreeing with the wave equation only for certain cases within certain ranges.

The Redtenbacher, Pacific Coast, and Canadian

Building Code Formulas seem to show the least agreement of the pile-driving formulas studied in this investigation.

Figures 32 through 34 illustrate the results of the Rankine Formula and indicate that they, like the *Engineering News* Formula are remarkably well banded and consistent, although they also are relatively inaccurate. However, it seems probable that constants could be applied to either the Rankine Formula or the *Engineering News* Formula in order to greatly increase their accuracy and usefulness, at least within the range of problems studied in this report.

The results obtained for the Gates Formula are illustrated in Figures 35 through 37. Although the results are not as closely grouped as some of the previously mentioned formulas, they are remarkably consistent considering the lack of variables accounted for, and it appears that it might be possible to modify the formula somewhat in order to obtain closer agreement with the wave equation.

Conclusions

For the cases shown, the *Engineering News* and Rankine Formulas are generally in better agreement with the wave equation than any of the other formulas studied in this report. For the cases analyzed, both formulas predicted bearing capacities around twice that given by the wave equation. There were several exceptions for the *Engineering News* Formula, these being for extremely large, heavy concrete piles driven by a Vulcan No. 1 hammer, and for long steel piles with extremely small cross-sectional areas, driven by Vulcan 80C hammer. The Rankine Formula, however, had exceptions only when driving long concrete piles of large cross-sectional area with the Vulcan No. 1 hammer.

Because the results obtained by these two formulas are quite consistent, the formulas can be multiplied by the factor (RU_W/RU_F) to bring the formulas into agreement with the wave equation. The factor RU_W/RU_F can be found from the figures presented. This factor, when applied to the formula in question, will produce a predicted safety factor of 1.0. For the *Engineering News* Formula, the equation with the appropriate constant becomes:

$$RU_F = \left(\frac{U}{S + C}\right)\left(\frac{RU_W}{RU_F}\right)$$

where the equation should be applied only to piles and hammers analyzed in this report.

Other simplified dynamic formulas can be treated in a similar manner. Note that an appropriate factor of safety should then be applied to the modified formulas.

It must be emphasized that throughout this report the authors have limited their considerations to pile capacity immediately after driving, (hence the term "resistance to penetration") since it is widely recognized that neither the wave equation nor any dynamic formula can account for time-dependent variables which might influence the capacity of a pile. Time effects can only be determined by the application of soil mechanics.

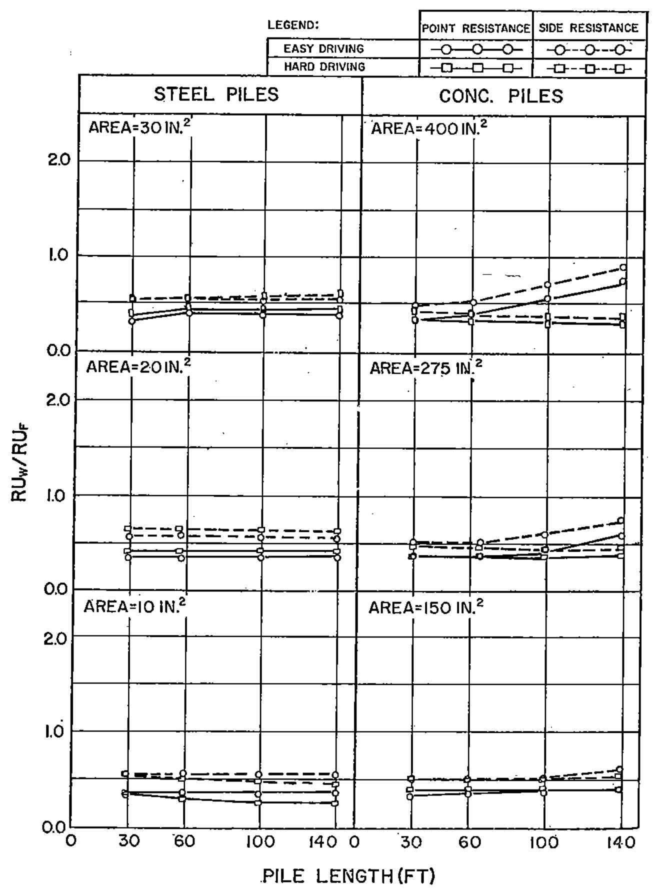

Figure 5. The Engineering News Formula vs the wave equation—Vulcan No. 1 hammer.

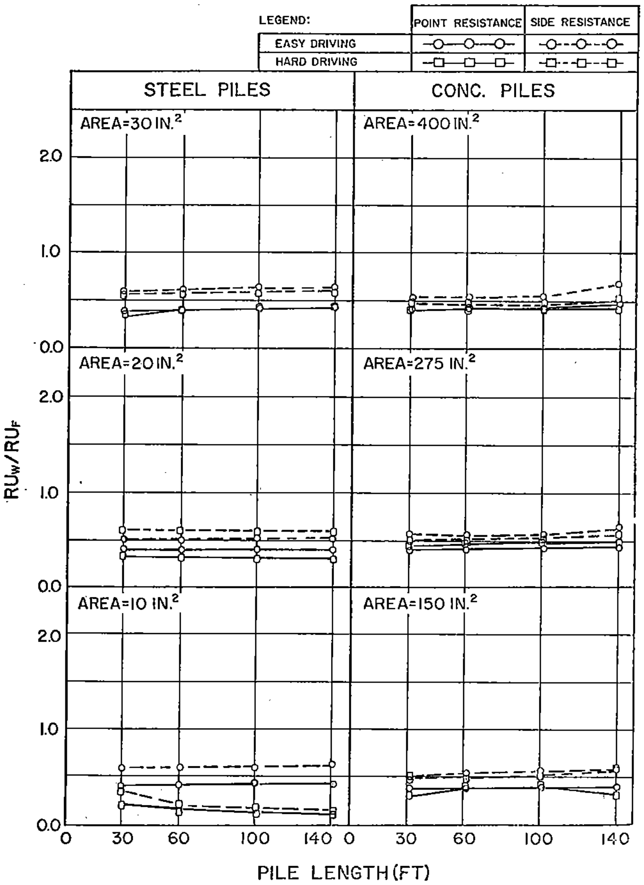

Figure 6. The Engineering News Formula vs the wave equation—Vulcan 80C hammer.

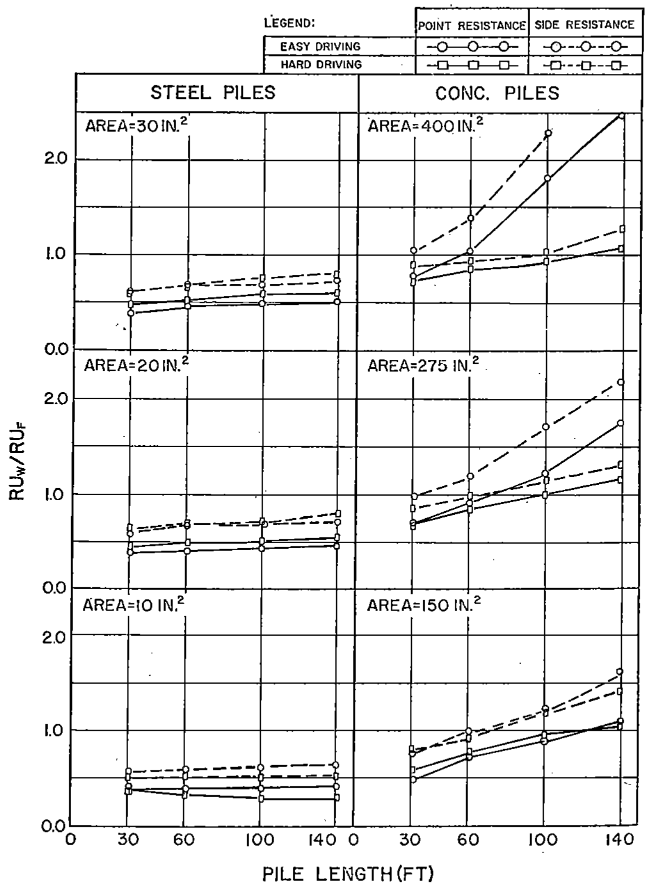

Figure 8. The Michigan Engineering News Formula vs the wave equation—Vulcan No. 1 hammer.

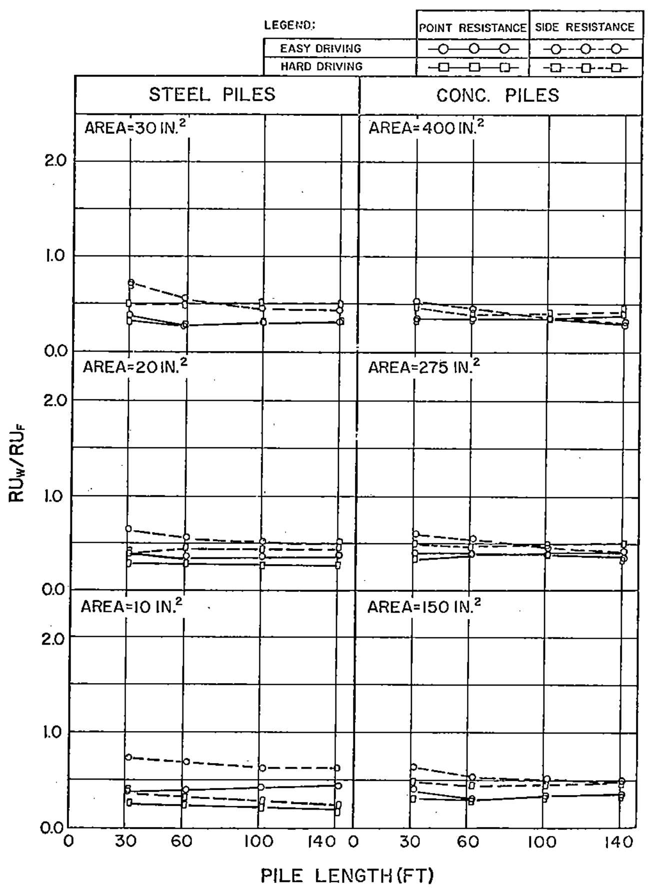

Figure 7. The Engineering News Formula vs the wave equation—Delmag D-22 hammer.

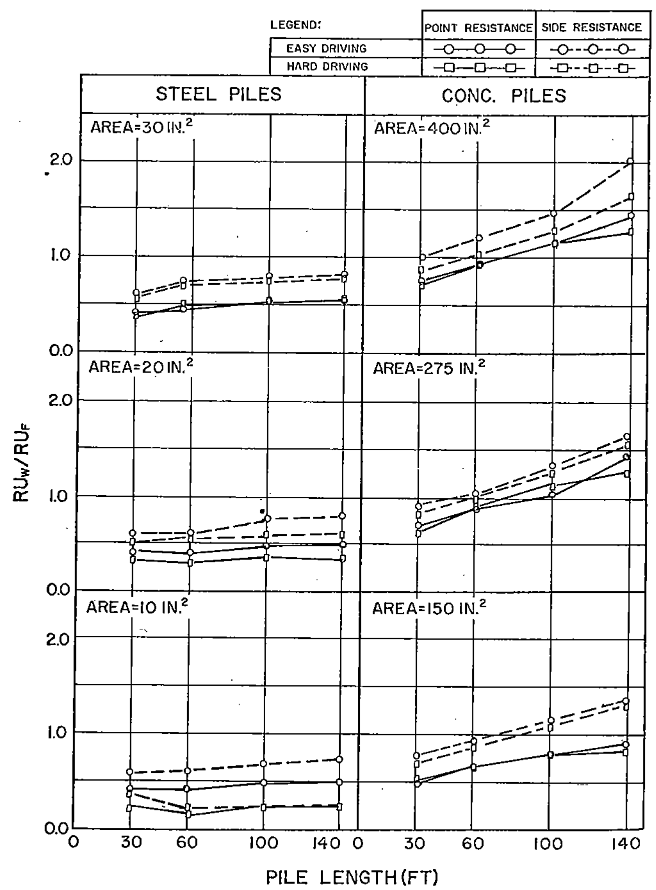

Figure 9. The Michigan Engineering News Formula vs the wave equation—Vulcan 80C hammer.

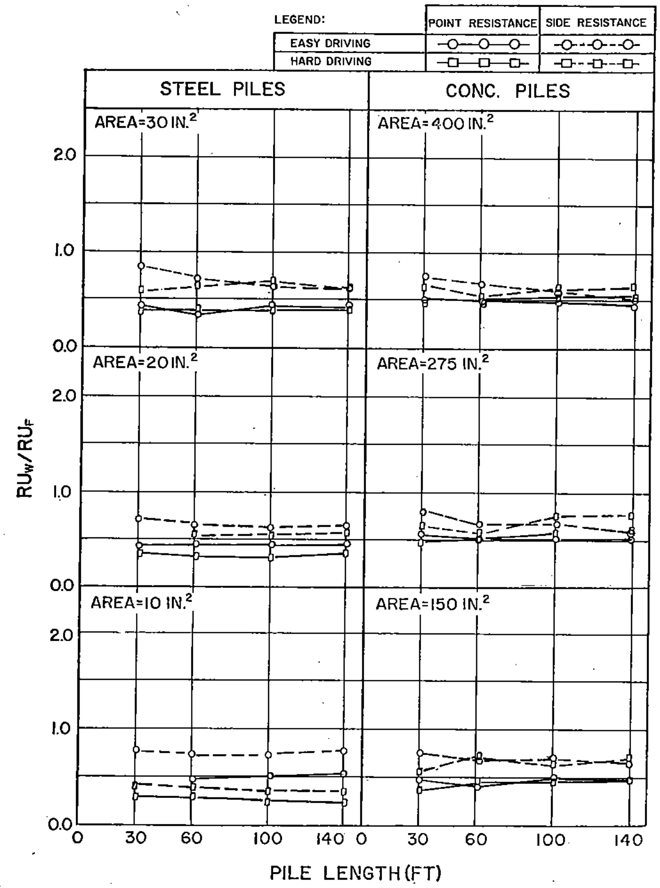

Figure 10. The Michigan Engineering News Formula vs the wave equation—Delmag D-22 hammer.

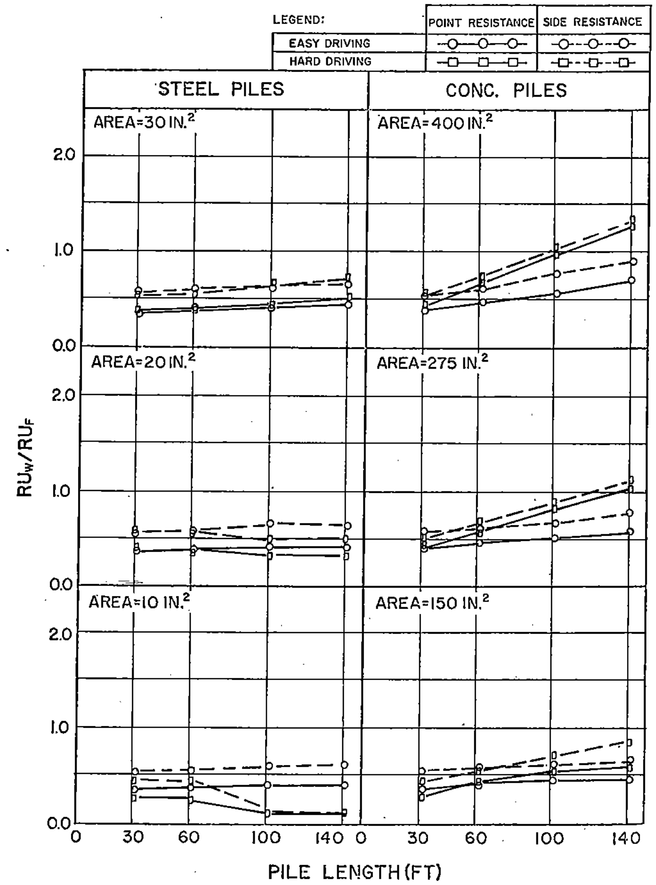

Figure 12. The Eytelwein Formula vs the wave equation—Vulcan 80C hammer.

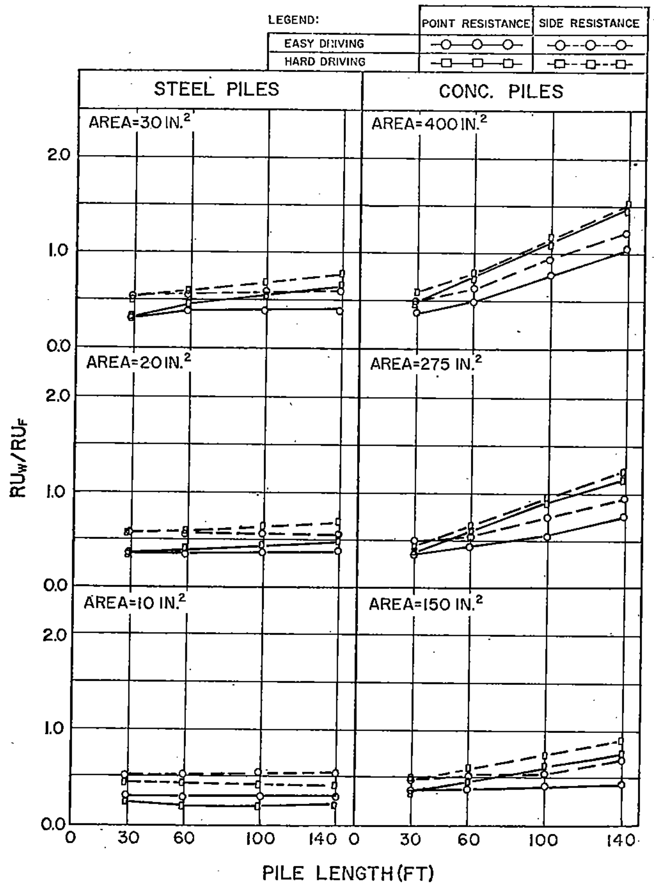

Figure 11. The Eytelwein Formula vs the wave equation—Vulcan No. 1 hammer.

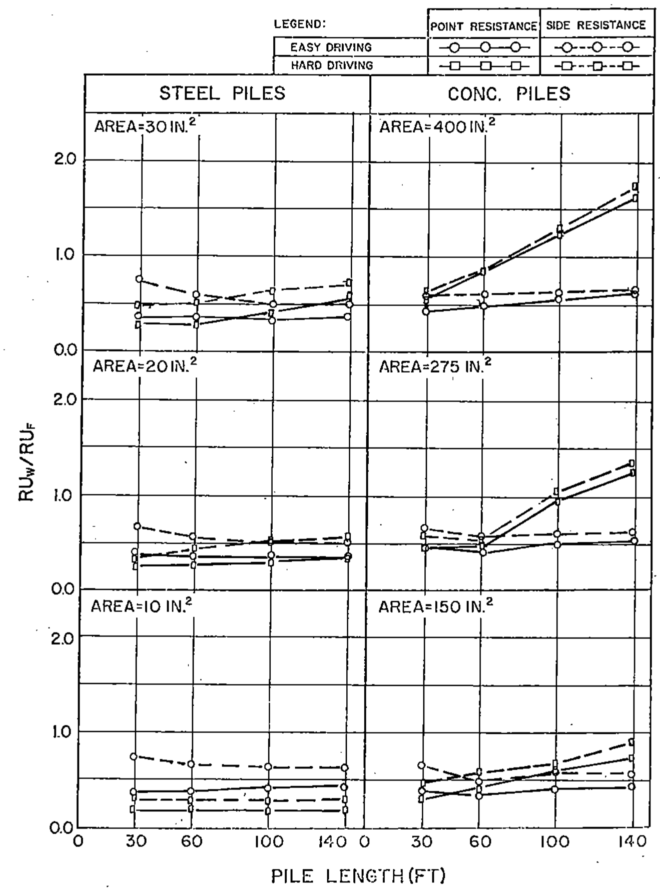

Figure 13. The Eytelwein Formula vs the wave equation—Delmag D-22 hammer.

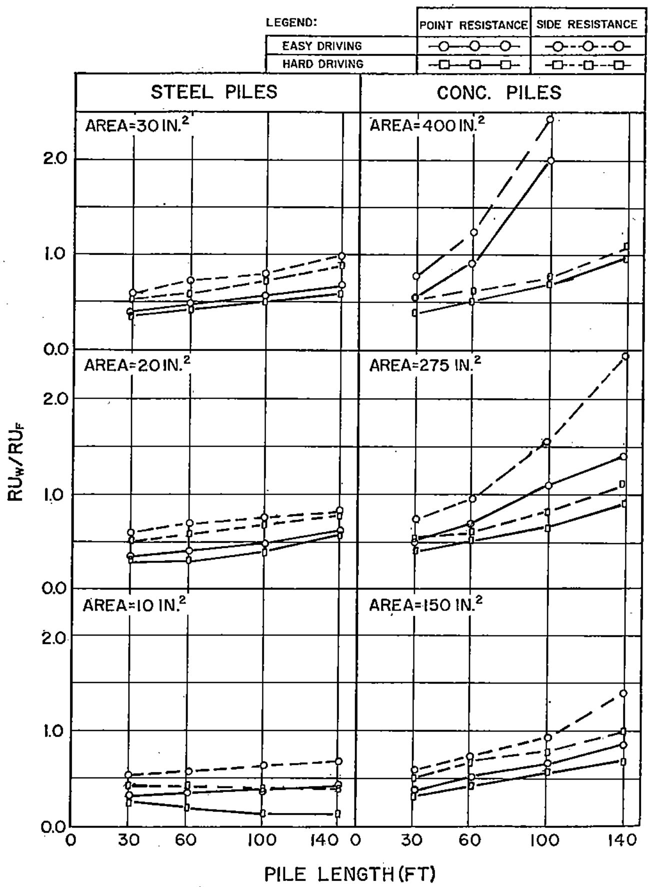

Figure 14. The Navy-McKay Formula vs the wave equation—Vulcan No. 1 hammer.

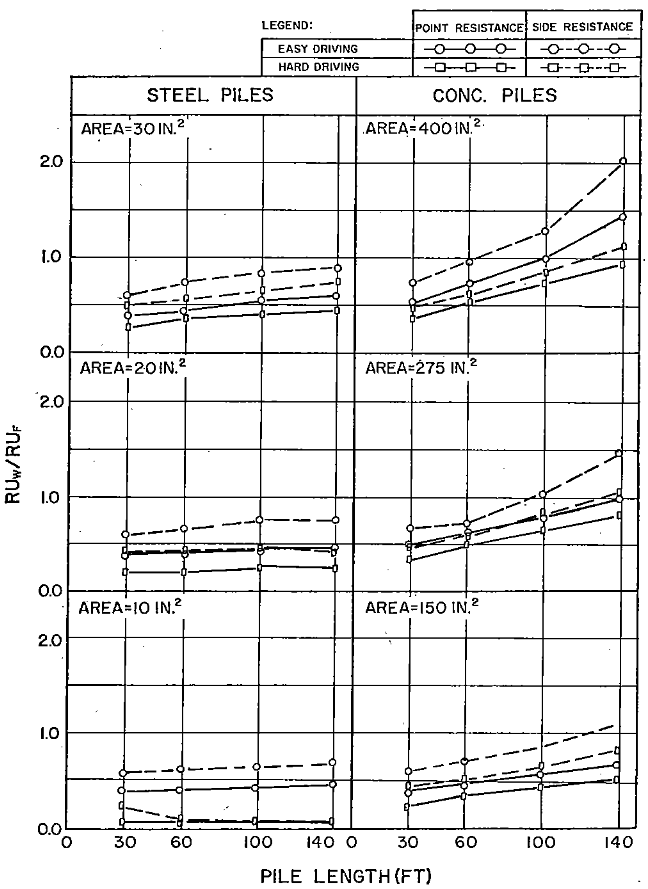

Figure 15. The Navy-McKay Formula vs the wave equation—Vulcan 80C hammer.

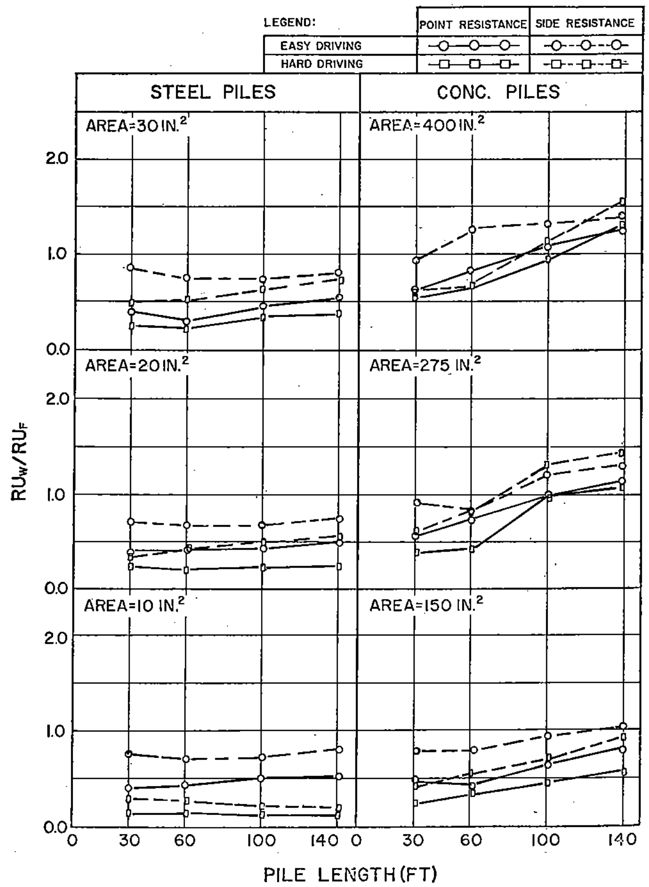

Figure 16. The Navy-McKay Formula vs the wave equation—Delmag D-22 hammer.

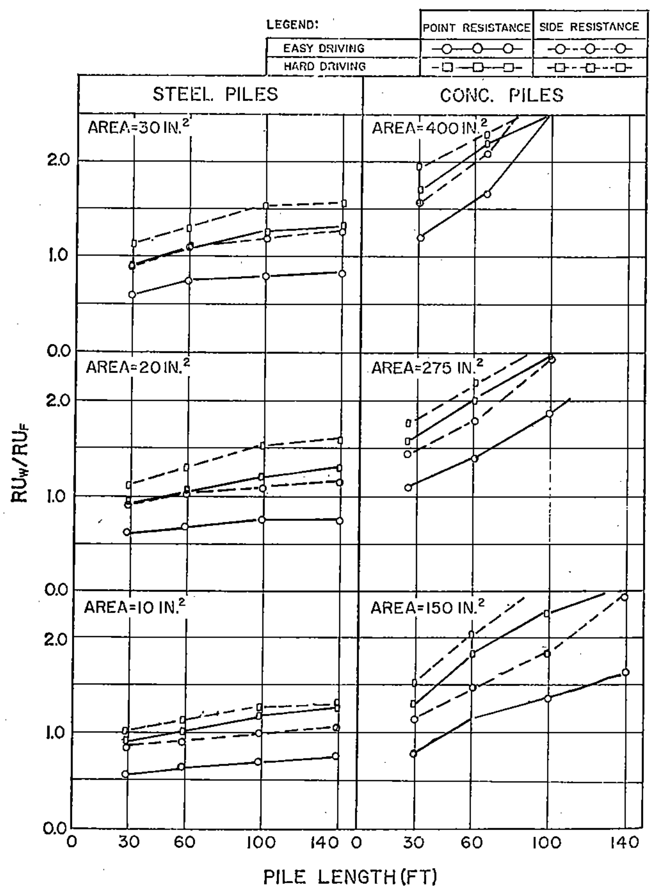

Figure 17. The Hiley Formula vs the wave equation—Vulcan No. 1 hammer.

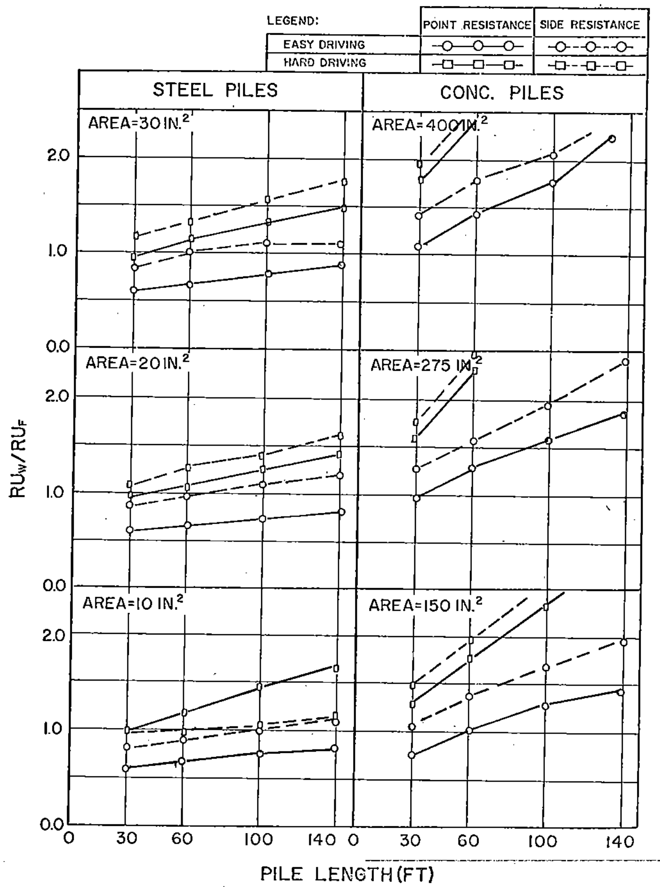

Figure 18. The Hiley Formula vs the wave equation—Vulcan 80C hammer.

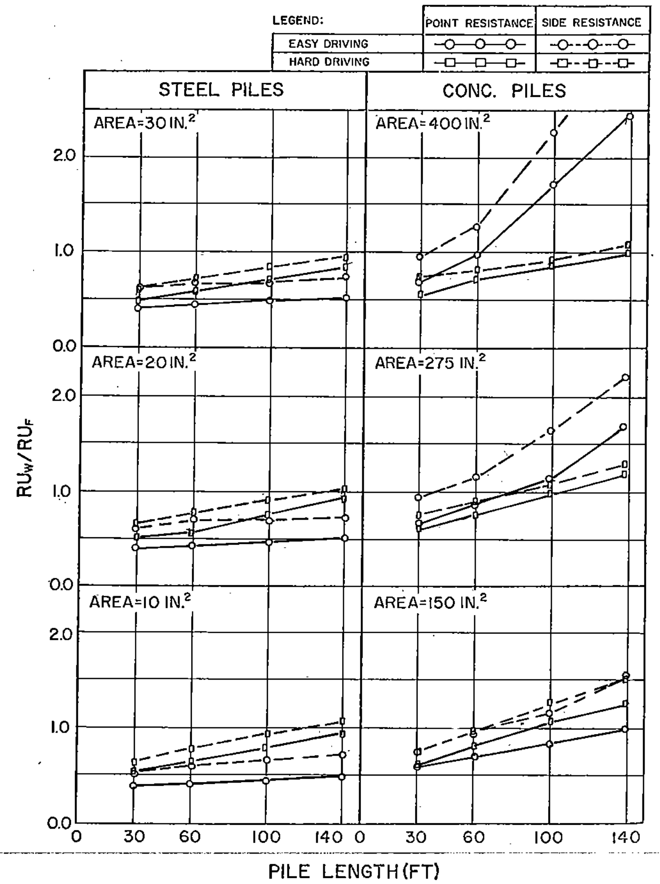

Figure 20. The Terzaghi Formula vs the wave equation—Vulcan No. 1 hammer.

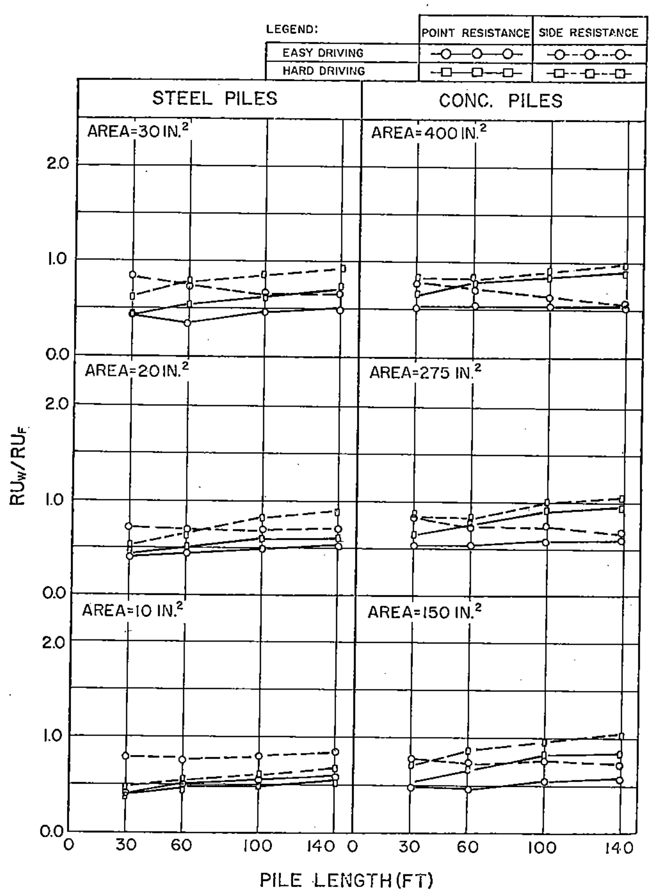

Figure 19. The Hiley Formula vs the wave equation—Delmag D-22 hammer.

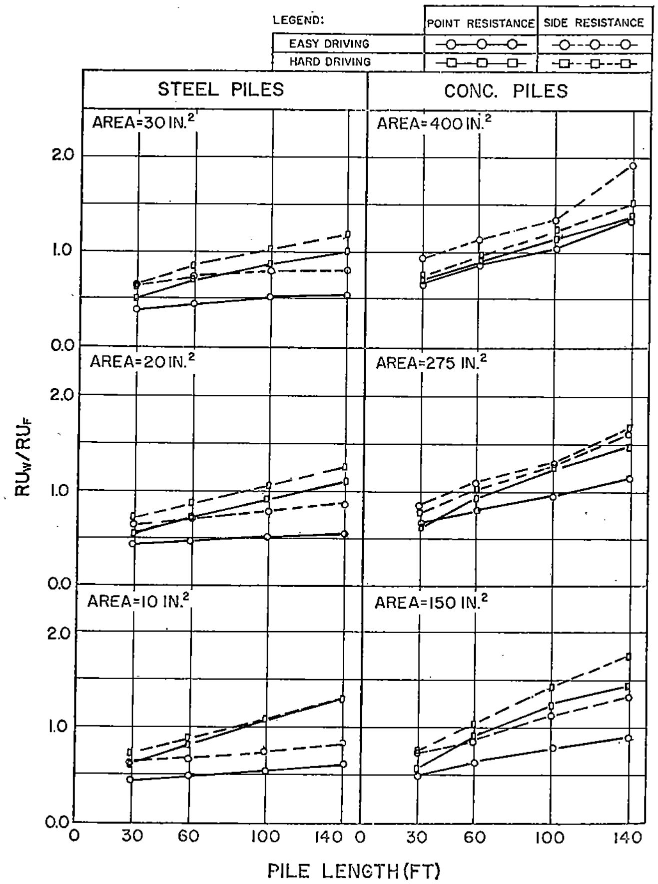

Figure 21. The Terzaghi Formula vs the wave equation—Vulcan 80C hammer.

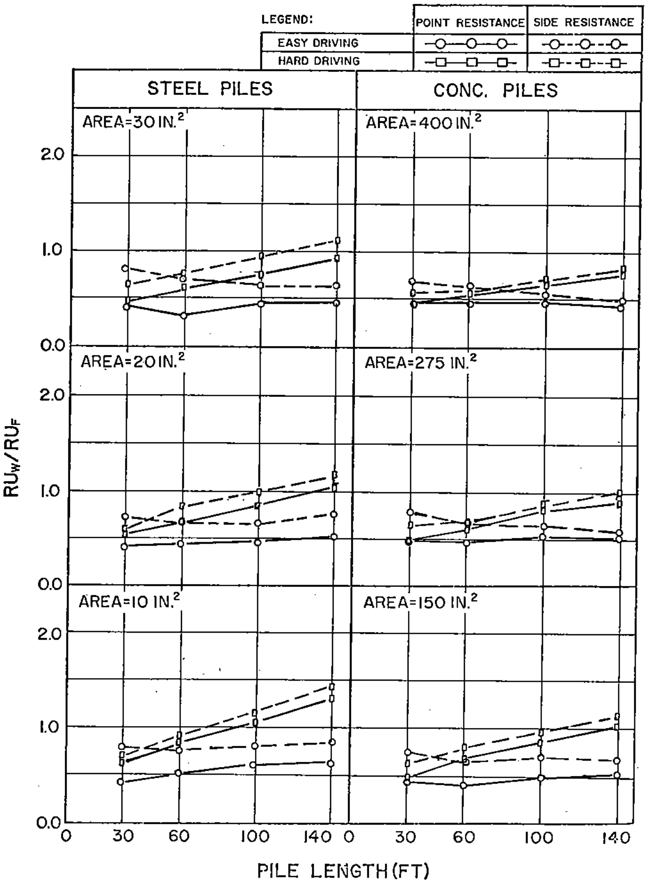

Figure 22. The Terzaghi Formula vs the wave equation —Delmag D-22 hammer.

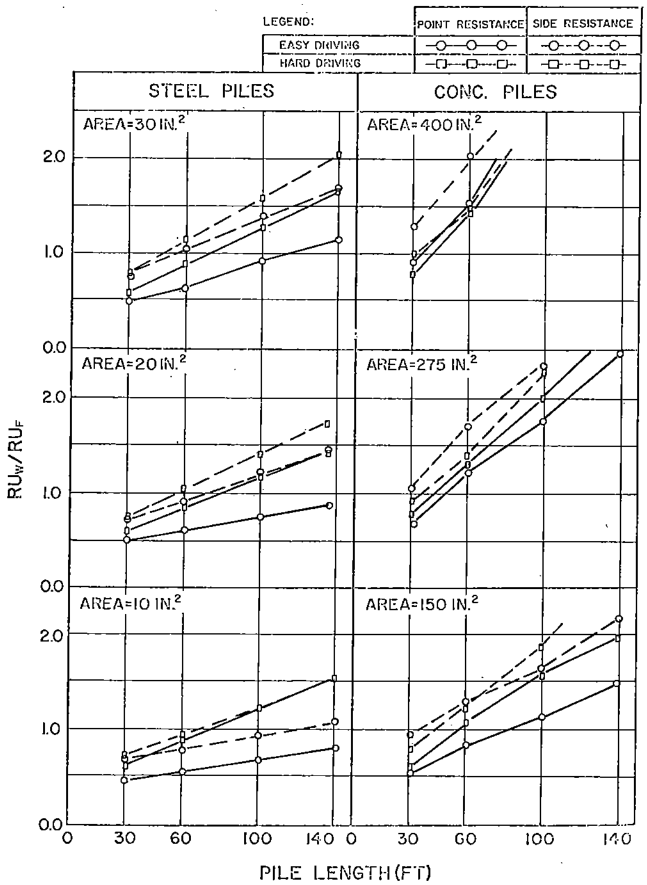

Figure 24. The Redtenbacher Formula vs the wave equation—Vulcan 80C hammer.

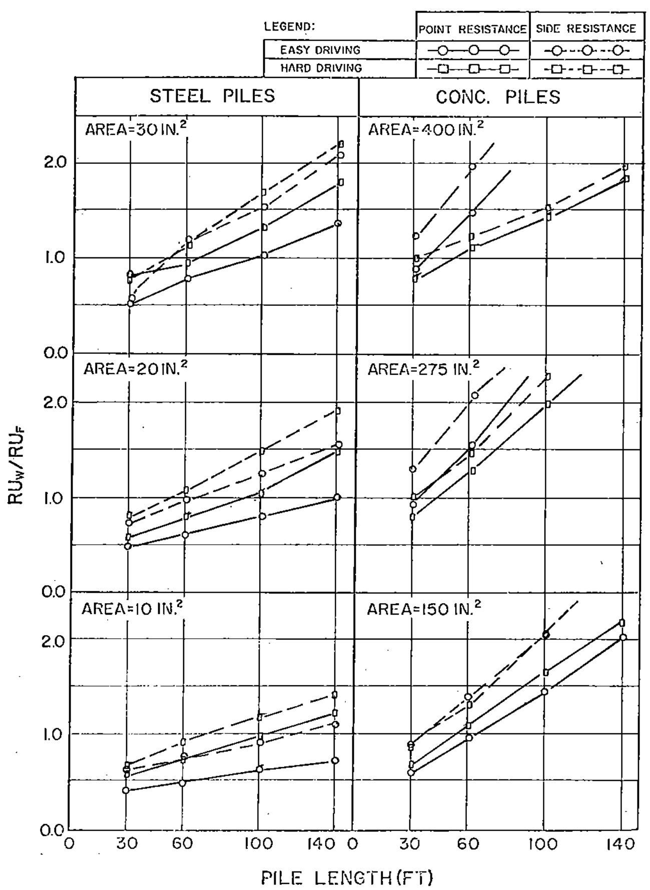

Figure 23. The Redtenbacher Formula vs the wave equation—Vulcan No. 1 hammer.

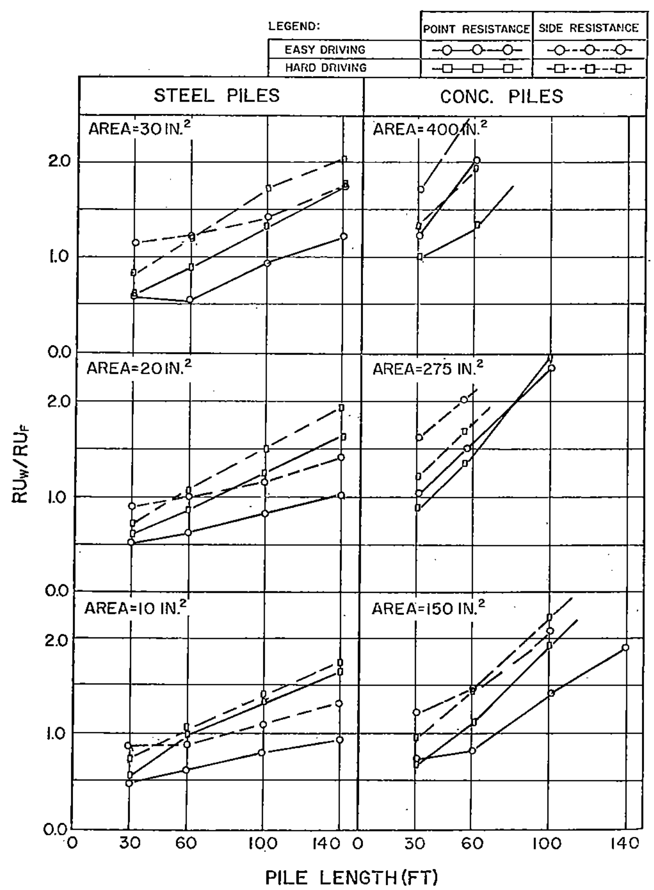

Figure 25. The Redtenbacher Formula vs the wave equation—Delmag D-22 hammer.

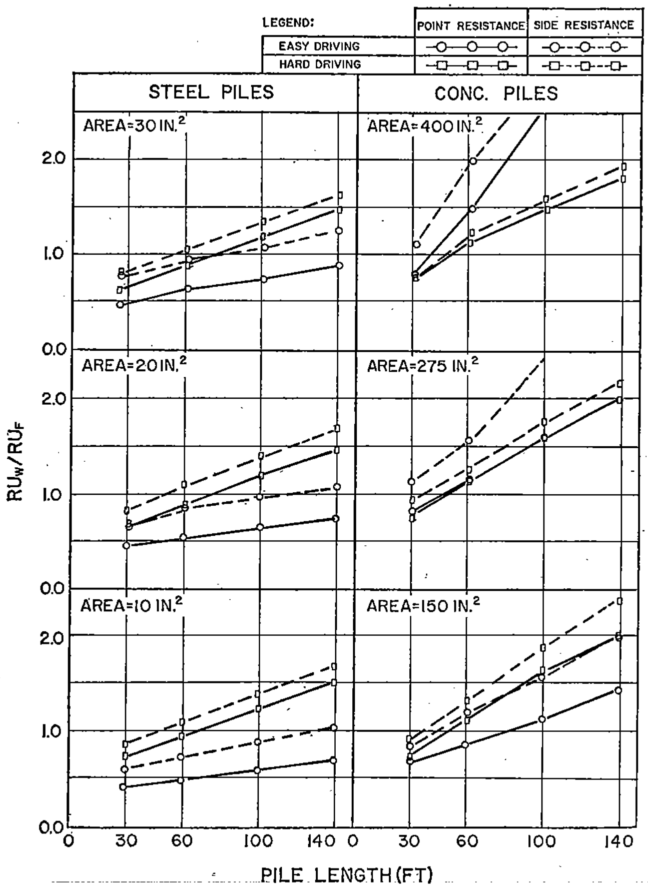

Figure 26. The Pacific Coast Formula vs the wave equation—Vulcan No. 1 hammer.

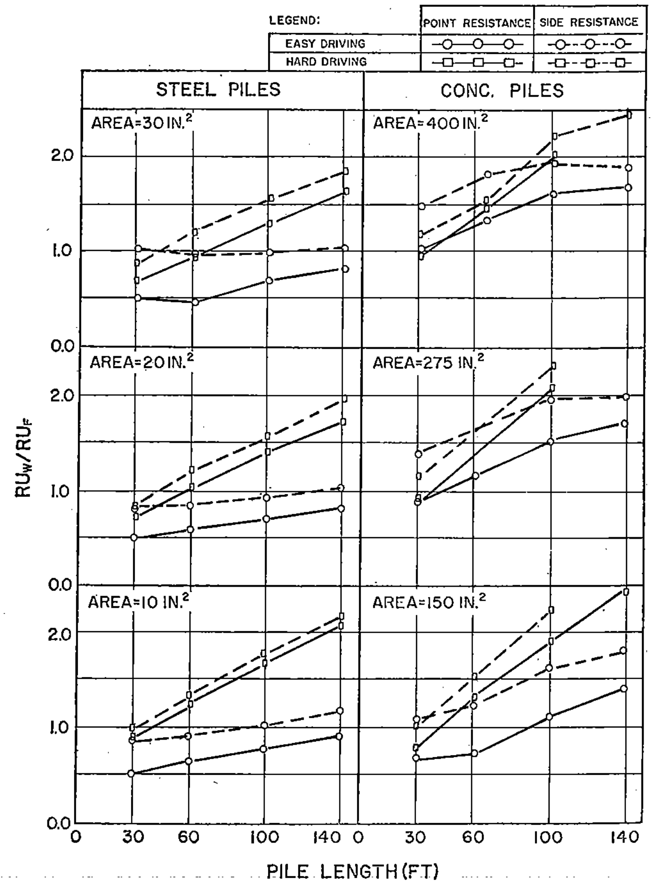

Figure 28. The Pacific Coast Formula vs the wave equation—Delmag D-22 hammer.

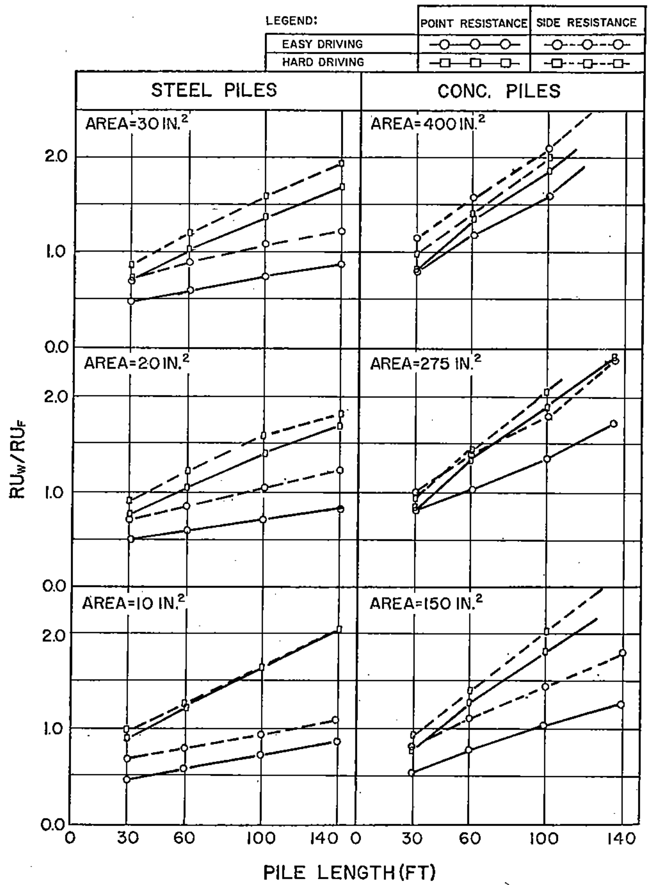

Figure 27. The Pacific Coast Formula vs the wave equation—Vulcan 80C hammer.

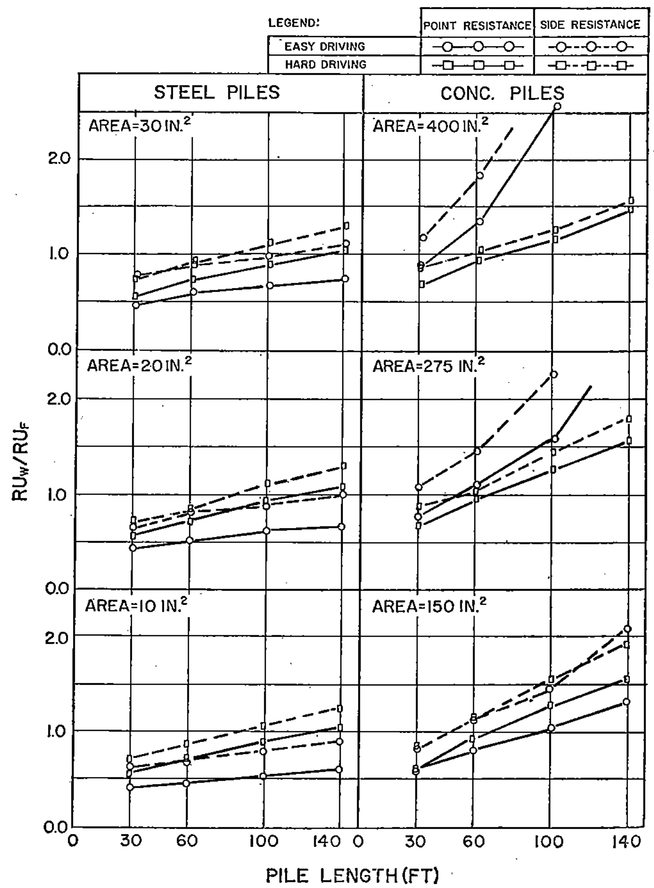

Figure 29. The Canadian Building Code Formula vs the wave equation—Vulcan No. 1 hammer.

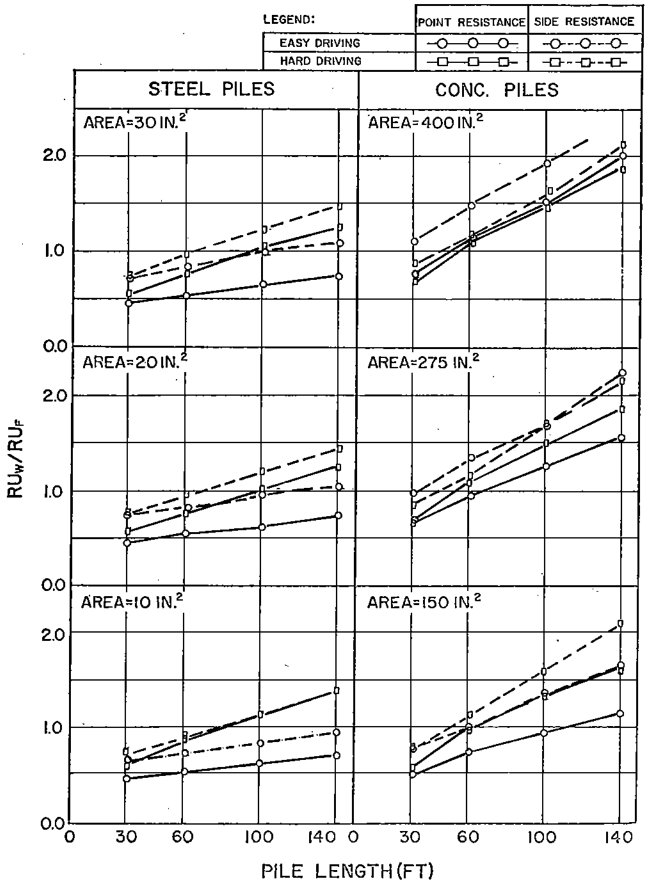

Figure 30. The Canadian Building Code Formula vs the wave equation—Vulcan 80C hammer.

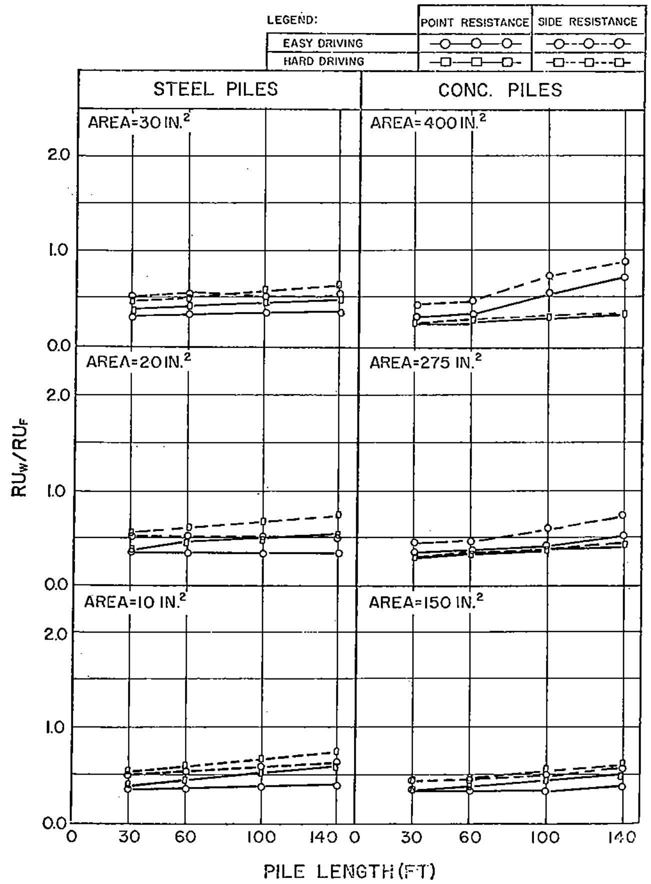

Figure 32. The Rankine Formula vs the wave equation —Vulcan No. 1 hammer.

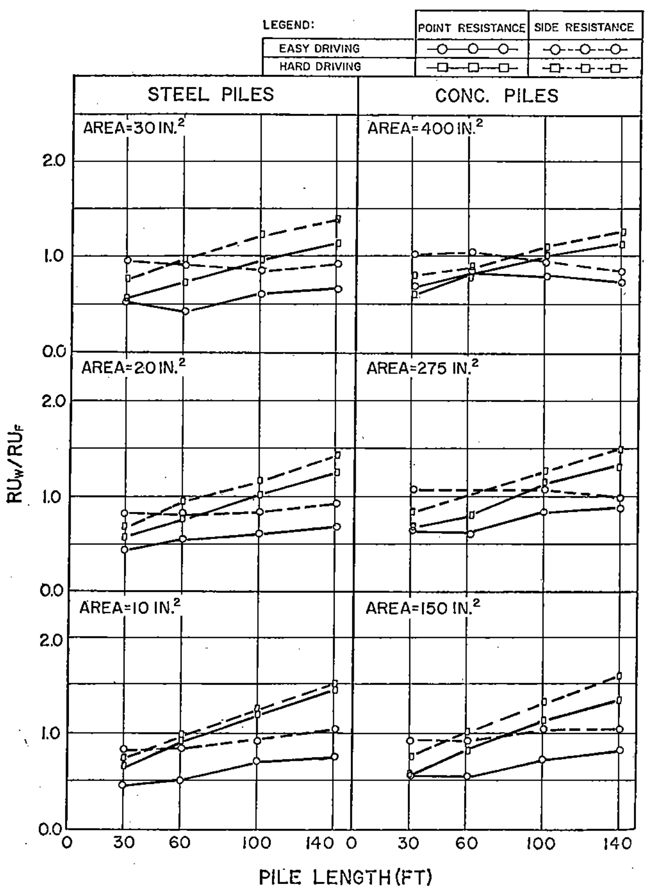

Figure 31. The Canadian Building Code Formula vs the wave equation—Delmag D-22 hammer.

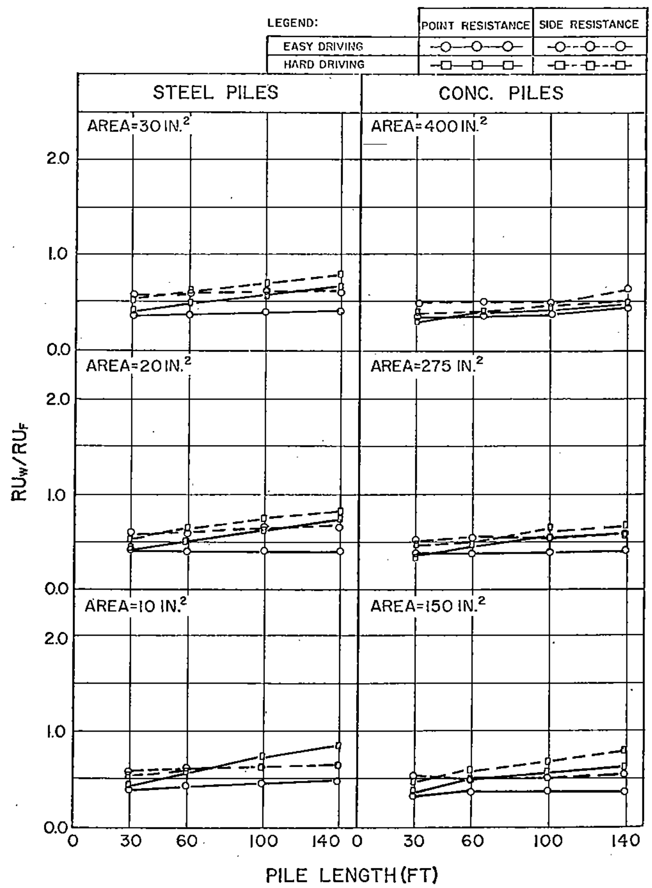

Figure 33. The Rankine Formula vs the wave equation —Vulcan 80C hammer.

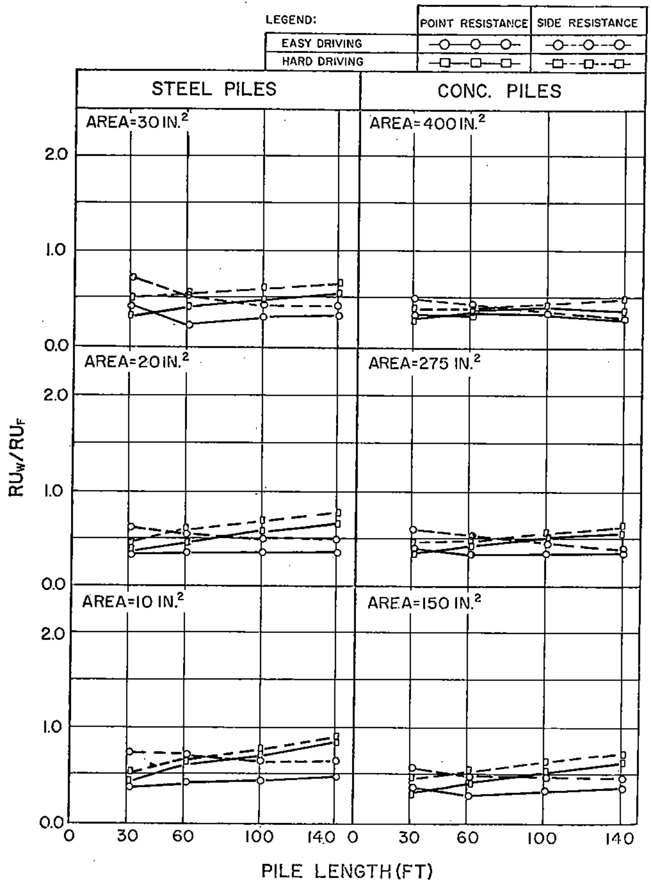

Figure 34. The Rankine Formula vs the wave equation—Delmag D-22 hammer.

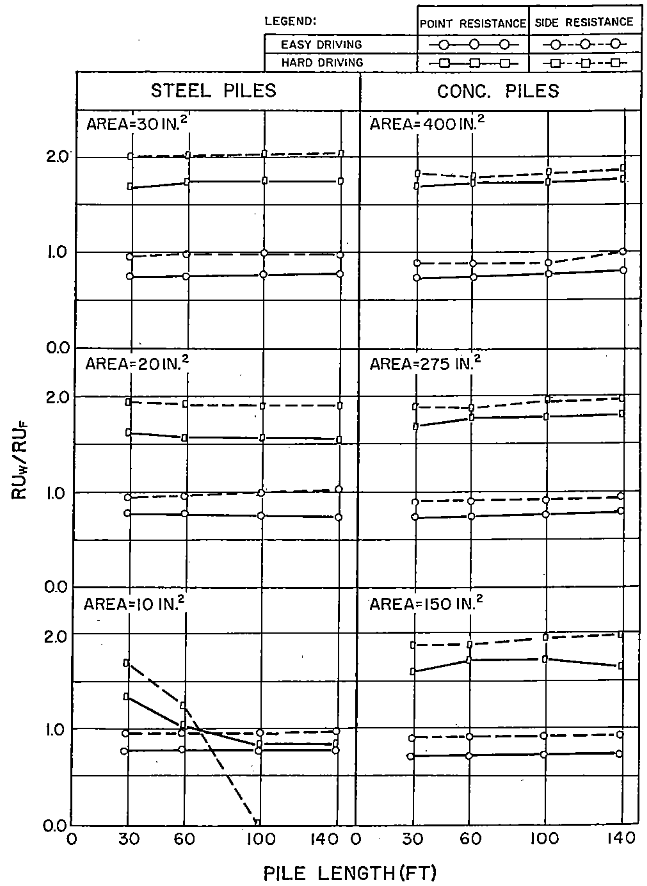

Figure 36. The Gates Formula vs the wave equation—Vulcan 80C hammer.

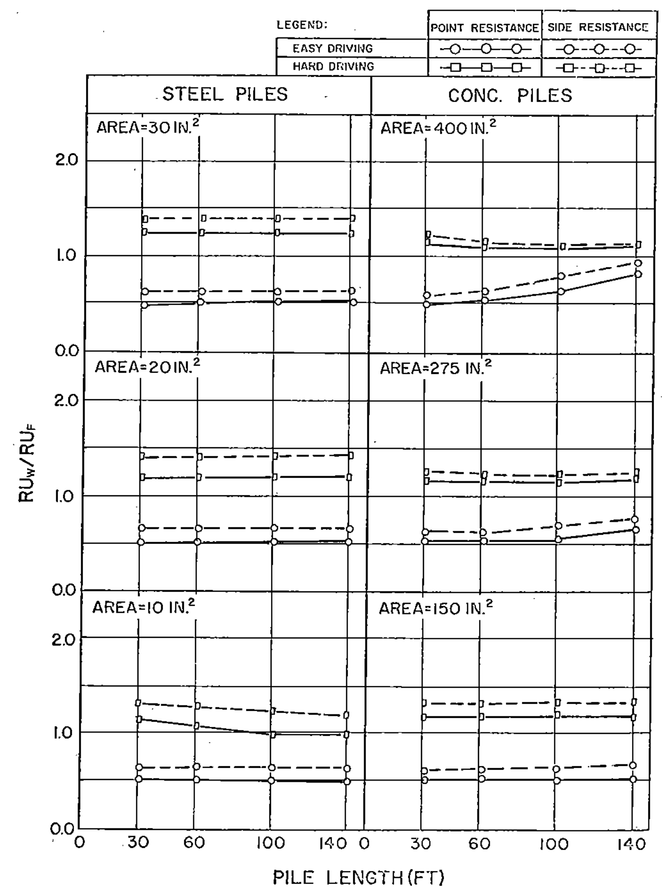

Figure 35. The Gates Formula vs the wave equation—Vulcan No. 1 hammer.

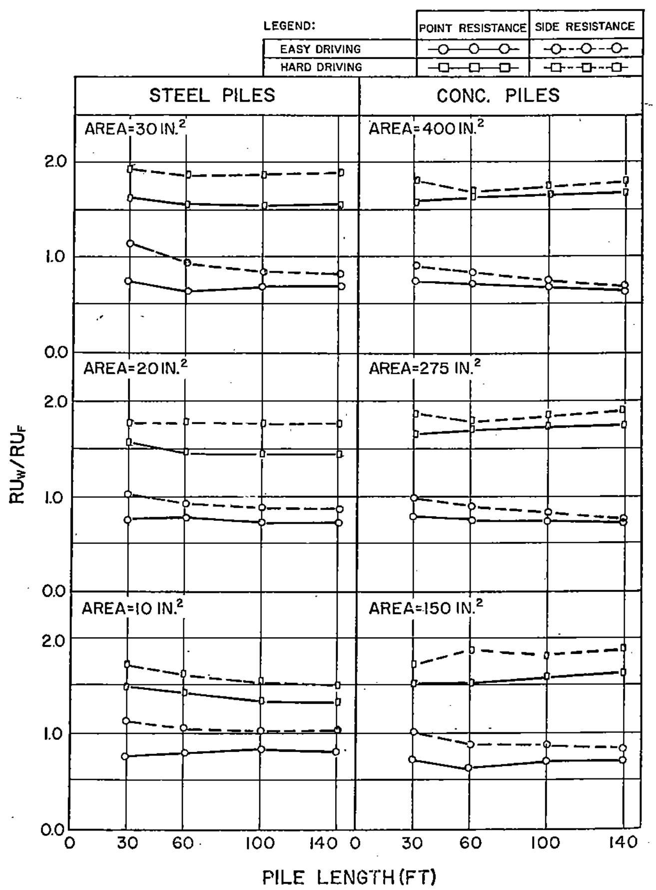

Figure 37. The Gates Formula vs the wave equation—Delmag D-22 hammer.

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Appendix

SUMMARY OF PERMANENT SETS PREDICTED BY THE WAVE EQUATION

TABLE A.1. PERMANENT SET OF PILE PER BLOW PREDICTED BY THE WAVE EQUATION—STEEL PILES

Area of Pile (in.)²	Length of Pile (ft.)	Vulcan No. 1 Hammer				Vulcan 80-C Hammer				Delmag D-22 Hammer			
		Soil Resistance at point (kips)		Soil Resistance on side (kips)		Soil Resistance at point (kips)		Soil Resistance on side (kips)		Soil Resistance at point (kips)		Soil Resistance on side (kips)	
		50	200	50	200	100	400	100	400	100	400	100	400
10	30	1.23	0.23	1.88	0.38	1.16	0.08	1.73	0.18	1.45	0.15	2.53	0.25
	60	1.22	0.17	1.93	0.35	1.19	0.01	1.75	0.05	1.41	0.14	2.50	0.21
	100	1.21	0.12	1.96	0.31	1.21	0.00	1.75	0.00	1.47	0.08	2.16	0.16
	140	1.22	0.13	1.80	0.28	1.20	0.00	1.82	0.00	1.45	0.08	2.18	0.17
20	30	1.19	0.28	1.88	0.42	1.12	0.15	1.70	0.30	1.24	0.26	2.19	0.18
	60	1.18	0.27	2.01	0.43	1.09	0.13	1.73	0.28	1.17	0.29	1.86	0.14
	100	1.20	0.27	1.87	0.44	1.08	0.13	1.83	0.27	1.09	0.13	1.64	0.28
	140	1.20	0.27	1.85	0.42	1.20	0.13	1.80	0.26	1.09	0.13	1.61	0.29
30	30	1.18	0.28	1.93	0.41	1.09	0.18	1.66	0.33	1.28	0.24	2.53	0.36
	60	1.32	0.30	1.96	0.40	1.08	0.21	1.76	0.34	1.18	0.17	1.938	0.35
	100	1.25	0.30	1.81	0.40	1.08	0.21	1.78	0.35	1.05	0.18	1.56	0.35
	140	1.30	0.28	1.93	0.37	1.15	0.22	1.70	0.36	1.05	0.18	1.46	0.30

TABLE A.2. PERMANENT SET OF PILE PER BLOW PREDICTED BY THE WAVE EQUATION—CONCRETE PILES

Area of Pile (in.)²	Length of Pile (ft.)	Vulcan No. 1 Hammer				Vulcan 80-C Hammer				Delmag D-22 Hammer			
		Soil Resistance at point (kips)		Soil Resistance on side (kips)		Soil Resistance at point (kips)		Soil Resistance on side (kips)		Soil Resistance at point (kips)		Soil Resistance on side (kips)	
		50	200	50	200	100	400	100	400	100	400	100	400
150	30	1.20	0.26	1.69	0.37	1.03	0.15	1.57	0.27	1.30	0.18	2.15	0.31
	60	1.23	0.27	1.71	0.35	1.09	0.20	1.58	0.28	0.96	0.19	1.75	0.31
	100	1.25	0.27	1.57	0.35	1.11	0.21	1.61	0.32	1.16	0.21	1.68	0.30
	140	1.34	0.27	1.93	0.33	1.12	0.17	1.65	0.31	1.18	0.21	1.51	0.34
275	30	1.21	0.23	1.74	0.31	1.09	0.18	1.51	0.27	1.32	0.22	2.04	0.34
	60	1.26	0.22	1.68	0.26	1.12	0.23	1.58	0.26	1.23	0.25	1.74	0.28
	100	1.36	0.23	1.85	0.27	1.13	0.23	1.52	0.29	1.50	0.26	1.89	0.31
	140	1.96	0.24	2.56	0.29	1.20	0.24	1.76	0.27	1.10	0.27	1.28	0.30
400	30	1.19	0.21	1.62	0.28	1.08	0.18	1.52	0.25	1.23	0.22	1.80	0.31
	60	1.34	0.19	1.79	0.22	0.10	0.21	1.49	0.23	1.16	0.22	1.57	0.24
	100	1.97	0.18	2.59	0.20	1.15	0.21	1.50	0.22	1.09	0.23	1.29	0.25
	140	2.70	0.20	3.23	0.23	1.38	0.22	1.94	0.24	1.06	0.24	1.07	0.26