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1. A survey of numerical methods in offshore piling

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A review of numerical methods used in the offshore piling industry is given. The purpose of these is to compute the static and dynamic behaviour of single piles and of pile groups at working load and at 'failure' for various types of loading, principally axial and lateral. Using discrete element methods (the 't-z' and 'p-y' types of calculation) it is possible to follow accurately the load transfer mechanism operating between single piles and the ground up to collapse, for monotonic and cyclic loading. The calculation is well within the scope of mini-computers, subject to adequate discretization of the ground's resistance. Back-analyses are essential. Using finite element methods, load transfer can be predicted, for single piles at working loads, using ground resistance parameters which are in everyday use in soil mechanics. It is within the scope of faster computers to continue such computations up to collapse, and this is a likely development. A particularly interesting case of 'failure' of single piles occurs during installation. Attempts to link driving resistance with ultimate static capacity still depend on the use of parameters which are not measured every day in soil mechanics laboratories. Drivability itself is reasonably predicted, particularly at soft sites, and back-analyses are encouraging. Flutter has been identified as an instability mechanism worthy of consideration. Analysis of pile group behaviour still rests heavily on the assumption of linear ground properties and on the principle of superposition. Errors of 20% or more are not uncommon in the computation of influence factors for single piles if the pile is inadequately represented. Effective stress analyses depend on a better knowledge of excess pore pressures due to driving, and on more realistic ground stresses after installation than can be measured or computed at present. Dynamic analyses of piles and groups in situ (as distinct from during driving) subject to wave and earthquake loading are at an early stage of development, but will clearly be pursued intensively in the future.

INTRODUCTION

Numerical methods have been widely used in the offshore piling industry for the past 25 years or so. This Paper cannot attempt to be a literature review, but merely sets out the current state of achievement and tries to point the way to future developments. In addition it draws together the various strands of work presented at this Conference. These fall naturally into four main subject areas; namely, quasi-static behaviour of piles and groups, drivability, pore pressure considerations and dynamics.

QUASI-STATIC BEHAVIOUR OF PILES AND PILE GROUPS

Single axially loaded pile

2. *Deflexion: the t-z method.* A basic problem is the calculation of the deflexion of a single axially loaded pile, subjected to prescribed load or displacement at the head. Even this problem is quite intractable without resort to numerical methods. For these purposes the pile can be discretized as a series of finite difference stations or as a series of line 'finite elements', and the ground as a series of discrete axial 'springs' or as some continuously distributed axial spring stiffness (Fig. 1). Although called springs, the ground resistance-displacement relationships can be as complicated as is necessary. These are usually called *t-z* curves and can be introduced into the computer program either as mathematical functions or, more usually, as a series of points between which linear interpolation is assumed (Fig. 2).

3. Various methods can be used in the computation to follow the prescribed *t-z* curves. The most modern and efficient are borrowed from genuine finite element analysis and work with a constant stiffness in each ground spring¹ (e.g., the slope of the first segment of the *t-z* plot (Fig. 3)). As loading proceeds, any excess force in a spring over and above the *t* which ought to be carried for that value of *z* is redistributed to the other springs by processes called 'initial stress' or 'viscoplastic strain' in finite element work. In these methods, the simultaneous equations have constant coefficients and are merely re-solved for varying loads. Displacements at the pile head rather than forces should be prescribed for two reasons. First, it is more efficient since fewer iterations are required in the numerical process (typically two before failure is approached) and secondly, displacement control is the only way of continuing the analysis beyond peak load on the pile. Some of the older finite difference algorithms are rather cumbersome by modern standards.

4. This type of calculation is well within the scope of mini-computers. Of course, the difficulty lies in selecting the *t-z* curves appropriate to various soil types and conditions. The suggestion has been made² of the dimensionless relationship

$$\frac{t}{t_{\max}} = 2 \left\{ \frac{z}{z_c} \right\}^{\frac{1}{2}} - \frac{z}{z_c}$$

for the side springs, where t_{\max} is the maximum soil resistance which is mobilized at a critical displacement z_c . For the end bearing spring, the corresponding suggestion is

$$\frac{t}{t_{\max}} = \left\{ \frac{z}{z_c} \right\}^{1/3}$$

5. Figure 4 shows how field data from test piles in sand and clay can be back-analysed using this approach.³ A

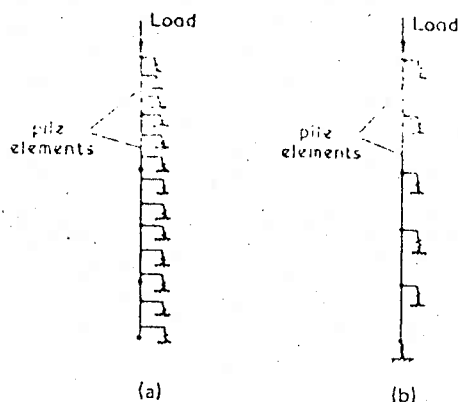


Fig. 1. Discretization of pile and ground: (a) continuous axial ground resistance; (b) discrete axial ground resistance

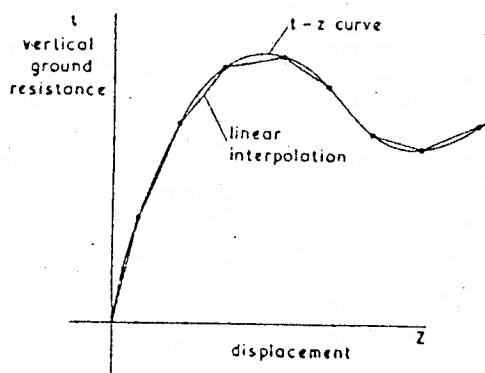


Fig. 2. The $t-z$ approximation

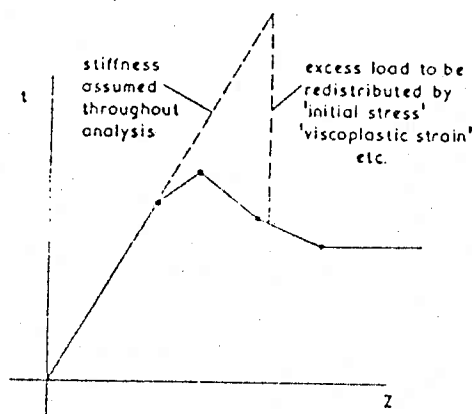


Fig. 3. Load redistribution process

large number of such fits would be necessary to build up confidence in the use of the method in new situations.

6. *Deflexion: finite element methods (Paper 11).* An improved representation of the ground is as a solid, in the simplest case an elastic solid bonded to the pile. However, mesh design problems arise when modelling pipe piles with open or closed ends. It is difficult to achieve both the right end bearing area and the right pile stiffness at that diameter when analysing equivalent solid piles.

7. The real benefits come when non-linear, stress-dependent properties are taken for the soil, together with slippage allowance between pile shaft and soil. For example, Desai⁴ originally showed that the load transfer in a pile in sand is quite non-uniform with depth, as shown in confirmatory calculations in Fig. 5. When a field test was back-analysed by this method, the load-displacement curve and load transfer profile could be rather well reproduced in such calculations.³ However, the non-linear elastic assumption for the soil and interfaces means calculation becomes unreliable when a large number of elements 'fail', so ultimate loads are best not computed in this way.

8. *Deflexion: boundary element methods (Paper 14).* Particularly when ground conditions are uniform and linear stress-strain properties can be assumed to prevail, boundary element methods can be superior to finite elements because fewer equations have to be solved. Poulos and Davis⁵ have provided widely used charts based on a simplified form of this method, subsequently somewhat refined by Butterfield and Banerjee.⁶ For layering and other forms of non-homogeneity, or when non-linear soil properties have to be considered, the method is less attractive.

9. *Failure.* The $t-z$ computations for load-deflexion can be continued to collapse, and by means of displacement control can take residual conditions into account. The non-linear elastic type of finite element calculation is not recommended for computing collapse. Instead, initial stress or viscoplastic strain algorithms⁷ should be used. Examples of displacement fields at collapse of deep foundations in cohesive and cohesive-frictional materials are shown in Fig. 6, together with load-displacement graphs for base pressure. By these means, the bearing capacity factors N_c , N_q and N_γ can be obtained numerically and the load transfer mechanism at failure identified.

10. Boundary element methods can of course in principle be used in this area, but have not so far found practical application.

11. *Cyclic loading (Paper 16).* An important feature of offshore loading conditions is their cyclic nature. It is well known that under (slow) cyclic loading, engineering materials degrade and become softer and weaker. Because of their particulate nature, clays are prone to the formation of low strength, slickensided rupture surfaces under large and repeated alternating displacements. The $t-z$ and finite element methods can cope with cyclic loading, given that the material behaviour can be defined.

12. For example, Fig. 7 shows a possible $t-z$ behaviour for side springs under cyclic loading.¹ Peak t is a function of N , the number of cyclic load applications, as is the ratio of peak t to displacement z at which it is attained. Fig. 8 shows typical results of this kind of computation for varying cyclic load (displacement) amplitude. At lower levels stabilization takes place but as the level increases, the pile fails in cyclic loading. Tip resistance can be ignored in tension and so on.

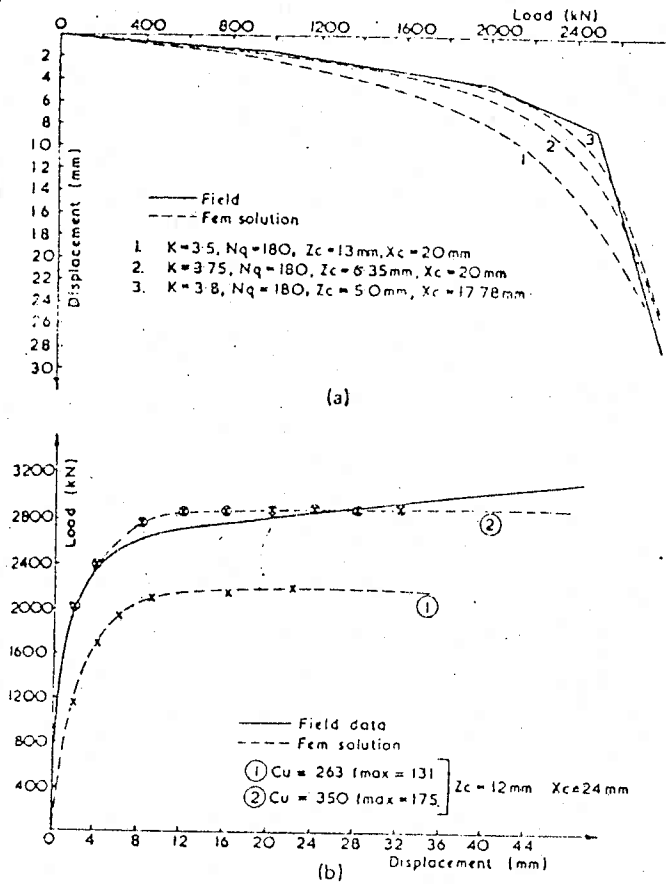


Fig. 4. Back-analysis of field results by t - z method: (a) closed-ended pile in sand; (b) pile with shoe in clay

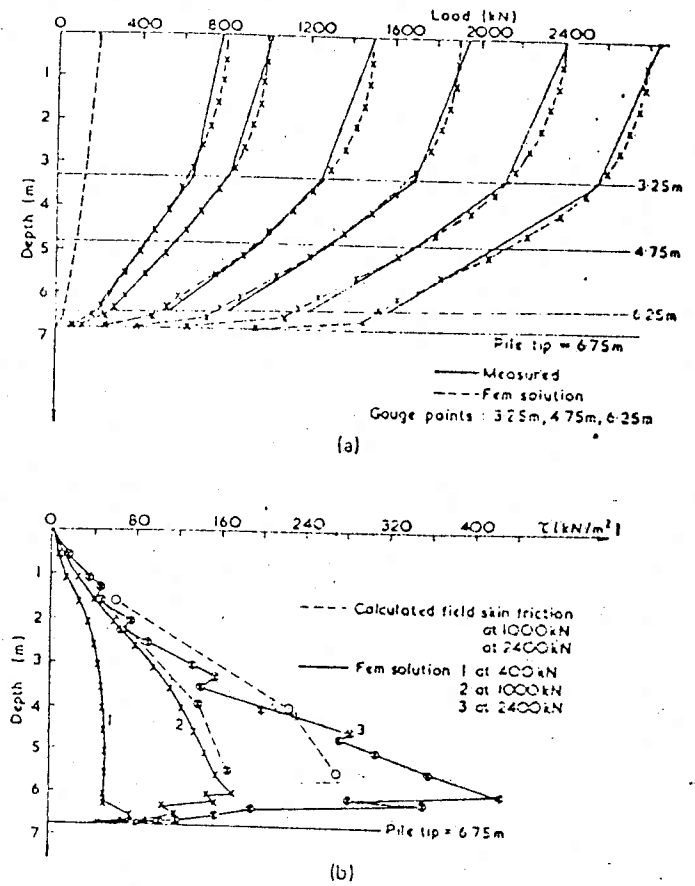


Fig. 5. Back-analysis of field results by finite element method: (a) load distribution of closed-ended pile; (b) graph of τ with depth for various load increments

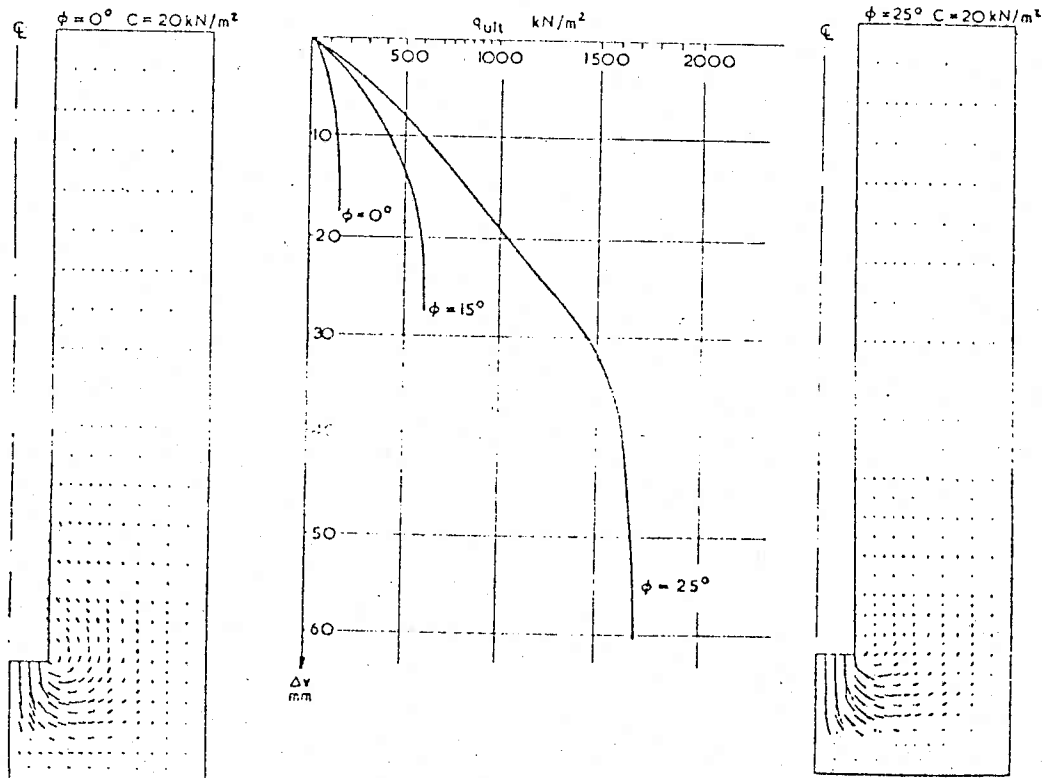


Fig. 6. Viscoplastic analysis of deep foundation in cohesive-frictional material

13. Another feature is the generation and dissipation of pore pressures during cycling. Finite element approaches can deal with this, but practical cases do not seem to have been solved yet.

Groups of axially loaded piles

14. *Deflexion (Papers 14 and 15).* For deflexion of groups of axially loaded piles, the boundary element methods come into their own. The t - z approach ignores interaction completely, and the three-dimensional nature of the problem makes finite element computations expensive, even for linear soils.⁸ Therefore linearity of the soil's stress-strain response is usually assumed for the purposes of interaction computations, and so boundary elements are attractive. Charts have been produced by Poulos and by Butterfield and Banerjee which enable group behaviour to be computed for various typical pile geometries. Sometimes the interaction factors from such a linear analysis are combined with t - z curves for single piles to yield an empirical non-linear group behaviour.

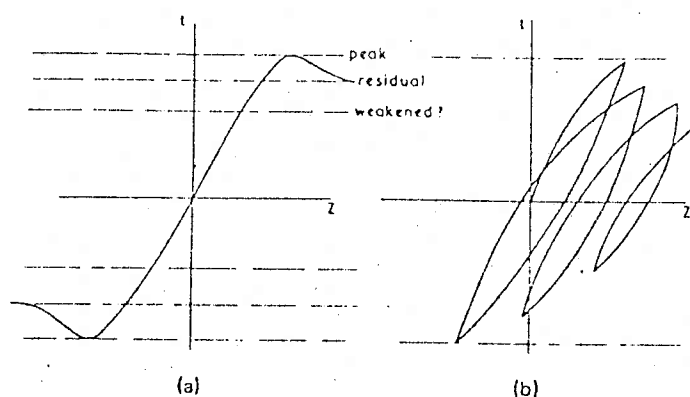
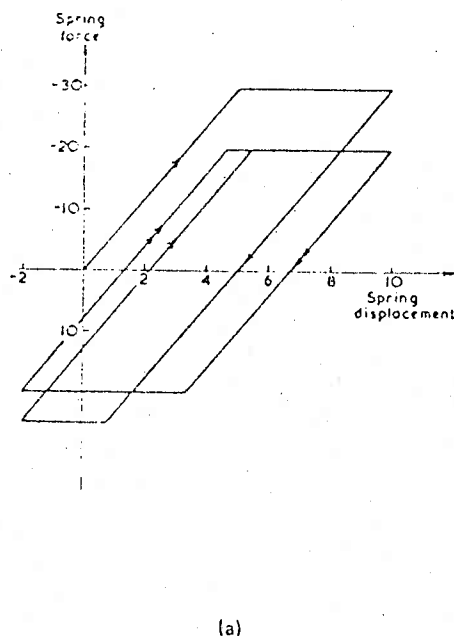


Fig. 7. Degradation of ground: (a) static loading; (b) cyclic loading



15. *Failure.* Three-dimensional finite element computations can be done, but are rather expensive. If an equivalent axisymmetric pier can be assumed, stability parameters follow.

16. *Cyclic loading.* The writer is not aware of work of this nature.

Single laterally loaded pile

17. *Deflexion: the p - y method (Paper 17).* The computations in the p - y method are entirely analogous to those of the t - z method, with p replacing t and y replacing z . As long as satisfactory curves can be specified this calculation is very quick and cheap.

18. *Deflexion: finite element methods (Paper 12).* For axisymmetric pipe piles the commonly used simplifications for non-axisymmetric structures under non-axisymmetric loads can be used. Displacements and so on are expanded in Fourier series so that the analysis becomes quasi-two-dimensional. As long as the geometry remains axisymmetric there is no difficulty in incorporating layered soils. Typical results for the deflexion of a pipe pile under lateral load⁹ are shown in Fig. 9, together with those originally published by Poulos,¹⁰ who assumed that the pile was a thin rectangular strip, and used a boundary element method. The latter can overestimate deflexions by 25% or more. The power of the finite element method is shown in Fig. 10, where deflexion profiles in layered soils are given. The radius of influence is computed to be typically 10 pile diameters for homogeneous soil and 6 pile diameters for soils increasing in stiffness with depth. The depth of influence is never greater than about 6 pile diameters. In the boundary element method (e.g., Banerjee and Davies¹¹), rather radical simplifications have to be made to cope effectively with arbitrary inhomogeneity. The finite element technique has been extended to consider non-linear soils.¹² The difficulties here are merely in storing enough information about the circumferential variations in properties.

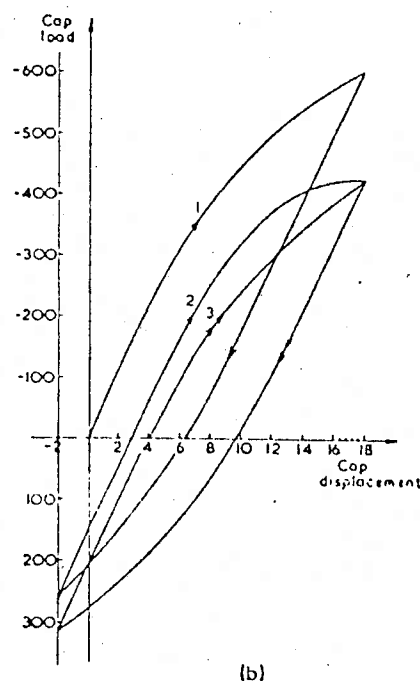


Fig. 8. Pile analysis by t - z method allowing for cyclic degradation: (a) load path of typical spring; (b) load-displacement response of pile

19. *Deflexion: boundary element methods (Paper 13).* With the provisos given above, the boundary element technique is quite suitable for analysis of linear problems, especially in homogeneous soils.

20. *Failure.* Because of the very high bending moments at the mudline carried by piles when laterally loaded, material failure in the piling is much more likely than soil failure for deep-driven offshore piles. Stiff, stubby piles could fail by a rotating mechanism but this does not seem to be of practical interest.

21. *Cyclic loading.* Comments are entirely analogous to those made about axial loading.

Groups of laterally loaded piles (Papers 13 and 18).

22. For groups of laterally loaded piles, the boundary element method has again seemed the natural choice. As was the case with axially loaded piles, empirical marriage of linear interaction factors with non-linear p - y response is often attempted to obtain practical solutions.

DRIVABILITY

The one-dimensional wave equation

23. Because of the difficulties of ordering equipment ahead of time, fleeting weather windows and so on, predictions of pile drivability have assumed great importance in offshore operations. The techniques are not widely used in on-land situations, in the UK at least. The original recommendations of E. A. L. Smith with respect to material parameters such as elastic compressions appear to be adhered to. Probably the major difficulty attaches to the estimate of the viscous component of resistance (Smith's parameter J). It has been pointed out that the success of the method in estimating pile set may well be due to the insensitivity of this quantity to the method of computation.¹³ Other factors, which are often measured in instrumented pile tests, are much more sensitive to the method of computation employed. It is also fair to say that predictive capacity has turned out to be much better in soft as opposed to hard sites, especially for clays and clay-sand mixtures.

Special problems of offshore piling (Papers 4-7)

24. Apart from the large scale of the operations, involving very massive piling and novel capacities of driving equipment, special problems concern, for example, gravity connectors. Offshore piles tend to be driven through a long follower at the end of which is a heavy connector. Thus in the wave propagation procedure there is a significant reflection back up the pile from the connector, and moreover a separation (no tensile stress transfer) occurs. Nevertheless, impressive back-analyses of driving records on some sites have been achieved.¹⁴ If this can be done, an obvious extension is to analyse the driving record in situ and hence to predict the ultimate static capacity of the pile, thus preventing costly over-driving. Considerable experience of these techniques has been built up on land sites in certain areas of the world¹⁵ and it remains to be seen how general the extrapolation procedure is. Again the difficulty in the drivability phase is the viscosity effect, and one would expect 'sands' to be more amenable to prediction than 'clays'. In addition the phenomenon of set-up due to pore pressure effects is clearly an additional difficulty in 'clays'.

Effect of curvature and/or kinks

25. Conductor piles can be deliberately driven with a curvature, the better to exploit the resources in a reservoir. Alternatively piles can be imperfectly welded so that there is an induced curvature or even a sharp kink between adjacent sections. Because of the great length of conductors particularly, concern has been expressed as to the effects of such disturbances on the drivability predictions, and on the stresses in the piling and forces on the guides.

26. The problem has been tackled in a rather mathematical way by Fischer,¹⁶ in the form of finite difference

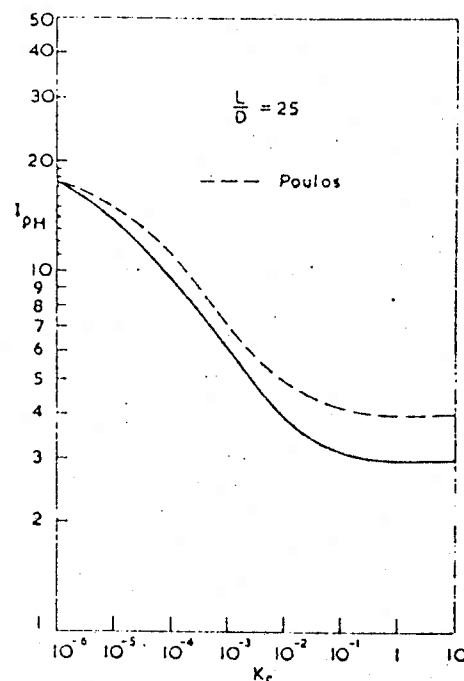


Fig. 9. Finite element analysis of laterally loaded pile in uniform ground; $L/D = 25$

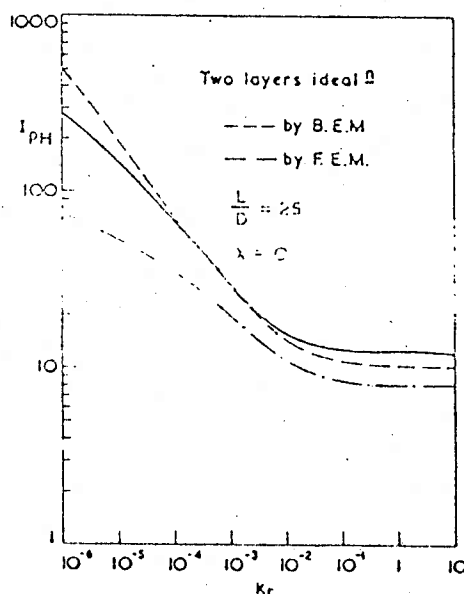


Fig. 10. Laterally loaded pile in layered ground; $L/D = 25$, $x = 0$

approximation to a rather obscure pair of coupled differential equations formulated by Isakovich and Komarova.¹⁷ A simpler method of attack, which shows immediately whether curvature and/or kinks have any great significance, involves finite element approximations of typical piles.¹⁸ As shown in Fig. 11, the pile elements can be genuinely curved, or curves can be approximated by a series of kinks. In either case there will obviously arise coupling between the compressional wave and the transverse motions of the pile. If the kinked pile is a reasonable approximation to the truly curved one, this representation is clearly preferable, since arbitrary kinks can readily be treated.

27. If curvature is to have any effect on the driving process it must be manifested in a shift of the eigenvalues of the curved pile relative to those of the straight pile. Table 1 shows the eigenvalues for three cases, namely straight, truly curved, and kinked piles. It can be seen that the differences between non-straight and straight piles are quite small, as are those between truly curved and kinked. On this basis one would expect the effects of curvature on drivability to be small for typical curvatures.

28. A second analysis involves the stresses in the piling (compressional plus flexural) during driving. Fig. 12 shows a representation of a pile which failed due to over-stressing during driving. The computed stress in the pile at a point close to the failure position is shown in Fig. 13, from which it can be seen that the additional stress due to flexure was a second-order effect and could not really have contributed much to the failure. Material imperfection is a more likely cause.

Flutter (Papers 2 and 3)

29. Recently, attention has been drawn to the nature of the soil forces which resist the penetration of piles during installation. It has been pointed out that these forces may be of the 'follower' type (i.e., they remain tangential to the pile rather than taking up some fixed (usually vertical) direction). This being so, instabilities of a type frequently encountered in aerodynamics can be met at load levels far short of those required to cause instability in the classical buckling sense. Fig. 14 shows typical results in terms of load combinations at which various types of instability occur.¹⁹ The finite element method proves to be a particularly simple means of solving these problems. So far, publications have merely indicated the possibility of

instability arising; they have not shown the effects on drivability of a tendency towards instability. This tendency again manifests itself as a shift in the eigenvalues of the pile-soil system and can readily be incorporated in drivability programs.

Three-dimensional effects

30. In hard driving, it seems quite likely that significant energy is expended in deforming the pile laterally against the sides of the hole. In addition, the presence or absence of a soil 'plug' inside the pipe can clearly affect the mechanisms of wave transmission. It is perfectly possible to analyse the influence of these factors using axisymmetric finite element representations of pile and soil in a dynamic calculation, but this does not yet seem to have been achieved.

PORE PRESSURE EFFECTS (Papers 19 and 20)

31. So far, soil resistance and strength have been represented exclusively in terms of total stresses. However, it is well known that, particularly for piles driven into normally consolidated, impermeable clays, large excess pore water pressures can be generated, the dissipation of which governs the pile's ability to resist loads applied at various times after driving.

32. Numerical methods have recently been applied in this area,²⁰⁻²² but the problem is a difficult one, and the writer doubts whether the state of excess pore pressure existing adjacent to piles driven into normally consolidated or overconsolidated clays and sand-clay mixtures, such as those in the North Sea, can be predicted with much confidence analytically. Field observation seems to be necessary here. However, rates of dissipation of the generated excess pore pressures should be perfectly adequately computed by present analytical techniques, given adequate values of permeability coefficients, obtained from tests in the field or on large specimens.

DYNAMICS (Papers 8-10)

33. In a cyclic loading environment the frequencies of the alternating forces and their relationship to the critical frequencies of the structure-soil system assume a decisive importance. The main types of dynamic loading experienced by offshore structures appear in the form of sea waves and/

Table 1. Natural frequencies of simply supported curved beam represented by straight and curved finite elements; radius of curvature 50 m, subtended angle 30°

Mode	Frequency, mHz	Percentage error for straight elements with kinks			Percentage error for curved elements		
		4	12	20	4	12	20
1	0.6345	+ 5.67	+5.48	+5.73	+ 1.50	1.4	1.46
2	2.6190	+ 2.4	+1.9	+2.10	+ 0.76	0.36	0.35
3	5.9269	+ 2.3	+0.57	+0.57	+ 2.01	0.20	0.16
4	10.5581	+11.6	+0.52	+0.50	+11.1	0.20	0.01
5	16.5125	+10.8	-0.11	-0.37	+13.0	0.80	0.01
6	19.362	+ 3.9	+0.14	+0.22	< 0.1	<0.1	<0.1
7	23.790	+23.0	+0.50	+0.15	24.1	0.5	0.09
8	32.391	+31.0	+0.89	+0.30	18.3	1.0	0.13
9	38.330	+15.6	+0.51	-0.30	15.5	<0.1	<0.1
10	42.315	+27.8	+1.63	+0.70	27.4	2.0	0.19

or earthquakes. These two forms of excitation are radically different, as has been pointed out in the context of gravity structures.²³ For piled structures, many of the same considerations apply.

Sea waves and earthquakes

34. Ocean wave loading is of low frequency and long duration, whereas earthquake loading is of high frequency and short duration. The magnitudes of total load imposed on the structure-soil system by the two excitations are, however, of the same order. In the case of excitation by earthquakes, it is usually assumed that shear waves propagate vertically through the soil from bedrock, and of course all of the soil and any deep-driven piles embedded in

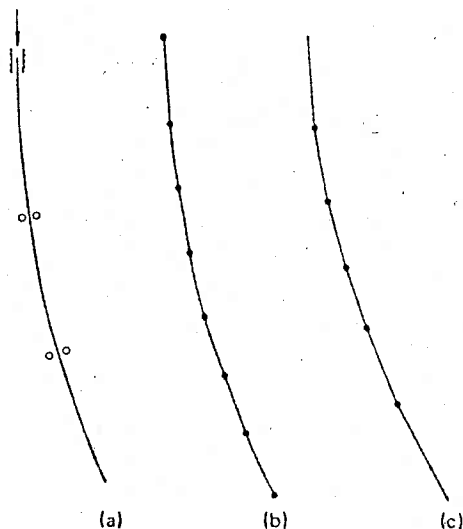


Fig. 11. Representation of curved conductors by finite elements: (a) curved pile; (b) curved elements; (c) straight elements

Vertical load factor 0.995
Horizontal load factor 0.01

208.5 m

Diameter = 1.524 m
Wall thickness = 0.06 m

150.4 m

120.1 m

89.41 m

46.4 m

0.0 m

$K = 1.0 \times 10^5 \text{ kN/m}$

Fig. 12. Idealization of failed pile

SURVEY OF NUMERICAL METHODS IN OFFSHORE PILING

it are subject to the earthquake motions. These lead to rather large shear strains everywhere in the soil mass and a great deal of 'primary' interaction between the soil and the piles. This interaction causes shaking of the structure which in turn exerts a 'secondary' interaction back through the piling into the soil. The descriptions primary and secondary dynamic interaction are borrowed from usage in the nuclear power plant industry and may not realistically express the relative importances of the two effects. When the excitation is by sea waves, the interaction is by the above definition totally secondary. This type of interaction is much more localized to the immediate vicinity of the structure, and in the case of typical deep-piled offshore structures, will not be felt at all by the ground below quite a shallow depth of a few metres below mudline.

Methods of analysis

35. Idealizations of piled foundations for dynamic analysis follow closely their static counterparts. Often the

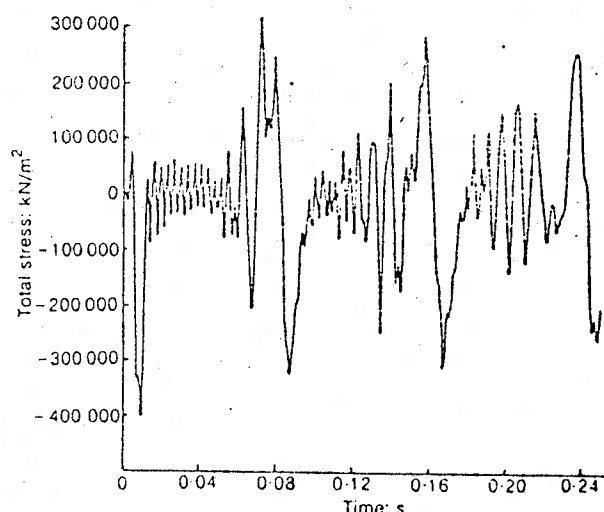


Fig. 13. Stress computations for failed pile

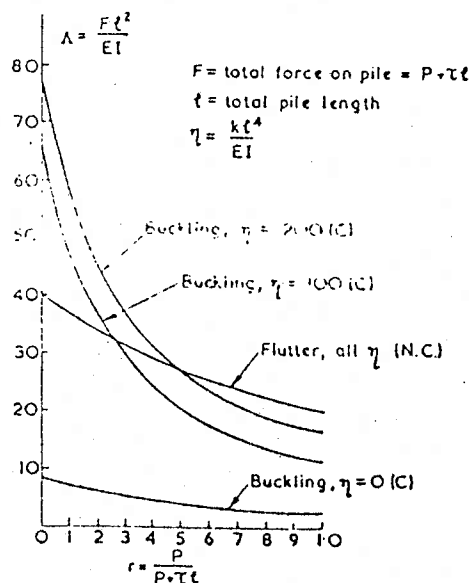


Fig. 14. Flutter analysis of piles by finite elements; F = total force on pile = $P + \tau l$, l = total pile length, $\eta = k l^4 / EI$

ground resistance is approximated by a set of springs, but this method is more questionable than in the static case because of the inability of the springs to cope effectively with a major source of energy dissipation, namely by geometric or radiation damping. This is particularly important in earthquakes where there is a lot of energy present throughout the excited ground. In order to include a measure of geometric damping, finite element or boundary element approximations can again be used,²⁴ and a few solutions for simple cases can even be produced analytically.²⁵

36. Among the various computational tactics which can be adopted, the most widely used are integration of the equations of motion in the time domain, which allows rather general energy dissipation mechanisms to be included. These tend to comprise the non-linear effects of friction, plasticity, viscosity and so on. Alternatively, linearized calculations can be made by the 'complex response' or 'impedance' methods, taking account of hysteretic damping only. Both methods have their advantages, and experience with the analysis of gravity platforms indicates that lateral stiffnesses and motions of the structure-soil system will probably be similarly predicted by both methods, given similar initial assumptions. However, the truly non-linear calculations produce some results, such as permanent displacements and sub-resonances, which cannot be present in any linearized analysis.²³ This is a fruitful field for further study.

CONCLUSIONS

37. Numerical methods in offshore piling are in many cases more sophisticated than the physical data which is input to the programs. What is often required is back-analysis from the field since full-scale tests are prohibitively expensive. Modelling, especially true scale modelling using centrifuges, can also contribute valuable data for calibration of the numerical results.

38. Areas where further numerical developments can be made include constitutive relationships for soil, especially during cyclic loading and penetrating of piles, three-dimensional effects and dynamics.²⁶

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