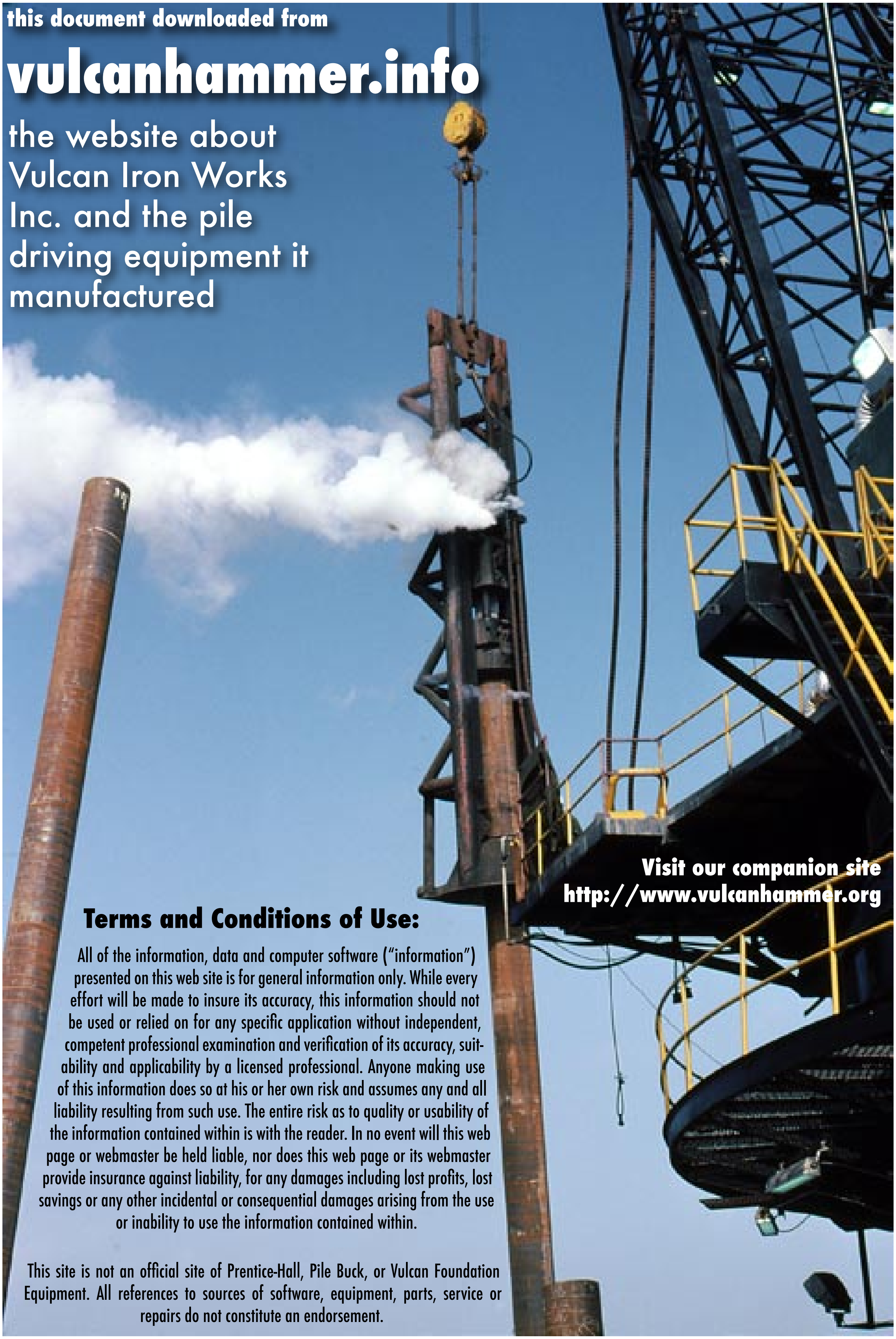

Estimating the flexibility of offshore pile groups

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1 INTRODUCTION

The overriding criterion in designing piles to support offshore structures is usually the required axial capacity of the pile. The number of piles, and frequently the diameter of each pile, may be fixed at an early stage of the design, while the final length of each pile is only settled after detailed site investigation and the application of a variety of design procedures for estimating the profile of ultimate skin friction. The stiffness of the final foundation must also be estimated accurately in order that the dynamic performance of the structure may be assessed. Modern methods of calculating the stiffness of a piled foundation involve first estimating the axial and lateral stiffness of a single, isolated, pile, and then using appropriate interaction factors and frame analysis techniques to arrive at a stiffness matrix for the complete pile group.

For a group of vertical piles, the final stiffness or flexibility matrix is a 6×6, sparsely populated matrix, relating the 6 degrees of freedom (3 orthogonal deflections, and 3 rotations) to the 6 applied forces and moments. A flexibility matrix F may be defined by

$$\begin{Bmatrix} v \\ u_x \\ \theta_x \\ u_y \\ \theta_y \\ \phi \end{Bmatrix} = \begin{bmatrix} F_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & F_{22} & F_{23} & 0 & 0 & 0 \\ 0 & F_{32} & F_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & F_{44} & F_{45} & 0 \\ 0 & 0 & 0 & F_{54} & F_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & F_{66} \end{bmatrix} \begin{Bmatrix} V \\ H_x \\ M_x \\ H_y \\ M_y \\ T \end{Bmatrix} \qquad (1)$$

or $\underline{\delta} = F \, \underline{P}$ (see Fig. 1).

For linear response, the matrix will be symmetric, so that $F_{32} = F_{23}$ and

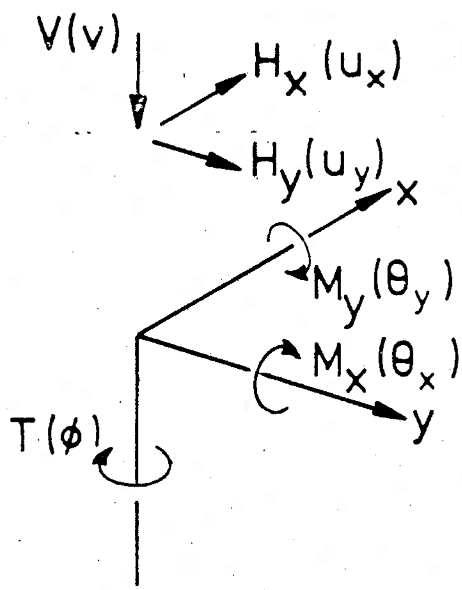

Figure 1: Applied loads with corresponding deformations

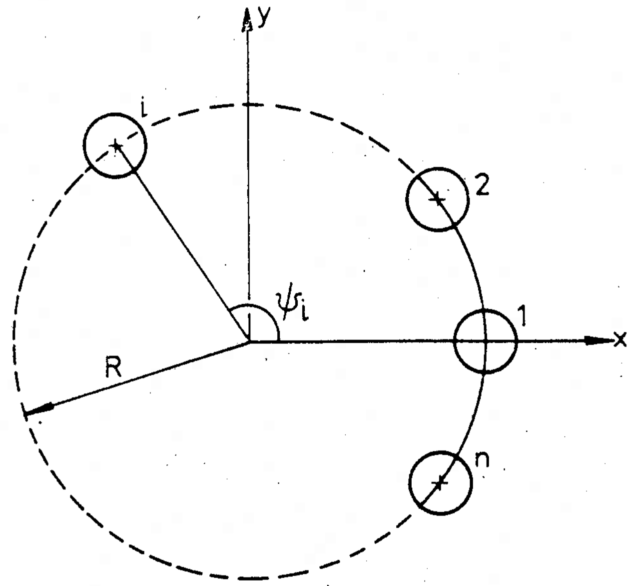

Figure 2: Typical configuration of piles in an offshore group

$F_{54} + F_{45}$. The straightforward configuration of most offshore pile groups - where the piles are approximately evenly spaced around a uniform pitch circle (see Fig. 2) - enables simple and reasonably accurate estimates to be made of the terms in the flexibility matrix F, without excessive computation. To a first approximation, the behaviour in any horizontal direction will be similar, so that F_{44}, F_{45} and F_{55} may be taken as equal to F_{22}, F_{23} and F_{33} respectively. This leaves only 5 unknown terms to be evaluated. The following section outlines how this evaluation may be achieved, based on flexibility coefficients for a single isolated pile, and appropriate interaction factors for axial and for lateral loading.

2 ANALYSIS

The starting point for the analysis is the flexibility coefficients for a single, isolated, pile; these will be denoted by f with appropriate subscripts. Thus, for axial (vertical) loading,

$$v = f_v V \tag{2}$$

while for lateral (horizontal) loading

$$\begin{aligned} u &= f_{uH} H + f_{uM} M \\ \theta &= f_{\theta H} H + f_{\theta M} M \end{aligned} \tag{3}$$

From symmetry, $f_{uM} = f_{\theta H}$ and the latter coefficient will be used for both quantities. In addition, for lateral loading, it is useful to refer to a 'fixed-head' mode of deformation, where the pile head is restrained from rotation. The relevant flexibility coefficient will be written f_{uf}; from equation (3), it may be shown that

$$f_{uf} = f_{uH} - f_{\theta H}^2/f_{\theta M} \ . \qquad (4)$$

The fact that the stiffness of a pile group is less than the combined stiffness of all the individual piles, is customarily quantified by the use of interaction factors α (Poulos, 1971). The interaction factor gives the fractional increase in deformation of a pile due to the presence of a similarly loaded neighbouring pile. Thus, for two piles each with a load V applied, the vertical deformation of each pile is

$$v = f_v(1 + \alpha_v)V \ . \qquad (5)$$

For lateral loading, if a load H is applied to each pile, then the deflection u, is given by

$$u = f_{uH}(1 + \alpha_{uH})H \qquad (6)$$

(or $u = f_{uf}(1 + \alpha_{uf})H$ for fixed-head piles).

The rotation θ is given by

$$\theta = f_{\theta H}(1 + \alpha_{\theta H})H \qquad (7)$$

Similar interaction factors α_{uM} and $\alpha_{\theta M}$ may be defined for two piles loaded by a moment M at the ground surface. Again, from symmetry, $\alpha_{uM} = \alpha_{\theta H}$, and the latter term will be used for both quantities.

The use of interaction factors may be extended to cases where the loads vary from pile to pile. A load P_j on the j^{th} pile is then seen as contributing a deformation δ_i to the i^{th} pile, where

$$\delta_i = f \ \alpha_{ij} \ P_j \ . \qquad (8)$$

This rule may be generalised to include the case i = j, by defining $\alpha_{jj} = 1$.

It is now possible to deduce the first term in the flexibility matrix F. For vertical loading, the symmetry of the offshore pile group entails that each pile will be similarly loaded. Thus, for an overall load V, each pile will carry a load V/n (for n piles in the group) and the deformation of each pile will be

$$v_i = \frac{V}{n} f_v \sum_{j=1}^{n} (\alpha_v)_{ij} \ . \qquad (9)$$

Thus F_{11} is given by

$$F_{11} = \frac{f_v}{n} \sum_{j=1}^{n} (\alpha_v)_{ij} \ . \tag{10}$$

For convenience, the summation limits and indices will be omitted in the expressions below.

The response of the pile groups to lateral load is complicated by the axial loads induced in the piles due to rotation of the group. A simple approach (Randolph, 1977) is to consider first the 'fixed-head' mode of deflection, where no rotation of the pile head occurs. Under an overall horizontal load H, it may be assumed that each pile carries a uniform load of H/n (Matlock et al (1980) have found that the variation of horizontal load around typical offshore pile groups is very small). Thus the fixed head deflection is

$$u_f = \frac{H}{n} f_{uf} \sum \alpha_{uf} \ . \tag{11}$$

The fixing moment M_f necessary at the head of each pile in order to restrain rotation may be calculated from

$$\theta = \frac{H}{n} f_{\theta H} \sum \alpha_{\theta H} + M_f \ f_{\theta M} \sum \alpha_{\theta M} = 0 \tag{12}$$

Thus $$M_f = -\frac{H}{n} \ \frac{f_{\theta H} \sum \alpha_{\theta H}}{f_{\theta M} \sum \alpha_{\theta M}} \ . \tag{13}$$

The overall moment applied to the group is now M (the actual moment) - nM_f (to counteract the fixing moments).

The applied moment will be shared between the 'push-pull' mode, where axial loads are induced in the outer piles as the group rotates, and moments at the head of each pile, in such a way that the rotation of each pile matches that of the group. The push-pull mode of rotation induces axial loads in the piles which may be assumed to vary as $V'\cos\psi$ (ψ being the angle of any particular pile from the x axis and V' being the axial load induced in pile 1 - see Fig. 2). Thus the axial deflection of pile 1 is

$$v' = V' f_v \sum \alpha_v \cos\psi \tag{14}$$

The group rotation is $\theta' = v'/R$ (where R is the pitch circle radius) and the moment absorbed is $\Sigma V'\cos\psi \times R\cos\psi = RV'\Sigma\cos^2\psi$. Thus the push-pull stiffness is

$$S_{pp} = \frac{R^2 \sum \cos^2\psi}{f_v \sum \alpha_v \cos\psi} \tag{15}$$

For a moment M' at the head of each pile, the rotation is given by

$$\theta' = M' f_{\theta M} \sum \alpha_{\theta M} . \tag{16}$$

Thus the rotational stiffness (for n piles) is

$$S_r = \frac{n}{f_{\theta M} \sum \alpha_{\theta M}} . \tag{17}$$

The overall moment will be shared between the push-pull mode and rotational mode in proportion to the respective stiffnesses. Thus the moment at each pile head, M', may be calculated as

$$M' = \frac{1}{n} (M - nM_f) \frac{S_r}{S_{pp} + S_r} . \tag{18}$$

$$= \frac{1}{n} (M - nM_f)/\chi$$

where $\chi = 1 + S_{pp}/S_r$.

The rotation of the group is given by

$$\theta = \frac{1}{n} (M - nM_f) f_{\theta M} \sum \alpha_{\theta M}/\chi \tag{19}$$

and the additional deflection by

$$u_r = \frac{1}{n} (M - nM_f) f_{\theta H} \sum \alpha_{\theta H}/\chi . \tag{20}$$

The final form of loading to be considered, is that of torsional loading. Analysis of pile groups under general loading using computer programs such as PIGLET (Poulos and Randolph, 1982), indicates that the torsional stiffness of the individual piles is negligible (∿ 5%) compared with the stiffness of the combined groups due to the enforced lateral movement of piles away from the axis of twist. An applied torque of T may thus be assumed to impart a horizontal load of $H' = T/nR$ to each pile in an offshore group. The component of H' which is parallel to pile 1 is $H'\cos\psi$, and thus the (fixed-head) deflection of each pile is

$$u' = \frac{T}{nR} f_{uf} \sum \alpha_{uf} \cos\psi \tag{21}$$

giving a rotation of the group of

$$\phi = \frac{u'}{R} = \frac{1}{n} \frac{T}{R^2} f_{uf} \sum \alpha_{uf} \cos\psi . \tag{22}$$

In summary, expressions for the coefficients in the flexibility matrix F may now be written down as

$$F_{11} = \frac{f_v}{n} \sum \alpha_v$$

$$F_{22} \simeq F_{44} = \frac{1}{n} \left\{ f_{uf} \sum \alpha_{uf} + \frac{(f_{\theta H} \sum \alpha_{\theta H})^2}{\chi f_{\theta M} \sum \alpha_{\theta M}} \right\}$$

$$F_{23} = F_{32} \simeq F_{45} = F_{54} = \frac{f_{\theta H}}{n} \sum \alpha_{\theta H}/\chi$$

$$F_{33} \simeq F_{55} = \frac{f_{\theta M}}{n} \sum \alpha_{\theta M}/\chi$$

$$F_{66} = \frac{f_{uf}}{nR^2} \sum \alpha_{uf} \cos\psi$$

where $\chi = 1 + S_{pp}/S_r = 1 + \dfrac{f_{\theta M}\ R^2 (\Sigma\alpha_{\theta M})(\Sigma\cos^2\psi)}{n\ f_v \sum \alpha_v\ \cos\psi}$.

3 ESTIMATION OF FLEXIBILITY COEFFICIENTS AND INTERACTION FACTORS

In order to calculate values for the components of F, some estimate must be made of the flexibility coefficients f and the interaction factors α. Since the main purpose of this simplified approach is as a preliminary design aid it is logical to turn to elastic theory for these coefficients. Numerical methods such as the integral equation (or boundary element) method have been used to calculate the flexibility of single piles under various loading conditions. Charts of values over a wide range of pile stiffnesses and geometry have been published by Poulos and Davis (1980). As an alternative, closed form expressions have been published by Randolph and Wroth (1978) for axial loading; and by Randolph (1981) for lateral loading. The latter expressions are particularly simple to use as they are cast in a form which is independent of the pile length. (The lateral response of piles is rarely affected by the overall length of the pile.)

Poulos and Davis (1980) also give charts showing interaction factors for axial and for lateral loading. It is worthwhile making a few observations concerning these interaction factors, and the manner in which they may be expected to vary with the pile spacing. Figure 3 shows values of α_v, taken from Poulos (1979a) for length to diameter ratios (ℓ/d) of 10, 25 and 50. Values are plotted for three values of the parameter ρ, defined as the ratio of the average shear modulus, G_{ave}, to the value at the level of the pile base, $G_{z=\ell}$ (Randolph and Wroth, 1978) and for stiffness ratios, $\lambda = E_{pile}/G_{z=\ell}$, of 3×10^2 and 3×10^3. The interaction factors are plotted on a logarithmic scale of spacing-to-diameter, s/d, and it is immediately evident that there is an approximately linear relationship between α_v and $\log(s/d)$. Values of the interaction factors tend to decrease as ρ decreases and also as λ decreases (and thus the pile becomes more compressible). The most striking feature of the straight line approximation is that they all pass through the point $\alpha_v = 0$ for $s/d = \ell/d$, regardless of the values of ρ and λ. For the range of stiffness ratios commonly encountered offshore, where $\lambda \sim 500$, a reasonable estimate of α_v is given by

$$\alpha_v \sim \frac{0.5 \, \ln(\ell/s)}{\ln(\ell/(d\rho))} \qquad \text{for } s \leq \ell \tag{23}$$

This expression gives reasonably good agreement with the curve deduced from experimental measurements by Cooke et al (1980), although tending to give higher values of α_v at pile spacings approaching the length of the piles.

For lateral loading, Randolph (1981) has suggested simple relationships, where the interaction factors are inversely proportion to the pile spacing. Such relationships give good agreement with values published by Poulos (1971). Defining G_c as the average value of shear modulus of the soil over the critical length, ℓ_c, of the pile (i.e. that part that deforms appreciably under lateral loading) and ρ_c as the ratio of $G_{z=\ell_c/4}$ to G_c, then the critical length may be estimated as

$$\ell_c \sim 2r_o(E_p/G_c)^{2/7} \tag{24}$$

where r_o is the radius of the pile (see Randolph, 1981). The interaction factors may be estimated as

$$\left.\begin{aligned} \alpha_{uf} &= 0.6\,\rho_c\,(E_p/G_c)^{1/7}\,\frac{r_o}{s}\,(1 + \cos^2\beta) \\ \alpha_{uH} &\simeq 0.4\,\rho_c\,(E_p/G_c)^{1/7}\,\frac{r_o}{s}\,(1 + \cos^2\beta) \\ \alpha_{\theta H} &\simeq \alpha_{uH}^2\,, \qquad \text{and} \qquad \alpha_{\theta M} \simeq \alpha_{uH}^3\,, \end{aligned}\right\} \tag{25}$$

where β is the angle between the direction of loading and the line between the pile centres (see Fig. 4). In order to avoid problems at very close

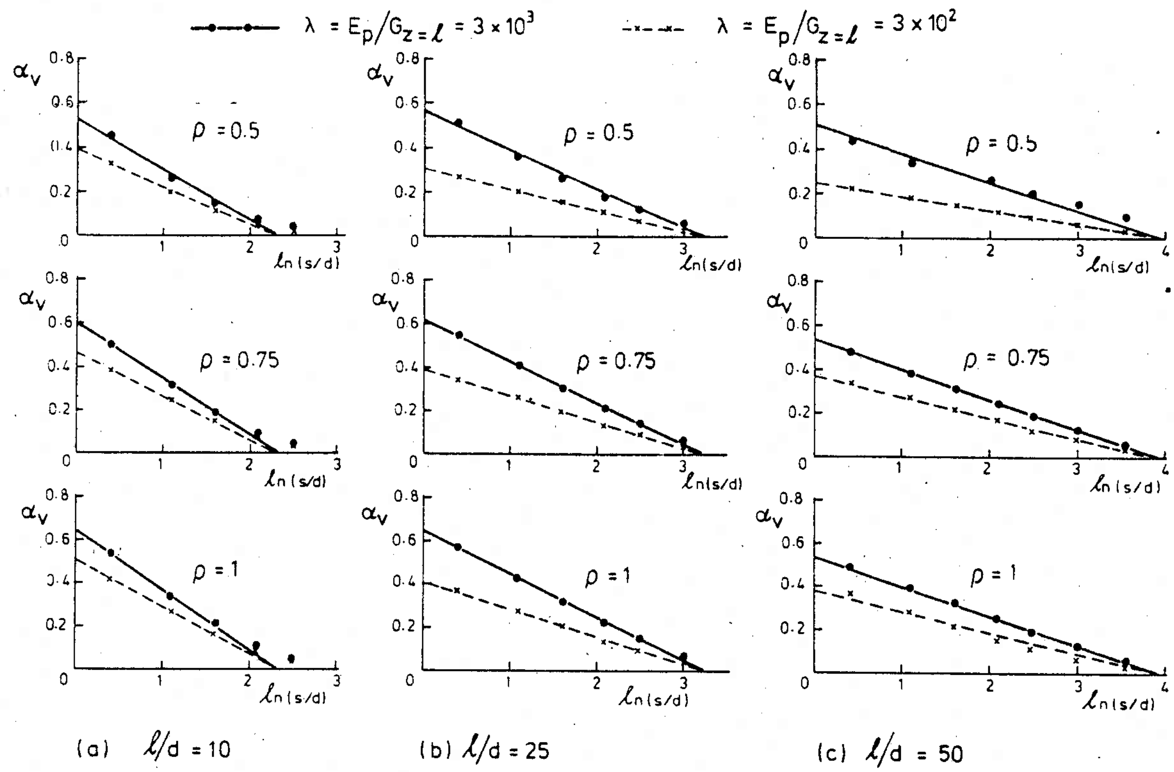

Figure 3: Interaction factors for vertically loaded piles

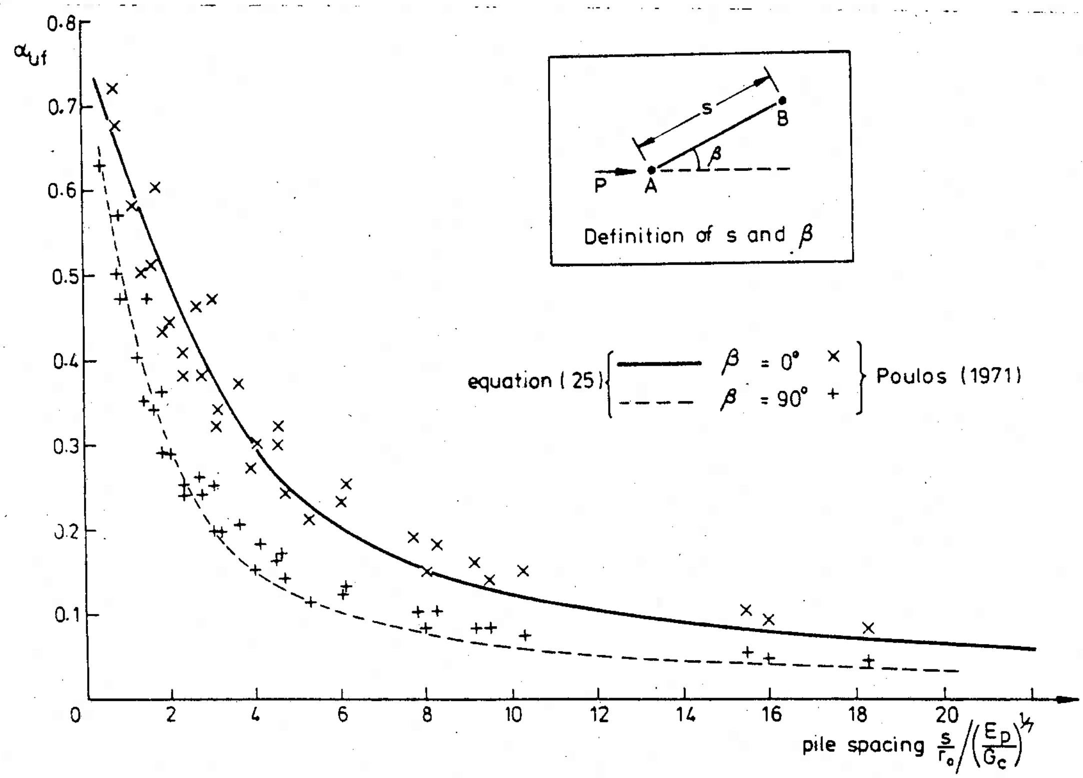

Figure 4: Interaction factors for horizontally loaded fixed-headed piles

spacings (where $s \to 0$ would imply $\alpha \to \infty$), a transformation may be introduced such that, where an interaction factor is calculated to be greater than 1/3, then the value is replaced by

$$\alpha = 1 - \frac{2}{(27\alpha)^{\frac{1}{2}}} . \tag{26}$$

This has the effect of smoothly transforming the hyperbola into a parabola for values of α greater than 1/3. (Note that equation (26) implies $\alpha \to 1$ as $s \to 0$.)

Figure 4 shows computed values of α_{uf}, compared with values published by Poulos (1971). The agreement is generally good, for both extreme values of $\beta = 0°$ and $\beta = 90°$. From equations (25), it should be noted that interaction factors for intermediate values of β may be calculated from the extreme values by

$$\alpha(\beta) = \alpha(0°) \cos^2\beta + \alpha(90°) \sin^2\beta \tag{27}$$

The form of variation of interaction factor with pile spacing given by equation (23) and by equations (25) reflects the manner in which the deflections decrease away from the pile - i.e. logarithmically in the case of axial loading, and inversely in the case of lateral loading. The coefficients have been chosen to give absolute values which are consistent with published values based on elastic solutions. Where appropriate, for example, if a zone of plastic yield has occurred closed to the pile, the absolute values may be decreased by adopting smaller coefficients, thus preserving the form of the variation of α with the spacing between the piles.

4 COMPARISONS OF SIMPLIFIED ANALYSIS WITH RIGOROUS ANALYSIS AND WITH EXPERIMENTAL RESULTS

In order to illustrate the degree of accuracy to be expected from the analysis outlined in section 2, two example analyses will be discussed. The first example concerns a typical, 11 pile offshore group that has been used previously be Poulos (1979b) as an example. The piles were of length 72 m, diameter 1.37 m and were arranged around a pitch circle of radius 5.4 m (giving a spacing to diameter ratio of 2.2 for neighbouring piles). The wall thickness of 50 mm gives an equivalent Young's modulus for a solid pile of $\sim$ 30,000 MN/m^2 and the soil stiffness has been assumed to vary from 18 MN/m^2 at the ground surface up to 30 MN/m^2 at the level of the pile bases (corresponding to $G \sim 150\ c_u$ - see Poulos (1979b)).

Analysis of a single pile in these soil conditions gives estimates of the flexibility coefficients of

$$f_u = 0.00109 \text{ m/MN} ; \qquad f_{uH} = 0.00568 \text{ m/MN} ;$$
$$f_{\theta H} = 0.00123 \text{ rad/MN} ; \qquad f_{\theta M} = 0.630 \times 10^{-3} \text{ rad/MNm}$$

The simplified analysis leads to a flexibility matrix for the groups of .

$$F = \begin{bmatrix} 3.21 \times 10^{-4} & 0 & 0 & 0 \\ 0 & 1.07 \times 10^{-3} & 1.33 \times 10^{-5} & 0 \\ 0 & 1.33 \times 10^{-5} & 5.58 \times 10^{-6} & 0 \\ 0 & 0 & 0 & 1.79 \times 10^{-5} \end{bmatrix}$$

For comparison, a full analysis of the group conducted using the program PIGLET yields a flexibility matrix of

$$F = \begin{bmatrix} 2.67 \times 10^{-4} & 0 & 0 & 0 \\ 0 & 1.08 \times 10^{-3} & 1.57 \times 10^{-5} & 0 \\ 0 & 1.57 \times 10^{-5} & 6.39 \times 10^{-6} & 0 \\ 0 & 0 & 0 & 1.56 \times 10^{-5} \end{bmatrix}$$

Apart from the term F_{11} there is very good agreement between the two matrices. The difference in F_{11} is $\sim$ 20% and reflects the different approach in estimating the interaction between piles under axial loading used in the program PIGLET. For comparison, analysis using the program DEFPIG (Poulos, 1979b) yields a value for F_{11} of 3.27×10^{-4} m/MN.

The second example is taken from a series of model tests conducted at the University of Sydney by Ferguson and Laurie (1980). Tests were conducted on groups of solid brass piles (E_p = 86,000 MN/m^2) of diameter 6.5 mm and length 164 mm embedded in kaolin clay. The kaolin was prepared in slurry form, placed in a pressure vessel 600 mm in diameter and 2.45 m deep, and consolidated to an overburden pressure of 210 kN/m^2. Details of the apparatus are given by Wiesner and Brown (1980). After completion of consolidation, the pressure was removed and the piles were jacked into the clay. The piles were arranged in groups of 8 and 12 piles around a pitch circle of radius 38.5 mm. The piles were built-in to rigid pile caps and loaded vertically and horizontally. In each case, vertical and horizontal load tests were also conducted on single piles in order to obtain flexibility coefficients for a single, isolated pile.

The load deformation response of the piles was non-linear and, in the comparison below, the flexibility of the group is that measured at a load factor of 2.5 against failure. Flexibility coefficients for the single pile have been obtained for the same load per pile.

For the tests on the eight pile group, the measured flexibility of the single pile was 2.88 mm/kN axially (this value was virtually constant for loads up to about 60% of ultimate). Horizontal loads were applied, and deflections measured, at a distance of 8 mm above the clay surface. At a load level of 40 N (corresponding to 1/(8 × 2.5) of the failure load of the 8 pile group) the measured flexibility was 8.03 mm/kN. Assuming homogeneous soil conditions, the analysis of lateral loading described by Randolph (1981) may be used to deduce flexibility coefficients for loads and deflections at the clay surface, which are consistent with the flexibility measured at 8 mm above the clay surface. The resulting coefficients are f_{uH} = 5.91 mm/kN, $f_{\theta H}$ = 0.110 rad/kN and $f_{\theta M}$ = 0.00496 rad/kNmm. These coefficients may be used in the analysis of section 2 to yield an overall flexibility matrix for the group of

$$F = \begin{bmatrix} 0.810 & 0 & 0 & 0 \\ 0 & 1.39 & 0.00743 & 0 \\ 0 & 0.00743 & 2.88 \times 10^{-4} & 0 \\ 0 & 0 & 0 & 3.99 \times 10^{-4} \end{bmatrix}$$

with units of mm, kN and radians. (The steps in evaluating the terms in F are given in detail in an Appendix to this paper.) Terms in this matrix may be compared with measured values of F_{11} = 0.888 mm/kN and F_{22} = 1.31 mm/kN. Both of these values are within 10% of the predicted values.

The corresponding tests on the 12 pile group gave flexibilities of the single pile of 2.88 mm/kN axially, and 6.64 mm/kN laterally (at a load level of 30 N applied 8 mm above the clay surface). The flexibility coefficients may be calculated as f_v = 2.88 mm/kN, f_{uH} = 4.78 mm/kN, $f_{\theta H}$ = 0.0959 rad/kN and $f_{\theta M}$ = 0.00462 rad/kNmm. The resulting flexibility matrix for the group is

$$F = \begin{bmatrix} 0.740 & 0 & 0 & 0 \\ 0 & 1.04 & 0.00555 & 0 \\ 0 & 0.00555 & 2.11 \times 10^{-4} & 0 \\ 0 & 0 & 0 & 2.96 \times 10^{-4} \end{bmatrix}$$

with units of mm, kN and radians. The measured vertical and horizontal flexibilities of the 12 pile group were $F_{11} = 0.761$ mm/kN and $F_{22} = 0.95$ mm/kN respectively. Again, the predicted and measured flexibilities agree to within 10%. Table 1 summarizes the comparisons between the measured and predicted values of F_{11} and F_{22}, and also gives values calculated from the programs DEFPIG and PIGLET which are in good agreement with those from the simplified method presented here.

	Flexibility Coefficient	Measured (mm/kN)	Predicted (mm/kN)		
			Simplified Analysis	DEFPIG	PIGLET
8 pile group	F_{11}	0.888	0.810	0.843	0.956
	F_{22}	1.31	1.39	1.23	1.25
12 pile group	F_{11}	0.761	0.740	0.785	0.865
	F_{22}	0.95	1.04	0.87	0.94

Table 1: Measured and predicted flexibility coefficients for model tests of Ferguson and Laurie (1980)

Additional horizontal load tests were also conducted on 8 and 12 pile groups with the piles inclined (parallel) at 7.5° to the vertical - simulating the typical arrangement for an offshore pile group. The flexibility matrix F may be modified to allow for batter of the complete pile group, by pre- and post-multiplying by the appropriate transformation matrix, assuming that the axial and lateral response of the group remains unaltered by small amounts of batter. Thus, for an angle of batter of μ, the new flexibility matrix is

$$F^* = T^T \, F \, T$$

where T^T is the transpose of T and

$$T = \begin{bmatrix} \cos\mu & \sin\mu & 0 & 0 \\ -\sin\mu & \cos\mu & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \cos\mu \end{bmatrix}$$

Under a horizontal load H, the horizontal deflection would thus be

$$u = (\sin^2\mu \; F_{11} + \cos^2\mu \; F_{22}) \; H$$

For $\mu = 7.5^\circ$, the modified horizontal flexibility of the battered pile groups is 1.38 for the eight-pile group and 1.03 for the 12 pile group. These figures are only marginally smaller than for the corresponding vertical pile group. In practice, however, the measured flexibilities were more markedly reduced by battering the piles. For the eight pile group, the flexibility was reduced to 1.04 mm/kN (compared with 1.31 for the vertical piles), while, for the twelve pile group, the measured flexibility was 0.92 mm/kN (compared with 0.95). It is not clear why the measured flexibility of the battered pile group, particularly the eight pile group, is so much less than that of the vertical pile group, although this tendency has been noted previously in relation to pile groups in sand (Poulos and Randolph, 1982). Further research in this area would be useful in order to investigate whether this experimental result is due to a chance variation in the soil properties.

5 CONCLUSIONS

It is common practice for the response of a pile group to be estimated by considering first the response of a single pile and then using appropriate interaction factors to allow for group effects. This approach allows different techniques to be used for the different stages. Thus the single pile response may be estimated from a subgrade reaction approach, using non-linear t - z or p - y curves, while interaction factors are usually estimated from elastic theory. The inconsistency of using different models of soil behaviour is, to some extent, balanced by the resulting ease with which the response of a complete pile group may then be calculated.

For configurations of piles which arise offshore, calculation of the overall flexibility matrix of the group becomes particularly straightforward. This Paper has described the way in which terms in the flexibility matrix may be deduced from flexibility coefficients for a single, isolated, pile and appropriate interaction factors. Within the accuracy with which soil stiffness parameters may be determined, it is sufficient to estimate interaction factors from simple expressions which are based on studies treating the soil as an elastic continuum.

Comparison between the approximate estimate of the group flexibility matrix obtained by this approach and a more rigorous analysis using programs such as PIGLET or DEFPIG shows reasonably good agreement. When applied to model tests of offshore pile groups, the use of measured single pile flexibilities, together with interaction factors given by equations (23) and (25), yields overall flexibilities of the group which are close to those actually measured.

The scale of pile foundations for offshore structures warrants the use of sophisticated analytical methods in order to assess the likely response of the foundations under working conditions. However, there is an important role, particularly in the early stages of a design, for simple methods, where the stiffness of the complete foundation may be rapidly estimated using minimal computation.

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APPENDIX

Calculation of the terms in the flexibility matrix F is illustrated here for the model eight pile group tested by Ferguson and Laurie (1980).

The properties of the group of piles are:

ℓ = 164 mm, d = 6.5 mm, R = 38.5 mm, n = 8 .

The single pile flexibility coefficients are:

$f_v = 2.88$ mm/kN, $f_{uH} = 5.91$ mm/kN, $f_{\theta H} = 0.110$ rad/kN,

$f_{\theta M} = 0.00496$ rad/kNmm, whence $f_{uf} = f_{uH} - f_{\theta H}^2/f_{\theta M} = 3.47$ mm/kN.

From equations (23) and (25) - (26) taking $\rho = \rho_c = 1$ and $E_p/G_c = 2.62 \times 10^4$ (consistent with the flexibility coefficients for lateral loading), the interaction factors may be tabulated for piles 2 to 8 (relative to pile 1) as

	Pile						
	2	3	4	5	6	7	8
Spacing, s	29.5	54.4	71.1	77.0	71.1	54.4	29.5
$\cos\psi$	0.707	0	-.707	-1	-.707	0	0.707
α_v	0.266	0.171	0.129	0.117	0.129	0.171	0.266
$\alpha_v \cos\psi$	0.188	0	-.092	-.117	-.092	0	0.188
α_{uf} (lateral)	0.324	0.230	0.217	0.217	0.217	0.230	0.324
$\alpha_{uf}\cos\psi$ (torsion)	0.331	0	-.095	-.108	-.095	0	0.331
α_{uH}	0.216	0.153	0.145	0.145	0.145	0.153	0.216
$\alpha_{\theta H}$	0.047	0.024	0.021	0.021	0.021	0.024	0.047
$\alpha_{\theta M}$	0.010	0.004	0.003	0.003	0.003	0.004	0.010

It should be noted that the interaction factor α_{uf} between any two piles will be different for lateral loading and for torsion loading. This is because the angle β is different for the two forms of loading. It may be shown that $\beta = \pi/2 - \psi/2$ for lateral loading, while $\beta = \psi/2$ for torsion loading.

The summation needed for the terms in F may now be evaluated (remembering the contribution of pile 1, where $\alpha = 1$), giving:

$\sum \alpha_v = 2.249$, $\sum \alpha_v \cos\psi = 1.075$, $\sum \alpha_{uf}$ (lateral) $= 2.759$,
$\sum \alpha_{uf} \cos\psi$ (torsion) $= 1.364$, $\sum \alpha_{\theta H} = 1.205$, $\sum \alpha_{\theta M} = 1.037$,
$\sum \cos^2\psi = 4$.

The parameter χ may be calculated as $\chi = 2.231$, giving the proportion of moment that is transmitted to the pile heads as $1/\chi = 0.45$.

Finally, the terms in F may be calculated, giving

$F_{11} = 0.810$ mm/kN, $F_{22} = 1.39$ mm/kN, $F_{23} = 0.00743$ rad/kN,
$F_{33} = 2.88 \times 10^{-4}$ rad/kNmm, $F_{66} = 3.99 \times 10^{-4}$ rad/kNmm.