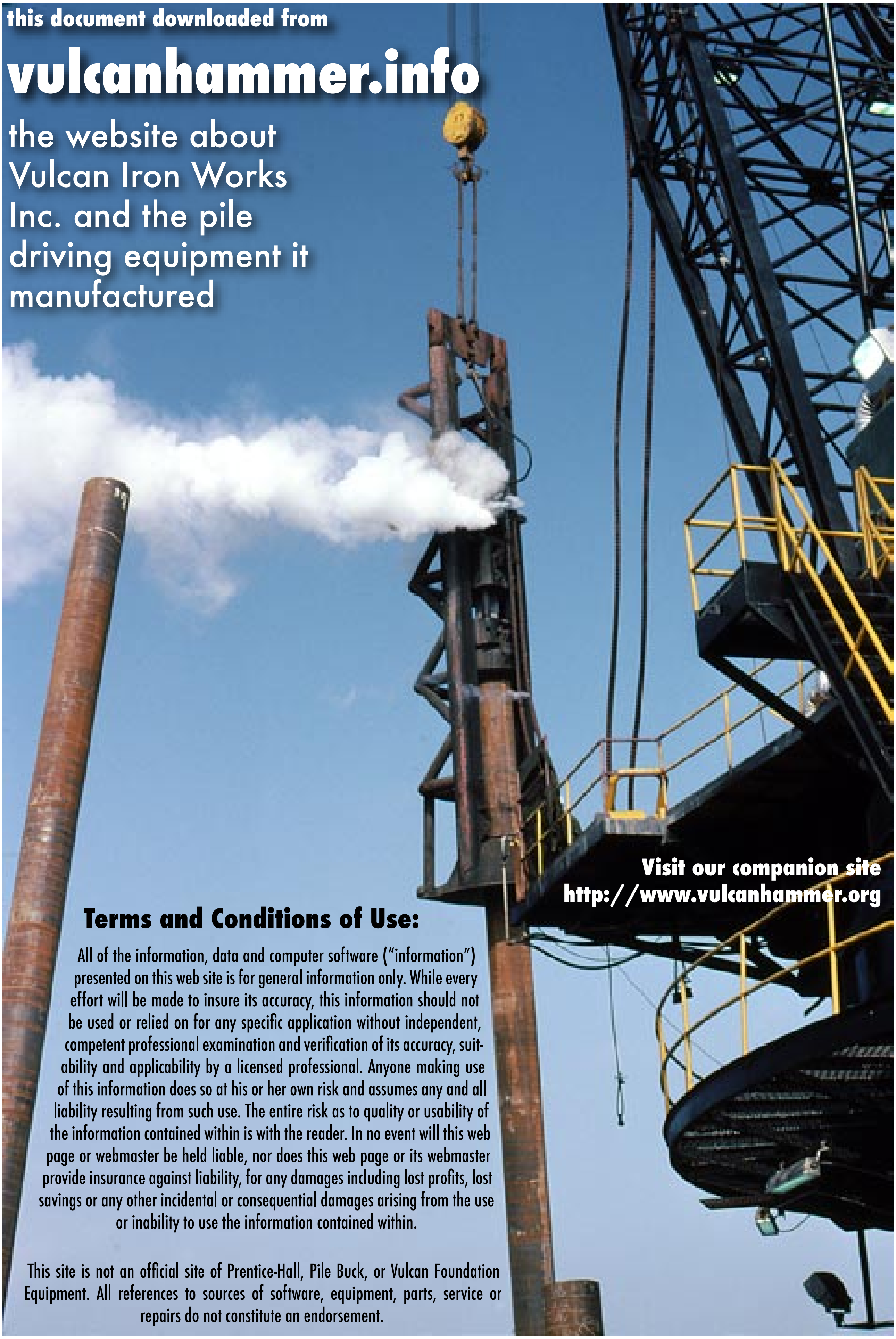

Sensitivity and accuracy of the pile wave equation

by GEORGE E. RAMEY* and ALAN P. HUDGINS§

THE DYNAMIC WAVE EQUATION provides a means of evaluating pile capacity that is mathematically well-founded[6], and probably provides the most realistic mathematical model available for depicting actual behaviour of the hammer-pile-soil system. Numerical integration of this equation, with the aid of a digital computer, appears to be the most rational analytical means of evaluating pile capacity. This has been accomplished by several investigators[1,3,4,5,6] and excellent correlations between predicted and measured failure loads have been observed.

A computer program solution of the wave equation was utilised in the investigation being reported to adjudge (*i*) the sensitivity of program generated *P–n* curves to the program input soil parameters, and (*ii*) the accuracy of the program in predicting pile capacity.

Wave equation—computer programme

Development of the pile wave equation resulted from a consideration of the internal forces and motion of a segment of a freely suspended prismatic bar that was subject to an impact at one end. For the case of a pile, the equation was modified to consider external resistance to the segment motion offered by the soil. Smith[6] converted the resulting partial differential equation to a finite difference equation and outlined a numerical procedure for its solution. His procedure accomplished an approximate solution of the real system by determining the displacement of each segment of the idealised system over a short time interval (e.g., 1/4 000 second). The algorithm begins at the impact of the hammer and the time is incremented in short intervals until the pile tip stops moving. The inverse of the displacement of the pile tip due to one hammer blow is the number of blows per foot that would be required during driving to develop the assumed pile static capacity. The interested reader is referred to Smith's article for a detailed description of his solution procedure.

Researchers at Texas A & M University[5] developed a computer program which executed a numerical solution of the wave equation as outlined by Smith. Their program increments time (in steps of Δt) until all movement of the pile tip due to the simulated single hammer blow has ceased and prints the resulting permanent set of the pile tip. The program repeats its series of calculations for as many input static capacities as desired, and hence can be used to generate pile *P–n* curves. These curves, in turn, can be used to predict pile capacity for any given blow count value. Fig. 1 provides a generalised pictorial summary of the fundamentals of the computer programme. This programme was slightly modified by the authors to facilitate I/O, and used in this investigation. Since the description of the computer program is well documented[4] its discussion here will be brief and limited to those items specifically considered in this investigation.

Program input data fall into the general categories of:

1. Pile characteristics and pile capacity desired,
2. Hammer characteristics, and
3. Soil properties and pile soil interactions.

Items (*1*) and (*2*) can normally be readily determined from the pile properties and hammer manufacturers' literature. However, the input data required for item (*3*), are not so readily determined. The specific information required for this is:

(*a*) Distribution of static capacity between point bearing and side friction
(*b*) Distribution of side friction along pile length
(*c*) Ultimate strain of the soil at the point and along the sides (Quake)
(*d*) Point and side soil damping coefficients.

This information is input in the program under the following variable names (and definitions).

PERCNT — The percentage of pile capacity that is developed by point bearing,
QPOINT — Quake or ultimate strain of the soil at the pile tip,
QSIDE — Quake or ultimate strain of the soil along the sides,
JPOINT — Damping coefficient of the soil at the pile tip, and
JSIDE — Damping coefficient of the soil along the sides.

Because of the potentially large variation of the values of these parameters, an abbreviated study was performed to adjudge the sensitivity of the program to these parameters.

Sensitivity and evaluation of wave equation pile-soil interaction parameters

Use was made of previous published results[2,3] which indicates linear relationships between QSIDE and QPOINT and between JSIDE and JPOINT, or more specifically,

$$\text{QSIDE} = \text{QPOINT} \qquad \text{JSIDE} = 1/3 * \text{JPOINT} \quad \ldots (1)$$

These relationships were assumed to be valid and thereby reduced the investigation of evaluating the sensitivity of the program generated *P–n* curves to the parameters, PERCNT, QPOINT, and JPOINT.

An abbreviated investigation was performed whereby each parameter was varied over its full realistic range while holding the other two parameters constant. Since the purpose of the study was to adjudge the sensitivity of the program solution to the pile-soil interaction parameters, the particular pile case considered was not deemed crucial. The case of a 15.24m steel UBP 254 × 254 × 63kg/m driven 13.72m into a relatively uniform sand with a Vulcan 06 hammer was utilised for the investigation.

Separate *P–n* curves were generated for each of the three parameter variations and these are shown in Figs. 2, 3 and 4. In each case, large variations in the soil parameters caused much smaller variations in predicted pile capacities, indicating the dampened effect of the soil parameters. This is desirable and means that errors in estimating these parameters will lead to errors of much smaller magnitude in predicted pile capacity. For example, looking at Fig. 2, one can see that a 200% error in estimating PERCNT leads to only a 13% error in the predicted pile capacity for a blow count of 30.

Results from several representative field

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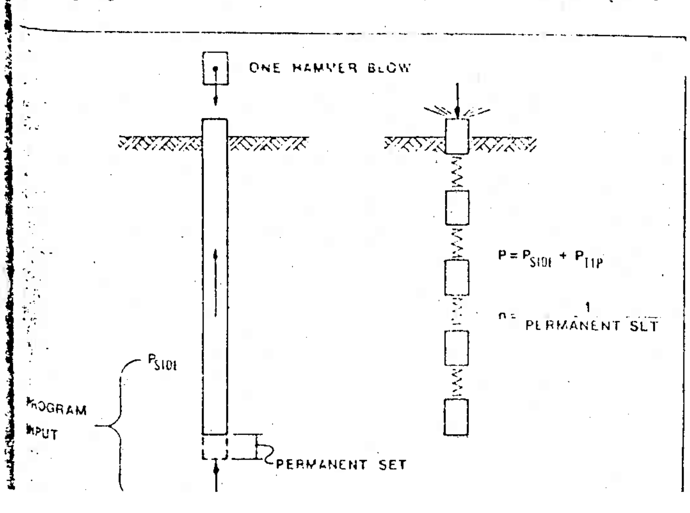

TABLE I. VALUES OF WAVE EQUATION PROGRAM SOIL PARAMETERS FOR TWO COMMON ALABAMA SOIL SETTINGS

Parameter	Sand	Stratified sand and clay with tip in sand
QPOINT	0.1	0.1
QSIDE	0.1	0.1
JPOINT	0.15–0.3	0.2–0.3
JSIDE	0.05–0.1	0.067–0.1
PERCNT	50–75	60–75

In each case, these parameters were adjusted to obtain a good fit of the generated curve to actual pile failure data points. Values of QSIDE and JSIDE were taken as indicated in Eq. (*1*). All of the pile failure data were from Alabama soil settings and the curve fitting procedures described on the preceding page were accomplished for the two soil settings indicated in Table I. The resulting parameter evaluations are summarised in that same table. These were the pile-soil interaction parameter values utilised in subsequent evaluation of pile capacities by the wave equation. Program parameter values utilised to depict pile and hammer characteristics were determined from the field test records and manufacturers' literature.

TABLE II. COMPARISON OF WAVE EQUATION AND DYNAMIC EQUATION PREDICTED CAPACITIES*

Pile type	Soil type§	Measured capacity	Wave equation	EN	MEN†	HILEY	GATES
STEEL H	S	48	[illegible]	125	99	89	61
	S	22	[illegible]	83	57	45	49
	S	34	[illegible]	85	110	96	70
	S	48	[illegible]	94	127	105	76
	S	58	[illegible]	106	156	118	85
	SS	58	[illegible]	180	124	91	69
	SS	53	[illegible]	180	122	87	69
	SS	27	44	143	92	64	64
	S	61	[illegible]	118	209	164	93
	S	39	[illegible]	82	106	93	68
	SS	118	[illegible]	179	108	68	70
	SS	107	112	249	145	76	[illegible]
	SS	140	140	315	177	78	57
	SS	87	[illegible]	157	95	63	67
	SS	91	73	151	103	72	67
	SS	136	122	266	162	89	82
	SS	209	91	202	114	63	73
	SS	98	57	127	80	59	61
	S	65	85	122	211	141	97
	S	67	65	110	162	117	88
	S	69	103	148	328	144	121
PRECAST CONCRETE	S	65	78	261	110	62	79
	S	67	54	192	81	52	70
	S	49	48	172	93	67	67
	S	50	50	167	100	77	68
	SS	81	83	354	197	118	91
	SS	82	80	303	154	94	80

*Dynamic equation capacities shown are ultimate capacities in tons (metric).
§S - Sand; SS - Stratified sand and clay with tip in sand
†Modified Engineer News.

Comparison of wave equation and actual pile failure loads

After evaluating the soil, pile, and hammer parameters as indicated above, the program was employed to generate families of *P–n* curves such as the one shown in Fig. 5. Each family was composed of piles of different length for various combinations of (*i*) pile type, (*ii*) soil type, and (*iii*) hammer type. These *P–n* curves were generated as design and construction aids as part of a research project and their discussion is not the object of this article.

Twenty-one steel H and six 30.5cm precast concrete piles ...

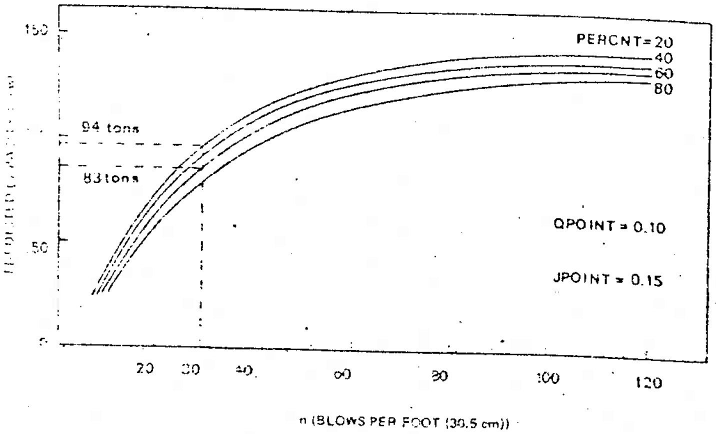

Fig. 2. Sensitivity of P–n curves to program input parameter PERCNT

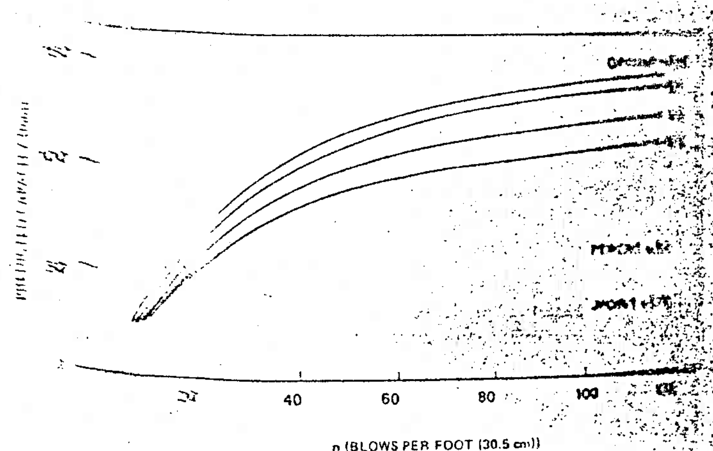

Fig. 3. Sensitivity of P–n curves to program input parameter QPOINT

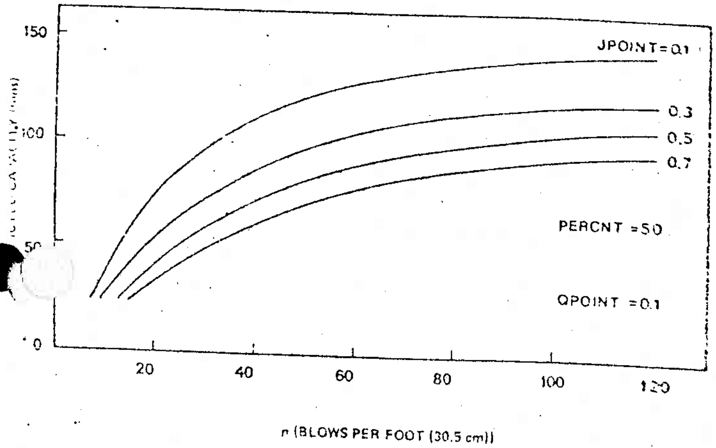

Fig. 4. Sensitivity of P–n curves to program input ...

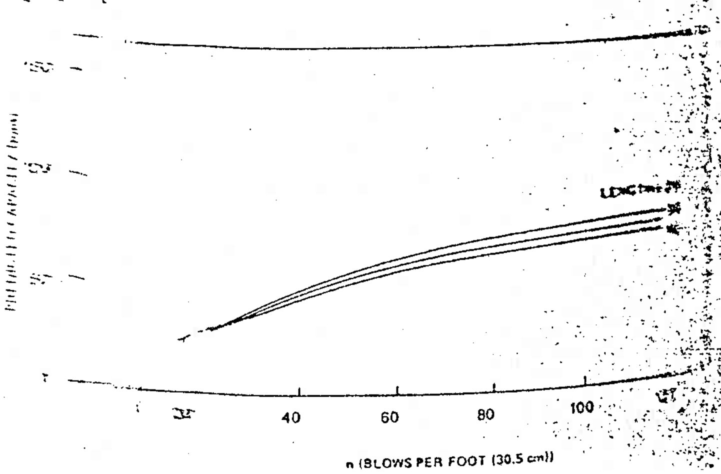

TABLE III. COMPARISON OF WAVE EQUATION AND DYNAMIC EQUATION $P_{predicted}/P_{failure}$ RATIOS

Pile type	Soil type	$P_{predicted}/P_{failure}$ ratio: Wave Eqn.	EN	MEN	HILEY	GATES	DANISH
	S	0.79	2.60	2.06	1.85	1.26	2.08
	S	1.88	3.79	2.62	2.08	2.25	2.98
	S	1.10	2.47	3.18	2.79	2.03	[illegible].21
	S	1.02	1.96	2.64	2.19	1.58	2.5[illegible]
	S	1.06	1.83	2.69	2.03	1.47	2.36
	SS	0.91	3.09	2.14	1.56	1.19	1.94
	SS	0.97	3.30	2.25	1.60	1.27	2.02
	SS	1.60	5.27	3.37	2.37	2.33	3.30
	S	1.28	1.94	3.43	2.70	1.52	2.75
	S	1.00	2.09	2.72	2.39	1.74	2.77
	SS	0.69	1.52	0.92	0.58	0.59	1.01
	SS	1.05	2.33	1.36	0.71	0.74	1.31
	SS	1.00	2.25	1.27	0.56	0.62	1.10
	SS	0.81	1.80	1.09	0.72	0.77	1.25
	SS	0.81	1.77	1.13	0.79	0.74	1.26
	SS	0.89	1.95	1.19	0.65	0.60	1.12
	SS	0.43	0.97	0.55	0.30	0.35	0.58
	SS	0.58	1.30	0.61	0.60	0.62	0.07
	S	1.30	1.86	3.24	2.15	1.49	2.43
	S	0.97	1.64	2.42	1.74	1.31	2.07
	S	1.49	2.14	4.76	2.09	1.75	2.46
	S	1.04	4.00	1.68	0.94	1.21	2.01
	S	0.81	2.86	1.20	0.77	1.04	1.66
	S	0.93	3.52	1.89	1.37	1.37	2.44
	S	1.00	3.34	2.00	1.54	1.36	2.40
	SS	1.03	4.38	2.44	1.46	1.12	2.24
	SS	0.98	3.71	1.89	1.16	1.06	1.90

TABLE IV. COMPARISON OF WAVE EQUATION AND DYNAMIC EQUATION STATISTICAL PARAMETERS

Pile type	Prediction equation	Number of tests	Average $P_{pred.}/P_{fail.}$	Standard deviation	Correlation coefficient
	WAVE	21	1.03	0.34	.725
	EN	21	2.28	0.95	.683
Steel H	MEN	21	2.18	1.09	.083
	HILEY	21	1.62	0.63	.259
	GATES	21	1.25	0.58	.165
	DANISH	21	1.97	0.81	.330
	WAVE	6	0.97	0.08	.929
	EN	6	3.64	0.53	.906
Concrete	MEN	6	1.85	0.41	.782
	HILEY	6	1.21	0.31	.600
	GATES	6	1.19	0.15	.904
	DANISH	6	2.11	0.31	.797

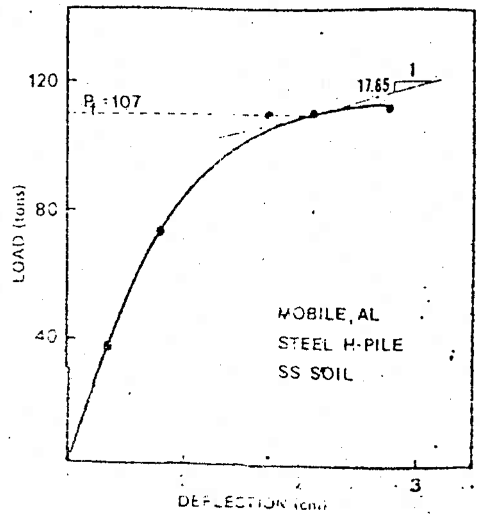

Fig. 6. Typical load test P-Δ curve

[illegible]ble pile failure test results fit the [illegible]tions of one of the generated P–n [illegible]s. For each of these cases, a wave [illegible]tion predicted capacity was deter[illegible] from the P–n curves and compared [illegible] the actual failure load.

[Be]fore making this comparison however, [it w]as necessary to define pile failure. [It wa]s felt that a condition of plunging [or ne]ar plunging was the preferred criter[ion]. Loads corresponding to a slope of [illegible]tons/cm on load-settlement curves [were f]elt to be at or near plunging (others [have] used a slope of 35.70tons/cm which [is do]uble the value used in this study) [and] were taken as the failure loads in [this st]udy. These loads were taken direct[ly fr]om the pile test load-settlement [curve]s as indicated on the sample P–Δ [curve] of Fig. 6.

[Th]e wave equation predicted and actual [failur]e loads are summarised in Table II. [Comp]arison of these loads indicates t[illegible] tests came from one geological setting. It is felt that the greater scatter exhibited by the steel H piles is due to variances in soil geological setting rather than inaccuracy of the wave equation.

Some popular dynamic impact equations were also employed for comparative purposes, and the results of their applications are also shown in Table II. Table III presents these same data as dimensionless ratios of $P_{predicted}/P_{failure}$. A comparison of these results indicates the superior accuracy of the wave equation for the failure test piles considered in this investigation.

Statistical analyses were performed on the Predicted vs. Actual Failure Load data sets and the results are summarised in Table IV. It can be noted from this table that the wave equation gave average $P_{predicted}/P_{failure}$ ratios much closer to unity, higher correlation coefficients, and writing pile-driving specifications, in interpolating between and extrapolating beyond load test results, etc. The authors believe that the most rational and accurate analytical approach in predicting pile capacity is through use of the wave equation.

For the failure test results of this investigation, wave equation analyses consistently gave better pile capacity prediction results than did the dynamic impact equations considered. Additionally, the accuracy of wave equation predicted capacity was very good in all but a few cases. Table IV indicates higher correlation coefficients, $P_{predicted}/P_{failure}$ ratios closer to unity, and lower standard deviations for the wave equation relative to the impact equations considered.

The abbreviated sensitivity study conducted in this work indicated a greatly dampened effect of uncertainty of pile-soil interaction parameter values on wave equation pile capacities. That is, errors in estimating these parameters lead to much smaller errors in the predicted pile capacities.

In using the wave equation program it is recommended that values of the pile-soil interaction parameters as given in Table I be used. These values have been evaluated to give a good fit of predicted-to-actual failure loads for the two common soil settings shown.

Acknowledgements

This study was conducted under the sponsorship of the State of Alabama Highway Department in co-operation with the US Department of Transportation, Federal Highway Administration.

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