Paper

Lateral motion of piles during driving

Professor T. J. Poskitt, PhD, DSc, CEng, FIStructE, FICE
Queen Mary College, University of London

Synopsis
Using an energy method, the lateral response of an initially curved raked pile to an axial blow from a piling hammer is derived. By treating the blow as an impulsive force, a considerable simplification results which enables a closed form solution to be obtained. The solution is in general terms and enables all boundary conditions of practical interest to be studied. The cases of cantilever and a propped cantilever pile are presented as being of most practical interest. Propping corresponds to supporting the hammer in leaders and results in lower bending stresses. The most critical case is the raked cantilever pile. This can experience significant bending stresses.

Introduction

When piles are over-driven or heavy hammers are being used, there is a risk of lateral oscillations being induced in the pile. These can be extremely vigorous and lead to a substantial loss of energy. This will significantly affect the set of the pile, so that, if the pile is being driven to a specific set/blow, overestimates of the driving resistance will be obtained[1]. The phenomenon is sometimes referred to as 'pile whip' and is particularly noticeable with raked piles.

Structural damage is also possible, as was revealed when a small accommodation jacket was installed by Conoco in the southern North Sea in the summer of 1987. Vibration from pile driving caused cracking in the attachments of the sacrificial anodes. To assess the scale of the problem the Department of Energy commissioned Lloyd's Register to set up a database in order that the significance of pile driving vibrations on structural damage and fatigue life could be assessed.

In addition to causing structural damage, lateral oscillations can result in a loss of bearing capacity in the pile. The lateral movement of the pile pushes the soil horizontally, and this results in a loss of soil pressure on the pile wall. In other words, the lateral movement produces an oversized cavity down which the pile shaft travels. For heavily over-consolidated clays or weakly cemented sands and silts, the horizontal soil pressure may not show any significant increase with time.

Lateral oscillations can also lead to considerable noise which, in environmentally sensitive locations, would be a nuisance. The factors which cause vibration are therefore of practical importance if speedy and efficient piling operations are to be achieved.

The problem of a vertical cantilever pile has been considered by Cugley et al[2]. These investigators developed a numerical solution to the problem. In the present treatment a more general closed form solution is given which covers this as a special case. In addition, the case where the piling hammer is restrained in leaders is also given.

General equations

An energy method is used to establish the equation of motion of the pile.

Referring to Fig 1 the kinetic, strain energy and potential energy due to *lateral* motion of the pile and hammer are:

$$T = \int_0^L \frac{m\,\dot{y}^2\,dx}{2} + \frac{W\,\dot{y}_L^2}{2} \qquad \dots (1)$$

$$U = \int_0^L \frac{EI}{2}\,(y'' - y_o'')^2\,dx \qquad \dots (2)$$

$$V = -\frac{Wg\cos\beta + P}{2}\int_0^L (y'^2 - y_o'^2)\,dx - \int_0^L mg\sin\beta\,(y - y_o)\,dx - Wg\sin\beta\,(y_L - y_{L_o}) \qquad \dots (3)$$

where
m is the mass/unit length of pile
W is the mass of hammer
EI is the flexural rigity of pile
$P\,(t)$ is the impulse applied by the ram
y_o is the shape of the pile as manufactured
Dots indicate time derivatives and primes x derivatives.

Using the normal Rayleigh-Ritz approach, a single-mode solution

$$y = f(t)\,F(x) \qquad \dots (4)$$

is sought. $F(x)$ is a function which satisfies the geometric boundary conditions. The following constants are defined

$$A = \int_o^L F''^2\,(x)\,dx \qquad \dots (5)$$

$$B = \int_o^L F'^2\,(x)\,dx \qquad \dots (6)$$

$$C = \int_o^L F^2\,(x)\,dx \qquad \dots (7)$$

$$D = \int_o^L F\,(x)\,dx \qquad \dots (8)$$

Substituting eqn. (4) into eqns. (1) to (3) gives:

$$T = \frac{\dot{f}^2}{2}\,(m\,C + W\,F_L^2)$$

or

$$T = \frac{\dot{f}^2}{2}\,M \qquad \dots (1(a))$$

M is the equivalent lumped mass of the system,

$$U = \frac{EI}{2}\,(f - f_o)^2 \int_o^L F''^2\,dx$$

$$= \frac{EI}{2}\,(f - f_o)^2\,A \qquad \dots (2(a))$$

and

$$V = -\frac{(Wg\cos\beta + P)}{2}\,(f^2 - f_o^2) \int_o^L F'^2\,dx - mg\sin\beta\,(f - f_o) \int_o^L F\,dx - Wg\sin\beta(f - f_o)F_L$$

$$= -\frac{(Wg\cos\beta + P)}{2}\,(f^2 - f_o^2)\,B - mg\sin\beta(f - f_o)\,D - Wg\sin\beta(f - f_o)F_L \qquad \dots (3(a))$$

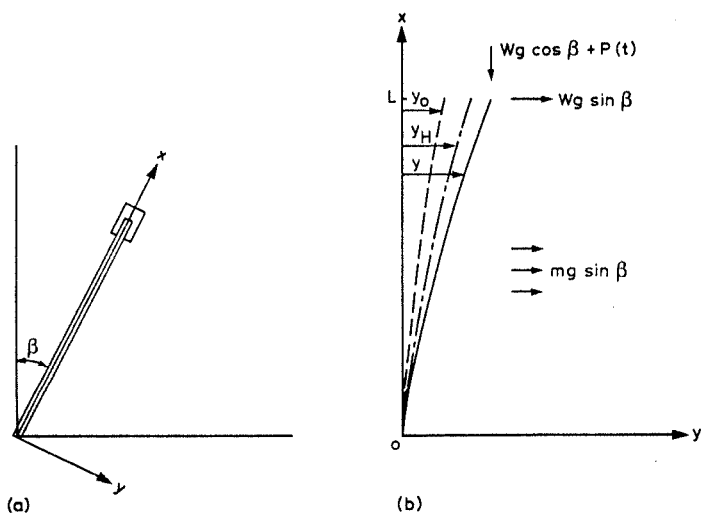$$Wg\cos\beta + P(t)$$
$$Wg\sin\beta$$
$$mg\sin\beta$$

(a) (b)

Fig 1. Model pile

Applying Lagrange's equations, the equation of motion is

$$M\ddot{f} + \{EIA - (Wg\cos\beta + P)\,B\}\,f = EIAf_o + (mD + WF_L)g\sin\beta \quad ....(9)$$

Mounting the pile and hammer
When this occurs, the pile deflects to the shape y_H (Fig 1(b)). From eqn. (9) the value of f then becomes

$$f_H = [EIAf_o + (mD + WF_L)g\sin\beta] \,/\, (EIA - Wg\cos\beta\,B) \quad ....(10)$$

In this expression $EIAf_o$ is due to lack of straightness in the manufacture of the pile and $(mD + WF_L)g\sin\beta$ is due to lateral loads caused by the rake. Taking the case considered by Cugley *et al*

$EI = 250E5\ N - m^2$; $L = 20m$; $W = 2000kg$; $m = 158kg/m$; $f_o = L/2000$ and taking $F = 1 - \cos\pi x/2L$, then

$EIAf_o = 95$.

If the pile were raked at $15°$,

$(mD + WF_L)g\sin\beta = 7990$.

From this it can be concluded that rake or accidental lack of verticality when pitching the pile is likely to be more significant than lack of straightness in the manufacture of the pile.

The blow impulse
Eqn. (9) must be solved for two stages; the first, which is extremely short covers the duration of the blow, while the second, which is much longer, covers the free oscillation of the pile after the blow has ceased. The first stage must be done numerically since $P(t)$ is an irregular function of time.

Considerable effort can be saved if P is treated as an impulsive force, i.e. a very large force acting for a very short time. Under these conditions $P \gg Wg\cos\beta$ and EIA and to a good approximation eqn. (9) becomes

$$M\ddot{f} - PBf = EIAf_o + (mD + WF_L)\,g\sin\beta \quad ....(11)$$

During the very short duration Δt of the impulse f departs very little from f_H, therefore

$$M\ddot{f}_H - PBf_H = EIAf_o + (mD + WF_L)\,g\sin\beta$$

which can be integrated to give

$$M\dot{f}_H - Bf_H\int_0^{\Delta t} P\,dt = \{EIAf_o + (mD + WF_L)\,g\sin\beta\}\,\Delta t$$

or

$$M\dot{f}_H - Bf_H\,Ip = 0 \quad ....(12)$$

where the right-hand side tends to zero as Δt diminishes, and

$$Ip = \int_0^{\Delta t} P\,dt \quad ....(13)$$

In experimental work, Ip can be measured directly. For analytical work, it is best if it is obtained from the wave equation analysis. This is a widely available program from which the force-time plot at the pile head can readily be obtained (see Fig 2). A small modification to the software will then give the area under the curve.

Fig 2 is a typical plot for a 310 UC 158 pile which is being driven by a Delmag D62 hammer. The pile stick-up is 20m and the hammer is experiencing hard driving. Under these conditions a long stress pulse, about 24ms, is obtained giving an impulse of 50000N-s.
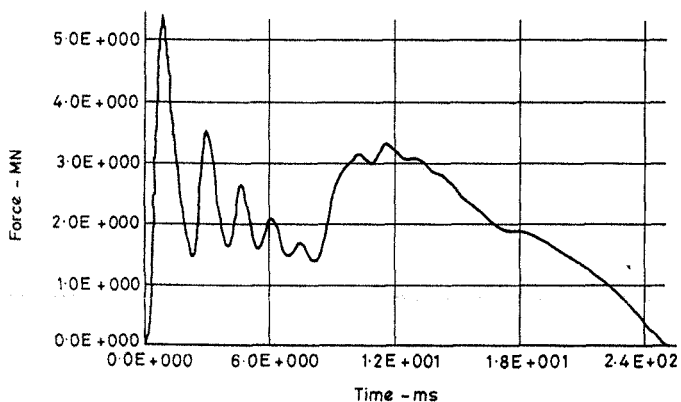

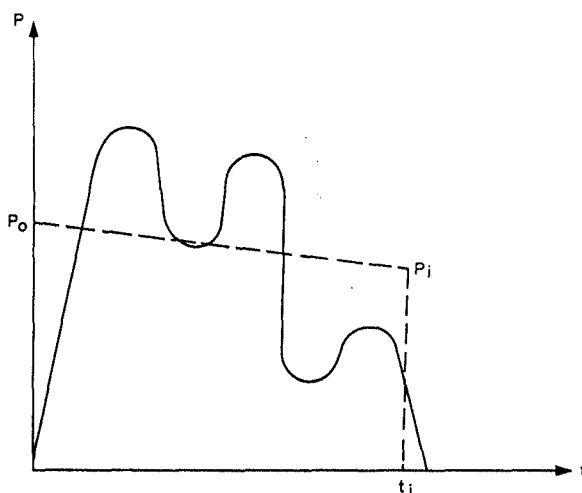*Fig 2. Stress pulse for 310 UC 158 pile driven by D62 hammer*

Fig 3. Cugley et al: *approximation to a stress pile*

Motion following the impulse
This is given by eqn. (9) with P omitted

$$M\ddot{f} + [EIA - Wg\cos\beta\,B]\,f = EIAf_o + (mD + WF_L)g\sin\beta \quad ....(14)$$

with the initial conditions $f = f_H$, and $\dot{f} = \dot{f}_H$ at $t = 0$.

For practical purposes it is the maximum value of f which is important in determining the greatest stress; this is given by

$$f_M/f_H - 1 = B\,Ip/[\,(mC + WF_L^2)\,(EIA - Wg\cos\beta\,B)]^{\frac{1}{2}} \quad ....(15)$$

Cantilever piles

Eqns. (10) and (15) give simple formula from which the magnification of the initial deflection can be obtained. For the case of a vertical cantilever pile, they can be compared with results given by Cugley *et al*. For a cantilever pile Cugley *et al* use

$$F = 1 - \cos(\pi x/2\,L) \quad ....(16)$$

Substituting eqn. (16) into eqns. (5), (6) and (7) gives

$$A = \pi^4/32\,L^3 \;;\; B = \pi^2/8\,L \;;\; C = (3/2 - 4/\pi)\,L$$

Substituting A and B into eqn. (10) gives

$$f_H = f_o/(1 - Wg/Pe) \quad ....(17)$$

Comparing eqn. (17) with eqn. (15) of Cugley it can be seen that the two are identical. $Pe = \pi^2EI/(4\,L^2)$

Cugley *et al* approximate the pulse shape by a trapezoid (Fig 3). For this

$$Ip = 0\cdot5\,(Po + Pi)\,t_1 \quad ....(18)$$

This must be substituted into eqn. (15).

The value of f_M/f_H given by eqn. (15) corresponds to the parameter n of Cugley. For the three pile sizes considered by Cugley, values of f_M/f_H have been evaluated and are entered in Table 1.

The general conclusion which can be drawn from Table 1 is that agreement is good, provided static buckling effects are small. As the axial load approaches the static buckling load, the differences increase. For the normal range of practical parameters, eqn. (15) overestimates by 10-20%. This is acceptable, bearing in mind the various approximations inherent in the theoretical model.

One of the sources of error in the results can be identified for the special case of a cantilever with a mass W at its end. Cugley *et al* use this for defining w given in their eqn. (10).

$$w^2 = 3\,EI/[33\,mL/140 + W]\,L^3 \quad ....(19)$$

By deleting P from eqn. (8) of Cugley, this provides a means of obtaining w

$$\pi^4/4\,EI/[mL\,(1/2 - 3/\pi + 4/\pi^2) + W]\,L^3 \quad ....(20)$$

Doing the same for eqn. (9) of the present paper gives

$$\pi^4/32\,EI/[mL\,(3/2 - 4/\pi) + W]\,L^3 \quad ....(21)$$

Taking $m = 158kg/m$, $L = 20m$ and $W = 2000kg$ and dividing eqns. (20) and (21) by eqn. (19) gives $0\cdot792$ for Cugley *et al* and $1\cdot025$ for the present work.

TABLE 1—Comparison of magnification factors

Pile	W(kg)	L(m)	Cugley ref.	n	f_M/f_H
310 UC 158 H	0	20	Fig 5	6·38	6·34
"	2000	20	"	3·56	3·94
"	4000	20	"	2·92	3·41
310 UC 283 H	2000	20	Fig 6	2·64	2·84
310 UC 97 H	2000	20	"	4·59	5·28
310 UC 283 H	2000	20	Fig 7	4·35	4·62
310 UC 97 H	2000	20	"	3·04	3·50
310 UC 158 H	2000	30	Table 2	3·86	4·74
"	"	28	"	3·81	4·56
"	"	26	"	3·75	4·40
"	"	24	"	3·70	4·24
"	"	22	"	3·63	4·09
"	"	20	"	3·56	3·94
"	"	18	"	3·49	3·79
"	"	16	"	3·41	3·63
"	"	14	"	3·32	3·47
"	"	12	"	3·22	3·30
"	"	10	"	3·11	3·12
"	"	8	"	2·97	2·91

The conclusion to be drawn from this is that eqn. (8) of Cugley *et al* and eqn. (9) of the present paper represent two different approximations to the same problem. The degree of approximation in the two treatments is different, and this partly explains the differences observed in Table 1.

Piles in leaders

For long, slender piles, it is common to support the hammer in leaders. Under these circumstances the pile is constrained against lateral movement at the top but is, however, free to rotate. The shape of the pile is then as shown in Fig 4. A simple function for representing this is

$$F = (x/L)^2 (1 - x/L) \qquad \dots (22)$$

Substititing eqn. (22) into eqns. (5), (6) and (7)

$$A = 4/L^3; \ B = 2/15L; \ C = L/105$$

Substituting A and B into eqn. (9) gives

$$f_H = f_o/[1 - W g/Pe] \qquad \dots (23)$$

where $Pe = 30 EI/L^2$ is an approximation to the buckling load of a strut encastré at its base and pinned at its upper end.

Supporting the hammer in leaders does not significantly affect the shape of the force pulse. Thus the pulses used for cantilever piles can be used when the pile hammer is in leaders.
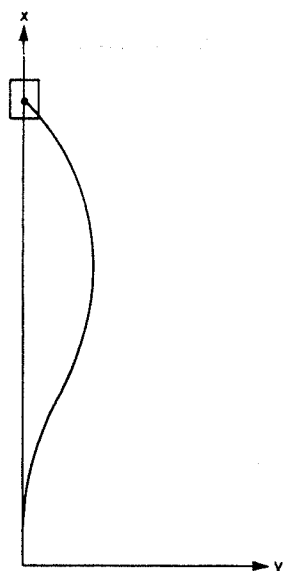

Fig 4. Pile with hammer in leaders

TABLE 2—Influence of leaders

W	Leaders	Cantilever
0	3·46	6·34
2000	3·47	3·94
4000	3·48	3·41

Taking the first case considered in Table 1 of a 310 UC 158 pile with axial loads of 0, 2000, and 4000 kg, the results given in Table 2 were obtained. Also given, for comparison, are the results for the cantilever pile taken from Table 1.

The most significant feature to emerge when the hammer is restrained by leaders is that the magnification factors do not vary much. This is because, in the loading range 0-4000 kg, buckling effects are negligible.

Conclusions

Lateral motions in piles caused by pile driving can be extremely vigorous. They waste energy, produce noise, may cause structural damage and lead to a loss of bearing capacity of the pile.

Because some of the factors affecting lateral motion are not well understood, simple formula have been developed from which the magnified displacement can be calculated.

References

1. Poskitt, T., & Ward, G.: 'The effect of stick-up on pile driveability', *Jnl. Soc. Underwater Tech.*, **10**, No.1, 1984, pp23-31
2. Cugley, R., Kennedy, B., & Melchers, R.: 'Dynamic stability of unrestrained piles during driving', *The Structural Engineer*, **63A**, No.6, June 1985