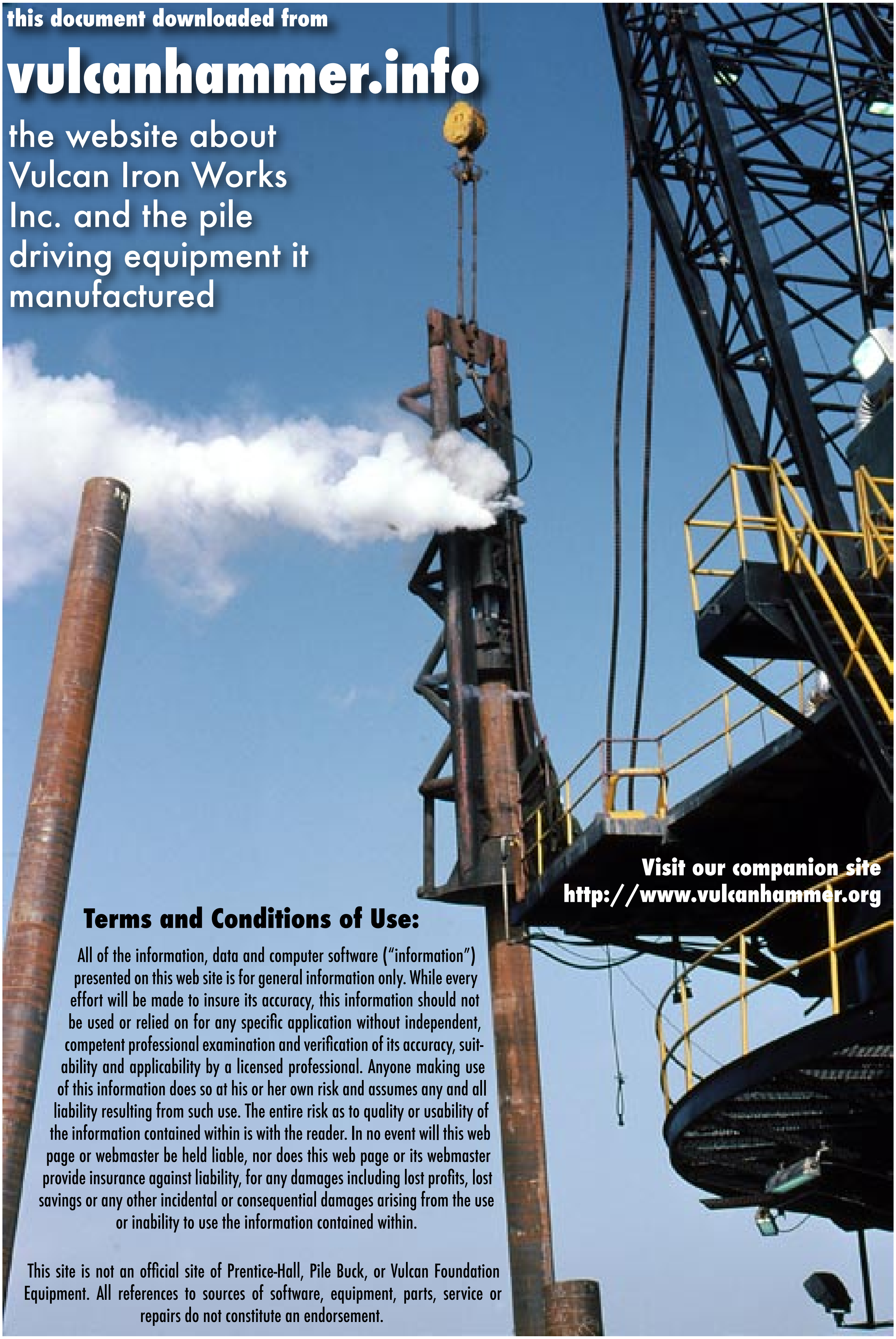

APPENDIX.—NOTATION

The following symbols have been adopted for use in this paper:

A = cross-sectional area of pile;
D = depth below ground surface;
d = outside diameter of pile;
E = modulus of elasticity of pile material;
e_o = initial void ratio;
f = skin friction;
K = coefficient of lateral earth pressure;
L = embedded pile length;
Q = load in pile at any point;
y = movement of pile at any point;
γ = unit weight of soil mass;
Δ = movement of pile at top;
δ = angle of wall friction;
$\bar{\sigma}$ = effective stress;
τ_f = soil shear strength; and
ϕ = angle of internal friction.

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PILE-DRIVING FORMULAS FOR FRICTION PILES IN SAND

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INTRODUCTION

Dynamic pile-driving formulas are widely used in predicting the load-carrying capacity of friction piles and in writing pile-driving specifications. Further, when load tests are available, dynamic pile-driving formulas are used to interpolate between, or extrapolate beyond, the load test results.

The accuracy of a pile-driving formula can be checked by comparing calculated pile capacities with capacities measured in the field. Such comparisons have demonstrated that the formulas do not generally apply to cohesive soils, especially soft cohesive soils,[3] and do not apply to piles acting as groups. They apply most accurately to individual piles driven into cohesionless soils. It has also been demonstrated that few, if any, of the existing dynamic pile-driving formulas are theoretically valid.[4] Most of the formulas were derived either using oversimplified assumptions or using empirical parameters that could be adjusted to bring the predicted capacities approximately into conformance with field measurements.

In addition to the theoretical errors, there are errors in many of the field measurements because of friction in the rams of hydraulic jacks or improper calibration of equipment. In addition, random variations in the measured capacity occur because of the problem of uniquely defining failure in the field. Thus, correlations between predicted pile capacities and capacities measured in the field are likely to involve considerable scatter.

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[3] Terzaghi, K. T., discussion of "Pile-Driving Formulas: Progress Report of the Committee on the Bearing Value of Pile Foundations," Proceedings, ASCE, Vol. 68, No. 2, Feb., 1942, pp. 311-323.

[4] Cummings, A. E., "Dynamic Pile Driving Formulas," Contributions to Soil Mechanics, 1925-40, Boston Society of Civil Engrs., Boston, Mass., pp. 392-413.

In the investigation reported herein, seven different dynamic pile-driving formulas (Table 1) were used to predict the capacities of 93 timber, precast concrete, or steel friction piles driven into sandy soils, and the predicted capacities were compared statistically with the measured values. A linear

TABLE 1.—PILE-DRIVING FORMULAS USED IN THIS INVESTIGATION[a]

Engineering News	$Q_c = \dfrac{e_h E_n}{s + e}$
Gow	$Q_c = \dfrac{e_h E_n}{s + e\,(W_p/W_h)}$
Hiley	$Q_c = \dfrac{e_h E_n}{s + 1/2\,(C_1 + C_2 + C_3)} \; \dfrac{W_h + n^2 W_p}{W_h + W_p}$
Pacific Coast Uniform Building Code	$Q_c = \dfrac{A\ E}{2\ L}\left[-s + \sqrt{s^2 \dfrac{4\ e_h E_n}{A\ E}\ \dfrac{W_h + n^2 W_p}{W_h + W_p}}\right]$
Janbu	$Q_c = E_n/k_u s$ $k_u = C_d\left(1 + \sqrt{1 + \dfrac{\lambda_e}{C_d}}\right)$ $\lambda_e = E_n L/A\ E\ s^2,$ $C_d = 0.75 + 0.15\,(W_p/W_h)$
Danish	$Q_c = \dfrac{e_h E_n}{s + \sqrt{\dfrac{e_h E_n L}{2 A E}}}$
Gates[b]	$Q_c = 5.6\sqrt{e_h E_n}\ \log\ (10/s)$

[a] As presented in this table, none of the formulas contains a factor of safety. A suitable factor of safety should be applied to obtain the design loads.

[b] In all but the Gates formula, any consistent set of units may be used. We have chosen to use inches and tons throughout. In the Gates formula, the constants contain parameters needed to make the formula dimensionally correct. Thus, any change of units from the ton-inch system necessitates changes in the constants. In the metric system of tons (1000 kg) and centimeters, the formula becomes:

$$Q_c = 4.0\sqrt{e_h E_n}\ \log\left(\frac{25}{s}\right)$$

relationship was assumed between the measured and computed pile capacities, and the reduced major-axis type of linear regression analysis was used to determine the slope and intercept of the regression line. Correlation coefficients were used as measures of the scatter about the regression lines. The statistical data were then used to adjust the formulas to improve their accuracy. Conclusions are drawn regarding the accuracy of the various formulas.

PILE-DRIVING FORMULAS USED IN THIS INVESTIGATION

The seven pile-driving formulas used in this investigation are presented in Table 1. In these formulas, $e_h E_n$ is the energy delivered to the cushion blocks. These energies should be multiplied by e_i, the efficiency of impact, to obtain the energy delivered to the pile. However, values of e_i were unk[illegible]. Eliminating e_i from the formulas has doubtless introduced a certain amount of scatter into the calculated capacities, but unknown types and condition of the cushion blocks preclude further refinement. In the special case of Janbu's formula, e_h and e_i were both set equal to 0.7 when the formula was originally developed[11] and were incorporated into the driving coefficient.

The Engineering News formula was developed by Wellington,[6] who deduced its general form by setting the applied energy equal to the energy obtained by graphically integrating the area under what he considered to be typical load-settlement curves for timber piles driven with drop hammers. He subsequently modified the formula for use with steam hammers. He stated that his formula "was first deduced as the correct form for a theoretically perfect equation of the bearing power of piles, barring some trifling and negligible elements to be noted; and I claim in regard to that general form that it includes in proper relation to each other every constant which ought to enter into such a theoretically perfect practical formula, and that it cannot be modified by making it more complex. . . ." It appears that the Engineering News formula achieved wide acceptance but has been less used in recent years as new formulas were introduced and the inability of the Engineering News formula to predict pile capacities with reasonable accuracy became better documented.[7] The formula was restricted to use with piles for which the average penetration per blow for the last few blows was not less than 0.25 in. and preferably not less than 0.5 in.

In the Gow formula, the denominator of the Engineering News formula was adjusted, based on intuition and experience, to account for the extra energy-absorbing characteristics of precast concrete piles.[8]

In an attempt to eliminate some of the errors associated with the theoretical evaluation of energy absorption by a pile-soil system during driving, Hiley[9] developed a formula in which the recoverable deformations of the pile cap and driving head, pile, and soil, were measured during driving and inserted into the formula as three constants, C_1, C_2, and C_3 respectively. The Hiley formula has been used extensively in the British Commonwealth and in Europe.

The Pacific Coast Uniform Building Code formula,[10] hereafter referred to as the PC formula, is typical of a number of rather cumbersome formulas in which attempts were made to include a variety of sources of energy loss. Cummings[4] objected to most of these formulas because of the inclusion of both the coefficient of restitution and a separate term for the energy loss caused by elastic compression of the pile, which is redundant.

Janbu[11] factored out of the conservation-of-energy equation a series of

[5] Johannsen, N., "Beregning av frittstaende pelers bruddlast og tillatte belastning," Teknisk Ukeblad, Oslo, Norway, No. 26, June 28, 1951, pp. 505-515.

[6] Wellington, A. M., discussion of "The Iron Wharf at Fort Monroe, Va.," by J. B. Duncklee, Transactions, ASCE, Vol. 27, Paper No. 543, Aug., 1892, pp. 129-137.

[7] Agerschou, H. A., "Analysis of the Engineering News Pile Formula," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 88, No. SM5, Proc. Paper 3298, Oct., 1962, pp. 1-11.

[8] Isaacs, D. V., "Reinforced Concrete Pile Formulae," Transactions, Inst. of Engrs., Sydney, Australia, Vol. 12, 1931, pp. 305-323.

[9] Hiley, A., "The Efficiency of the Hammer Blow, and Its Effects with Reference to Piling," Engineering, London, June 2, 1922, p. 673.

[10] "Uniform Building Code," Pacific Coast Building Officials Conf., Los Angeles, Calif., Vol. 1, 1955, pp. 207-208.

[11] Janbu, N., "Une Analyse Energetique du Battage des Pieux a l'aide de Paramètres sans Dimension," Publication No. 3, Norwegian Geotechnical Inst., Oslo, Norway, 1953.

variables that could not usually be evaluated and associated them together as his "driving coefficient," C_d. The driving coefficient included terms representing the efficiency of the pile hammer, the difference between the dynamic and static pile capacities, and the rate of transferral of pile load into the soil with respect to depth. It also included the length and cross-sectional area of the pile, Young's modulus for the pile, and both the pile capacity and the set. Janbu correlated his driving coefficient with the ratio of the weight of the pile to the weight of the falling parts of the hammer, W_p/W_h (Table 1).

The Danish formula[12] was developed using dimensional analysis and by simplifying some of the more complicated formulas. A total of 78 load tests was used in correlating the predicted capacities with capacities measured in the field.

The Gates formula[13] was developed by greatly simplifying the form of existing formulas and then applying a statistical adjustment which was based on approximately 100 pile load tests. The data on which the study was based were not presented and there was no indication of the amount of scatter. Apparently all types of soil were included in the study.

These formulas were selected to be representative of various types of formulas and to include most of the formulas in common use at the present time.

FIELD DATA

The field test results used in this study have been reported by Flaate.[14] He collected the results of 116 load tests on timber, precast concrete, and steel piles driven into sandy soils. In this analysis, only the timber piles with measured capacities less than 100 tons, and steel and concrete piles with measured capacities less than 250 tons were used. It seems unlikely that piles would be designed for higher loads without requiring load tests. Flaate[14] reported the following data: (1) The type of pile, (2) the approximate subsoil conditions, (3) the length, cross-sectional area, and weight of the pile, (4) the type of hammer, (5) either the weight and height of fall of the hammer or its reported energy, (6) the average penetration of the pile under the final few blows, (7) the reported capacity, and (8) the source of the information.

Attempts to apply dynamic pile-driving formulas to the piles used in these tests involve a number of uncertainties, including the following:

1. The efficiencies of the various pile hammers were not reported. The values tabulated by Chellis[15] were used in this study. The actual field values of e_h depend greatly on the condition of the hammer at the time of driving and may differ significantly from the values used in this analysis.

2. The cushion blocks used on top of the piles were not usually specified.

[12] Sorensen, T., and Hansen, B., "Pile Driving Formula—An Investigation Based on Dimensional Considerations and a Statistical Analysis," Proceedings, 4th Internatl. Conf. on Soil Mechanics and Foundations, held in London in 1956, Vol. 2, 1957, pp. 61-65.

[13] Gates, M., "Empirical Formula for Predicting Pile Bearing Capacity," Civil Engineering, Vol. 27, No. 3, Mar., 1957, pp. 65-66.

[14] Flaate, K. S., "An Investigation of the Validity of Three Pile Driving Formulae in Cohesionless Material," Publication No. 56, Norwegian Geotechnical Inst., Oslo, Norway, 1964, pp. 11-22.

[15] Chellis, R. D., Pile Foundations, 2nd ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1961, pp. 28-33.

They doubtless varied widely in type and condition. These blocks exert great influence on the shape of the load pulse applied to the pile[16] and thus influence the energy actually delivered to the pile.

3. The coefficients of restitution were not known. The PC Code was followed by using n^2 = 0.25 for steel piles and 0.10 for concrete and timber piles.

4. The dynamic compression and recovery of the piles during driving were not generally reported. Thus, values for the constants C_1, C_2, and C_3 in Hiley's formula were taken from the tabulations in Chellis.[17]

5. The elastic modulus of the material in each pile was not reported. Average values for the static elastic modulus[14] were used in this study. The dynamic elastic modulus may exceed the static modulus, especially for timber piles. In addition, the moduli of the timber piles vary with the type of wood, storage conditions, and driving conditions.

6. Only about ten of the piles were driven entirely through cohesionless soils. At many of the sites the piles were driven through soft cohesive soils into underlying sands. In other cases the sand was interstratified with clay, silt, and sometimes organic soil, or was described as silty sand or clayey sand.

7. The capacities of most of the piles were reported without presenting the actual load-settlement diagrams. It is believed that a scatter of perhaps 15% has resulted from the use of different failure criteria.

8. Based on recent field studies[18] it is believed that many of the measured pile capacities are in error by 10% or more because of friction in hydraulic loading jacks, and improper calibration of equipment.

Attempts to account for the various sources of error by adjusting the field data were not considered desirable because: (1) The adjustment procedures would be too complex for normal field use; (2) data were not available for making most of the corrections; and (3) arbitrary choices involved in making such adjustments would introduce bias.

STATISTICAL METHODS

As expected, the measured and computed pile capacities did not correlate perfectly. It was convenient, therefore, to apply certain simple forms of statistics to the interpretation of the data. The equations used in the statistical study are presented here for convenience of reference.

The mean $(\overline{X})$ and estimated standard deviation (S_X) of N observations (X_i) of the variable X are given by

$$\overline{X} = \frac{1}{N} \sum X_i \quad \text{. . . . .} \quad (1)$$

[16] Housel, W. S., "Michigan Study of Pile Driving Hammers," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 91, No. SM5, Proc. Paper 4483, Sept., 1965, pp. 37-64.

[17] Chellis, R. D., op. cit., pp. 505-506.

[18] Davisson, M. T., "Summary of Knowledge Gained from Tests on Instrumented Piles," presented at ASCE Metropolitan Section, Seminar on Pile Foundations, New York, N. Y., 1966.

$$\text{and} \quad S_X = \sqrt{\frac{\sum (X_i^2) - \frac{1}{N} (\Sigma X_i)^2}{N - 1}} \qquad (2)$$

If there are two variables, X and Y, and Y is assumed to be linearly related to X according to

$$Y = AX + B \qquad (3)$$

then linear regression analysis can be used to estimate the most probable values of A and B. In comparing the measured and computed pile capacities, it was assumed that significant errors existed in both, and thus it was necessary to minimize the square of the deviations measured perpendicular to the regression line, i.e., the reduced major-axis technique of linear regression analysis applied. The pertinent equations are

$$A = \frac{S_Y}{S_X} \qquad (4)$$

$$\text{and} \quad B = \overline{Y} - A\overline{X} \qquad (5)$$

In addition to the parameters A and B, it would be desirable to have one or more parameters to describe the scatter of the observations relative to the regression line. Of the various available parameters, it appears that the correlation coefficient, r, is the most satisfactory because it is single-valued and varies between fixed limits of plus one for a perfect positive correlation through zero for no correlation to minus one for a perfect negative correlation (standard errors of slope and intercept, and standard deviations could also be used). The correlation coefficient is defined as

$$r = \frac{\sqrt{\sum X_i Y_i - \frac{1}{N} (\Sigma X_i)(\Sigma Y_i)}}{\sqrt{\sum (X_i^2) - \frac{1}{N} (\Sigma X_i)^2} \quad \sqrt{\sum (Y_i^2) - \frac{1}{N} (\Sigma Y_i)^2}} \qquad (6)$$

MODIFICATION OF JANBU'S FORMULA

There is no apparent reason why Janbu's driving coefficient should correlate with the ratio of the weight of the pile to the weight of the hammer. To check this correlation, the load tests reported by Flaate[14] were used to back-calculate the driving coefficients. These driving coefficients are plotted in Fig. 1. It is apparent that there is no significant relationship between C_d and W_p/W_h for the tests used in this study. The average values of C_d for timber, precast concrete, and steel piles, were 0.92, 1.06, and 1.07, respectively, with an average for all piles except S49, C15, and T20, of 1.02. Therefore, Janbu's formula was simplified by substituting $C_d = 1$.

Separate statistical studies of the pile data were performed using Janbu's original formula, using the modification with a driving coefficient of one, and using another modification in which the efficiency terms were not incorporated into the driving coefficient. The studies showed that the modification with $C_d = 1$ was both the most accurate and the simplest. Thus, in all subsequent sections, any statement regarding Janbu's formula will refer to the modification in which the driving coefficient is one.

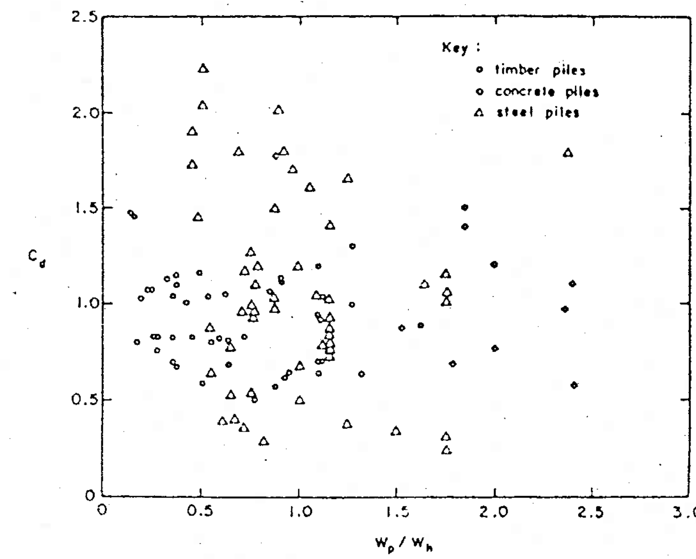

FIG. 1.—INFLUENCE OF THE RATIO W_p/W_h ON JANBU'S DRIVING COEFFICIENT

COMPARISON OF MEASURED AND COMPUTED PILE CAPACITIES

Presentation of Data.—The capacities of the piles studied by Flaate[14] were calculated using the seven formulas presented in Table 1, except that the modified form of Janbu's formula was used. The measured pile capacities and the values calculated using the seven formulas are reported in Table 2 for timber, precast concrete, and steel piles. These data were used to prepare diagrams in which the measured pile capacity, Q_m, was plotted as the ordinate and the calculated capacity, Q_c, as the abscissa. The reduced major-axis type of linear regression analysis was then used to find the best linear relationship between Q_m and Q_c according to

$$Q_m = A\, Q_c + B \qquad (7)$$

For a perfect pile-driving formula and uniquely defined values of Q_m, the measured and computed capacities would be identical for each pile and the statistical parameters would be $A = 1$, $B = 0$, and $r = 1$.

Regression analyses were performed for each type of pile separately using each pile-driving formula. Representative Q_m - Q_c diagrams for timber, precast concrete, and steel piles, are presented in Figs. 2 through 4, respectively. The statistical parameters are tabulated in Table 3. Cumulative frequency curves are presented in Figs. 5 through 7.

Evaluation.—For timber piles, Janbu's formula is clearly superior to the others (Table 3). The regression line has a slope of nearly 45°, a small inter-

TABLE 2.—COMPARISON OF MEASURED AND COMPUTED CAPACITIES OF PILES

Test Number (1)	Q_m, in tons (2)	Q_c, in tons EN (3)	G (4)	Ga (5)	Da (6)	PC (7)	HI (8)	J (9)
(a) Timber								
1	38	30	31	46	50	35	29	34
2	38	33	35	51	60	42	35	42
3	35	26	32	41	37	29	26	25
4	34	28	35	43	41	32	29	28
5	23	21	28	33	25	22	22	17
6	22	25	38	40	33	28	30	23
7	107	90	134	84	117	88	108	75
8	117	90	209	110	167	113	205	123
9	40	77	76	46	40	26	26	33
10	40	62	61	42	35	22	23	28
11	40	68	67	44	37	24	24	31
12	40	152	147	60	55	33	48	44
13	43	86	87	48	40	26	27	34
14	25	40	41	31	25	20	20	20
15	26	43	44	32	27	23	25	22
16	71	155	177	60	64	53	68	58
17	37	85	103	43	43	37	39	39
18	45	113	131	49	51	41	41	47
19	187	200	183	134	258	151	184	192
20	—	—	—	—	—	—	—	—
21	124	126	141	102	170	69	124	127
22	162	138	175	114	191	127	159	144
23	98	38	48	78	91	55	64	67
24	96	38	43	78	96	55	57	71
25	50	49	37	55	68	37	30	45
26	53	63	55	65	72	45	47	53
27	45	12	12	15	8	4	4	6
28	28	39	39	29	23	15	14	19
29	44	82	85	46	42	32	33	37
30	40	92	96	48	46	36	35	41
31	68	74	123	70	104	94	105	72
32	93	90	165	84	138	116	144	102
33	68	63	104	61	84	82	88	56
34	105	84	160	79	125	112	131	90
35	68	55	97	62	84	77	83	58
36	74	85	120	74	85	66	94	63
37	53	64	60	65	69	44	46	50
38	43	101	115	47	53	41	41	48
39	59	73	81	41	39	34	34	35
40	37	91	104	45	46	37	38	44
41	47	101	113	47	54	42	40	49
42	54	161	189	61	80	75	83	73
43	68	161	158	61	76	69	78	70
44	66	188	228	65	100	93	101	90
(b) Concrete								
1	77	53	63	66	103	74	73	74
2	77	39	48	64	101	70	67	75
3	99	41	37	68	125	80	64	93
4	132	42	33	71	129	75	60	97
5	121	42	29	71	148	87	60	110
6	138	42	25	71	142	77	55	106
7	70	195	201	64	135	96	68	118
8	34	29	16	45	50	23	18	33
9	35	26	17	41	42	21	17	27
10	60	37	21	73	133	68	55	100
11	73	37	21	71	128	66	54	96
12	121	75	39	113	183	86	85	134
13	132	90	42	96	188	91	66	141

TABLE 2.—*CONTINUED*

Test Number (1)	Q_m, in tons (2)	Q_c, in tons EN (3)	G (4)	Ga (5)	Da (6)	PC (7)	HI (8)	J (9)
(b) Concrete (continued)								
14	296	455	452	116	235	110	120	210
15	95	692	398	115	259	83	158	221
16	290	316	316	154	370	133	130	312
17	138	245	245	98	227	88	93	207
(c) Steel								
1	56	37	34	50	65	46	46	41
2	41	37	34	50	65	46	46	41
3	45	40	36	53	72	52	51	46
4	55	40	37	54	75	55	53	48
5	67	42	38	56	82	60	59	54
6	45	45	40	59	91	66	66	61
7	79	45	40	59	91	66	66	62
8	45	45	49	60	98	73	76	65
9	67	45	49	60	98	73	76	65
10	89	49	43	65	109	79	80	77
11	79	51	61	69	131	103	107	93
12	110	51	45	69	121	87	90	87
13	155	174	212	116	236	178	213	175
14	124	119	146	97	182	139	150	133
15	220	131	202	110	215	163	210	161
16	118	63	70	78	170	131	110	122
17	107	61	75	75	149	115	107	106
18	203	114	105	106	286	205	165	210
19	244	140	127	129	365	234	241	275
20	337	115	167	108	238	179	209	178
21	233	71	70	101	209	162	145	342
22	297	127	254	180	282	199	307	456
23	305	129	262	136	291	202	327	494
24	296	129	239	136	308	210	317	515
25	310	131	246	141	320	215	327	559
26	140	600	423	111	179	99	129	145
27	150	705	174	121	189	101	135	151
28	140	725	482	122	196	102	135	152
29	90	150	344	99	191	93	113	49
30	80	470	355	100	163	94	115	38
31	150	468	571	93	121	123	119	106
32	120	450	546	94	120	78	119	105
33	99	197	220	61	102	75	55	99
34	24	26	19	44	44	26	19	26
36	290	378	106	92	166	128	100	152
37	285	154	182	98	202	151	118	186
38	135	259	275	79	136	113	115	122
39	350	790	913	122	245	181	126	461
40	315	625	684	111	229	193	137	385
41	300	467	468	118	306	229	124	500
42	360	1395	1395	163	301	171	112	263
43	170	643	740	109	107	64	52	90
44	113	693	882	115	102	61	55	85
45	258	500	499	115	175	96	59	148
46	307	1395	1384	163	300	173	143	252
47	173	302	296	84	159	115	108	144
48	462	1170	1169	163	333	166	132	280
49	44	257	319	73	142	131	143	147
50	63	265	321	74	137	116	135	125
51	55	150	164	59	87	76	74	77
52	42	128	129	56	83	64	38	70
53	27	100	101	50	63	49	41	52
54	54	180	183	63	99	75	68	89
55	60	186	185	60	98	68	60	90

cept, and a high correlation coefficient. The data are shown graphically in Fig. 2(*d*). The Danish and Gates formulas have equally high values of the correlation coefficient but less satisfactory values of A and B. The PC and Hiley formulas are somewhat less accurate, and the Engineering News and Gow formulas are considerably less accurate.

Consideration of the data for precast concrete piles is made difficult by the fact that a total of only fifteen tests was available. For the available data, Hiley's formula yields the value of A closest to one but the low correlation

TABLE 3.—COMPILATION OF STATISTICAL PARAMETERS

Pile (1)	Formula (2)	N (3)	A (4)	B, in tons (5)	r (6)
Timber	Engineering News	37	0.45	16	0.28
	Gow	37	0.37	18	0.43
	Hiley	37	0.64	19	0.77
	Pacific Coast	37	0.80	14	0.74
	Janbu (C_d = 1)	37	0.98	9	0.86
	Danish	37	0.71	9	0.86
	Gates	37	1.30	-17	0.86
Concrete	Engineering News	15	0.20	72	0.11
	Gow	15	0.32	69	0.12
	Hiley	15	1.08	24	0.43
	Pacific Coast	15	1.57	-19	0.75
	Janbu (C_d = 1)	15	0.66	23	0.64
	Danish	15	0.60	11	0.69
	Gates	15	1.62	-27	0.65
Steel	Engineering News	41	0.28	43	0.37
	Gow	41	0.28	42	0.38
	Hiley	41	1.14	-10	0.76
	Pacific Coast	41	1.07	0	0.79
	Janbu (C_d = 1)	41	0.91	7	0.83
	Danish	41	0.89	-16	0.82
	Gates	41	2.34	-83	0.84
All	Engineering News	93	0.33	37	0.29
	Gow	93	0.32	37	0.36
	Hiley	93	0.92	7	0.72
	Pacific Coast	93	1.04	2	0.76
	Janbu (C_d = 1)	93	0.87	10	0.81
	Danish	93	0.77	-2	0.81
	Gates	93	1.81	-48	0.81

coefficient indicates considerable scatter [Fig. 3(*b*)]. This scatter is also evident in the cumulative frequency curve (Fig. 6). The A-values are farther from one for the PC, Danish, Janbu, and Gates formulas, but the correlation coefficients are higher. The cumulative frequency curves in Fig. 6 suggest that the formula yielding values of Q_m/Q_c closest to one is Janbu's formula. The low correlation coefficients obtained when the Engineering News and Gow formulas were used indicate that there is essentially no correlation between the measured and computed pile capacities [Fig. 3(*a*)].

For steel piles, the PC formula yields excellent values of A and B but the correlation coefficients are slightly higher for the Danish, Janbu and Gates formulas. The Hiley formula is slightly less accurate than these and, again, the Engineering News and Gow formulas are considerably less accurate.

To obtain a measure of the over-all average accuracy of the formulas, the weighted average values of A, B, and r were calculated (Table 3). The corre-

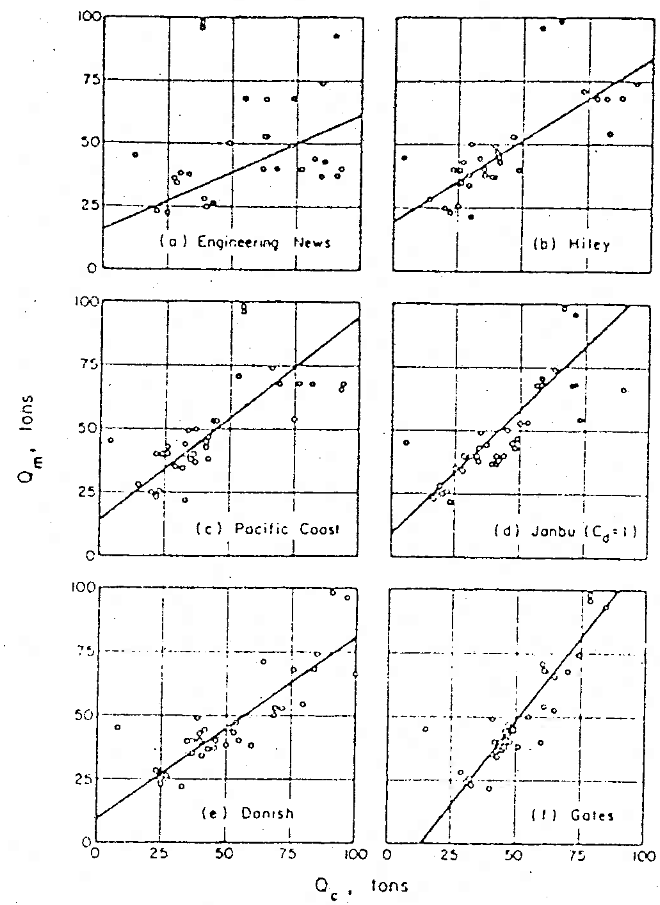

FIG. 2.—RELATIONSHIPS BETWEEN MEASURED AND COMPUTED CAPACITIES FOR TIMBER PILES

lation coefficients are highest, and equal, for the Gates, Danish, and Janbu formulas, and are slightly less for the PC and Hiley formulas. They are so low for the Engineering News and Gow formulas that these formulas can be eliminated from further consideration. When the values of A and B are taken into account, the Gates formula can be eliminated. The PC formula yields the best average values of A and B but it does so, in part, by underestimating the capacities of timber piles and overestimating the capacity of most concrete piles. It appears that valid arguments can be used in support of the Danish, PC, Hiley, or Janbu formulas; no one of the four is clearly superior to the others.

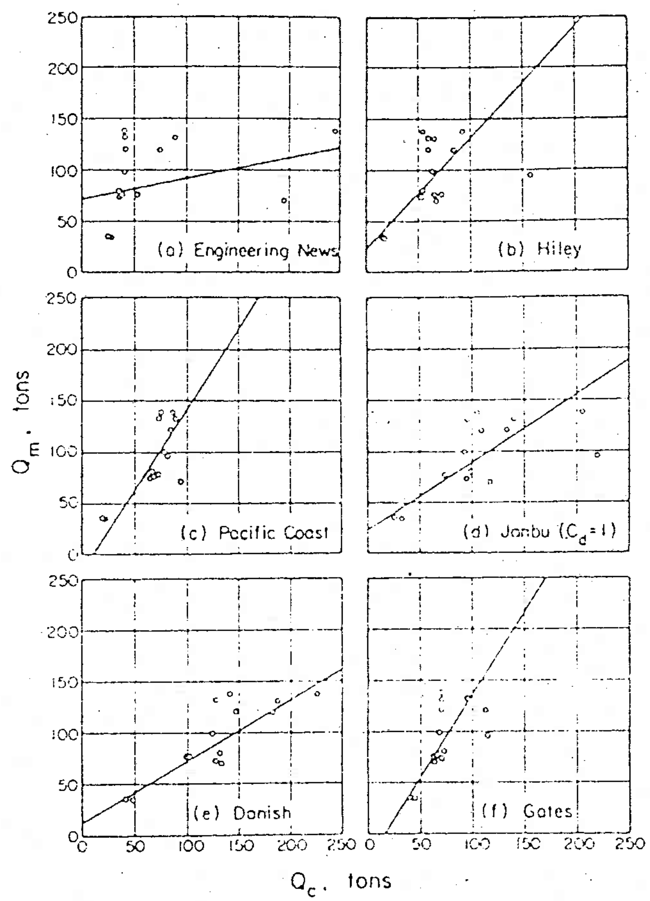

FIG. 3.—RELATIONSHIPS BETWEEN MEASURED AND COMPUTED CAPACITIES FOR CONCRETE PILES

Adjustment of the Formulas.—None of the formulas used in this study has any claim to theoretical rigor. Their usefulness is determined solely by the degree to which they successfully predict pile capacities in the field. Any adjustment to the formulas that improves their correlation with measured capacities is justified on pragmatic grounds.

All of the formulas can be improved if a new calculated capacity, Q_c^*, is defined using the equation

$$Q_c^* = A\,Q_c + B$$

in which the parameters A and B are tabulated in Table 3. If the values of A and B are taken separately for each type of pile, and the new calculated capacities are compared with the measured capacities, the regression line must yield $A = 1$ and $B = 0$. Thus, the accuracy of the adjusted formula depends on the correlation coefficient.

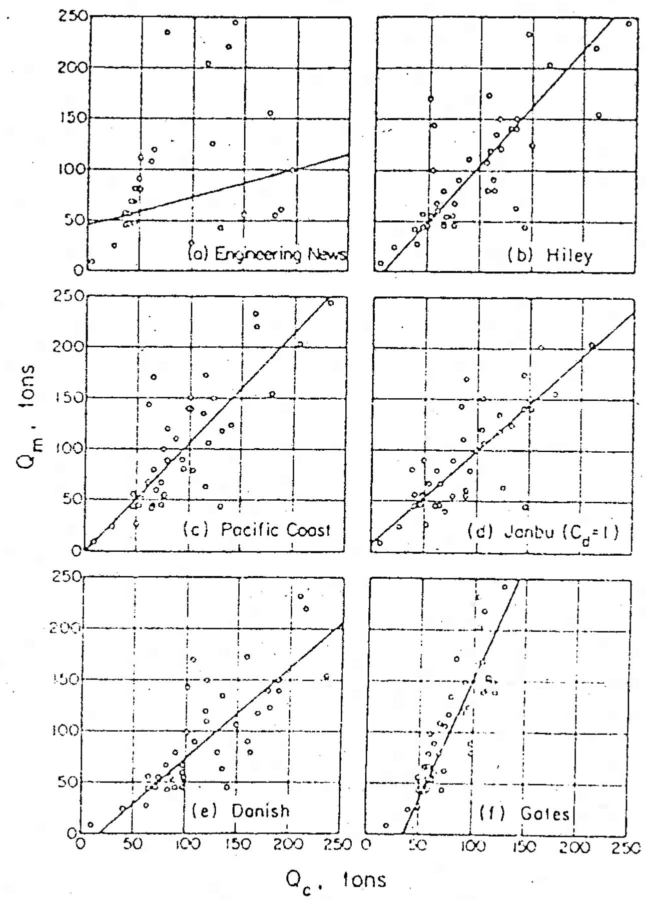

FIG. 4.—RELATIONSHIPS BETWEEN MEASURED AND COMPUTED CAPACITIES FOR STEEL PILES

In terms of high average correlation coefficients and simplicity of use, the Gates formula must rank first, followed closely by the Danish and Janbu formulas. The PC formula is less accurate and considerably more cumbersome to apply. The Hiley formula might be more accurate if the values of C_1, C_2, and C_3 were measured in the field, as Hiley recommended, but the formula

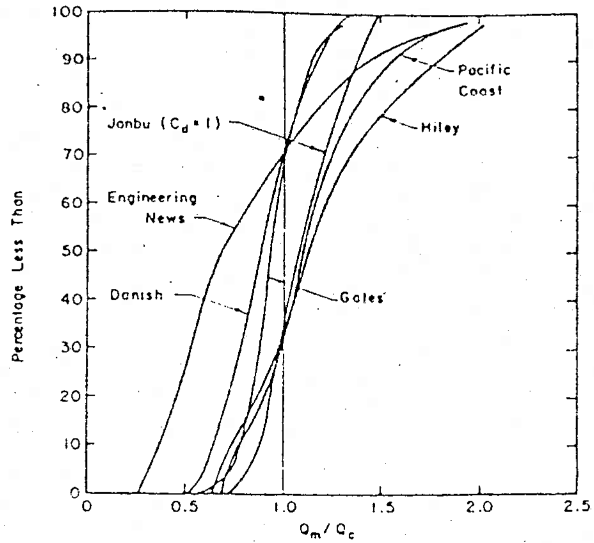

FIG. 5.—CUMULATIVE FREQUENCY CURVES FOR TIMBER PILES

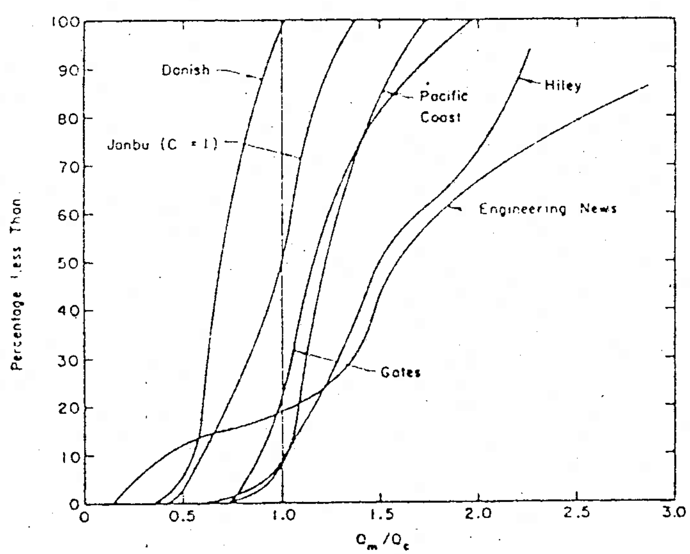

FIG. 6.—CUMULATIVE FREQUENCY CURVES FOR PRECAST CONCRETE PILES

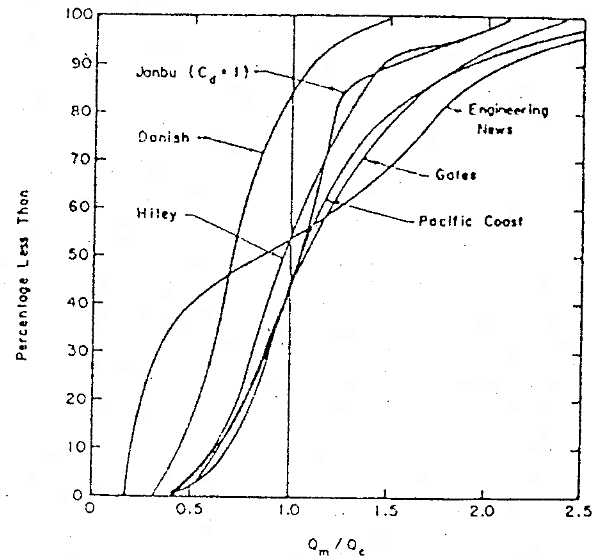

FIG. 7.—CUMULATIVE FREQUENCY CURVES FOR STEEL PILES

ranks fifth among the seven studied, when the average values of these constants are used.

If separate formulas are used for timber, precast concrete, and steel piles, the adjusted forms of the Gates formula become

$$\left.\begin{array}{ll} \text{Timber} & Q_c = 7.2\sqrt{e_h E_n}\,\log(10/s) - 17 \\ \text{Precast Concrete} & Q_c = 9.0\sqrt{e_h E_n}\,\log(10/s) - 27 \\ \text{Steel} & Q_c = 13.0\sqrt{e_h E_n}\,\log(10/s) - 83 \end{array}\right\} \quad \ldots\ldots\ (9)$$

in which Q_c is in tons, E_n is in in.-tons, and s is in inches.

The pile capacities were all recalculated using these three formulas. The measured and recomputed capacities are compared in Fig. 8. The equation of the regression line was $Q_m = Q_c$ and the correlation coefficient was 0.83, indicating a good correlation. The ratio Q_m/Q_c was calculated for each pile. The average value of the ratio was one and the standard deviation was estimated (Eq. 2) to be 0.28 (pile S35, which was driven with a small hammer, yielded a negative capacity and was not included in the calculation). Thus, approximately two-thirds of the piles had measured capacities within 28% of the calculated value.

If a single pile formula is desired for use with all three types of piles, then the average A and B factors (Table 3) can be used to adjust any of the formulas. It seems apparent from their high correlation coefficients and simplicity of use, that the Gates, Danish, or Janbu formulas should be recommended. As an example of the adjustment, a complete set of calculations was performed using the modified form of Janbu's formula with $A = 0.87$ and $B = 10$ tons. For all 93 piles, the regression line relating the measured capacities to the adjusted, computed capacities yielded $A = 1$, $B = 0$, and $r = 0.85$. The mean value of Q_m/Q_c was one and the coefficient of variation was 35%, i.e., about two-thirds of the piles had calculated capacities within 35% of the measured values. Similar correlations would be obtained if a single adjusted Danish or Gates formula were used.

When applying the revised pile-driving formulas in practice, use of a factor of safety of about three is recommended. There are several reasons for

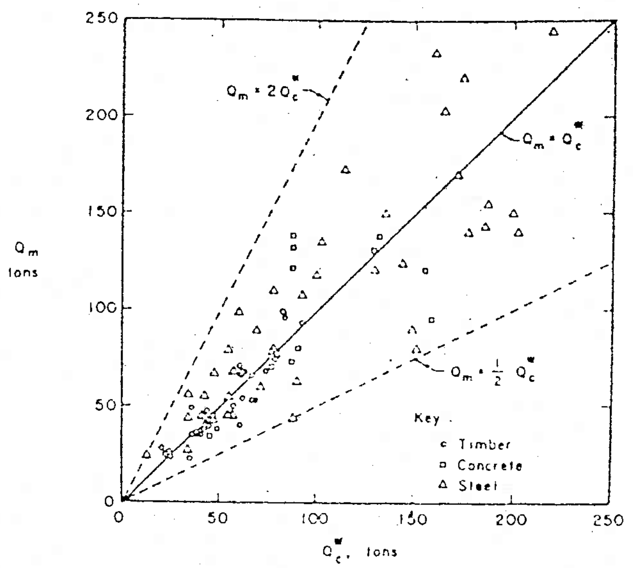

FIG. 8.—COMPARISON OF MEASURED PILE CAPACITIES AND THE CAPACITIES CALCULATED USING THE STATISTICALLY ADJUSTED FORMS OF GATES' FORMULA (EQ. 9)

choosing three. First, a factor of two is needed to account for inaccuracy in calculations (Fig. 8). Second, additional capacity is needed to account for normal uncertainties in loading. Third, the factor of safety should be increased further to ensure relatively small settlements under the design load. The factor of safety actually used in design may be adjusted to larger or smaller values depending on the consequences of excessive settlement.

Validity of the Adjustment Procedure.—A question may be raised regarding the validity of the adjusted forms of the pile-driving formulas because of the fact that they were adjusted, statistically, to fit certain data and then were compared with the same data. Can the adjusted formulas be applied to other piles with reasonable confidence? The answer would appear to be yes, provided that the data used in this study are representative of data generally obtained in the field. Whether the data are, in fact, representative can be established only by the collection of a large number of additional load-test results which would then be used to refine the values of A and B. It appears the number of tests used in this study for timber and steel piles was sufficient to allow moderate confidence to be placed in the values of A and B presented in Table 3. However, the small number of tests using precast concrete piles, and the large amount of scatter, would suggest that more data are needed to establish values for A and B for these piles. No data are included for cast-in-place concrete piles where the casing is driven with a steel mandrel, but it appears probable that the formulas used for steel piles would apply.

In any case, it seems certain that the adjusted formulas are more accurate than the original ones. If the original formulas were good enough to use, then the adjusted formulas are likely to be better.

CONCLUSIONS

The measured capacities of 93 piles driven into sandy soils were compared with the capacities predicted using the Engineering News, Gow, Hiley, PC Code, Janbu ($C_d = 1$), Danish, and Gates formulas. The accuracy of the formulas varied with the type of pile. Janbu's formula was the most accurate for timber and steel piles. None of the formulas was clearly best for the precast concrete piles. In all cases, however, the Engineering News and Gow formulas were clearly inferior to the others and were eliminated from further consideration.

The three formulas that yielded the highest average correlation coefficients were the Danish, Janbu ($C_d = 1$), and Gates formulas. These formulas were adjusted statistically to fit the observed capacities. The adjusted formulas are believed to be more accurate in predicting the field capacities of timber and steel piles than any previous formulas. For greatest accuracy and simplicity of use, it is recommended that the adjusted forms of Gates' formula, Eq. 9, be used.

Adjusted forms of the Janbu ($C_d = 1$) or Danish formulas are also recommended. They yield essentially the same results as the Gates formula but are slightly more difficult to use.

Single adjusted formulas may be used for all types of piles but the predicted capacities are likely to be slightly less accurate than when a different adjusted form of the formula is used for each type of pile.

APPENDIX.—NOTATION

The following symbols are used in this paper:

A = slope of a regression line (dimensionless); cross-sectional area of a pile, in square inches;
B = intercept of a regression line, in tons;
C_1 = recoverable deformation of the pile cap and head, in inches per blow;
C_2 = recoverable deformation of the pile, in inches per blow;
C_3 = recoverable deformation of the soil, in inches per blow;
E = Young's modulus for the pile material, in tons per square inch;

E_n = nominal energy of the pile hammer, in inch-tons per blow;
e_h = efficiency of the hammer (dimensionless ratio);
e_i = efficiency of impact between the hammer and the cushion block (dimensionless ratio);
H = height of fall of the ram, in inches;
L = length of the pile, in inches;
N = number of observations;
Q_c = pile capacity calculated using a dynamic pile-driving formula, in tons;
Q_c^* = calculated pile capacity after the application of a statistical adjustment, in tons;
Q_m = measured pile capacity, in tons;
r = correlation coefficient (dimensionless number ranging from 1 to -1);
S_X = estimated standard deviation of X;
S_Y = estimated standard deviation of Y;
s = average penetration of the pile per hammer blow for the final ten blows, also known as the set, in inches per blow;
W_h = weight of the falling part of the hammer, in tons;
W_p = weight of the pile, in tons;
X = a variable;
$\overline{X}$ = mean value of X;
X_i = a particular value of X;
Y = a variable;
$\overline{Y}$ = mean value of Y; and
Y_i = a particular value of Y.

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VOLUME CHANGES IN TRIAXIAL AND PLANE STRAIN TESTS

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INTRODUCTION

Studies of the strength of sands under high confining pressures have shown that the volume change a specimen exhibits during shear is dependent on the magnitude of the consolidation pressure. If the density of a sand is above a certain minimum density, the sand will expand during shear under low confining pressures. When high confining pressures are used, the sand grains are crushed and, during shear, volume contraction takes place. At a certain value of the confining pressure, depending on the magnitude of the initial void ratio, no volume changes take place. The confining pressure which assures no volume change during shear has been termed the "critical confining pressure."[4]

The magnitude of the pore-water pressure that occurs in sand during shear depends in part on the kind and amount of volume change; a tendency to volume expansion induces negative pore pressure leading to increased strength in undrained shear, while a tendency to volume contraction leads to positive pore pressures and reduced strength. The shear strength of saturated sand in undrained shear under high confining pressures will, therefore, depend on the relation between the actual confining pressure and the critical confining pressure. At confining pressures less than critical, expansion tends to occur with a resulting decrease in pore-water pressures, but at pressures greater than critical, a tendency to contraction and positive pore pressures will develop.

To date most high-pressure tests on soils have been under conventional triaxial test conditions. Yet in practice the soil may deform under plane

Note.—Discussion open until April 1, 1968. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, Vol. 93, No. SM6, November, 1967. Manuscript was submitted for review for possible publication on December 13, 1966.

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[4] Seed, H. Bolton, and Lee, Kenneth L., "Drained Strength Characteristics of Sands," Journal of the Soil Mechanics and Foundation Division, ASCE, Vol. 93, No. SM6, Proc. Paper 5561, Nov. 1967.