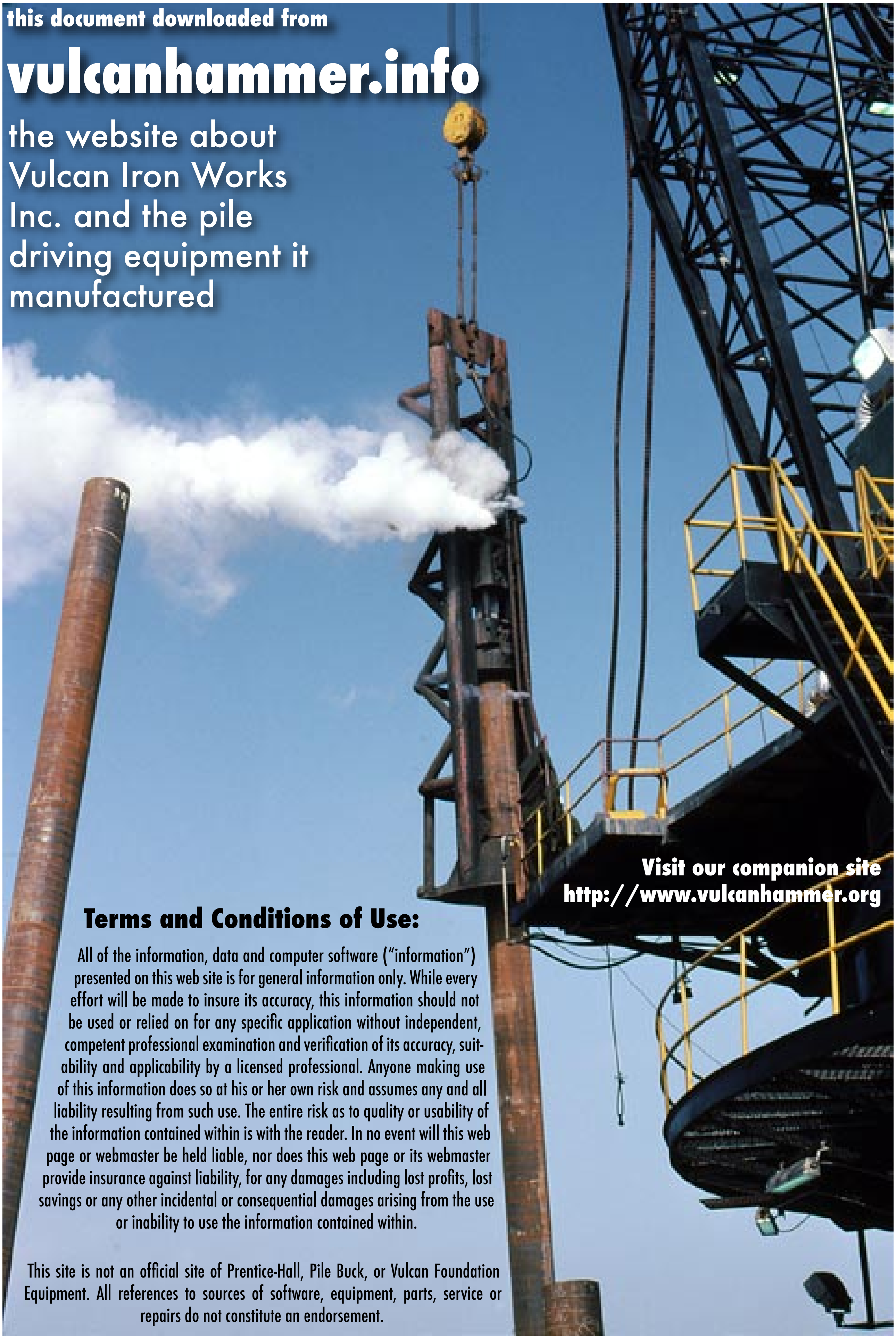

DYNAMIC RESPONSE OF PILES AND PILE GROUPS

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The paper reviews the results of theoretical and experimental reasearch into dynamic behaviour of piles and pile groups conducted at The University of Western Ontario. The importance of soil layering is experimentally demonstrated and an approximate theory to account for it is outlined. Basic features of dynamic behaviour of pile groups are discussed.

INTRODUCTION

In recent years there has been a considerable increase in interest in dynamic behaviour of piles and pile groups. Much of the research into this subject was prompted by energy related designs of large, expensive structures such as offshore towers, nuclear power plants, compressor stations for pipelines etc. A number of approaches have been developed for dynamic analysis of single piles. They employ the lumped mass models (14,25), the continuum models (10,17,19,22,30,31) and the finite element method (5,11). Each of these methods has its advantages and disadvantages. A detailed comparison of the approaches available was presented by Roesset (28). Dynamic response of pile groups is affected by interaction of the piles with soil and also by the interaction between individual piles in the group. The latter effect, known as the group effect or pile-soil-pile interaction, can be quite dramatic but most of its studies were limited to static loads (4,6,26,27). Dynamic behaviour of pile groups was studied only recently using the finite element method (32-35), approximate continuum approaches (7,15,16,29) and the boundary integral procedure (2).

This paper describes the theoretical and experimental studies of dynamic pile behaviour conducted at The University of Western Ontario. The emphasis is on the comparison between theory and experiments.

SINGLE PILES

Stiffness and damping of single piles can be analyzed using various techniques. A prominent feature of the modern techniques is that they account for the propagation of elastic waves in soil. This phenomenon modifies the stiffness of the pile, makes it frequency dependent and generates radiation (geometric) damping. For a homogeneous soil medium and small amplitudes compatible with the assumption of linearity, the finite element technique and the continuum approaches yield pile stiffness and damping constants that agree very well. This was shown by Blaney et al. (5), Kuhlemeyer (11), Roesset (28), the author (17) and Aubry and Chapel (2). This agreement promotes confidence in the theoretical correctness of the modern approaches and makes their experimental verification very desirable.

Comparison of Theory With Experiments

Many of the theoretical approaches developed are based on the assumption of soil homogeneity. This assumption is made quite often because it is convenient and facilitates the solution.

Comparison with experiments, however, indicates that the assumption of soil homogeneity is not very suitable for practical applications because it can result in considerable overestimation of pile stiffness reaching even hundreds of per cent and similar errors in the prediction of damping. The main reasons for these discrepancies are the variation of soil properties with depth and particularly the reduction of shear modulus toward ground surface or the mudline and the lack of bond between the pile and soil which may result in pile separation (Fig. 1a).

Both the shear modulus of soil and the bond between the pile and soil diminish with confining pressure which is controlled primarily by the depth of the overburden. As the confining pressure decreases towards ground surface so does soil shear modulus and the pile losses its support. This can reduce pile stiffness very dramatically, especially in the horizontal direction in which the pile derives most of its stiffness from the topmost layers of soil where the shear modulus is most reduced (Fig. 1b). This problem cannot be allivated by using a lower effective shear modulus than experiments,with wave propagation, would indicate. Such a lower effective shear modulus, derived for example from a static test, may make it possible to match the stiffness (natural frequency) but it also changes the damping and may increase the difference between the theoretical and experimental resonant amplitudes. This is shown in Fig. 2 based on the theory presented in (17) and experiments with small pile foundations described in (21). The response curve A was computed with a constant shear wave velocity, $\overline{V}_s = V_t$, derived by means of the steady-state vibration technique in the field. With this value, the resonant frequency was over-

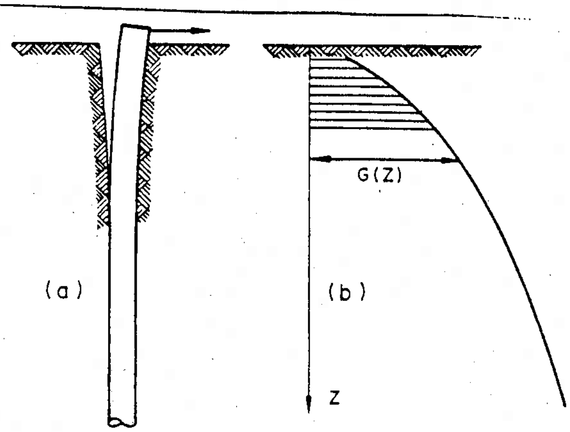

Fig. 1 **Pile Separation and Soil Shear Modulus Variation With Depth**

estimated by a factor of about two. A static test revealed a much lower effective shear wave velocity, V_{st}, equal to about 0.26 V_t. With this value, the theoretical response curve B matches the experimental resonant frequency very well but the difference between the experimental and theoretical resonant amplitudes increases. Only the response calculated with a parabolically varying shear wave velocity, as is discussed further herein, gave satisfactory results (curve C in Fig. 2). The same difficulty appeared even more dramatically with a group of four piles as shown in Fig. 3. (The piles were relatively far apart and thus, the group effect was not very strong). The resonant frequency was overestimated by a factor of about three when the constant wave velocity from wave propagation tests was used (curve A). The effective shear wave velocity, V_{st}, derived from the static test, gave a better estimate for the resonant frequency but not for the resonant amplitude (curve B). Obviously, the assumption of soil homogeneity makes it difficult if not impossible to match both pile stiffness and damping and seriously limits the applicability of the theories based on this assumption. The theory should incorporate the variation of shear modulus with depth which implies soil nonhomogeneity.

Soil Nonhomogeneity

Soil nonhomogeneity is caused by the variation of confining pressure and soil layering. The dependence of soil shear modulus on confining pressure is well recognized and best understood with sands but it is just as important in cohesive soils. The decrease in shear modulus with decreasing confining pressure is illustrated in Fig. 4 in which laboratory

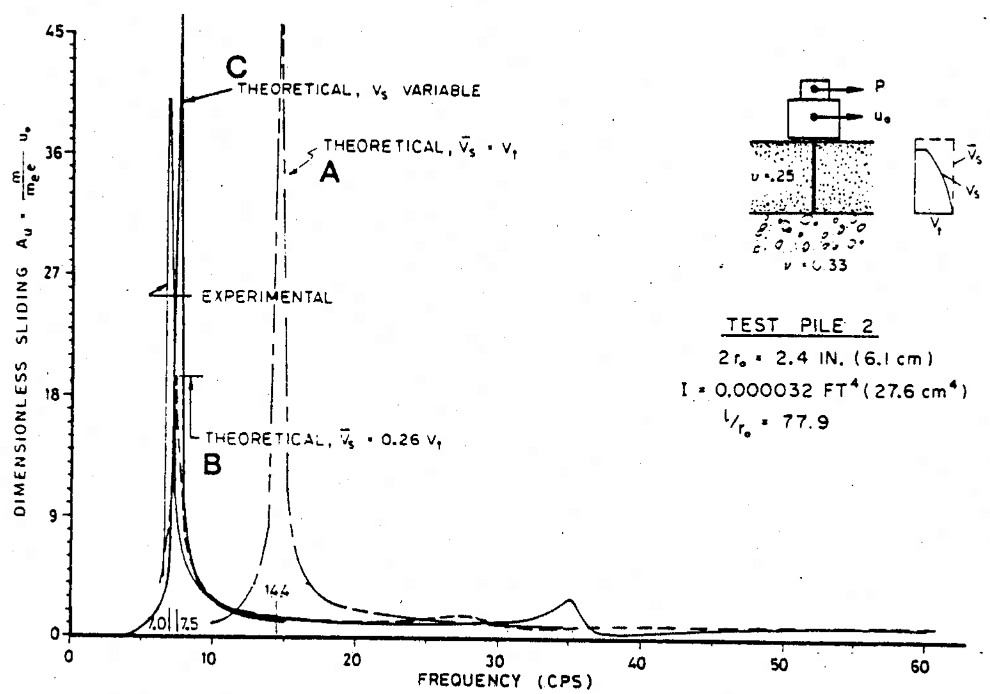

Fig. 2 Comparison of Experimental and Theoretical Response of Pile to Horizontal Excitation

results are plotted for seven different cohesive soils. This behaviour of soils indicates that the variation of shear modulus with depth is inevitable even in a seemingly homogeneous deposit or layer and is further substantiated by natural layering of soil.

Soil layering can readily be considered when using the finite element method to solve the dynamic soil-pile interaction problem (32-25) but a rigorous analytical solution is difficult and is not available. An approximate analytical solution was formulated by Takemiya and Yamada (30) who extended the single layer solution due to Nogami and the author (22). The approximation of these solutions stems from the omission of the vertical component of the soil motion in the solution of the horizontal response and the omission of the horizontal component in the solution of the vertical response. Even with these simplifications, the solution is quite involved.

Plane Strain Soil Stiffness

A much simpler solution for piles in nonhomogeneous soil can be based on soil stiffnesses calculated under the assumption of plane strain. Such soil stiffnesses are mathematically accurate for an infinitely long rigid pile undergoing uniform harmonic vibration in an infinite homogeneous layer but can be used as approximate for flexible piles and layered media. This implies that a soil reaction generated at a certain depth is assumed to be independent of those acting at other stations. Such a soil medium behaves as a kind of Winkler's medium in which stress waves propagate only in the horizontal direction. The plane strain soil stiffnesses are available for all vibration modes in a closed form (24)

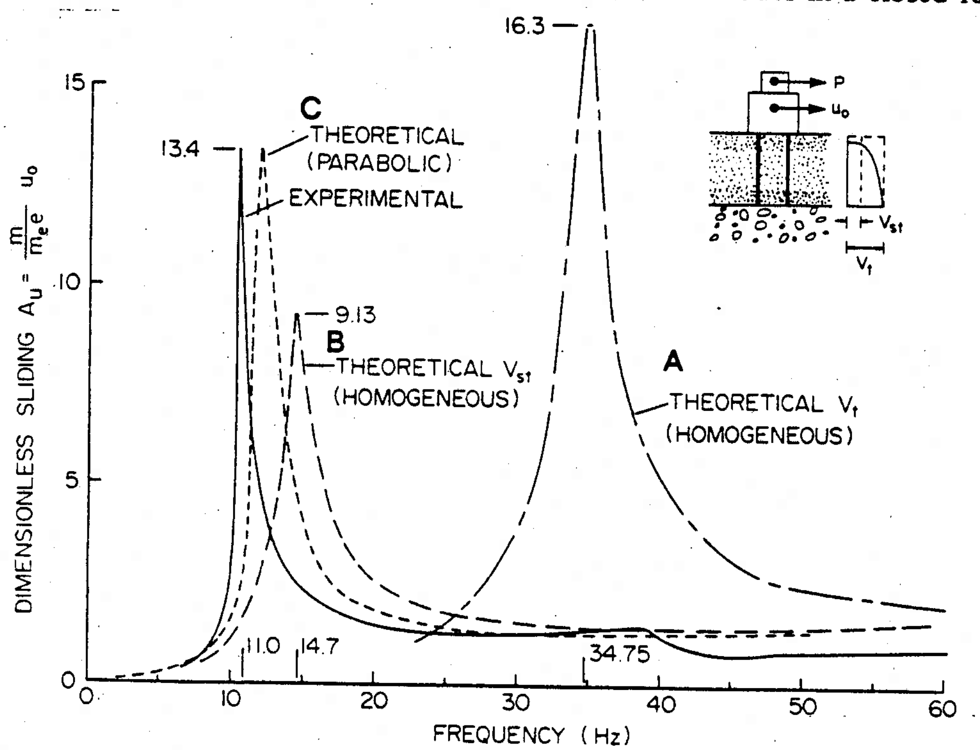

Fig. 3 Comparison of Experimental and Theoretical Response of Group of Four Piles to Horizontal Excitation

and have an in-phase stiffness term and an out-of-phase (imaginary) damping term. For example, for horizontal vibration, the complex soil stiffness per unit length of the pile can be written as

$$k_u = G [S_{u1}(a_o , \nu, D) + iS_{u2}(a_o , \nu, D)] \tag{1}$$

in which G is soil shear modulus and S_{u1}, S_{u2} are dimensionless stiffness and damping parameters. These parameters depend on the dimensionless frequency $a_o = r_o \omega / \sqrt{G/\rho}$, Poisson's ratio ν and material damping D; r_o = pile radius, ω = circular frequency and ρ = mass density of soil. Material damping is of the frequency independent, hysteretic type and is defined as

$$D = \frac{G'}{G} = \tan \delta = 2\beta$$

where δ = the loss angle, β = the material damping ratio and G' = the imaginary part of the complex shear modulus

$$G^* = G (1 + iD) \tag{2}$$

With the soil reactions described by Eq. 1 and similar expressions for other vibration modes, stiffness and damping of piles can be calculated with any soil layering without

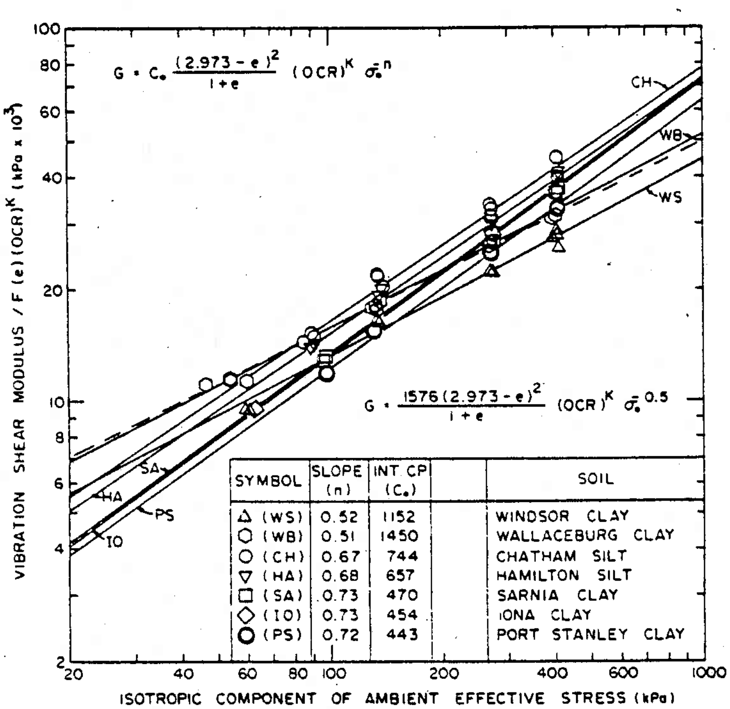

Fig. 4 Variation in Shear Modulus of Cohesive Soils With Confining Pressure (9)

difficulty using the dynamic stiffness method. Continuous variation of the shear modulus with depth can be replaced by a stepwise variation. This type of approach was first formulated for one homogeneous layer (17) and then extended to multi-layered media (19,20). All types of piles can be treated, i.e. end bearing piles, floating piles or Franki piles. The approach is very versatile and applicable to general situations such as that indicated in Fig. 5 with negligible computing costs. The plane strain soil reactions are also useful for the analysis of the so-called sleeved piles (8), underground pipes and embedded footings. The analytical approach based on the plane strain reactions compares very well with the finite element solutions (5,8,11) as well as the more rigorous continuum solutions (22, 30) and its accuracy increases with frequency.

The agreement with experiments can improve greatly when the variation of soil shear modulus with depth is accounted for. In the experiment shown in Fig. 2, best results were obtained with a parabolic variation of shear modulus with depth which complied with a more detailed measurement of shear wave velocity. In addition, the top layer, two pile diameters thick, was considered as void to account for the gap between the footing base and the ground surface and for pile separation. The response calculated under these assumptions is shown as curve C in Fig. 2. The agreement with the experimental curve is very good considering that the horizontal response of a single pile is most difficult to predict because it depends very much on the behaviour of the topmost layers of soil.

The only drawback of the plane strain soil reactions is that, at very low dimensionless frequencies, the stiffness diminishes to zero when the frequency approaches to zero. However, this drawback can be corrected. It has been shown in (17) and in a number of further studies such as (5,8,22,30) that pile dynamic stiffness remains almost constant for frequencies a_o smaller than about 0.3 and very close to static stiffness. Thus, the plane strain soil stiffness (or pile stiffness) for very low frequencies can be considered as constant and equal to the dynamic stiffness calculated for a_o = 0.3 or so (Fig. 6). (This

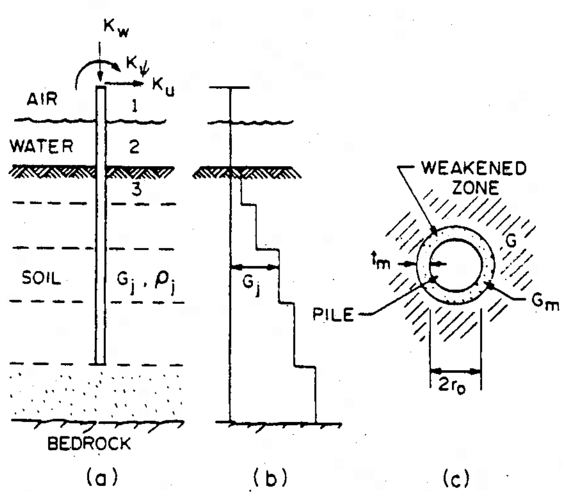

Fig. 5 Pile in Layered Medium

approximation is not needed for torsional and rocking stiffnesses for which the plane strain approach yields static stiffnesses).

In the case of a shallow layer overlying bedrock, another correction may be desirable for geometric damping at low dimensionless frequencies. The more rigorous analytical solutions (22) and the finite element solutions (5) indicate that below the first resonance of the soil layer no geometric damping (no progressive wave) is generated. (Only a very weak progressive wave is generated if material damping is present). Thus, soil material damping is the principle source of energy dissipation in this frequency region. This damping can be evaluated from Eq. 1 by ignoring the imaginary part, $S_{u2} = 0$, taking $D = 0$ in S_{u1} and replacing G by the complex shear modulus G^* (Eq. 2) in the sence of the correspondence principle. Taking the stiffness parameter as frequency independent , the complex horizontal soil stiffness for frequencies lower than the first natural frequency of the layer can be described as

$$k_u = G^* S_{u1} = G\,(1 + iD)\, S_{u1}\,(\nu) \qquad (3)$$

or

$$k_u = G\,(S_{u1} + i\,D\,S_{u1}) \qquad (4)$$

Thus the soil damping below the first layer resonance can be taken as constant and equal to GDS_{u1} where S_{u1} is the stiffness parameter at low frequencies (Fig. 6). The accuracy of this correction was found to be quite sufficient by Kaynia and Kausel (8) who compared it with their finite element solution and is also confirmed by the exact solutions available for torsion and rocking (24); these vibration modes are suitable for such a verification because they feature very little geometric damping at low frequencies.. The first natural frequency of the soil layer below which the damping may be adjusted is

$$f_H = \frac{V_s}{4H} \qquad (5a)$$

for horizontal vibration and

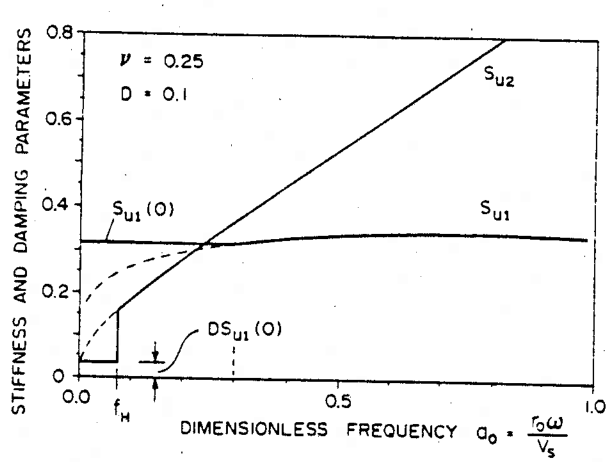

Fig. 6 Adjustments of Plane Strain Soil Stiffness for Low Frequencies

$$f_V = \frac{V_s}{4H} \sqrt{\frac{2(1-\nu)}{1-2\nu}} \qquad (5b)$$

for vertical vibration. In Eqs. 5, V_s = shear wave velocity, H = layer thickness and ν = Poisson's ratio.

The possible lack of soil geometric damping at frequencies lower than the first natural frequency of the layer would result in a reduction of pile damping. Another reason for the reduction of pile damping may be the behaviour of the contact between the pile and soil and soil nonlinearity.

Contact Effects of Piles

Most theoretical solutions are based on the assumptions that the soil behaves in a linear fashion and is perfectly bonded (welded) to the pile. However, the bond is not perfect and slippage or even separation often occurs between the pile and the soil. In addition, the soil immediately adjacent to the pile can be exposed to high strain and consequently behave in a nonlinear fashion. It is very difficult to account for these phenomena in a rigorous way. To account for them approximately a theory was formulated in which the pile is surrounded by a composite, linear, viscoelastic medium comprising two parts; an outer infinite region and an inner cylindrical zone (Fig. 5c). Soil nonlinearity, the lack of bond and slippage all effectively weaken the soil around the pile and are presumed to be accounted for by a reduced shear modulus and increased material damping in the inner (weakened) zone. In this approach, the weakening is limited to the part of the medium in which it is likely to occur which seems preferable to the often used measure whereby the correction for soil nonlinearity is applied to whole layers.

The dynamic stiffnesses of this composite medium can be established for all vibration modes using the assumption of plane strain (23) and written again in the form of Eqs. 1 or 4. The dimensionless parameters S now depend also on the ratios t_m/r_o (thickness of the weakened zone/pile radius) and G_m/G (shear modulus of the weakened zone/shear modulus of the outer region). The inclusion of the weakened zone affects both the stiffness and damping parameters $S_{1,2}$ and also their ratio and hence, the phase shift between the motion of the pile and the soil reaction. The stiffness is usually somewhat reduced but the geometric damping is reduced more dramatically and the reduction of the damping increases with frequency. The degree of these effects depends on the magnitude of the ratios G_m/G , t_m/r_o and frequency a_o .

The resultant effect of the weakened zone on response to dynamic loads can be seen in Fig. 7 where the rotational (rocking) component of the coupled response to horizontal excitation is shown. The excitation is harmonic and proportional to the square of frequency. In the example shown, the piles are far apart and the group effects are not considered. The weakened zone (Case B) affects the stiffness only slightly and therefore the resonance frequencies do not change very much. The damping is reduced by the weakened zone quite markedly particularly at higher frequencies and therefore the resonant amplitudes are increased much more in the second resonance region than in the first. These trends have been observed in some experiments with piles and embedded foundations (23). Thus, the inclusion of the weakened zone increases the versatility of the plane strain approach and may improve the agreement between the theory and experiments. The weakened zone was included in the computer program PILAY2. The concept of the composite medium was further explored by Lakshmanan and Minai (12).

Pile-Water Interaction

When a pile vibrates in water (Fig. 5a), it meets the resistance of the water to the motion of the pile. This water resistance can be described in a way analogous to Eq. 1 for soil stiffness. Then, the fluid reaction force acting on a unit length of the pile vibrating in water with a unit amplitude and frequency ω can be written as

$$k_w = - M\omega^2 (H_1 + iH_2) \tag{6}$$

in which $M = \rho_w \pi r_o^2$ is the mass of the displaced water and ρ_w = water density; H_1 relates to the added mass and H_2 to viscous damping. Using the solution by Chen et al. (3), H_1 and H_2 are respectively the real and imaginary parts of the complex function

$$H = 1 + \frac{4 K_1(a)}{a K_o(a)} \tag{7}$$

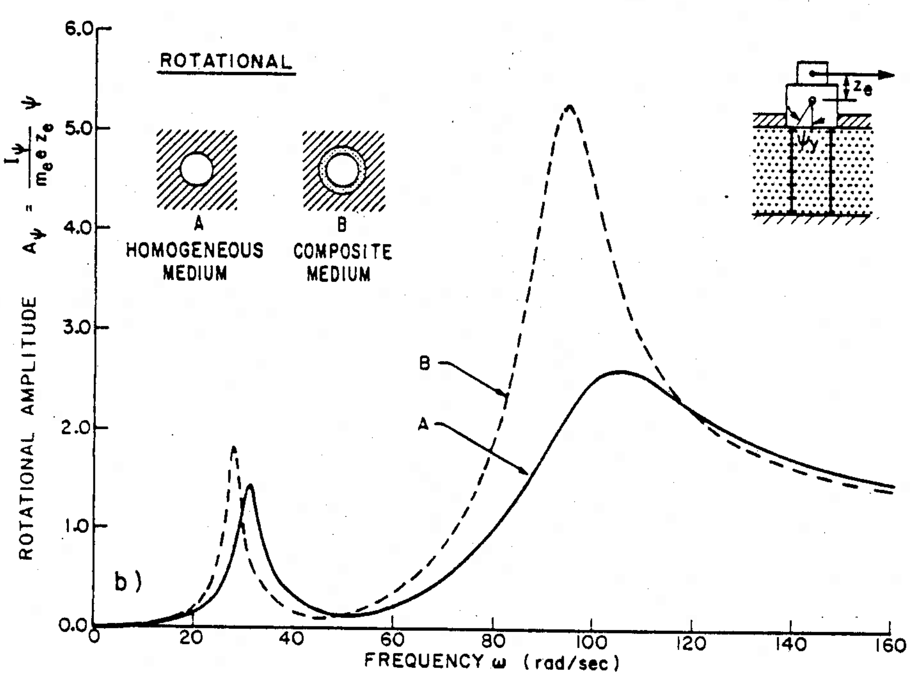

Fig. 7 Rotational Response of Pile Supported Footing to Horizontal Harmonic Excitation

in which $a = r_o \sqrt{i\omega/\nu}$, ν = kinematic viscosity of the water and K_0, K_1 are modified Bessel functions. The complex stiffnesses of a pile passing through water are evaluated quite readily by replacing the soil reaction, Eq. 1 by the water reaction, Eq. 6, for the submerged pile element. The submerged length of the pile may be taken as one element or a few elements.

By way of example the horizontal impedance was evaluted for the partially submerged steel pile shown in Fig. 8a. Eqs. 1 and 7 were used for soil and water resistance, respectively. The complex stiffness referred to the pile head can be written as

$$K_u = \frac{E_p I}{r_o^3} \left(f_{u1} + i \frac{r_o \omega}{V_s} f_{u2}\right)$$

in which $E_p I$ is the bending stiffness of the pile and f_1, f_2 are dimensionless stiffness and damping parameters shown in Fig. 8b. The water reduces pile stiffness more and more as the frequency increases; this is caused by the inertia (added mass) effect. The effect of water on pile damping appears negligible. This suggestion may be acceptable for small amplitudes. A more general description of water reactions should account for an increase in water damping due to surface effects (three-dimensionality) and amplitude effects (nonlinearity).

PILE GROUPS

When the piles of a group are widely spaced they do not affect one another and the group behaves as an assembly of individual piles. Consequently, the group stiffness and

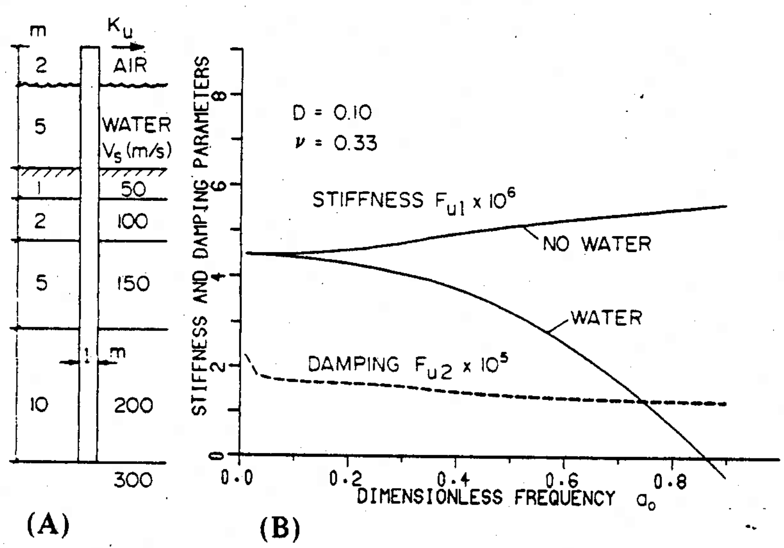

Fig. 8 **Partially Submerged Pile and Its Stiffness and Damping**

damping (impedance) can readily be established using the stiffness and damping of individual piles considered in isolation (17). About the only complication is that group response to horizontal loads and moments in the vertical plane engages horizontal impedance as well as vertical impedance of individual piles. This poses no difficulty in linear analysis but may considerably complicate nonlinear analysis.

If the piles are closely spaced, they affect one another and pile-soil-pile interaction occurs. Most of the studies conducted so far have been limited to linear elasticity and static loads for which considerable amount of information has been acquired particularly through the research of Banerjee (4), Butterfield (6) and Poulos (26,27). The static studies indicate that the vertical and horizontal stiffnesses of the group are always reduced by pile-soil-pile interaction but the torsional and rocking stiffness are either reduced or increased. The increase may occur only in small groups (18).

The studies of dynamic pile-soil-pile interaction are only recent and few in number. Various linear approaches have been used: the finite element approach (32-36), approximate analytical solutions (7,15,16,29), a semi-analytical solution (33) and the boundary integral procedure (2). These studies suggest a number of observations: dynamic group effects are profound and differ considerably from static group effects; dynamic stiffness and damping of pile groups vary with frequency and these variations are more dramatic than with single piles; and, finally, group stiffness and damping can be either reduced or increased by pile-soil-pile interaction. To examine some other effects such as the lack of bond the authors formulated an approximate analytical solution based on the extension of the plane strain approach to pile groups.

Plane Strain Approach to Pile Groups

The plane strain approach facilitates a simple approximate solution which accounts for soil layering, wave propagation and the lack of bond between the piles and soil as well as the weakening of the soil around the piles due to high strain. The latter factors are allowed for by means of the weakened cylindrical zone around the piles as shown in Fig. 5c.

The mathematical model of the group is shown in Fig. 9. The soil consists of horizontal layers featuring an outer zone stretching to infinity and an inner cylindrical zone around the pile. The properties of the inner zone can be different from those of the outer region. The piles and the soil are linearly elastic with hysteretic material damping. The soil reactions acting on pile tips are derived from a viscoelastic half space. The model is somewhat similar to that employed by Nogami (16) who, however, did not include the weakened zone, used a soil column to model the tip reaction and used a different analytical procedure.

The principal step of the solution is the derivation of the complex soil stiffness matrix related to the nodal points of the piles. This stiffness matrix is obtained by the inversion of the flexibility matrix which follows from the displacement field of the composite soil medium. The soil stiffness matrix is combined with that of the piles and the system is solved for prescribed loads or displacement. In this way, group response or impedance functions are established. The solution is described in more detail in (29). The drawback of using the plane strain is that the soil cross stiffnesses are generated only within individual layers. However, the success of this approach with single piles indicates that the approach may be worth exploring for groups as well. The theory was efficiently programmed and used to study dynamic behaviour of pile groups.

The effect of pile-soil-pile interaction on group stiffness and damping can be evaluated by means of the group efficiency ratio (GER) defined as the stiffness (damping) of the group divided by the sum of stiffnesses (damping) of individual piles considered in isolation. For a group of two end bearing piles, the variation of the group efficiency ratio with spacing is shown in Fig. 10. While the static group efficiency is always smaller than unity, the dynamic group efficiency oscillates and can be smaller or greater than unity depending on pile spacing, frequency and tip condition. The increase in damping can be particularly dramatic as was also observed by Wolf and von Arx (34) and Nogami (15). The weakened zone around the pile reduces the group effect on stiffness but may increase the group effect on damping. (This does not mean an increase in the magnitude of group damping). However, the effects of the weakened zone depend strongly on frequency, the thickness of the zone, t_m, and the shear modulus ratio, G_m/G.

Another feature of dynamic pile interaction is the sharp peaks which may occur in group stiffness and damping. For a group of four concrete piles shown in Fig. 11, these peaks can be seen in Fig. 12 (cases 4 and 5) in which group stiffness and damping are plotted for five different conditions. The weakened zone (case 5) suppresses the peaks but the impedance functions and the response to harmonic loads (Fig. 11) remain very sensitive to the assumed soil profile, properties of the weakened zone and pile interaction. For the

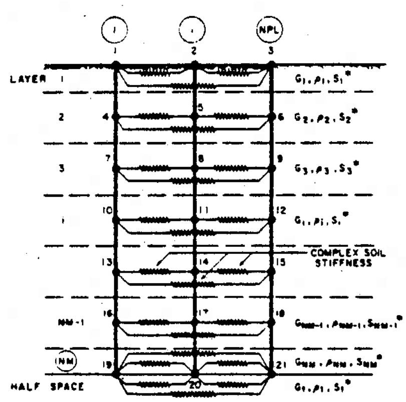

Fig. 9 Plane Strain Model for Dynamic Analysis of Pile Groups

pile foundation shown in Fig. 11, the resonant amplitude is amplified due to pile interaction and the weakened zone (case 5) because the damping is reduced at the resonant frequency of about 130 s^{-1}. However, if the same piles supported a footing whose weight would bring the resonant frequency down to about 50 s^{-1}, pile interaction would produce an opposite effect and reduce the resonant amplitudes because the damping is increased at that frequency (Fig. 12b).

An increase in damping due to pile interaction was indicated in experiments with a test footing described in detail in (29). The footing weighing 3606 lb (1635 kg) was supported by four steel piles 11 feet (3.4 m) in length and 2.38 in (6.03 cm) in diameter (Fig. 13). The measured response to harmonic excitation displays only small resonant amplification. The theory overestimates the resonant amplitudes by a factor of about 3 if pile interaction is neglected. The approximate theory of pile interaction outlined above

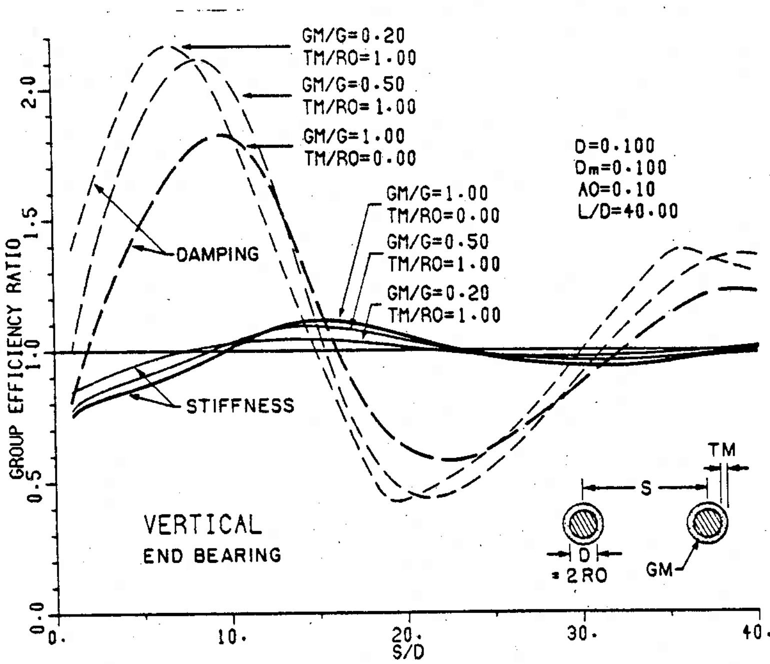

Fig. 10 **Group Efficiency Ratio of Two End Bearing Piles Culculated With And Without Weakened Zone Around Piles**

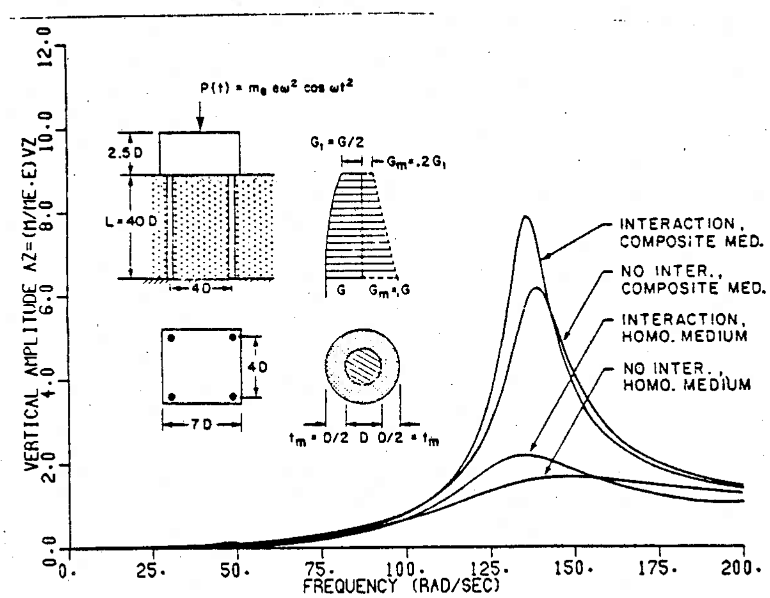

Fig. 11 **Dynamic Response of Pile Supported Foundation Calculated With and Without Group Effects And Weakened Zones Around Piles (V_s = 200 ft/s = 60.96 m/s; γ = 100 lb/cu.ft = 1592 kg/m^3, footing weight = 147000 lb = 66678 kg, D = 2 ft = 0.61 m)**

gives much better agreement with the experiment (Fig. 13). Nevertheless, more experimental research is needed to prove or disprove the existence of the peaks in group impedance functions and the oscillatory nature of group efficiency.

Simplified Approaches

The finite element method and the other approaches mentioned are capable of dynamic analysis of pile groups but they are rather complex and the incorporation of nonlinearity, though desirable for large amplitudes, would make the analysis even more involved. On the other hand, experimental evidence of the more peculiar features of dynamic group interaction is lacking. In addition, not every situation justifies a very complex approach. Thus, it appears that some simplified approaches should also be investigated.

One simplified approach can be based on the limitation of compatibility between piles to pile heads. This approach gives results that are qualitatively similar to those obtained by the other methods.

Another simplified approach could utilize the concept of the equivalent pier. This concept, used with static loads for decades, is extremely simple because it replaces the group by a single equivalent pier (a composite pile) whose cross sectional properties are established similar to reinforced concrete. This concept was explored by Aboul-Ella (1) who obtained encouraging results. The equivalent pier concept is limited to closely spaced piles and cannot yield the peaks in impedance functions and the oscillatory character of group

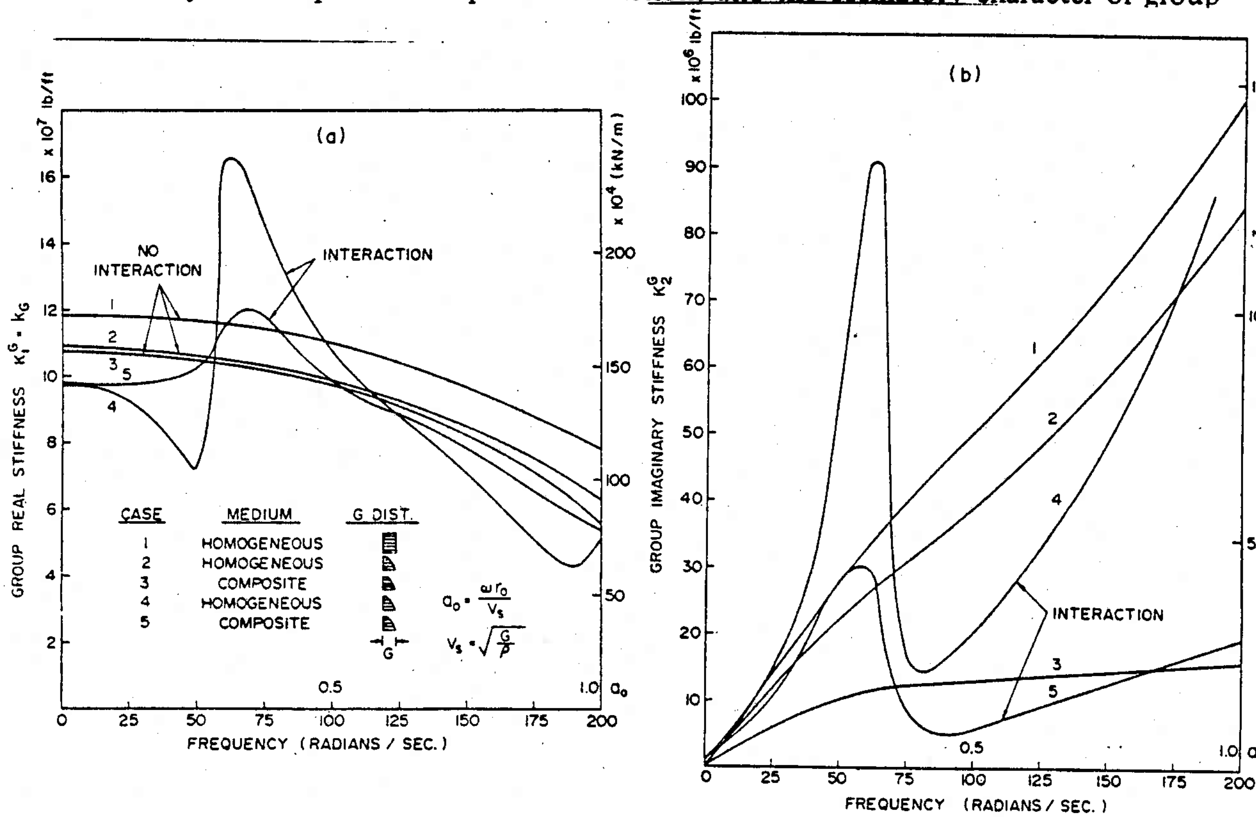

Fig. 12 Vertical Stiffness of Group of Four Concrete Piles vs. Frequency; (a) – Real Stiffness, (b) – Damping

efficiency with frequency, however.

For the experiment shown in Fig. 13, the equivalent pier gave a better prediction of damping than obtained ignoring pile interaction.

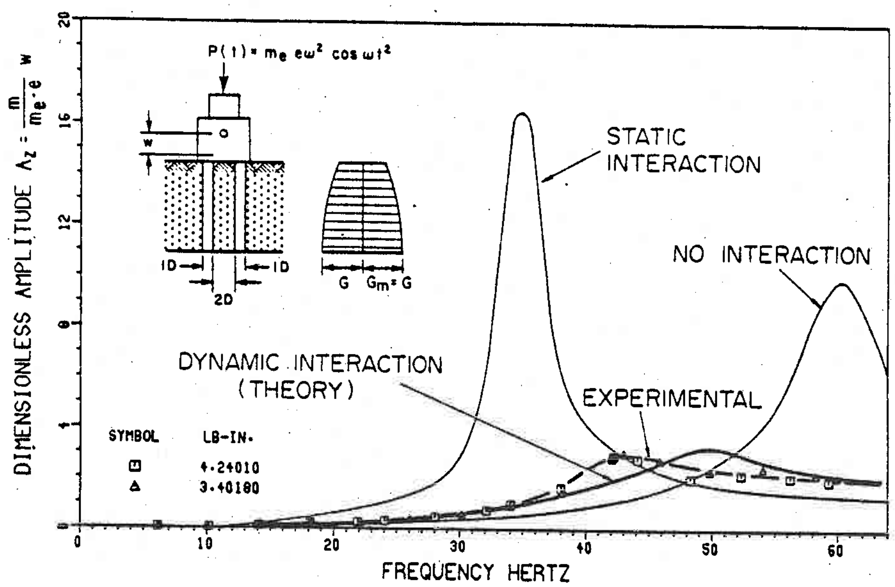

Fig. 13 Vertical Response of Model Pile Foundation and Its Comparison With Theoretical Predictions

CONCLUSIONS

The theoretical and experimental research into dynamic behaviour of piles and pile groups suggest the following conclusions:

1. Dynamic behaviour of piles is strongly affected by the variation of soil stiffness with depth and the lack of bond between the pile and soil.
2. Plane strain soil reactions facilitate dynamic analysis of piles and give satisfactory results.
3. Dynamic characteristics of pile groups differ considerably from static characteristics and are more frequency dependent than characteristics of single piles.
4. Stiffness and damping of pile groups can either be reduced by pile-soil-pile interaction or increased depending on frequency, spacing between piles and the weakened zone around the piles.
5. There is a need for experimental research into dynamic behaviour of pile groups and the examination of simplified approaches.

ACKNOWLEDGEMENTS

The study was supported by a grant-in-aid-of research from the Natural Sciences and Engineering Research Council of Canada.

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