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The primary purpose of this report is to document four pile analysis-related finite difference computer programs (COM62, PX4C3, MAKE, and BENT1) and a structural analysis program (BMCOL51), all developed at the University of Texas, Austin, Texas, under the guidance of Professors L. C. Reese and H. Matlock. These programs have been converted to run on the U. S. Army Engineer Waterways Experiment Station (WES) G-635 computer system. Basic theory to explain the

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20. ABSTRACT (Continued).

methods used in the computer programs is also included in the report. COM62 solves laterally loaded pile problems using an iterative scheme that considers nonlinear soil resistance versus pile movement curves. PX4C3 is a computer program written for the analysis of an axially loaded pile accounting for nonlinear soil properties. MAKE is a program that generates lateral soil resistance-pile movement curves from laboratory soil testing data based on predefined criteria. BENT1 analyzes group pile problems, again accounting for nonlinear soil behavior under both axial and lateral loads. BMCOL51 is a computer program based on the discrete element theory. Some of the uses of BMCOL51 can be in obtaining general solutions for linear beam-columns, moving load problems, beam on elastic foundation problems, variable beam-size problems, and buckling problems. Each of the five computer programs has been documented completely with a general introduction, listing of program, flow charts, guide for data input, and example problems with input-output data. Programs COM62, PX4C3, MAKE, and BENT1 run on the time-sharing mode while program BMCOL51 runs on the batch/card-in mode on the WES G-635 computer system.

PREFACE

The report presented herein documents five soil-structure interaction finite difference computer programs obtained from the University of Texas (UT), Austin, Texas. The computer programs are designed to analyze a wide variety of problems involving laterally and axially loaded single piles, group pile foundations, and complex beam-column structural members.

Professors L. C. Reese and H. Matlock, Civil Engineering Department, UT, are gratefully acknowledged for giving permission to use these computer programs developed under their guidance. Special thanks also go to the Center for Highway Research at UT for permission to use their reports referred to extensively in this documentation.

Funds for this work were authorized by the Lower Mississippi Valley Division (LMVD), Corps of Engineers, as part of the analysis support provided by the U. S. Army Engineer Waterways Experiment Station (WES) Automatic Data Processing Center (ADPC). Mr. D. R. Dressler, Technical Engineering Branch, LMVD, was the contact engineer and also reviewed the content and format of this report.

The assistance given by several people in the Computer Analysis Branch (CAB), ADPC, is greatly appreciated. These include: Mr. D. W. Walters for converting the programs from the CDC 6600 computer to the G-440 system; Messrs. H. W. Jones and R. L. Hall for their help in documenting the codes; and Miss A. M. Wade for her help in converting the programs from the G-440 to the G-635 system.

The work was accomplished during the period July 1972 through April 1974 under the immediate supervision of Messrs. J. B. Cheek, Jr., Chief of CAB, and H. H. Ulery, Chief of the Pavement Design Division, Soils and Pavements Laboratory (S&PL). General supervision was provided by Messrs. D. L. Neumann, Chief of ADPC, and J. P. Sale, Chief, S&PL. The report was prepared by Drs. N. Radhakrishnan, CAB, and F. Parker, Jr., Pavement Design Division, S&PL.

BG E. D. Peixotto, CE, and COL G. H. Hilt, CE, were Directors of WES during the course of the work and the preparation of this report. Mr. F. R. Brown was Technical Director.

CONTENTS

Page

PREFACE 1

CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT 4

PART I: INTRODUCTION 5

Need for Pile Analysis 5
Group Pile Behavior 5
Methods of Group Pile Analysis 6
Nonlinear Interaction Curves 7
Beam-Column Programs 7
Purpose and Scope 8

PART II: FINITE DIFFERENCE APPROXIMATIONS FOR LATERALLY LOADED PILES 9

Formulation of Finite Difference Approximations 9
Development of Equations of Bending for Laterally Loaded Pile 11
Formulation of Finite Difference Approximations for Equations of Bending of Laterally Loaded Piles 18
Solution of the Difference Equations 19

PART III: FINITE DIFFERENCE APPROXIMATIONS FOR AXIALLY LOADED PILES 26

Mechanics of Axial Load Transfer 26
Development of Difference Equations 28
Development of Mechanical Model and Equations for Model . . 32
Solution Procedure for Equations 32

PART IV: PILE GROUP THEORY 34

Coordinate Systems and Sign Conventions 34
Relations Between Foundation Movements and Pile-Head Movements 37
Relations Between Foundation Forces and Pile Reactions . . 39
Relations Between Pile-Head Movement and Pile Reaction 39
Equilibrium Equations 42
Computational Procedure for Solution of Equilibrium Equations 45

PART V: CRITERIA FOR DEVELOPING SOIL-PILE INTERACTION CURVES 50

Laterally Loaded Pile 50
Soil Criteria 51
Axially Loaded Pile 59
Criteria for Clay 60
Criteria for Sand 63
Summary 67

CONTENTS

	Page
PART VI: DISCRETE ELEMENT THEORY FOR BEAM-COLUMNS	68
Discrete Element Representation of the Response of a Simple Beam	69
Discrete Element Representation of the Response of a Generalized Beam-Column	73
Recursive Solution Technique	79
REFERENCES	82
APPENDIX A: NOTATION	A1
APPENDIX B: USER'S GUIDE FOR PROGRAM COM62	B1
General Introduction	B1
Program Listing	B3
Flow Charts	B11
Guide for Data Input	B13
Example Problems	B15
APPENDIX C: USER'S GUIDE FOR PROGRAM PX4C3	C1
General Introduction	C1
Program Listing	C3
Guide for Data Input	C9
Example Problems	C12
APPENDIX D: USER'S GUIDE FOR PROGRAM MAKE	D1
General Introduction	D1
Program Listing	D2
Guide for Data Input	D9
Example Problems	D13
APPENDIX E: USER'S GUIDE FOR PROGRAM BENT1	E1
General Introduction	E1
Flow Chart	E3
Glossary of Notation	E4
Program Listing	E7
Guide for Data Input	E25
Example Problems	E32
APPENDIX F: USER'S GUIDE FOR PROGRAM BMCOL51	F1
General Introduction	F1
Program Listing	F6
Flow Chart	F30
Guide for Data Input	F31
Example Problems	F37

CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U. S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	By	To Obtain
inches	2.54	centimeters
feet	0.3048	meters
kips (mass)	453.5924	kilograms
pounds per cubic inch	27,679.90	kilograms per cubic meter
pounds (force)	4.448222	newtons
pounds per inch	175.1268	newtons per meter
pounds per square inch	6894.757	pascals
pounds per square foot	47.88026	pascals
inch-pounds	0.1129848	newton-meters
inch-kips	112.9848	newton-meters
inch-pounds/inch	4.448222	newton-meters/meter
degrees	0.01745329	radians

BACKGROUND THEORY AND DOCUMENTATION OF FIVE UNIVERSITY OF TEXAS SOIL-STRUCTURE INTERACTION COMPUTER PROGRAMS

PART I: INTRODUCTION

Need for Pile Analysis

1. Pile foundations are frequently used for structures when the soil immediately below the base will not provide adequate bearing capacity. The purpose of the piles is to transfer the load from the structure to soil strata which can sustain the applied loads. The Lower Mississippi Valley Division is interested in the analysis and design of pile foundations for a variety of structures.

Group Pile Behavior

2. If the structure is supported on vertical piles and if all loads from the structure are also vertical, then the loads transmitted to the piles will all be principally axial. If some horizontal component of load is present, a lateral force will also be transmitted to the piles. If some of the piles are battered, an axial and lateral force will be transmitted to the piles regardless of the direction of the applied load. For most structures both horizontal and vertical components of load are present. In some instances, the horizontal component may be small and can be neglected. However, for many structures, such as offshore drilling platforms, tall bridge bents, or hydraulic structures, wind and wave action will produce significant horizontal forces. Therefore, for a complete analysis of a pile foundation, the behavior of the piles must be analyzed for both lateral and axial loads.

3. When a pile is subjected to any load, deformation will occur. For small loads, the deformation may be proportional to the load; however, the load-deformation relationship becomes increasingly nonlinear as the load increases. This nonlinear load-deformation relationship is

principally due to the nonlinear load-deformation characteristics of the soil.

Methods of Group Pile Analysis

Hrennikoff's method

4. One of the most popular methods of analysis of a group pile foundation is due to Hrennikoff.[1] Implicit in this method of analysis is the assumption that the load-deformation relationships for soil are linear. In other words, the soil is represented by a series of linear springs in the analysis. Since the soil behavior under load is generally nonlinear, this method of analysis poses some limitations. An excellent review and comparison of various methods of analyses of group pile foundations is given by Robertson.[2]

University of Texas (UT) method

5. In the UT method of group pile analysis, nonlinear deflection-reaction curves are used to depict soil behavior. The axial pile-soil interaction is obtained from a nonlinear load-deformation curve. The lateral interaction is specified by a set of nonlinear deflection-reaction curves. These curves, referred to as p-y curves, establish the relationship between the deflection of the pile and the reaction exerted by the soil. The equilibrium position for the pile-supported structure is found by an iterative process that ensures the compatibility between the behavior of soil and the piles and between the piles and the structure.

Advantages of the UT method

6. The method of Hrennikoff and the UT method are somewhat similar in their approach. However, the UT method introduces two major improvements. Probably the most important of these is the use of nonlinear pile movement-soil resistance relationships. The second major improvement is that it permits the rotational stiffness of the structure or the pile-head restraint to be included in the analysis (Hrennikoff's method allows only for completely fixed or hinged conditions).

Nonlinear Interaction Curves

7. If families of curves that will simulate the nonlinear interaction between the pile and surrounding soil are available, existing procedures for numerical computation can be used to predict the response of individual piles. The response of individual piles may then be combined to predict the behavior of a foundation supported by these piles. This is the basis of the UT method of analysis. A detailed knowledge of the behavior of the foundation and of the individual piles will allow a superior design, which will usually be more economical than is possible with a less rational procedure.

8. The family of curves describing the behavior of the soil around an axially loaded pile will give axial soil reaction versus axial pile movement for a number of locations along the pile. For a given location, a curve would show the axial force per unit area transferred to the soil for a given axial movement of the pile.

9. Similarly, the family of curves describing the behavior of the soil around a laterally loaded pile will give lateral soil reaction versus lateral pile movement for a number of locations along the pile. For a given location, a curve would show the lateral force per unit length transferred to the soil for a given lateral movement.

10. Unless procedures are available to develop soil interaction curves based on available data, the UT method of analysis loses one of its principal advantages. There are semi-empirical procedures available for predicting the interaction curves for both axial and lateral behavior of piles. However, these procedures must be used with caution; the applicability of these techniques for the problem in hand must be fully examined before use. Some of the procedures used in the computer programs documented in this report are summarized in Part V.

Beam-Column Programs

11. A series of computer programs, developed under the guidance of Prof. H. Matlock, are available at UT at Austin to solve structural

and soil-structure interaction problems. These programs are very versatile and can be used for analysis of a variety of problems. One of the earlier beam-column programs, BMCOL51, developed by Matlock and Taylor,[3] is documented in this report. BMCOL51 is a discrete element program and can be used for obtaining general solutions for linear beam-columns, movable load problems, beam on elastic foundation problems, variable beam size problems, buckling problems, etc.

Purpose and Scope

12. The primary purpose of this report is to document four pile analysis-related finite difference computer programs (PX4C3, COM62, BENT1, and MAKE) and a structural beam-column program (BMCOL51), all developed at UT under the guidance of Professors L. C. Reese and H. Matlock. The subject area covered is rich in technical literature, and no attempt is made here to discuss the details of the methods of analyses. However, enough theory to explain the basis of the methods used in the computer programs is presented.

13. Finite difference approximations for laterally loaded piles (basis for program COM62) are presented in Part II and for axially loaded piles (basis for program PX4C3) in Part III. The UT pile group theory (basis for program BENT1) is discussed in Part IV. Some criteria for mathematically describing soil-structure interaction (basis for program MAKE) is presented in Part V. Part VI explains the discrete element theory used in BMCOL51 program. All five computer programs described in the report are documented with a general introduction, listing of program, flow charts, input data guide, and example problems with input-output data in Appendixes B, C, D, E, and F.

14. Most of the material presented herein has been covered in a number of earlier reports from the Center of Highway Research as well as other departments at UT. This report brings together the material needed to appreciate the power and limitations of the five computer programs selected. Liberal use of subject matter from References 2, 3, 15, 16, 22, and 31 and lecture notes on "Soil-Structure Interaction Courses" of Professors H. Matlock and L. C. Reese is gratefully acknowledged.

PART II: FINITE DIFFERENCE APPROXIMATIONS FOR LATERALLY LOADED PILES

15. The computer code COM62 utilizes central difference approximations for describing the load-deformation response of laterally loaded piles. In addition, the code BENT1 (that predicts the load-deformation response of a pile-supported foundation) has COM62 as a subroutine for predicting the lateral load-deformation response of the individual piles in the foundation. In this part, central difference approximations for describing the elastic curve of a laterally loaded pile will be derived and used in formulating a set of simultaneous equations for describing the load-deformation response of a laterally loaded pile.

Formulation of Finite Difference Approximations

16. The finite difference approach to the solution of laterally loaded piles was first suggested by Gleser.[4] This idea was further extended by a number of investigators including Reese and Matlock.[5,6]

17. The first step in the formulation is the derivation of the central difference approximations for the elastic curve (Figure 1). It can be seen from this figure that the slope of the curve at sta i may be approximated by the secant drawn through the points on the curve of the two adjacent stations. Mathematically this step is expressed as

$$\left(\frac{dy}{dx}\right)_i \approx \frac{y_{i+1} - y_{i-1}}{2h} \tag{1}$$

where h* denotes the increment length. For higher derivatives, the process could be repeated by taking simple differences and dividing by $2h$ each time. However, to keep the system more compact, temporary sta j and k are considered and the slopes at these points computed on the basis of the deflection of the station on either side. The second

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* For convenience, symbols and unusual abbreviations are listed and defined in the Notation (Appendix A).

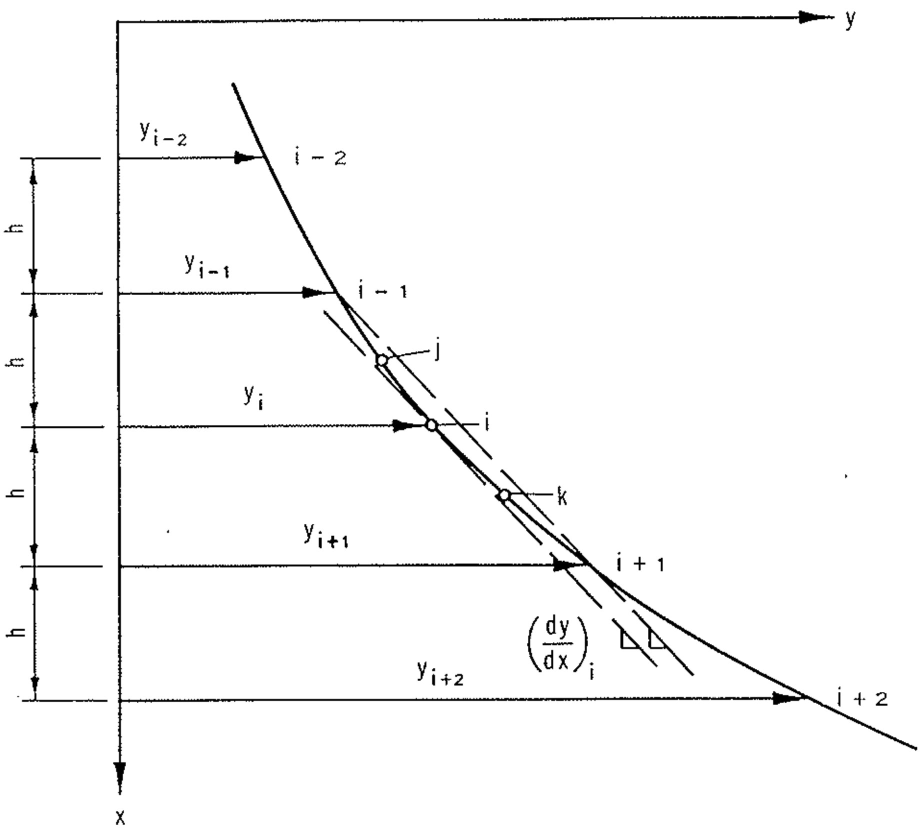


Figure 1. Geometric basis for central-difference approximations

derivative for each permanent station is then written as the difference between these slopes divided by one increment length in the following equation:

$$\left(\frac{d^2y}{dx^2}\right)_i = \frac{\left(\frac{dy}{dx}\right)_k - \left(\frac{dy}{dx}\right)_j}{h}$$

$$= \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} \tag{2}$$

Proceeding in a similar way, the third derivative is expressed as

$$\left(\frac{d^3y}{dx^3}\right) = \frac{\left(\frac{d^2y}{dx^2}\right)_{i+1} - \left(\frac{d^2y}{dx^2}\right)_{i-1}}{2h}$$

$$= \frac{y_{i+2} - 2y_{i+1} + 2y_{i-1} - y_{i-2}}{2h^3} \tag{3}$$

and the fourth derivative as

$$\left(\frac{d^4y}{dx^4}\right)_i = \frac{\left(\frac{d^3y}{dx^3}\right)_k - \left(\frac{d^3y}{dx^3}\right)_j}{h}$$

$$= \frac{y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}}{h^4} \tag{4}$$

Development of Equations of Bending for Laterally Loaded Pile

18. The second step in the formulation is the derivation of the differential equations for bending of a laterally loaded pile, and the substitution of the central difference approximations for the exact derivatives in the resulting differential equations. The differential equations are derived by considering an element of the pile (Figure 2). The sign of all forces, deflections, and slopes shown are positive. It should also be noted that the axial load is constant over the length of the pile. For piles this assumption is not consistent with observed behavior, since it is known that some of the applied axial load is transferred to the soil by skin friction along the shaft. The validity of this assumption is based on the fact that the errors introduced will be insignificant. Considering the problem from a physical standpoint,

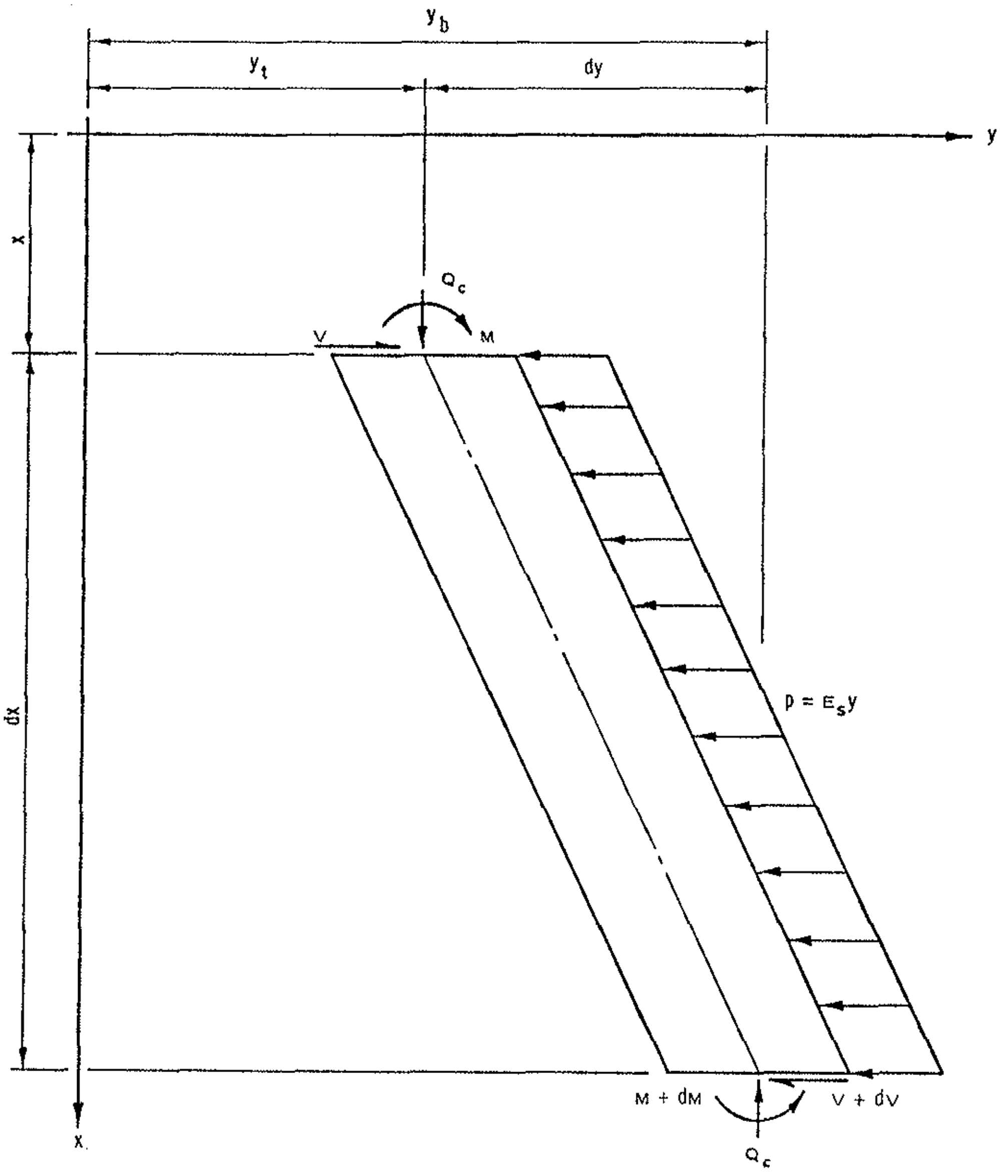


Figure 2. Generalized beam-column element

it is known that for most cases the axial load transferred to the soil increases with depth. This, plus the fact that any lateral movement will cause a decrease in axial load transfer, leads to the conclusion that the axial load removed by the skin friction in the upper portion of the pile is small. Since the maximum deflection and moment occur in the top portion of the pile, and since it is the deflection of the pile top which is of interest, the assumption of constant axial load will not significantly affect the results of interest.

19. The reason for making the assumption of axial load being constant on the top of the pile is one of convenience. The addition of a variable axial load could have been handled analytically, but the effort required to obtain a solution would not be warranted because of uncertainties involved in obtaining the nature of the variation.

20. Referring to Figure 2, the equilibrium equations for the element may be written as

$$\frac{dM}{dx} - V + Q_c \frac{dy}{dx} = 0 \tag{5}$$

and

$$\frac{dV}{dx} = -p = -E_s y \tag{6}$$

where

M = bending moment

x = distance along axis of pile

V = shear

Q_c = in pile axial load constant

y = lateral deflection

p = lateral soil reaction per unit length

E_s = soil modulus; lateral soil reaction (p) divided by lateral deflection (y)

By combining Equations 5 and 6 and differentiating, the following equation is obtained:

$$\frac{d^2M}{dx^2} + E_s y + Q_c \frac{d^2y}{dx^2} = 0 \tag{7}$$

The equation for shear is written as

$$V = \frac{dM}{dx} + Q_c \frac{dy}{dx} \tag{8}$$

Consider that the deformation of the pile is caused only by the bending moment. The following expression for moment can then be written:

$$M = EI \frac{d^2y}{dx^2} = R \frac{d^2y}{dx^2} \tag{9}$$

where

E = modulus of elasticity of the pile

I = moment of inertia of pile section

R = EI (flexural rigidity)

Equation 9 is the basic expression for bending which states that the bending moment in the pile is equal to the product of the curvature of the elastic curve and the stiffness of the section.

21. At this point, the mechanics of the transfer of lateral load from a pile to the surrounding soil will be considered before proceeding further with the development of the finite difference equations of bending for a pile. In Equation 6 this load transfer is represented by the expression $p = E_s y$.

22. When a lateral load is applied to the top of a pile, the load is transferred to the soil surrounding the pile as illustrated in Figure 3. A thin slice through the pile and surrounding soil is shown at a depth of x_1 below the ground surface. Before any lateral load is applied to the pile, the pressure distribution on the pile will be similar to Figure 3b. For this condition, the resultant force on the pile, obtained by integrating the pressure around the segment, will be zero. If, however, the pile is given a lateral deflection of y_1 at depth x_1 , the pressure distribution will be similar to Figure 3c. The integration of the pressure around the segment, for this condition, will yield a resultant force P_1 per unit length of pile, as shown in the above figure. The same procedure may be applied for a series of deflections, resulting in a corresponding series of forces which may be combined into a p-y curve. In a similar manner, p-y curves for any depth may be defined, resulting in a set of curves (Figure 4).

23. Implicit in the development thus far are the assumptions that the soil pressure is a linear function of deflection, the relationship being defined by the constant E_s , and that the pressure at a

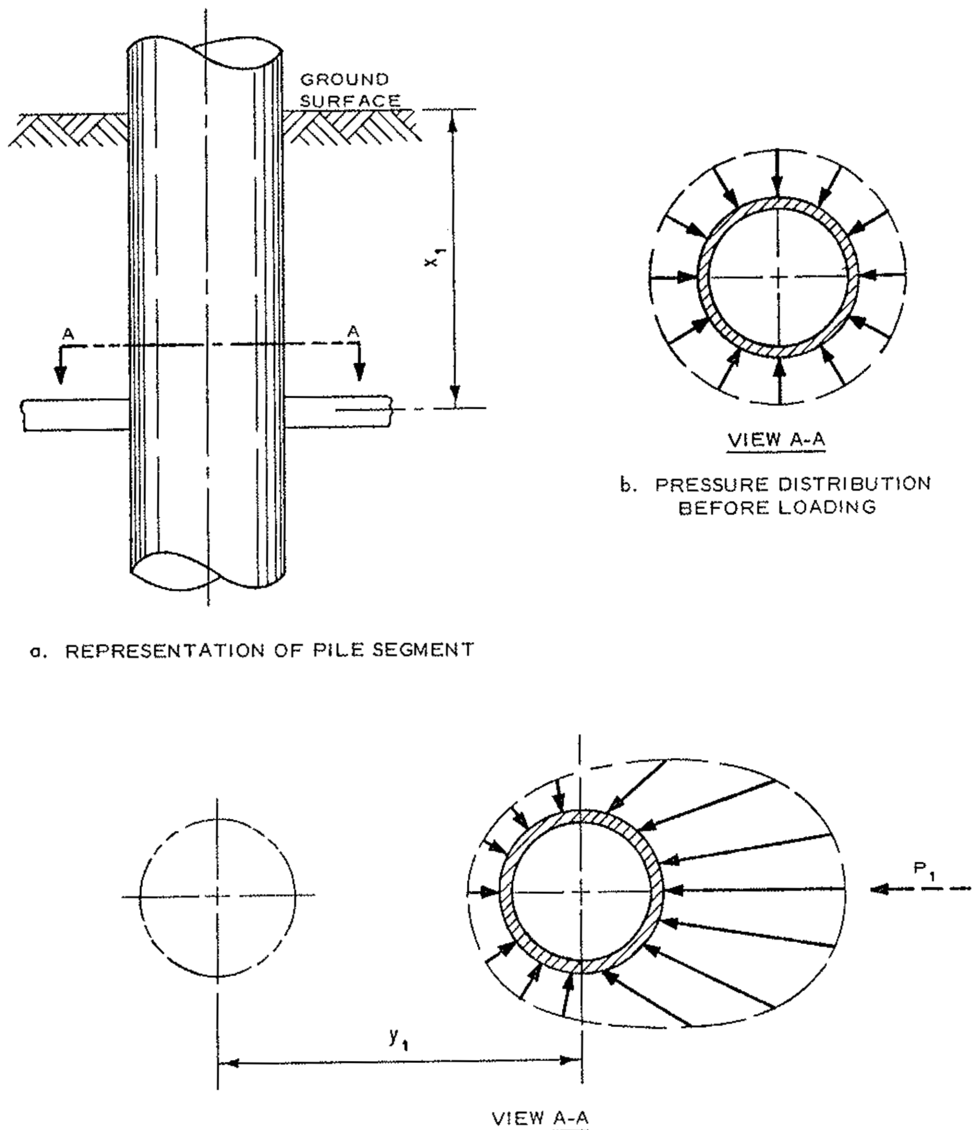


Figure 3. Illustration of lateral load transfer

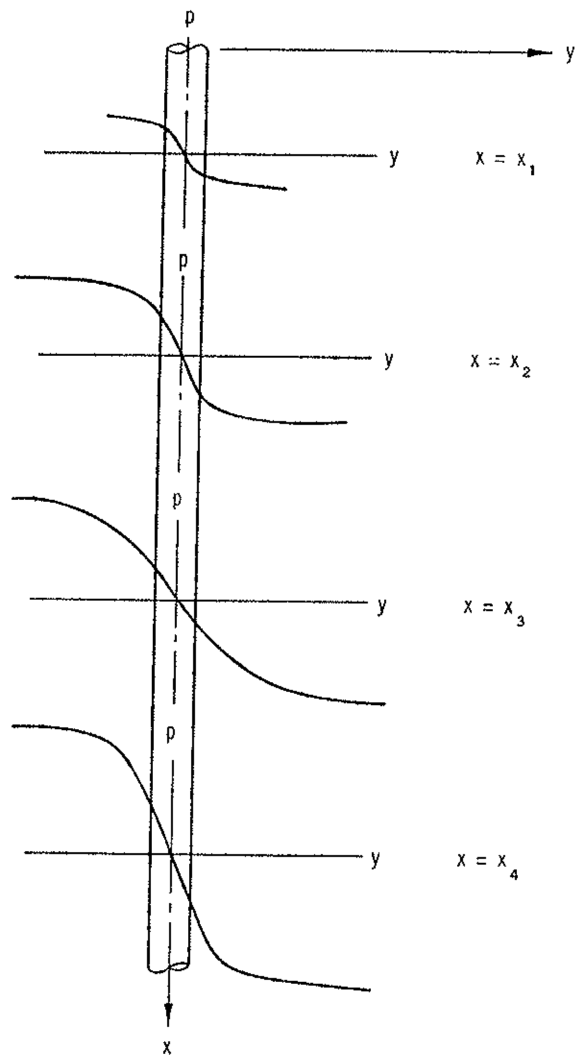


Figure 4. Family of p-y curves

particular point is independent of the deflections at all other points on the pile. Nonlinear soil behavior can be handled by relying on repeated applications of elastic theory where the constant coefficient of soil reaction is replaced by a secant modulus value. Figure 5 illustrates the secant modulus concept.

24. The second assumption leads to the representation of the soil by a set of independent springs as proposed by Winkler[7] in 1867. If the effects of the soil pressure is considered to be concentrated at a finite number of points along the pile then, the pile-soil system can be represented by the model shown in Figure 6. This model is compatible with the finite difference equations which will be developed.

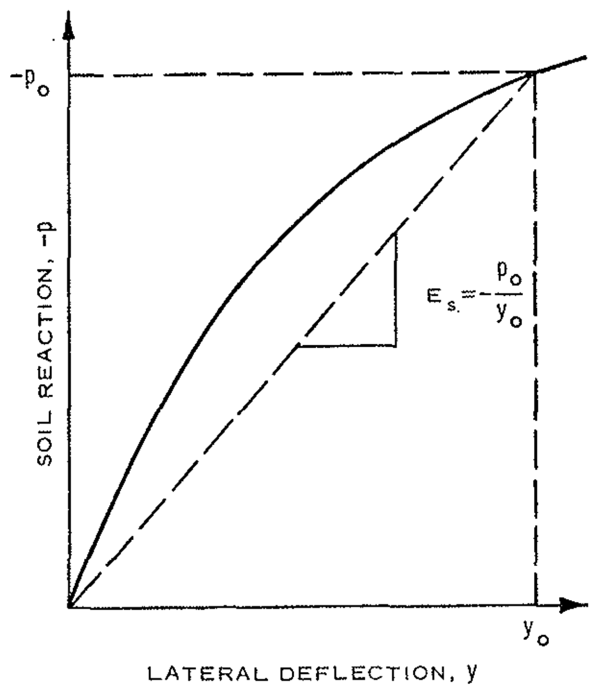


Figure 5. Definition of secant modulus

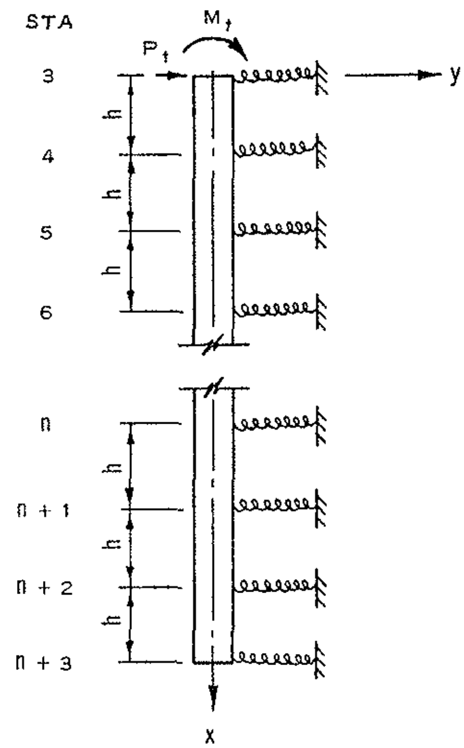


Figure 6. Model of a laterally loaded pile

Formulation of Finite Difference Approximations for Equations of Bending of Laterally Loaded Piles

25. Equations 7, 8, and 9 may be written in finite difference by using the central-difference approximations for the first and second of the elastic curve. The equations will be written for a general point referred to as sta i. Station numbering increases from top to bottom of piles. The equations obtained for sta i are as follows:

$$y_{i+2}(R_{i+1}) + y_{i+1}\left(-2R_{i+1} - 2R_i + Q_c h^2\right) + y_i\left(R_{i+1} + 4R_i\right.$$

$$\left. + R_{i-1} - 2Q_c h^2 + E_{si} h^4\right) + y_{i-1}\left(-2R_i - 2R_{i-1} + Q_c h^2\right)$$

$$+ y_{i-2}(R_{i-1}) = 0 \qquad (10)$$

$$V_i = \frac{1}{2h^3}\left[y_{i+2}(R_{i+1}) + y_{i+1}\left(-2R_{i+1} + Q_c h^2\right) + y_i(R_{i+1} - R_{i-1})\right.$$

$$\left. + y_{i-1}\left(-Q_c h^2\right) + y_{i-2}(-R_{i-1})\right] \qquad (11)$$

$$M_i = R_i \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} \qquad (12)$$

26. In the development of the equations, no consideration was given to the assumptions regarding the variation in pile bending stiffness (EI = R). For the case of pure bending and constant bending stiffness, the second derivative of moment is usually written as

$$\frac{d^2M}{dx^2} = EI \frac{d^4y}{dx^4} \qquad (13)$$

For the case of pure bending and a variable bending stiffness, the second derivative of moment is expressed as

$$\frac{d^2M}{dx^2} = EI\,\frac{d^4y}{dx^4} + \frac{2d(EI)}{dx}\,\frac{d^3y}{dx^3} + \frac{d^2(EI)}{dx^2}\,\frac{d^2y}{dx^2} \tag{14}$$

However, in formulating the difference equations, the assumption was made that the moment was a smooth continuous function of x and that the second derivative of moment could be approximated by the expression

$$\frac{d^2M}{dx^2} \approx \frac{M_{i+1} - 2M_i + M_{i-1}}{2h} \tag{15}$$

where M_{i+1}, M_i, and M_{i-1} are the moment at joints $i+1$, i, and $i-1$, respectively. The use of Equation 15 gives the same expression as does Equation 13 but, for a variable stiffness, is a somewhat cruder approximation than Equation 14. However, it permits the bending stiffness to vary from station to station since Equation 12 can be substituted directly into Equation 15.

Solution of the Difference Equations

27. The final step is the formulation of a set of simultaneous equations which when solved yield the deflected shape of the pile. The solution requires the application of four boundary conditions since Equation 7 is actually a fourth order differential equation in terms of the dependent variable y. With values of deflection known, moment, shear, and soil reaction may be obtained for any location along the pile by back substitution of appropriate values of deflection into appropriate equations.

28. The pile is divided into n increments of length h (Figure 7). In addition, two fictitious increments are added to the top and bottom of the pile. The four fictitious stations are added for formulating the set of equations, but they will not appear in the solution or influence the results. The coordinate system and numbering system used is illustrated in the same figure.

29. The procedure used is to write Equations 10, 11, and 12 about sta $n+3$. This results in three equations involving five unknown

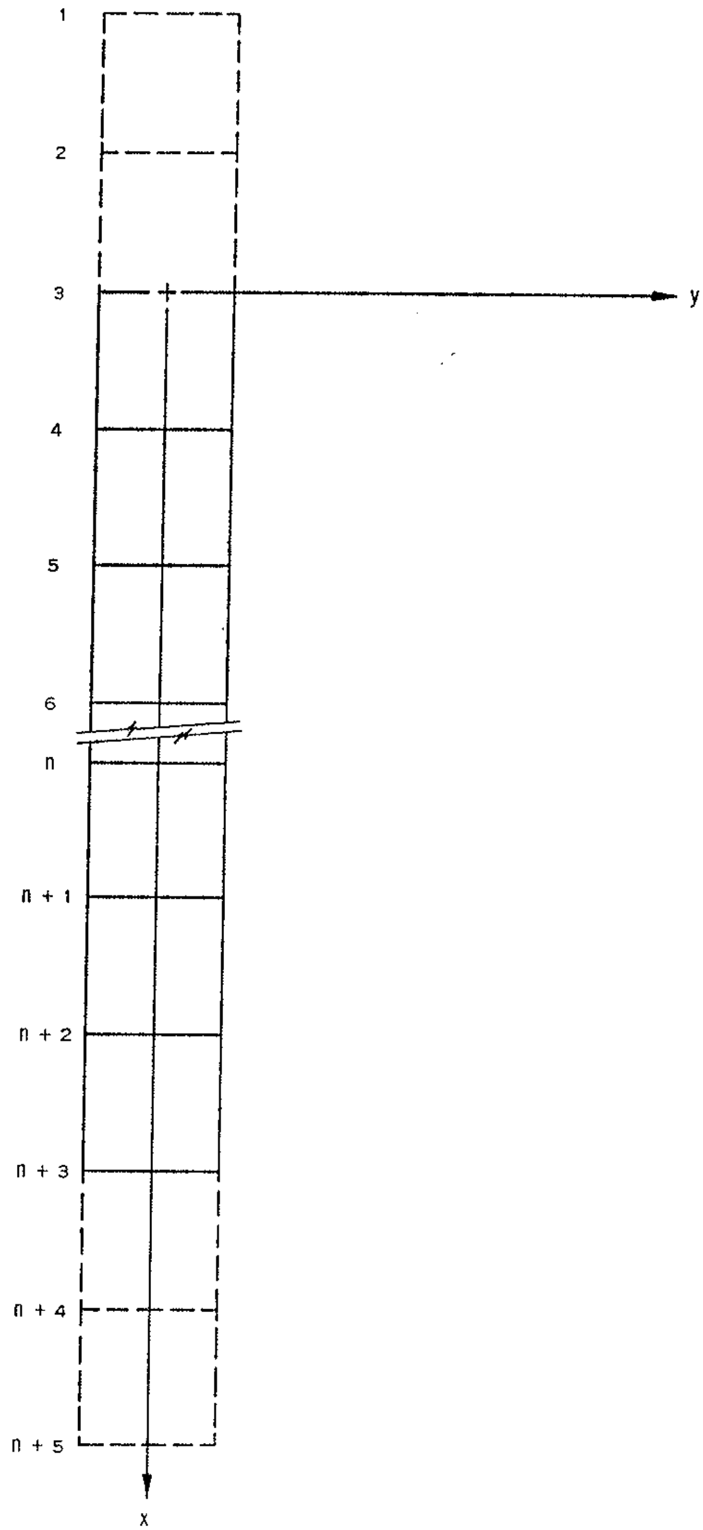


Figure 7. Finite difference representative of pile

deflections (y_{n+5} , y_{n+4} , y_{n+3} , y_{n+2} , y_{n+1}). Two boundary conditions, $V_{n+3} = 0$ and $M_{n+3} = 0$, are applied at sta n+3. The deflections for the fictitious sta n+4 and n+5 are eliminated from the three equations, and the deflection for sta n+3 is found in terms of the deflection at sta n+2 and n+3. The equation obtained may be written as

$$y_{n+3} = a_{n+3}y_{n+2} - b_{n+3}y_{n+1} \tag{16}$$

where

$$a_{n+3} = \frac{4R_{n+2} - 2Q_c h^2}{2R_{n+2} - 2Q_c h^2 + E_{s(n+3)}h^4} \tag{17}$$

and

$$b_{n+3} = \frac{2R_{n+2}}{2R_{n+2} - 2Q_c h^2 + E_{s(n+3)}h^4} \tag{18}$$

Equation 10, written for sta n+2, can be combined with Equations 11 and 12 for sta n+3 and with Equation 16 to determine the deflection for sta n+2. The deflection y_{n+2} is found in terms of the deflection of sta n+1 and n. The equation obtained is as follows:

$$y_{n+2} = a_{n+2}y_{n+1} - b_{n+2}y_n \tag{19}$$

where

$$a_{n+2} = \frac{2R_{n+1} + \left(2R_{n+2} - Q_c h^2\right)(i - b_{n+3})}{R_{n+1} + \left(2R_{n+2} - Q_c h^2\right)(2 - a_{n+3}) + E_{s(n+2)}h^4} \tag{20}$$

and

$$b_{n+2} = \frac{R_{n+1}}{R_{n+1} + \left(2R_{n+2} - Q_c h^2\right)(2 - a_{n+3}) + E_{s(n+2)}h^4} \tag{21}$$

The deflection for sta n+1 may be found in a similar manner. From sta n+1 to the top of the pile the expressions for the deflection have the same form. The general form of the equation is as follows:

$$y_i = a_i y_{i-1} - b_i y_{i-2} \tag{22}$$

where

$$a_i = \frac{2R_{i-1} + R_i(2 - 2b_{i+1}) + R_{i+1}(a_{1+2}b_{1+1} - 2b_{i+1}) - Q_c h^2(1 - b_{i+1})}{c_i} \tag{23}$$

$$b_i = \frac{R_{i-1}}{c_i} \tag{24}$$

and

$$c_i = R_{i-1} + R_i(4 - 2a_{i+1}) + R_{i+1}(a_{i+1}a_{i+2} - b_{i+2} - 2b_{i+1} + 1) - Q_c h^2(2 - a_{i+1}) + E_{si}h^4 \tag{25}$$

The terms, a_i , b_i , and c_i , are recursive coefficients and are defined for all stations along the pile during the solution procedure.

30. With the general expression, the deflection of each station may be expressed as a function of the deflection of the two stations immediately above it. If the deflections for sta 3, 4, and 5 are written, a set of three equations involving five unknown deflections will be obtained. If two boundary conditions are introduced, the deflections for the fictitious stations may be eliminated and the equations solved for the deflections. Once the deflections for sta 3 and 4 are found, the deflections for the remainder of the pile may be obtained by back substitution into the equations derived for the deflection of a station in terms of the deflection of the two stations directly above it.

31. The expressions obtained for y_3 and y_4 will depend on the boundary conditions applied to the top of the pile. Three sets of

boundary conditions are used resulting in three sets of equations.

32. For the first case, the following boundary conditions are applied:

$$M_3 = M_t \tag{26}$$

$$V_3 = P_t \tag{27}$$

where M_t and P_t are the moment and lateral load, respectively, applied to the top of the pile. The application of these boundary conditions results in the following expressions for y_3 and y_4 :

$$y_3 = \left\{\lambda_1 \left[R_4(2a_5b_4 - 4b_4) + R_3(2 - 2b_4) + 2Q_c h^2 b_4\right] + \lambda_2 \nu_2\right\} \Big/ \left\{\nu_1 \left[R_3(2b_4 - 2) + R_4(4b_4 - 2a_5b_4) - 2Q_c h^2 b_4\right] + \nu_2 \left[R_3(4 - 2a_4) + R_4(2a_4a_5 - 2b_5 - 4a_4 + 2) + Q_c h^2(-2 + 2a_4) + E_{s(3)} h^4\right]\right\} \tag{28}$$

$$y_4 = y_4 \left(a_4 - \frac{B_4 \nu_1}{\nu_2}\right) - \frac{b_4 \lambda_1}{\nu_2} \tag{29}$$

where the boundary condition coefficients are defined as follows:

$$\lambda_1 = \frac{M_t h^2}{R_3} \tag{30}$$

$$\lambda_2 = 2P_t h^3 \tag{31}$$

$$\nu_1 = 2 - a_4 \tag{32}$$

$$\nu_2 = 1 - b_4 \tag{33}$$

33. The second set of boundary conditions applied is as follows:

$$V_3 = P_t \tag{27 bis}$$

$$\left(\frac{dy}{dx}\right)_3 = \frac{y_4 - y_2}{2h} = \Omega_t \tag{34}$$

where Ω_t is the slope of the pile top. These boundary conditions result in the following expressions for y_3 and y_4 :

$$y_3 = \left\{\lambda_2 \ (1 + b_4) + \lambda_3 \left[2R_4(2b_4 - a_5b_4) + 2R_3(b_4 - 1) - 2Q_c h^2 b_4\right]\right\} \Big/ \left\{2R_4 \left[a_4a_5 - b_5 - b_4b_5 - 2a_4 + 1 + b_4\right] + 4R_3 \ (1 - a_4 + b_4) + 2Q_c h^2(a_4 - b_4 - 1) + E_{s(3)}h^4\right\} \tag{35}$$

$$y_4 = y_3 \left(\frac{a_4}{1 + b_4}\right) + \frac{b_4\lambda_3}{1 + b_4} \tag{36}$$

where

$$\lambda_3 = 2\Omega_t h \tag{37}$$

34. The third set of boundary conditions applied is as follows:

$$V_3 = P_t \tag{27 bis}$$

$$\frac{M_3}{\Omega_3} = \frac{M_t}{\Omega_t} \tag{38}$$

These boundary conditions result in the following expressions for y_3 and y_4 :

$$y_3 = \lambda_2 \left[1 - b_4 + \lambda_4(1 + b_4)\right] \bigg/ \Big\{ 2\lambda_4(2R_3 + 2R_3b_4$$

$$- 2R_3a_4 + R_4a_4a_5 - R_4b_4b_5 - 2R_4a_4$$

$$+ R_4 + R_4b_4) + 2R_4(a_4a_5 - 2a_5b_4 - b_5$$

$$+ b_4b_5 - 2a_4 + 3b_4 + 1) + 2Q_ch^2(a_4 - b_4 - 1$$

$$+ a_4\lambda_4 - \lambda_4 - b_4\lambda_4) + E_{s3}h^4 \left[1 - b_4 + \lambda_4 (1 + b_4)\right] \Big\} \quad (39)$$

$$Y_4 = \left[a_4 - \frac{b_4(2 - a_4 + a_4\lambda_4)}{(1 + \lambda_4 - b_4 + b_4\lambda_4)}\right] y_3 \quad (40)$$

where

$$\lambda_4 = \frac{M_t}{\Omega_t}\left(\frac{h}{2R_3}\right) \quad (41)$$

35. The values of y_3 and y_4 are used to begin the back substitution procedure to calculate deflections for the remainder of the stations along the pile. With the values of deflection thus established, values of moment, shear, and soil reaction may be computed for any station along the pile.

PART III: *FINITE DIFFERENCE APPROXIMATIONS* FOR AXIALLY LOADED PILES

36. The computer code PX4C3 utilizes finite difference approximations for describing the load-deformation response of axially loaded piles. In addition, the code BENT1 (that predicts the load-deformation response of a foundation supported by a group of piles) requires the top load deformation curves of the individual piles in the foundation which is the particular response that is computed by PX4C3.

Mechanics of Axial Load Transfer

37. An axial load applied to the top of a pile is resisted by the shearing resistance developed along the shaft of the pile and the pressure on the base of the pile. The transfer of load from the pile to the soil is illustrated in Figure 8[8] and may be stated mathematically by the equation

$$Q_t = \int_{x=o}^{x=L} F \, dx + Q_b \tag{42}$$

where

Q_t = axial load applied to the top of a pile

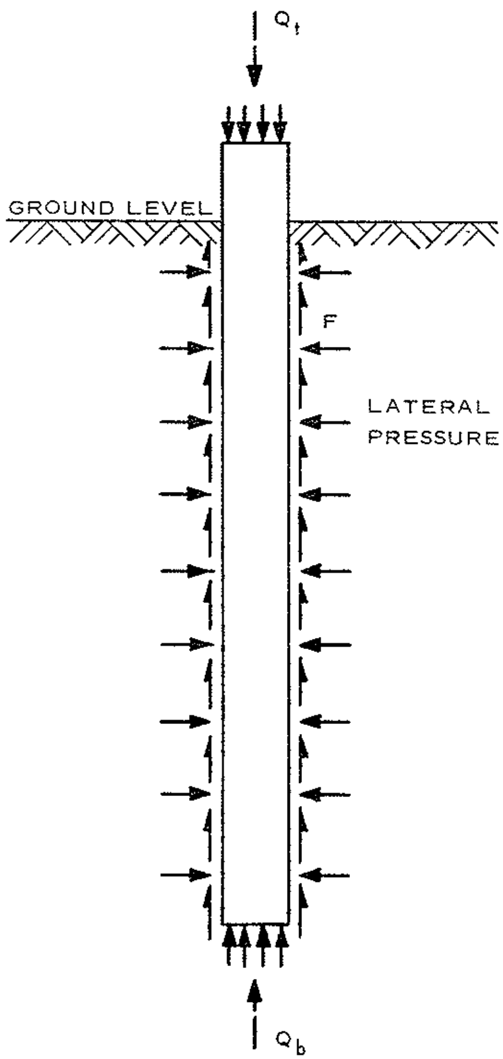
L = length of pile

F = shear force per unit length transferred to the soil as a function of the location along a pile

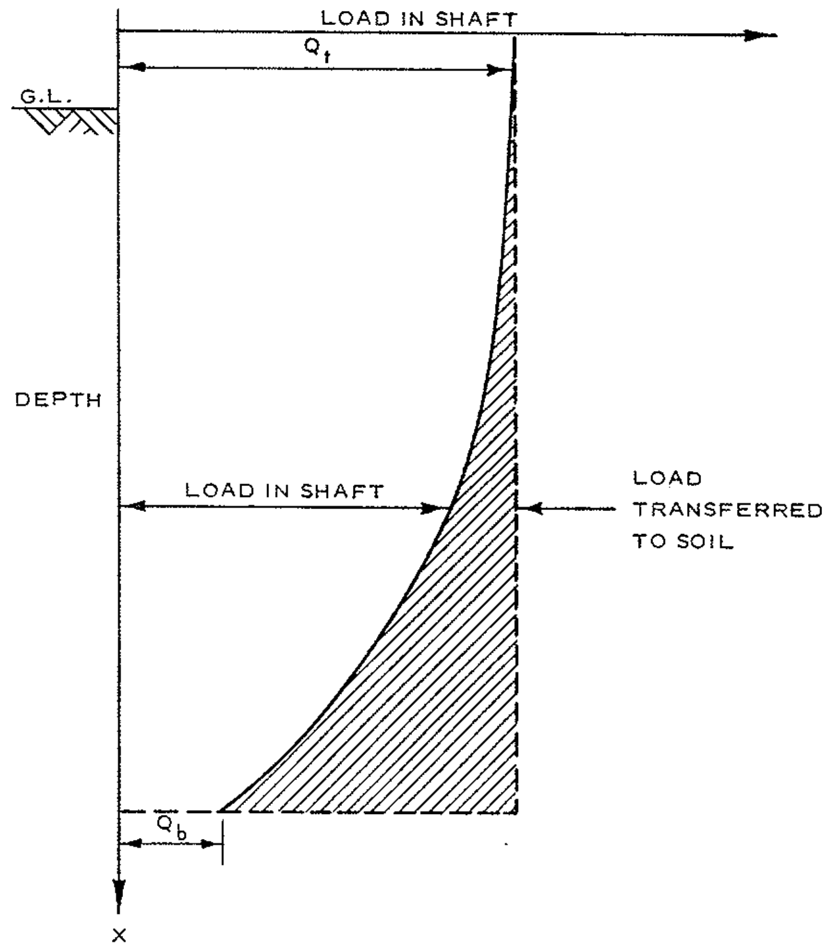
x = distance along axis of pile

Q_b = load due to the normal pressure on the base of a pile

This equation involves only statics, and its solution will only assure that the forces on the pile are in equilibrium. It provides no insight into the deformation pattern that is necessary to produce the base pressure and shear transfer along the shaft for equilibrium. For the ultimate strength approach, this equation is sufficient since the deformations are not considered, and the assumption is made that the maximum base pressure and maximum shear transfer occur simultaneously. If,



a. FORCES ACTING ON A PILE



b. TYPICAL CURVE FOR LOAD DISTRIBUTION

Figure 8. Illustration of axial load transfer in a pile

however, the load-deformation behavior of the pile is to be considered, the compatibility between loads and deformations must be considered. To represent this compatibility condition, another mathematical expression must be formulated relating load and deformation.

Development of Difference Equations

38. The derivation of an analytical expression for this purpose is suggested by Seed and Reese[9] and expanded by Reese.[10] Considering a segment of an axially loaded pile as shown in Figure 9, the expression

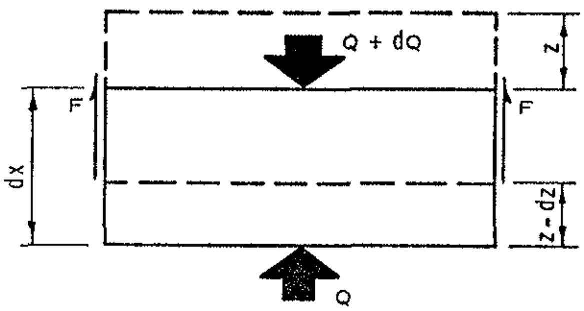


Figure 9. Element from an axially loaded pile

for the strain in the pile at depth x is given by

$$\frac{dz}{dx} = \frac{Q}{EA} \tag{43}$$

where

z = axial movement of pile

Q = axial load in pile

A = cross-sectional area of pile

This equation may be rearranged to yield

$$Q = EA\,\frac{dz}{dx} \tag{44}$$

Differentiation of Equation 44 with respect to x , assuming EA constant, yields

$$\frac{dQ}{dx} = EA \frac{d^2 z}{dx^2} \tag{45}$$

Summing forces on the pile segment, shown in Figure 9, yields the equilibrium expression

$$\frac{dQ}{dx} = F \tag{46}$$

The shear force per unit area is defined as

$$f = \frac{F}{C} \tag{47}$$

where

f = shear force per unit area transferred to the soil as a function of the location along a pile

C = pile circumference

Equation 46 may now be written as

$$\frac{dQ}{dx} = fC \tag{48}$$

If ψ is a function which relates the shear stress to the relative deflection between the pile, and soil so that

$$f = \psi z \tag{49}$$

then Equation 48 may be written as

$$\frac{dQ}{dx} = \psi z C \tag{50}$$

Equations 45 and 50 may be equated for $\frac{dQ}{dx}$ yielding

$$EA \frac{d^2 z}{dx^2} = \psi z C \tag{51}$$

which is the desired compatibility expression. To obtain a solution for

Equation 51, the function ψ and two boundary conditions must be known. For realistic problems, considering nonlinear soil behavior, the function ψ usually cannot be defined analytically, and a numerical solution is necessary.

39. A numerical solution to the nonlinear differential Equation 51 is suggested by Seed and Reese,[9] Reese,[10] and Coyle and Reese.[11] The first step in obtaining a solution is to write Equation 51 in finite difference form. Referring to Figure 10, the difference form of the equation for sta i may be written as

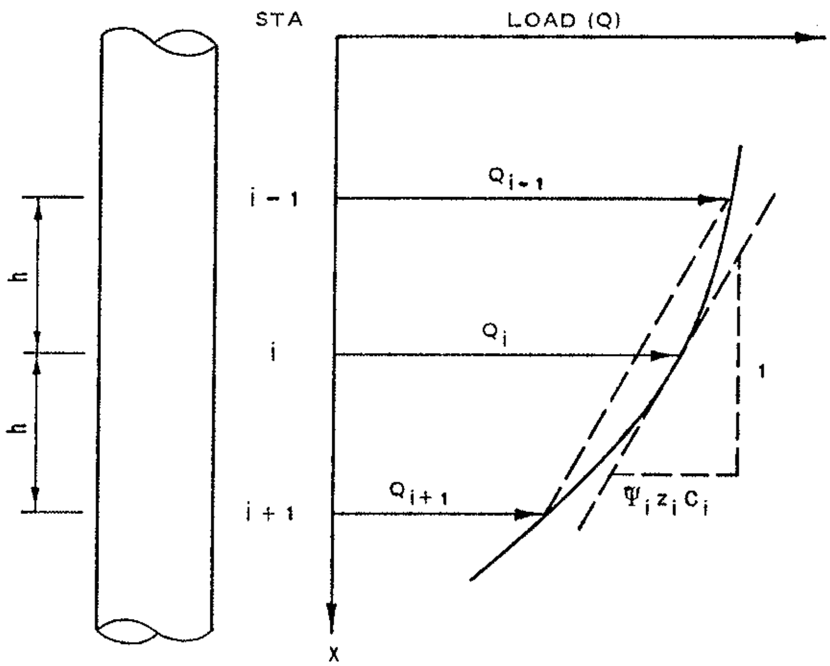


Figure 10. Load distribution along an axially loaded pile

$$\frac{\left(\frac{dz}{dx}\right)_{i-1} - \left(\frac{dz}{dx}\right)_{i+1}}{2h} = \frac{\psi_i z_i C_i}{EA} \tag{52}$$

Substituting Equation 43 into Equation 52 and simplifying yields

$$Q_{i-1} - Q_{i+1} = 2h\psi_i z_i C_i \tag{53}$$

which is the desired form of the equation. Equation 44 can also be written in difference form as

$$\frac{Q_i}{EA} = \frac{z_{i-1} - z_{i+1}}{2h} \tag{54}$$

or by arranging as

$$\frac{2hQ_i}{EA} = z_{i-1} - z_{i+1} \tag{55}$$

40. Equation 53 is simply a statement that the difference between the forces in the pile at sta i+1 and i-1 is equal to the load transferred to the soil between these two points. Equation 55 is simply a statement that the deformation that occurs in the pile over a segment 2h in length can be computed from the strain at the midpoint of the segment which is equal to the load in the pile at the midpoint of the segment divided by the product of the pile area and modulus of elasticity. Furthermore, the load distribution within the pile is assumed to be linear between these two points. The slope of the straight-line load distribution is approximated by the rate of load distribution at the midpoint between sta i+1 and i-1. This procedure results in a concentration of the shear force, $h\psi_i z_i C_i$, at sta i. The physical significance of this assumption leads to the mechanical model of an axially loaded pile, illustrated in Figure 11. The mechanical model can be used to develop equations which are analogous to Equations 54 and 55 and to formulate a procedure for solving the equations, which will yield the desired

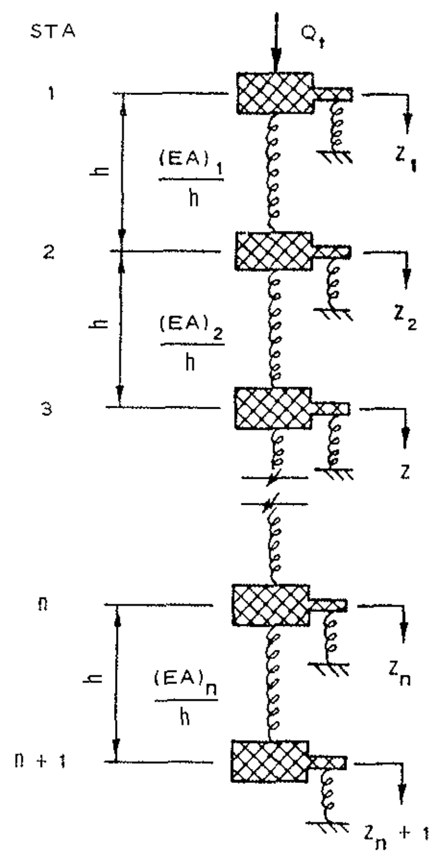


Figure 11. Mechanical model of an axially loaded pile

load-deformation response of an axially loaded pile.

Development of Mechanical Model and Equations for Model

41. The mechanical model represents the pile by n springs, of length h, connected by rigid joints (Figure 11). The springs, representing the pile, are linear and have a spring constant as shown. The nonlinear springs, representing the load transfer to the soil, are attached to the rigid joints. The spring attached to joint 1 will represent the load transferred from the ground surface to a depth of $h/2$. The spring attached to joint n+1 will represent the load transferred to the soil through the pressure on the pile base rather than through shear along the pile shaft as it is for all other springs. The spring attached to joint n will represent the load transferred from the pile base to a distance of $3h/2$ above the base. The interior springs represent the load transferred over a distance of $h/2$ above and below the joint. The concentration of the shear transfer for an arbitrary interior joint represented by force SF is illustrated in Figure 12. Summing forces on a joint yields:

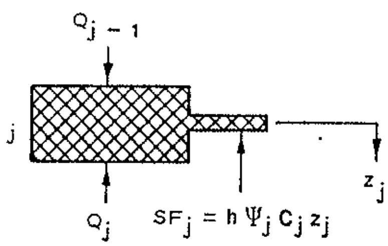


Figure 12. Joint j of the mechanical model of an axially loaded pile

$$Q_{j-1} - Q_j = SF_j = h\psi_j C_j z_j \tag{56}$$

This equation is the same as Equation 53 except it considers only the load change or transfer over one increment rather than two.

Solution Procedure for Equations

42. If curves are available describing the load transfer, Equation 56 can be used to obtain the load-deformation behavior of the pile. The solution procedure may be formulated by considering the mechanical

model in Figure 11. If a load Q_t is applied to joint 1, the model will deform in such a way that conditions of equilibrium and compatibility are satisfied. The first step in the procedure is to assume a deflection of the pile base. From the nonlinear spring at joint n+1, the force SF_{n+1} may be found for the assumed deflection. The force Q_n may now be found by considering the equilibrium of joint n+1, and solving Equation 56 at sta n+1. Solution of this equation yields $Q_{n+1} = SF_{n+1}$. With the force Q_n known, the deflection z_n may be obtained by considering the deformation in the linear spring between sta n and n+1. The deflection is expressed mathematically as

$$z_n = z_{n+1} + \frac{Q_n h}{(EA)_n} \tag{57}$$

With z_n and Q_n known, Equations 56 and 57 may be solved for each joint and spring until the top of the pile is reached. This procedure will yield a top load Q_t and a top deflection z_1 . Additional values may be assumed for the base deflection, and the procedure repeated until a complete load-deflection curve is obtained for the top of the pile.

43. It should be noted that in the derivation of Equations 54 and 55 it was assumed that EA was constant. With the mechanical model and for Equations 56 and 57, this assumption was not necessary. It is only necessary that EA be constant over each increment length.

PART IV: PILE GROUP THEORY

44. The computer program BENT1 provides a method for analyzing foundations which are supported on pile groups consisting of vertical and batter piles. The procedures are similar to those described by Reese and Matlock.[12-14] In this part, equations for describing the load-deformation response of a pile-supported foundation will be developed.

Coordinate Systems and Sign Conventions

45. Two types of coordinate systems are established as shown in Figure 13. A horizontal axis u and a vertical axis v are

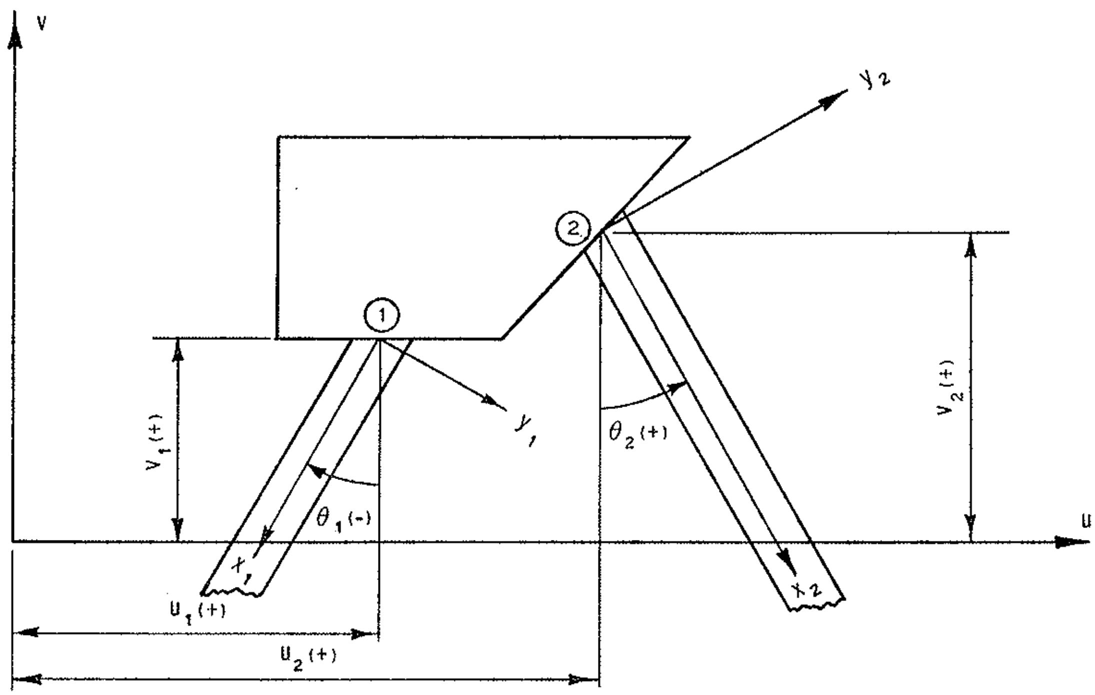


Figure 13. Geometry of foundation

established relative to the foundation. Foundation movements, forces, and dimensions are related to these axes. The location of this system is completely arbitrary, but proper location will simplify calculations for most foundations.

46. For each pile an x-y coordinate system is established. The x axis is parallel to the pile and the y axis is perpendicular to the pile. Subscripts are used to indicate the particular pile. Pile deflection and forces are related to these systems.

47. The coordinates of the pile heads as related to the u-v axes are all positive for the example (Figure 13). The batter of the piles is positive counterclockwise from the vertical and negative clockwise from the vertical as shown. The variable θ will be used to denote the angular measure of pile batter.

48. The external loads on the foundation are resolved into a vertical and horizontal component through the origin of the structural coordinate system and a moment about the origin. The sign convention established is illustrated in Figure 14.

49. The external loads M_e , P_v , and P_u will cause the foundation to move. If the u-v coordinate system is considered to be rigidly attached to the foundation, the movement of the foundation may be related to the movement of the coordinate system. These

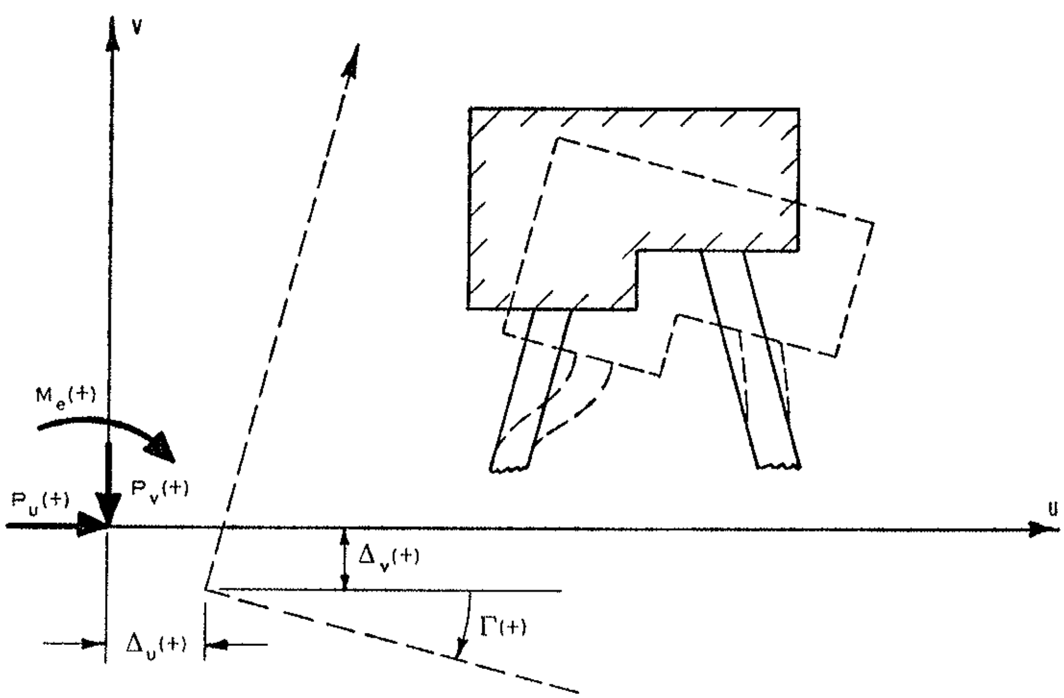


Figure 14. Sign convention for foundation forces and movements

movements (Δ_v , Δ_u , and Γ) are shown with positive signs (Figure 14).

50. Due to the movement of the foundation, forces will be exerted on the foundation by the piles. The sign convention for these forces is illustrated in Figure 15 in two ways: (a) conventions consistent with

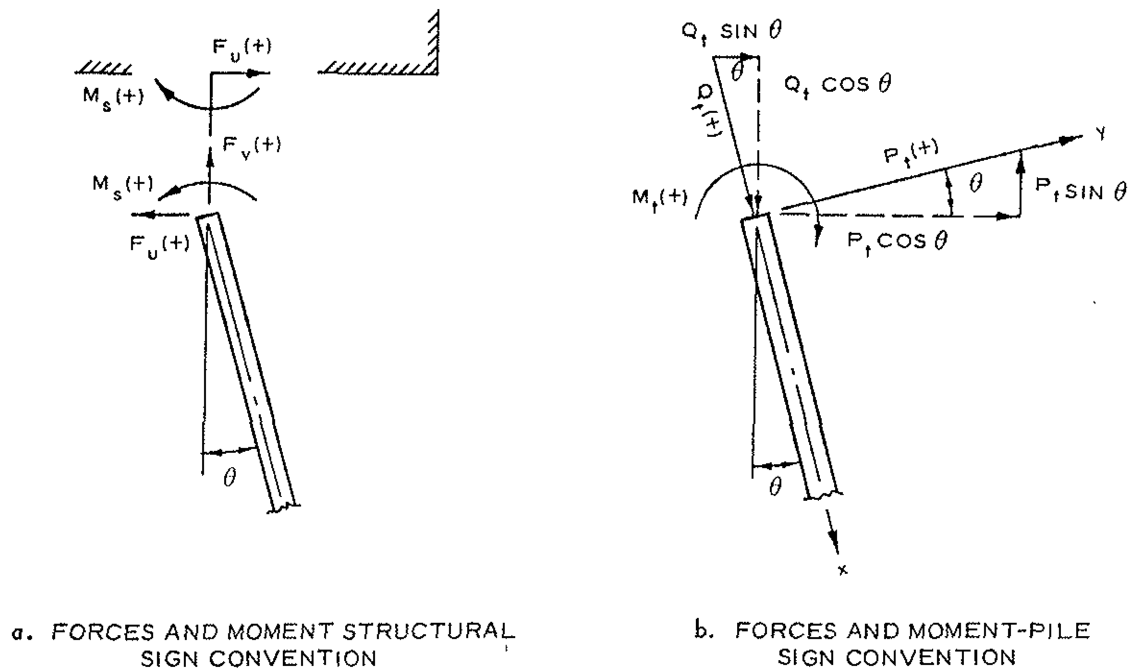


Figure 15. Forces and moment on pile head

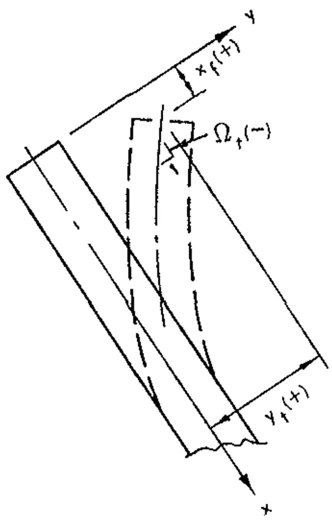


Figure 16. Pile-head movements on the x-y coordinate system

those established previously for the structure; and (b) conventions consistent with those established in the solution of laterally loaded piles. The differences should be carefully noted. The inconsistencies are taken care of when the relations between foundation forces and pile forces are developed.

51. The sign conventions for movements of the pile head (Figure 16) are consistent with the x-y coordinate system. A movement in the positive x direction, which constitutes an axial compression, is considered as a positive movement. A movement in the positive y direction

is considered as a positive movement. A rotation of the pile head will cause a change in the slope at the top of the pile. The sign convention for slope is consistent with the usual manner in which slope is defined.

Relations Between Foundation Movements and Pile-Head Movements

52. When the structure moves, the pile heads move. Two assumptions are made in order to relate structure movement to pile-head movements. The first assumption is that the foundation is rigid so that the pile heads maintain the same relative positions before and after movement. Because of this assumption the approximation

$$\Gamma \approx \tan \Gamma \tag{58}$$

is valid.

53. In Figure 17a, diagrams are given of the lineal movements at the pile head of a given pile in terms of the structural movements. The movement of the structure is defined by the shift of the u-v axes to the position indicated by the u'-v' axes. The total movement of the pile head is resolved into a component parallel to the u axis $(\Delta_u + v\Gamma)$ and a component parallel to the v axis $(\Delta_v + u\Gamma)$.

54. Figure 17b illustrates the resolution of the horizontal and vertical components of movement into components parallel and perpendicular to the direction of the pile. These movements are designated as x_t and y_t. From the same figure, the axial component of pile-head movement may be written as

$$x_t = (\Delta_u + v\Gamma) \sin \theta + (\Delta_v + u\Gamma) \cos \theta \tag{59}$$

and the corresponding lateral movement as

$$y_t = (\Delta_u + v\Gamma) \cos \theta - (\Delta_v + u\Gamma) \sin \theta \tag{60}$$

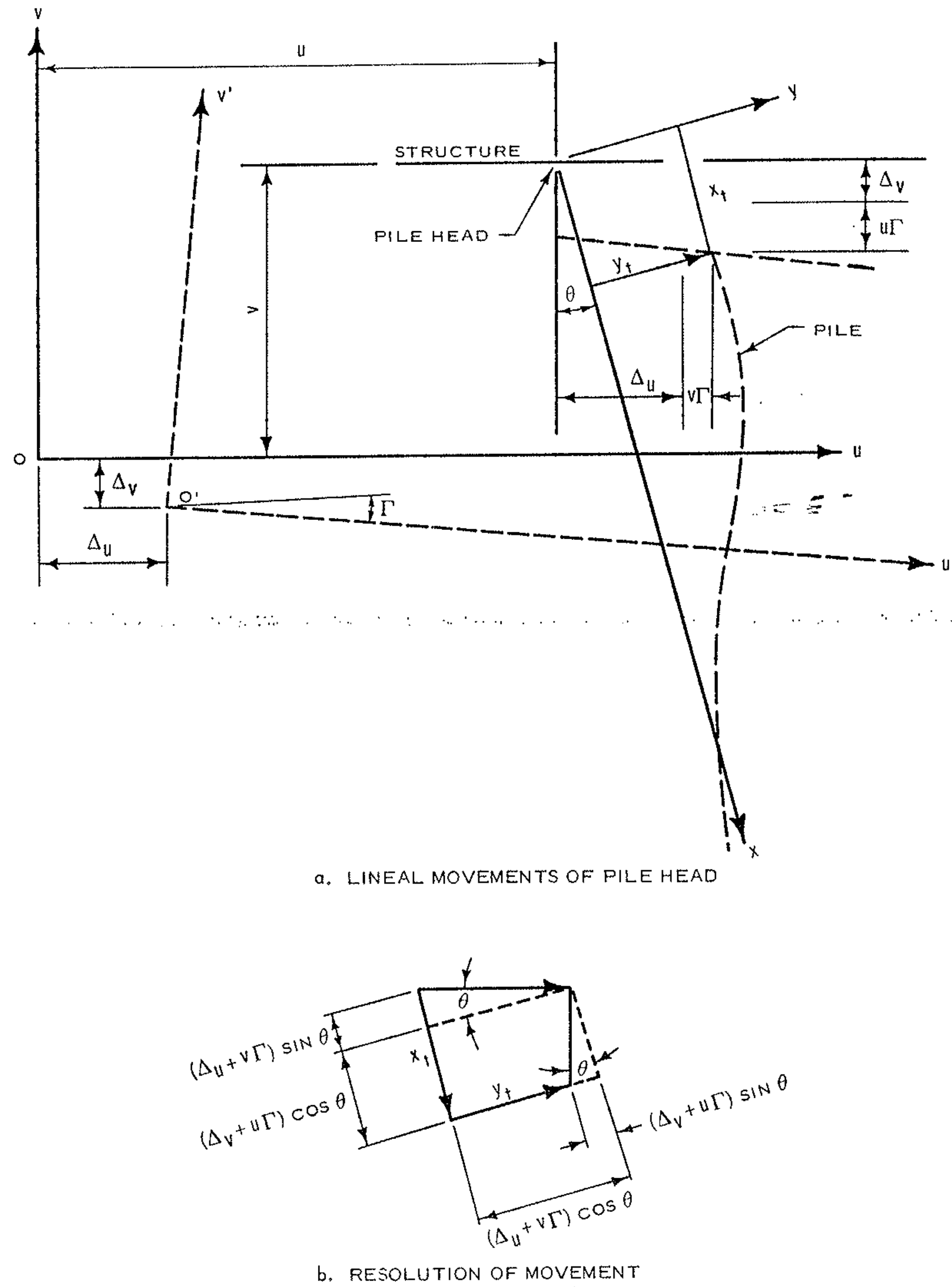


Figure 17. Movements of pile-head structural coordinate system

55. In addition to the lineal displacements of the pile head, the change in slope of a tangent to the elastic curve will be considered. The change in the slope will depend on the manner in which the pile is attached to the foundation. If the pile is fixed to the structure, then the change in slope will be equal to the rotation of the foundation. For the restrained case the change in slope will depend on the moment applied to the pile top. For a pinned connection the slope will depend on the deflected shape of the pile.

Relations Between Foundation Forces and Pile Reactions

56. The forces acting on the foundation and pile are illustrated, along with sign convention, in Figure 15. It has been noted that inconsistencies in the sign conventions are present. However, these will be corrected while deriving the relations between the forces.

57. From Figure 15, the relationship between moments on the structure and moment on the pile may be expressed as

$$M_s = -M_t \tag{61}$$

The relations between forces are obtained by resolving the forces on the pile into components in the horizontal and vertical directions. With the sign conventions considered, the components are summed as follows:

$$F_v = P_t \sin \theta - Q_t \cos \theta \tag{62}$$

$$F_u = -Q_t \sin \theta - P_t \cos \theta \tag{63}$$

Relations Between Pile-Head Movement and Pile Reaction

58. In the preceding paragraphs the movement of the pile head and the forces acting on the pile head have been defined. Relations between

pile reaction and movement will be developed below.

59. For computational purposes the pile shown in Figure 18a may be simulated by the set of springs as shown in Figure 18b. The springs

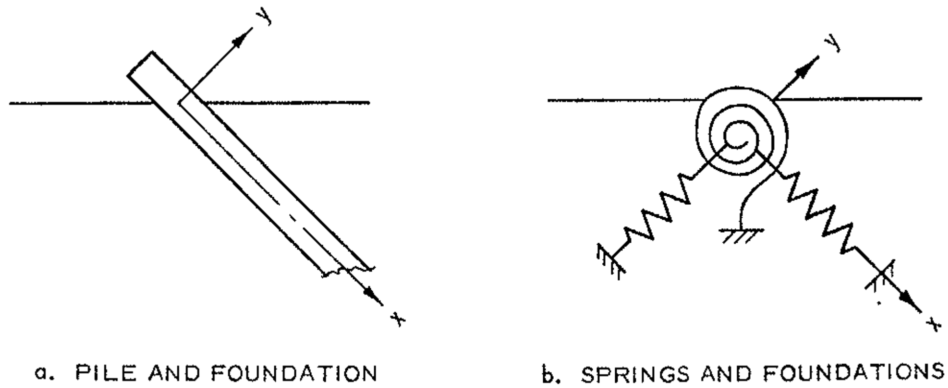


Figure 18. Spring representation of pile

will produce a force parallel to the pile axis, Q_t , and a force acting perpendicular to the pile axis, P_t . The rotational spring will yield a moment about the pile top, M_t .

60. The forces produced by the springs will depend on the deflection of the springs. Since the springs are nonlinear, the movement and reaction are not related by a single constant. It is assumed that curves can be obtained which show spring reaction as a function of deflection. In Figure 19, a hypothetical set of load-deflection curves are drawn for a set of springs. If the curves are single valued, then the spring reactions may be calculated for a particular deflection by

$$Q_t = J_x x_t \tag{64}$$

$$P_t = J_y y_t \tag{65}$$

$$M_t = J_m y_t \tag{66}$$

where J_x , J_y , and J_m are the secant modulus values as illustrated in the figure.

61. It should be noted that the moment produced by the rotational

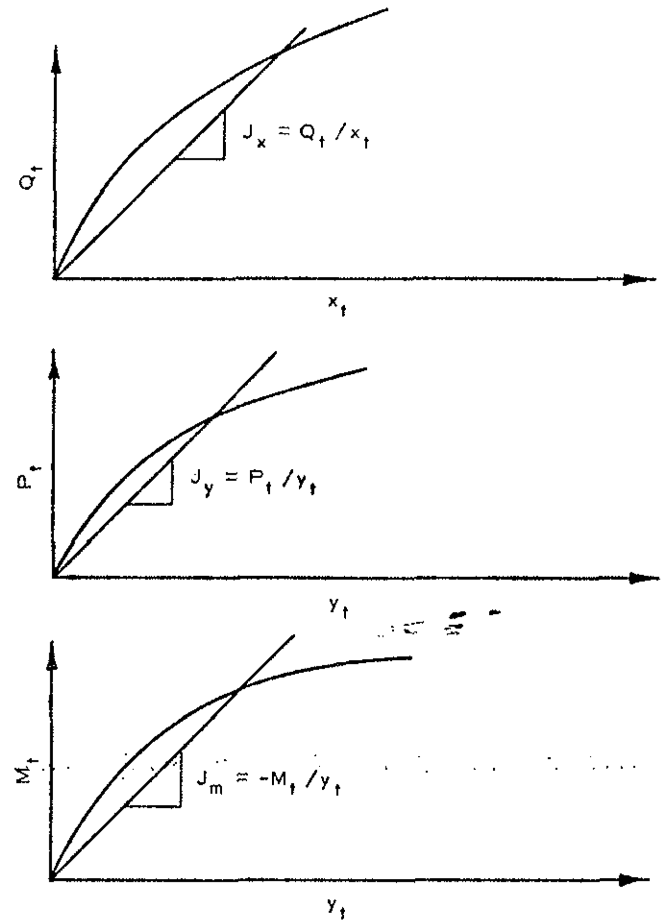


Figure 19. Hypothetical spring load-deflection curves

spring is proportional to the lateral deflection rather than the rotation. For a rotational spring this procedure is inconsistent with usual concepts. This concept is used because it provides a convenient means for deriving and solving the equilibrium equation for the structure.

62. The curves in Figure 19 do not adequately explain the behavior of a pile. It is not necessary that the exact nature of the curves be known. The representation shown is only for the formulation of the equilibrium equations. The procedure for calculating values for J_x , J_y , and J_m will be discussed in the following paragraphs. However, for the formulation of the equilibrium equations, Equations 64-66 are sufficient, since they will be applicable no matter what kind of relationship exists between the loads on the pile tops and the resulting displacements.

Equilibrium Equations

63. The relations between forces and movements for the structure and the pile, previously developed, will now be combined to form three equations of equilibrium for the structure. The form of the equations is such that an iterative type solution may be used. This is necessary since the system is nonlinear.

64. Consider a foundation supported by n piles. The coordinate system and the i^{th} pile are shown in Figure 20. The external loads

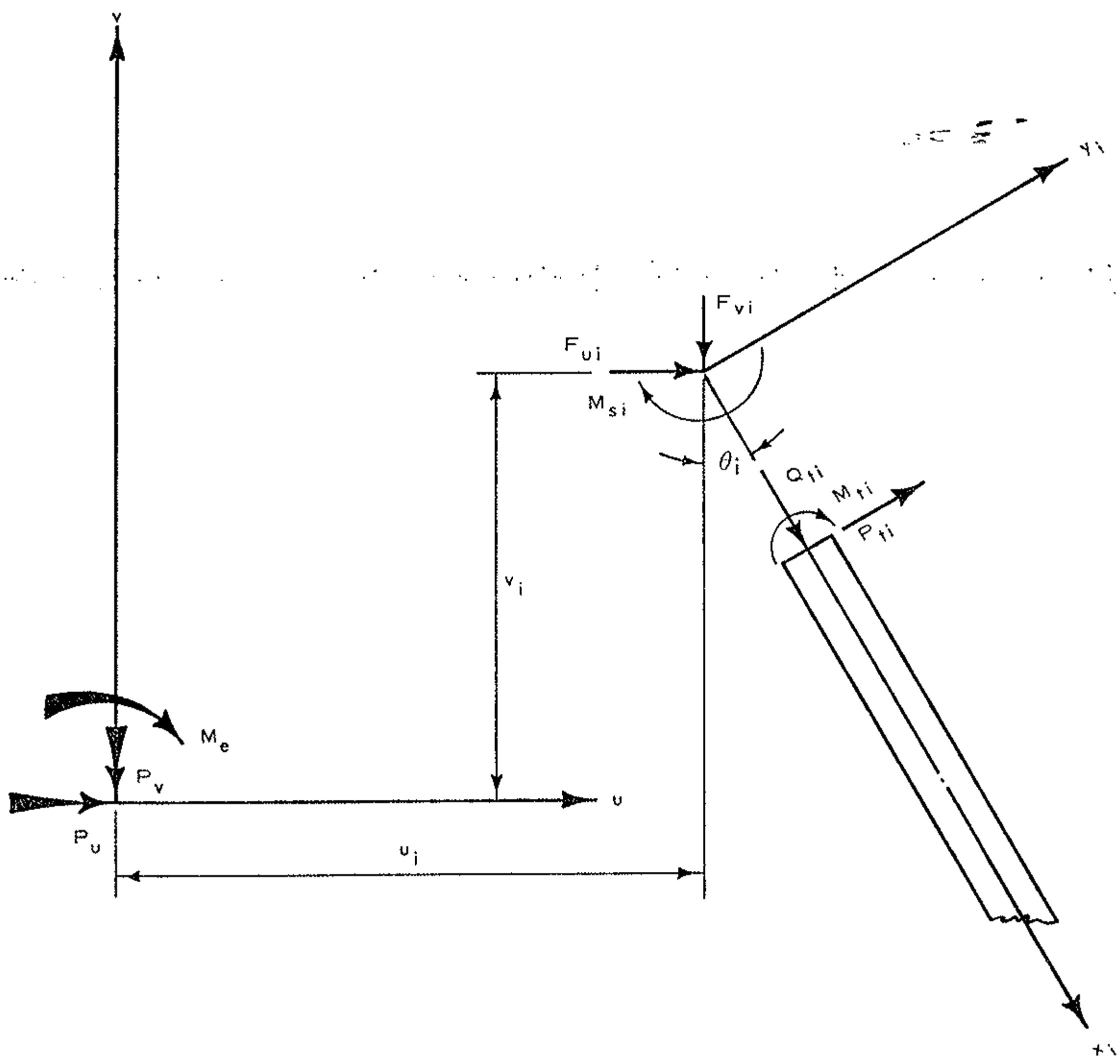


Figure 20. Forces on the piles and foundation

applied to the foundation are resolved into the forces and moment through and about the origin of the coordinates. The forces and moment exerted by each pile are designated as F_{vi} , F_{ui} , and M_{si} in the figure. The three equations are obtained by summing forces in the horizontal and vertical directions and by summing moments about the origin of the u-v coordinate system. Performing these operations the equilibrium equations may be written as

$$\sum_{i=1}^{n} F_{vi} + P_v = 0 \tag{67}$$

$$\sum_{i=1}^{n} F_{ui} + P_u = 0 \tag{68}$$

$$\sum_{i=1}^{n} (M_{si} + u_i F_{vi} + v_i F_{ui}) + M_e = 0 \tag{69}$$

where M_e , P_u , and P_v symbolize external moment horizontal force, and vertical force applied to the foundation at the origin of u-v coordinate system. Substituting Equations 61-63 into Equations 67-69 and rearranging

$$P_v = \sum_{i=1}^{n} (Q_{ti} \cos \theta_i - P_{ti} \sin \theta_i) \tag{70}$$

$$P_u = \sum_{i=1}^{n} (P_{ti} \cos \theta_i + Q_{ti} \sin \theta_i) \tag{71}$$

$$M_e = \sum_{i=1}^{n} \Big[M_{ti} + u_i (Q_{ti} \cos \theta_i - P_{ti} \sin \theta_i) + v_i (P_{ti} \cos \theta_i + Q_{ti} \sin \theta_i) \Big] \tag{72}$$

Substituting Equations 64-66 into Equations 70-72 the equilibrium equations may be expressed as

$$P_v = \sum_{i=1}^{n} (J_{xi} x_{ti} \cos \theta_i - J_{yi} y_{ti} \sin \theta_i) \tag{73}$$

$$P_u = \sum_{i=1}^{n} (J_{yi} y_{ti} \cos \theta_i + J_{xi} x_{ti} \sin \theta_i) \tag{74}$$

$$M_e = \sum_{i=1}^{n} \Big[-J_{mi} y_{ti} + u_i (J_{xi} x_{ti} \cos \theta_i - J_{yi} y_{ti} \sin \theta_i) + v_i (J_{yi} y_{ti} \cos \theta_i + J_{xi} x_{ti} \sin \theta_i) \Big] \tag{75}$$

The equations are modified further by substituting equations 59 and 60 into Equations 73-75 and rearranging to obtain

$$P_v = \sum_{i=1}^{n} \Big\{ \left(J_{xi} \cos^2 \theta_i + J_{yi} \sin^2 \theta_i\right) \Delta_v + \left[(J_{xi} - J_{yi}) \sin \theta_i \cos \theta_i\right] \Delta_u + \left[u_i \left(J_{xi} \cos^2 \theta_i + J_{yi} \sin^2 \theta_i\right) + v_i (J_{xi} - J_{yi}) \sin \theta_i \cos \theta_i\right] \Gamma \Big\} \tag{76}$$

$$P_u = \sum_{i=1}^{n} \Big\{ \left[(J_{xi} - J_{yi})(\sin \theta_i \cos \theta_i)\right] \Delta_v + \left(J_{yi} \cos^2 \theta_i + J_{xi} \sin^2 \theta_i\right) \Delta_u + \left[u_i (J_{xi} - J_{yi}) \sin \theta_i \cos \theta_i + \left(v_i J_{yi} \cos^2 \theta_i + J_{xi} \sin^2 \theta_i\right)\right] \Gamma \Big\} \tag{77}$$

$$\begin{aligned} M_e = \sum_{i=1}^{n} \Big\{ & \Big[J_{mi} \sin \theta_i + u_i \left(J_{xi} \cos^2 \theta_i + J_{yi} \sin^2 \theta_i\right) \\ & + v_i (J_{xi} - J_{yi}) \sin \theta_i \cos \theta_i \Big] \Delta_v + \Big[-J_{mi} \cos \theta_i \\ & + u_i (J_{xi} - J_{yi}) \sin \theta_i \cos \theta_i + v_i \left(J_{yi} \cos^2 \theta_i + J_{xi} \sin^2 \theta_i\right) \Big] \Delta_u \\ & + \Big[J_{mi} (u_i \sin \theta_i - b_i \cos \theta_i) + u_i^2 \left(J_{xi} \cos^2 \theta_i + J_{yi} \sin^2 \theta_i\right) \\ & + v_i^2 \left(J_{yi} \cos^2 \theta_i + J_{xi} \sin^2 \theta_i\right) + 2(J_{xi} - J_{yi})(\sin \theta_i \cos \theta_i) u_i v_i \Big] \Gamma \Big\} \end{aligned} \tag{78}$$

65. Equations 76-78 constitute a complete set of equilibrium equations for a foundation. The loads on the foundation, the distance to the pile tops, and the batter of the piles are known quantities. If the spring modulus values are known, the three equations may be solved simultaneously for Δ_v , Δ_u , and Γ . However, since the system is nonlinear, J_m , J_x , and J_y will not be constants. Thus, an iterative solution is required. The procedure utilized for solving the equilibrium equations is described in the following paragraphs.

Computational Procedure for Solution of Equilibrium Equations

66. The iterative procedure used for the solution of the equilibrium equations is illustrated in Figure 21. The iterative procedure is necessary for establishing the deflected position of the foundation so that equilibrium and compatibility are satisfied.

67. To begin the procedure, values of Δ_v , Δ_u , and Γ are set equal to zero. In addition, the deflections of each pile top (x_{ti} , y_{ti}) are set equal to one. These values are used only for starting the iterative procedure and have no bearing on the final solution.

68. Values of x_{ti} are used directly with load-deflection curves for the individual piles to obtain values of J_{xi} . A typical load-deflection curve and the procedure for computing J_{xi} are shown in Figure 22. The use of a unique single valued curve for the axial load-deflection response of a pile is based on the assumption that the axial behavior of a pile is unaffected by any lateral effects. That is to say, the axial load on the pile is dependent only on the axial deflection of the pile. This is not rigorously correct since it is known that lateral forces on the pile top will cause lateral movement which will decrease the axial load carrying capacity of the pile. However, for realistic situations the influence of the lateral forces on the axial response will be small and thus is ignored in this procedure.

69. Values of y_{ti} are used with a lateral loaded pile

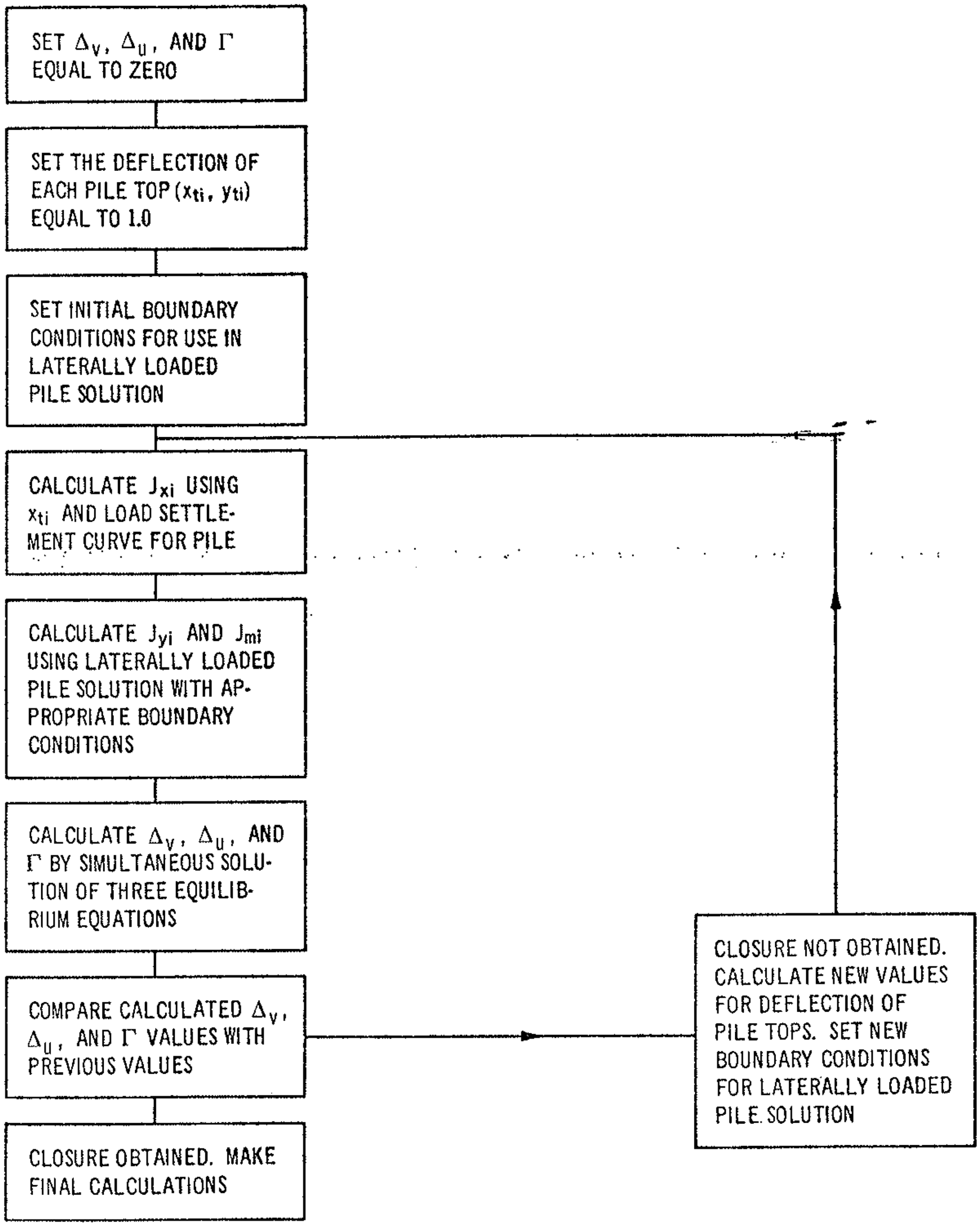


Figure 21. Block diagram for iterative solution

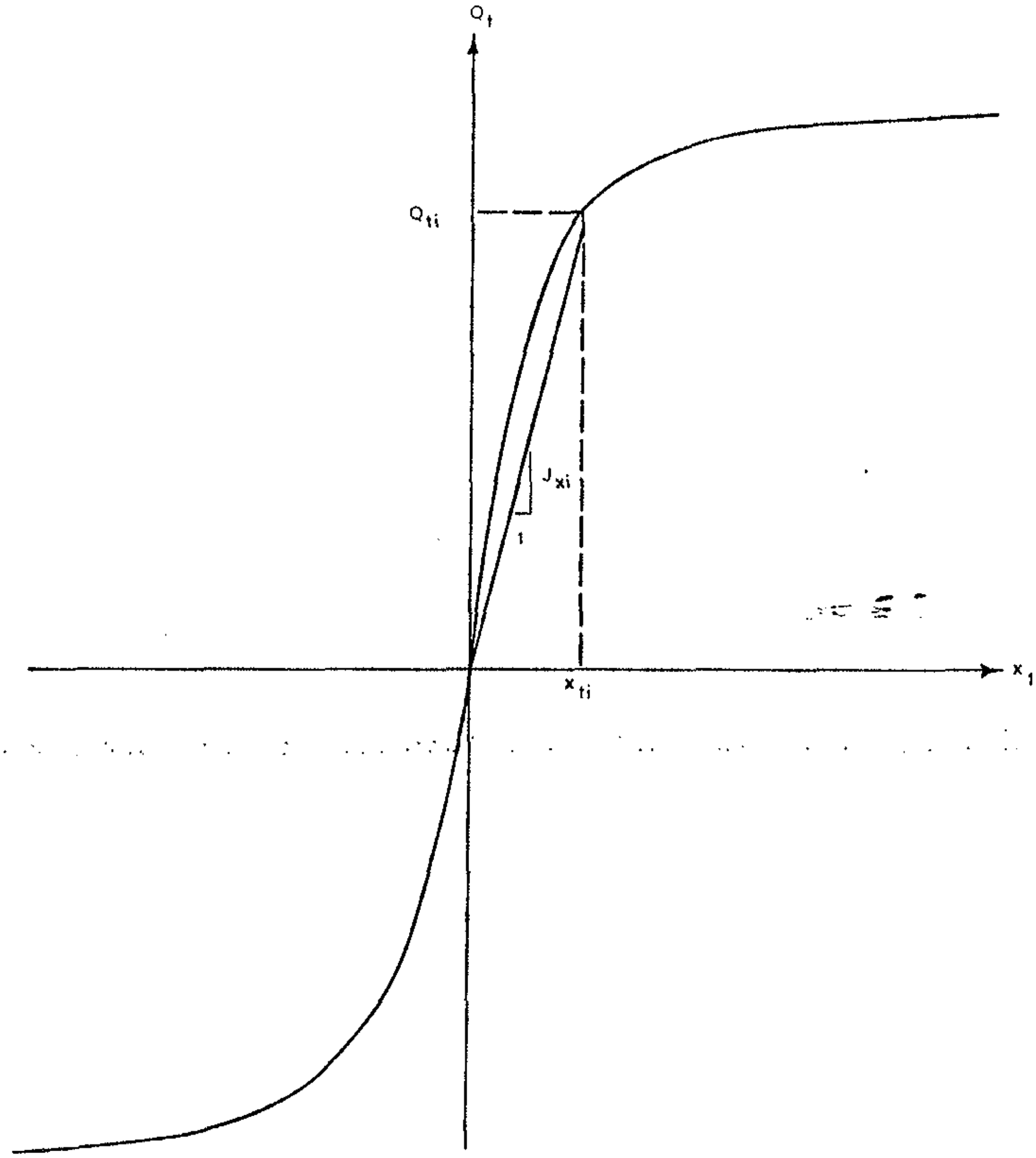


Figure 22. Axial load-settlement curve

subroutine, similar to COM62, to obtain values of J_{yi} and J_{mi}. The subroutine requires two boundary conditions for the top of the pile. For the initial iteration, one of these boundary conditions is that the lateral deflection of the top of the pile is 1 in.* The second boundary

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* A table of factors for converting U. S. customary units of measurement to metric (SI) units is presented on page 4.

condition will depend on the manner in which the pile is connected to the foundation and is set equal to zero. For pinned connections this means that the moment at the pile top is zero, for fixed connections the slope at the pile top is set equal to zero, and for restrained connections the stiffness of the rotational restraint spring is set equal to zero. In addition to the above boundary conditions, the axial forces applied to the top of the piles are obtained directly from the axial load-deformation curves. The axial forces are included because it is felt that these forces significantly affect the lateral response of the piles. However, this is contrary to what is assumed for the effects of lateral forces on the axial response. The reasoning for these assumptions are as follows:

- a. The majority of the axial load is transferred to the soil in the lower part of the pile. For practical problems the lateral forces only cause significant lateral movement in the upper part of the pile where the axial load transfer is small--hence, the assumption that lateral forces have little influence on the axial response.
- b. The majority of the lateral load is transferred to the soil in the upper part of the pile. This means that the maximum deflections, bending moments, and lateral soil reactions occur near the top of the pile. Because the axial forces in the pile may be quite large near the top, the effects on the lateral behavior may be significant--hence, the inclusion of the effects of axial load on lateral behavior.

70. With the initial boundary conditions, the finite difference equations for the piles are solved and values of moment and shear at the pile top computed. With the moment and shear at the pile top known, values of J_{yi} and J_{mi} are computed by dividing the moment and shear by the top deflection which is 1 in. for the initial iteration.

71. With spring moduli for each pile, the equilibrium equations for the foundation movement are solved for Δ_v, Δ_u, and Γ. The new values of Δ_v, Δ_u, and Γ are then used to start the second iteration.

72. To start the second and each preceding iteration, deflections of the pile top (x_{ti} and y_{ti}) are computed using current values of Δ_v, Δ_u, and Γ. New values of J_{xi} are computed directly from

the load deflection curves for the individual piles as was done for the initial iteration. To calculate new values of J_{yi} and J_{mi} , it is necessary to establish boundary conditions for the top of the pile as was done for the initial iteration. One boundary condition is the lateral load at the top of the pile. The lateral load is found by multiplying J_{yi} from the previous iteration by y_{ti} . The second boundary condition will depend on the manner in which the pile is connected to the foundation. For pinned conditions the second boundary condition is that the top moment is zero. For fixed connections the slope at the top of the pile is set equal to the rotation of the structure (Γ). For restrained connections the second boundary condition is the stiffness of the rotational restraint spring. The axial forces are computed by multiplying J_{xi} by x_{ti} . With the boundary conditions established the remainder of the procedure is the same as for the initial iteration.

73. The values of Δ_v , Δ_u , and Γ are compared with values from the previous iteration. The correct solution is obtained when the movements agree within the specified allowable tolerance. If closure is not obtained, the procedure is repeated. If closure is obtained, a control is set and the forces and moments exerted by each pile on the foundation are computed. In addition, the deflected shape, moment distribution, and soil reaction for each pile are calculated.

74. A computer program GROUP, developed by Dr. Katsuyuki Awoshika and Prof. L. C. Reese[15] at the University of Texas, is currently available. GROUP can perform the same type of analysis as BENT1 but is considered more general and efficient.

PART V: CRITERIA FOR DEVELOPING SOIL-PILE INTERACTION CURVES

75. The UT method of analysis for single piles (loaded laterally or axially) and group of piles (discussed previously) requires the development of nonlinear soil resistance-pile movement curves for both lateral and axial loading conditions. This subject is quite complex, and no attempt will be made in this report to review the work that has been done in this area. A concise presentation on this topic is given by Awoshika and Reese[15] and Parker and Cox;[16] the material in this part is principally extracted from these two reports. The discussions here will be limited to the establishment of criteria that are used internally in the computer codes documented in this report.

76. The actual pile-soil systems are quite complex and the interaction will be affected by a number of parameters, such as time effect on soil behavior, disturbance of soil due to pile driving/placing, cyclic loading of soil, settlement of the soil surrounding the pile due to negative skin friction, and interference of adjacent piles. The criteria presented have been derived for static, short-term loading conditions and are based on semi-empirical considerations.

77. Soil criteria for lateral and axial loading conditions are developed separately. Also, the criteria are developed separately for two common but extreme soil types, clay and sand. One may expect other soils to exhibit characteristics somewhere between those for clay and sand.

Laterally Loaded Pile

78. In Part III, the effect of the soil on a laterally loaded pile was shown as a distributed reaction p . The soil reaction p was defined as

$$p = E_s y \qquad (79)$$

where E_s is the soil modulus and y is the lateral deflection. The soil modulus values vary generally with p and y in a nonlinear manner. The subsequent paragraphs discuss the methods to obtain these p-y curves.

79. The p-y curves will depend on the soil properties. For most cases the properties of the soil in a profile are not constant with depth, the usual case being that the strength of the soil increases with depth. A typical variation of shear strength of soil with depth is shown in Figure 23a. Since the strength of the soil will affect the p-y curves obtained, a variation similar to that illustrated in Figure 23b might be expected. It should be pointed out that the shear strength is not the only parameter which will affect the p-y curve, although it does have considerable influence. The purpose of the variation shown in Figure 23b is only to illustrate the variability of the p-y relation.

Soil Criteria

80. Soil resistance movement for both clays and sands are constructed assuming that the p-y curves can be divided into two segments. These two segments are designated as O-A and A-B in Figure 24. The segment O-A represents the early part of the curve, and the segment A-B , the ultimate part of the curve. Because of this division, the construction of p-y curves may be carried out in two steps. First, the ultimate soil resistance is calculated and then the shape of the early part of the curve is obtained. Secondly, the horizontal line representing the ultimate soil resistance and the early part of the curve are then joined to form a continuous curve. In the following paragraphs the procedure will be explained first for clay and then for sand.

Criteria for clay

81. For clay two methods are employed to obtain p-y curves. If stress-strain data are available, the method proposed by McClelland and Focht[17] is used, with one modification. For this method stress-strain

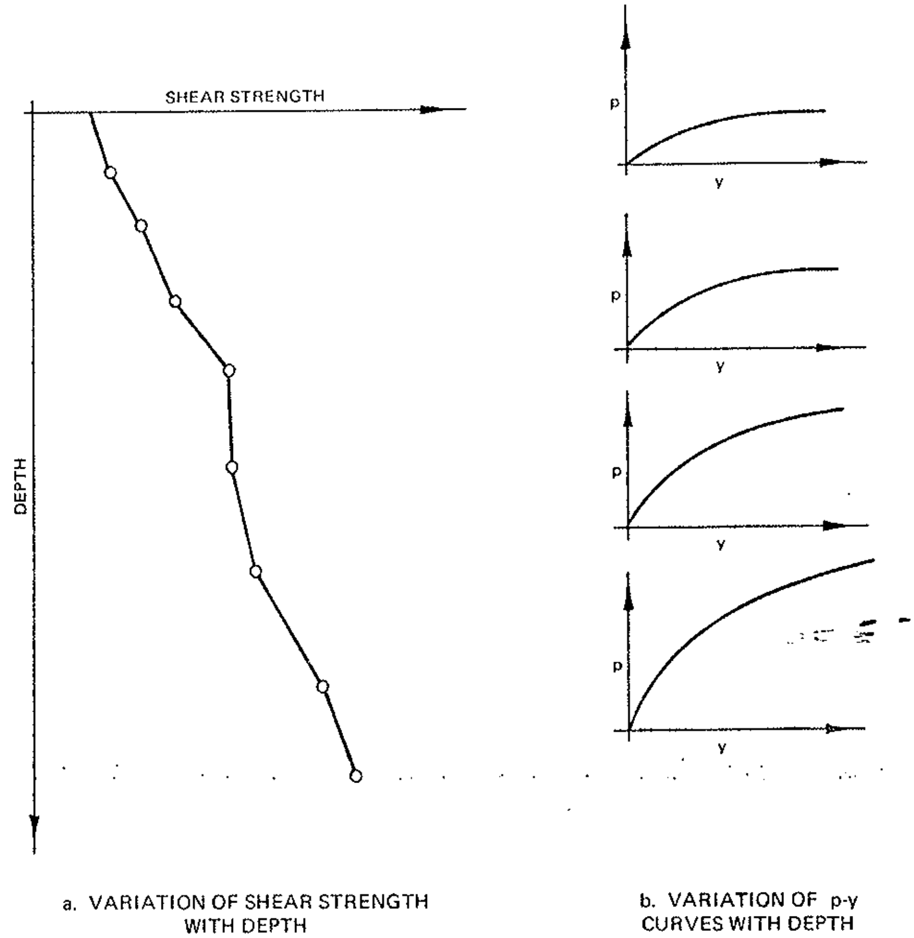


Figure 23. Variation of soil properties with depth

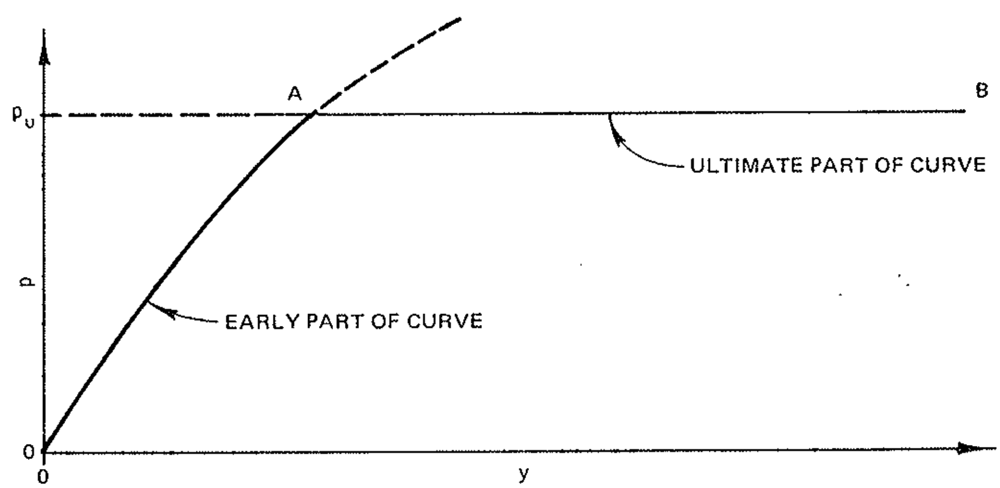


Figure 24. Construction of p-y curve

curves, similar to the one shown in Figure 25, are required. The curve is obtained from a triaxial test in which the confining pressure σ_3

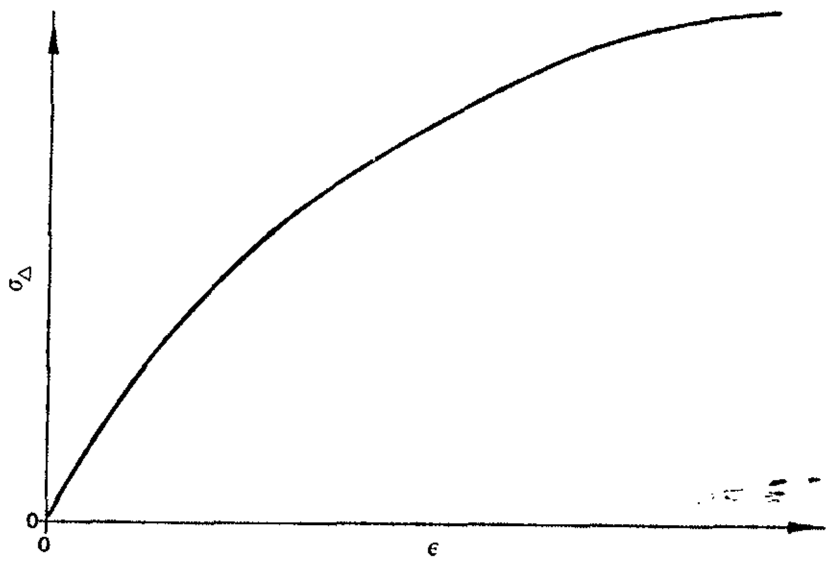


Figure 25. Stress-strain curve

is as close as possible to the confining pressure on the soil in the field. McClelland and Focht recommend that the p-y curve be obtained by using the following relations:

$$p = 5.5d\sigma_\Delta \tag{80}$$

and

$$y = 1/2d\varepsilon \tag{81}$$

where

d = diameter of pile or equivalent diameter

σ_Δ = soil deviator stress $(\sigma_1 - \sigma_3)$ from triaxial compression test in psi

ε = axial soil strain from triaxial compression test

82. Skempton[18] has suggested the following relationship for calculating deflections of footings:

$$y = 2d\varepsilon \tag{82}$$

An average value to use for deflection would be one between the values calculated using Equations 81 and 82. The equation suggested is

$$y = d\varepsilon \tag{83}$$

Using Equations 80 and 83 and the stress-strain curve, a corresponding p-y curve may be obtained.

83. It is assumed that the test is run until failure is obtained. That is, the maximum value for σ_Δ obtained will represent the ultimate value which may be carried by the soil. Consequently, the value for p calculated using the ultimate value of σ_Δ is considered to be the ultimate soil resistance.

Reese's criteria

84. If no stress-strain curves are available, but the shear strength and unit weight are known, p-y curves can be obtained. Two expressions are available for calculating the ultimate soil resistance for clay. These equations suggested by Reese[19] are as follows:

$$p_u = \gamma dX + 2cd + 2.83cX \tag{84}$$

and

$$p_u = 11cd \tag{85}$$

where

γ = effective unit of soil

X = depth from soil surface

c = cohesion of clay

Equations 84 and 85 are usually plotted (Figure 26), and the smaller of the two values is used in constructing p-y curves. Equation 84 will control near the surface since it is based on the occurrence of a wedge-type failure, and Equation 85 will control at depth since it is based on

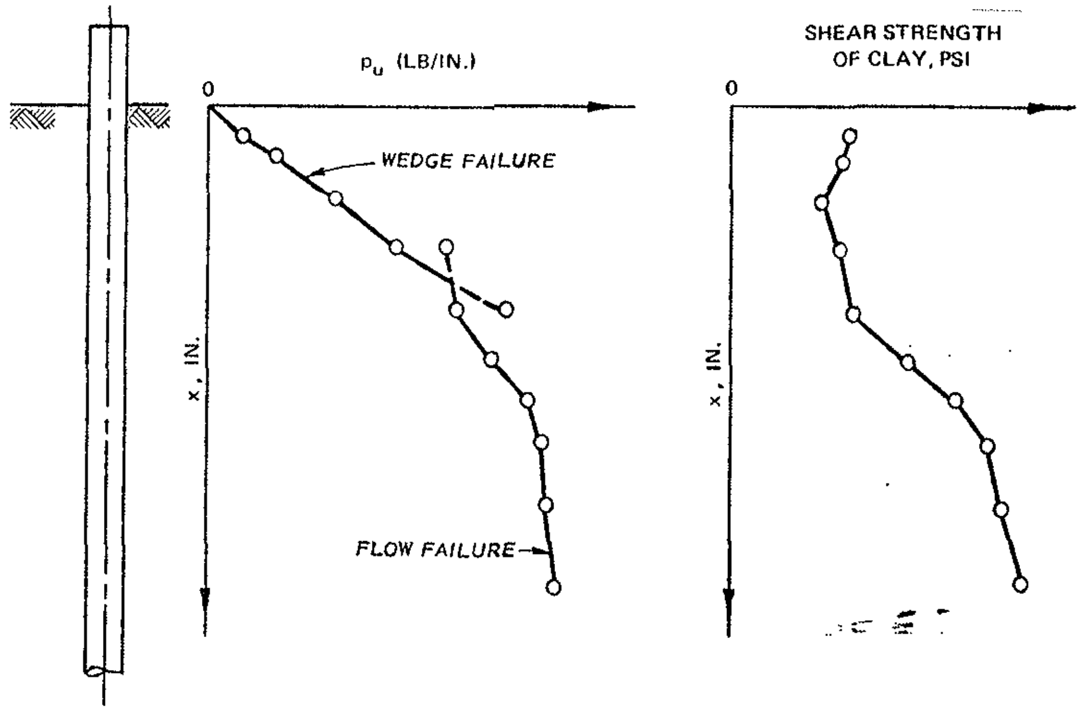


Figure 26. Ultimate soil resistance for clays

the soil failing by flowing around the pile. At such depths, there is sufficient restraint to prevent the upward movement of the soil.

Early part of curve

85. The early part of the curve is obtained by Equations 80 and 83. Since no stress-strain curve is available, values of σ_Δ and ε must be found. These are found by approximating the stress-strain curve. The following assumptions are made for drawing approximate stress-strain diagrams:

$$\sigma_{\Delta 50} = c = q_u/2$$

$$\varepsilon_{50} = 0.005 \text{ (brittle or stiff clays)}$$

$$\varepsilon_{50} = 0.02 \text{ (soft plastic clay)}$$

$$\varepsilon_{50} = 0.01 \text{ (no consistency data available)}$$

where

$\sigma_{\Delta 50}$ = deviator stress corresponding to 50 percent strain

q_u = unconfined compressive strength

ε_{50} = 50 percent of the maximum axial strain from triaxial compression test

The values of $\sigma_{\Delta 50}$ and ε_{50} are plotted as shown in Figure 27. A

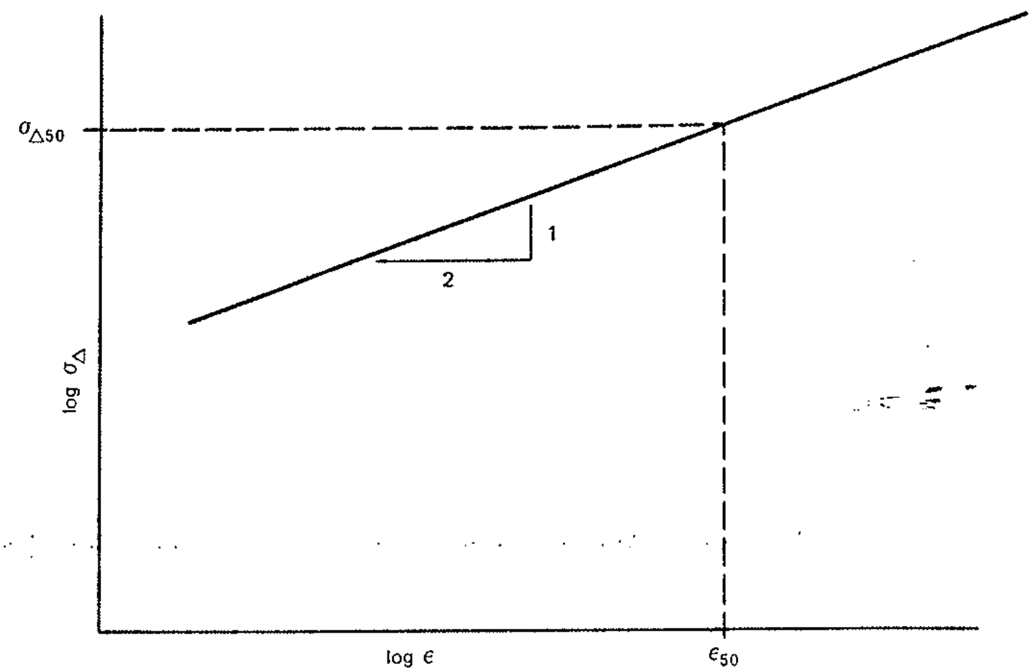


Figure 27. Approximate log-log plot of stress-strain curve

straight line with a slope of 0.5 is drawn through this point to represent the stress-strain curve for the soil. With this curve the early part of the curve may be obtained by applying Equations 80 and 83.

<u>Criteria for sand</u>

86. For sand the following two equations for calculating the ultimate soil resistance[19] are used:

$$p_u = \gamma dX \left[\frac{\tan \beta}{\tan(\beta - \phi)} - K_A\right] + \gamma X^2 \left[\frac{\tan^2\beta \tan \alpha}{\tan(\beta - \phi)} + \frac{K_o \sin \beta \tan \phi}{\cos \alpha \tan(\beta - \phi)} + K_o \tan \beta \tan \phi \sin \beta - K_o \tan \beta \tan \alpha\right] \quad (86)$$

and

$$p_u = \gamma dX \left\{ \left(\tan^2 45° - \frac{\phi}{2}\right) \left[\tan^8\left(45° + \frac{\phi}{2}\right) - 1\right] + K_o \tan \phi \tan^4\left(45° + \frac{\phi}{2}\right)\right\} \tag{87}$$

where

$\beta = 45° + \phi/2$

ϕ = angle of internal friction of sand

K_A = active earth pressure coefficient

K_o = coefficient of earth pressure at rest

$\alpha = \begin{cases} \phi/2 \text{ to } \phi/3 \text{ (loose sand)} \\ \phi \text{ (dense sand)} \end{cases}$

Equation 86 is for wedge-type failure (near surface), and Equation 87 is for flow around failure (at depth). Equations 86 and 87 are shown plotted in Figure 28a. The lower of the two values obtained from the equations will be used in constructing the p-y curves.

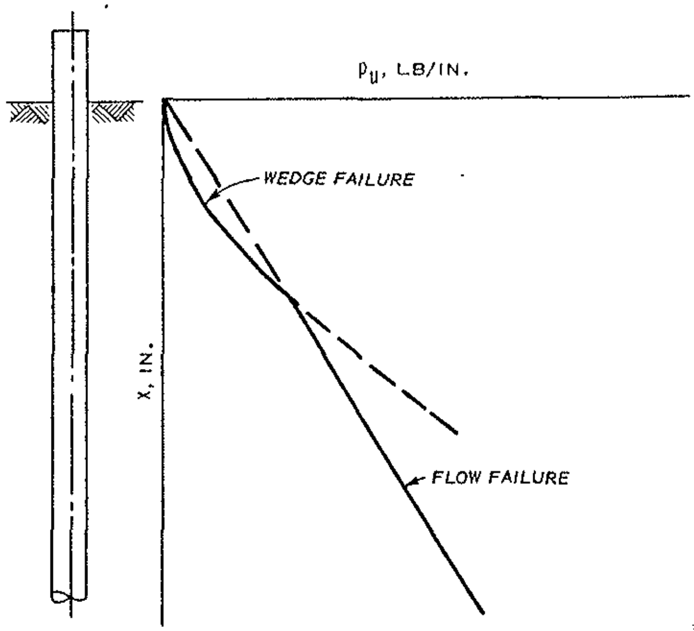
Early part of curve

87. The early part of the curve is obtained by applying theory developed by Terzaghi.[20] This results in a linear variation between p and y, with the slope defined as

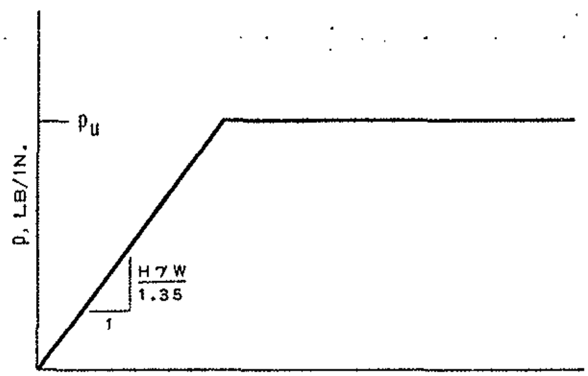
$$\text{Slope} = \frac{H\gamma X}{1.35} \tag{88}$$

where H is the constant depending on relative density of sand. Suggested values for H are 200 for loose sand, 600 for sand with medium density, and 1500 for dense sand. The unit weight used is the effective unit weight.

88. If the slope of the early part of the curve is known, the p-y curve can be constructed by connecting a straight line through the origin, with a slope (expressed by Equation 88) to the horizontal line defined by the ultimate soil resistance. This results in a p-y curve which consists of two straight lines (Figure 28b). When one considers the behavior of a sand, it will be noted its behavior is not linear.



a. ULTIMATE LATERAL SOIL RESISTANCE, p_u



b. BILINEAR p-y CURVE

Figure 28. Reese's criteria for p-y curves in sands

As a result, the p-y curve obtained should be considered as an approximation.

Program MAKE

89. A computer program MAKE, documented in Appendix B, is programmed for computing Equations 80 through 88. The program can also produce p-y curves at various depths and various size piles embedded in clays or sands.

Other criteria for
computing p-y curves

90. There are various other criteria for computing p-y curves for laterally loaded piles in clays and sands. Worthy of particular mention are Matlock's[21] criteria for constructing p-y curves for static and cyclic loading in soft clay and Parker and Reese's[22] criteria for developing p-y curves in sands. A summary of these two and other criteria is contained in Reference 15.

General comments on p-y curves

91. It must be emphasized that the procedures explained herein to develop p-y curves are based on semi-empirical relations. This points out that these procedures need to be carefully evaluated with regard to the problem environment before being used in analyses. Perhaps the most important consideration regarding p-y curves is whether or not there are validating experimental results. The oil industry has funded several experimental (both laboratory and field) programs to obtain confidence in the methods employed for constructing these curves. When the results become available in the public domain, the level of confidence in the techniques proposed is likely to increase.

Axially Loaded Pile

Type of interaction curves needed

92. The mechanics of the axially loaded pile problem described in Part II requires the determination of a set of load transfer curves along the pile and the point resistance curve at the tip of the pile. The load transfer curve refers to a relationship between the skin friction developed on the side of a pile and the absolute axial displacement of a pile section. The point resistance curve expresses the total axial soil resistance on the base of the pile tip in terms of the pile-tip movement.

Factors affecting interaction curves

93. The properties of soil which determine the load transfer curve and the point resistance curve may be considerably affected by

pile driving. In the case of clays, Seed and Reese[9] reported that soon after the pile driving a loss in shear strength was observed in clays adjacent to the pile equal to 70 percent of that for total remolding. They also observed that the recovery of shear strength with the passage of time resulted in a five-fold increase in the load-carrying capacity of a pile, even in insensitive clays. As is pointed out by Kishida,[23] the pile driving in a loose sand results in the increase in the relative density and in the confining pressure, both of which are major factors affecting the load transfer curves and the point resistance curve. The action of arching observed in sands around a pile (Robinsky and Morrison[24]), may be another important factor to be considered.

94. In spite of all these complex factors, presently available soil criteria are based only on the soil properties before pile driving. In view of the fact that the effect of different methods of pile installation on the soil properties with the passage of time are excluded from the soil criteria, in the following paragraphs the soil criteria described must be regarded as tentative.

Criteria for Clay

Coyle and Reese's criteria for load transfer curves

95. To develop soil criteria for the load transfer curves for a pile in clays, Coyle and Reese[11] proposed (after Woodward, Lundgren, and Boitano[25]) a reduction factor K to express the relationship between the cohesion of a clay and the shear strength that can be assumed to be effective in resisting axial load on a pile. Figure 29 shows that the reduction factor K is less than unity if the shear strength of a clay is over 1000 psf.

96. Coyle and Reese expressed the rate of load transfer developed on the side of a pile as a function of absolute pile movement. Curves were given for various depths (Figure 30).

97. The procedure for developing a load transfer curve for the side of a pile is summarized as follows:

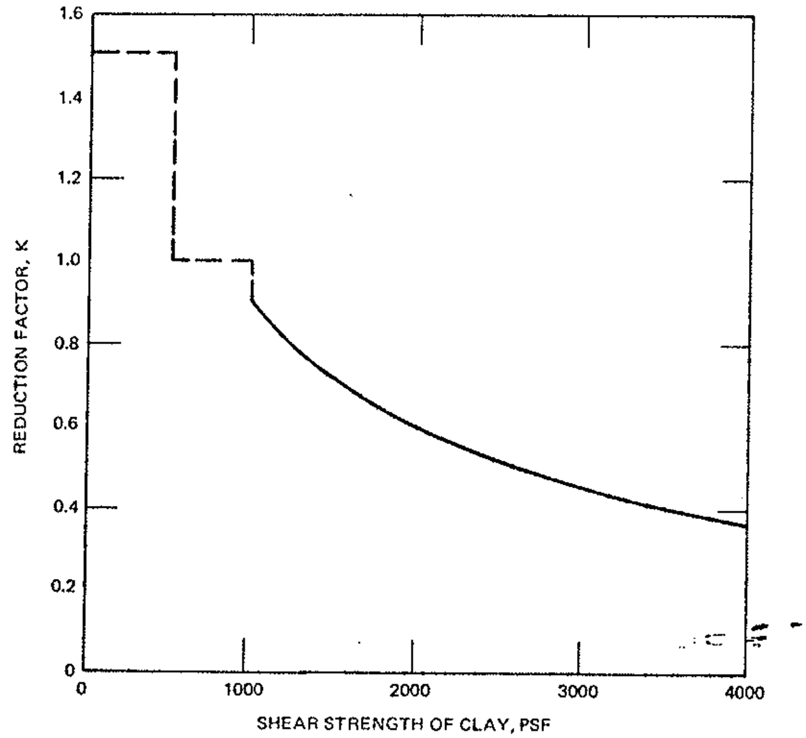


Figure 29. Reduction factor for maximum adhesion

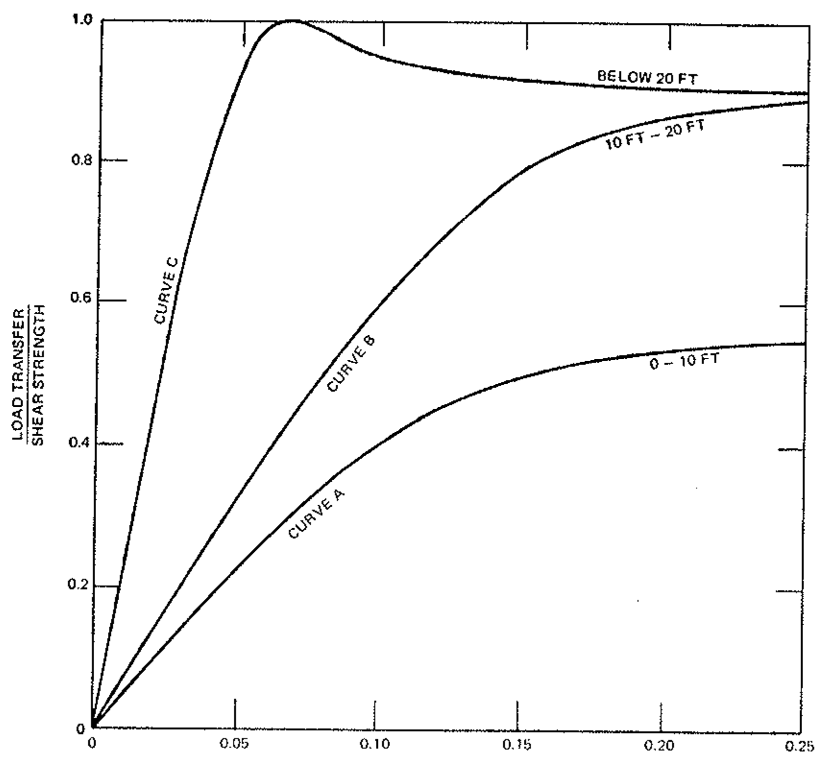


Figure 30. Nondimensional load transfer curves of a pile in clays

Step 1. Estimate the distribution of cohesion of the clays along the length of the pile from available soil data.

Step 2. Compute the effective shear strength as a function of depth from Figure 29 using the reduction factor K .

Step 3. Obtain the distance from the ground surface to the midpoint of the section where the load transfer curve needs to be developed.

Steo 4. Select the curve A, B, or C in Figure 30 depending on the depth.

Step 5. Choose a pile movement; obtain the ordinate from the selected curve in Figure 30.

Step 6. Compute load transfer for the selected pile movement by multiplying the ordinate obtained in step 5 by the effective shear strength obtained in step 2 and the circumferential area of the pile.

Step 7. Repeat steps 5 and 6 for other pile movements to construct the entire load transfer curve at that depth.

Step 8. Repeat steps 2 through 7 for varying depths to obtain a set of load transfer curves along a pile.

Skempton's criteria
for tip resistance curves

98. The point resistance curve for a pile in a clay may be generated by Skempton's[18] criteria. Starting with the theory of elasticity, Skempton found a correlation between the load-settlement curve of the shallow foundation and the stress-strain curve for the undrained triaxial compression test. The validity of the same correlation for a deep foundation was attested by examining the effect of the foundation depth on the pertinent variables in the basic equation. The correlation for piles can be expressed by

$$z_b = 2A_b\varepsilon \qquad (89)$$

$$q_b = m\sigma_\Delta A_b \qquad (90)$$

where

z_b = axial movement of base of pile

A_b = area of base of pile

q_b = normal pressure on base of pile

m = coefficient that can be taken as 5.0 to 5.5

99. If a stress-strain curve from undrained triaxial test is available, it is readily transformed to a point resistance curve as described for a laterally loaded pile earlier in this part. If no stress-strain curve is available, the procedure shown in Figure 27 can be followed to develop approximate stress-strain curves.

Criteria for Sand

100. Limited studies have been made for sands to establish generally applicable soil criteria for generating a set of load transfer curves along a pile and a point resistance curve at the tip of the pile. Two soil criteria are described below.

Coyle and Sulaiman's criteria

101. Coyle and Sulaiman[26] experimentally investigated the load transfer curves of a pile in sand. The ultimate shear transfer or skin friction on the side of a pile wall is expressed in the simplest form by

$$f_u = K\gamma X \tan \delta \qquad (91)$$

where

f_u = maximum shear transfer in psi

K = horizontal earth pressure coefficient at pile-soil interface whose value may lie somewhere between the active earth pressure coefficient K_A and the passive earth pressure coefficient K_P

δ = friction angle between the pile and the surrounding sand

102. Assuming that the earth pressure coefficient K is equal to one and the friction angle is equal to the angle of internal friction of the sand before disturbance, Coyle and Sulaiman found the relationship between the load transfer of a pile in a sand and the pile displacement.

103. Their conclusion, however, does not agree with the

experimental observation by Parker and Reese.[22] Coyle and Sulaiman state that at shallow depth there is a considerable increase in the actual maximum load transfer over that calculated by Equation 91 with the assumption of constant K and constant δ throughout the length of the pile. They further state that the maximum load transfer is reached at the lower portion of the pile with smaller pile displacement than at the upper portion. The observation by Parker and Reese indicated that the actual maximum load transfer at shallow depth is close to that obtained from Equation 91 with the same K and δ at all depths. Parker and Reese also found that the pile displacement necessary to reach the maximum load transfer increases linearly with depth.

Parker and Reese's criteria

104. Empirical criteria were established by Parker and Reese[22] for generating a set of load transfer curves along a pile in sand. The criteria correlates the load transfer curve with the stress-strain curve of a triaxial compression test. Their criteria includes a recommendation for the estimation of a point resistance curve.

105. The description of the procedure for generating a set of load transfer curves and a point resistance curve is given as follows:

Step 1. Determine the relative density of sand and the stress-strain curve of a triaxial test with the ambient pressure equal to the overburden pressure.

Step 2. Obtain the correction factor for the maximum load transfer as a function of the relative density of sand (Figure 31).

Step 3. Obtain modified correlation coefficients, which relate the deviator stress in the triaxial test with the load transfer on the side of the pile. The modified correlation coefficient for uplift loading is calculated by dividing the value obtained by Equation 92 with the correction factor (step 2).

$$U_t = \frac{D_t}{\tan^2(45° + \phi/2) - 1} \qquad (92)$$

where

U_t = correlation coefficient for uplift loading

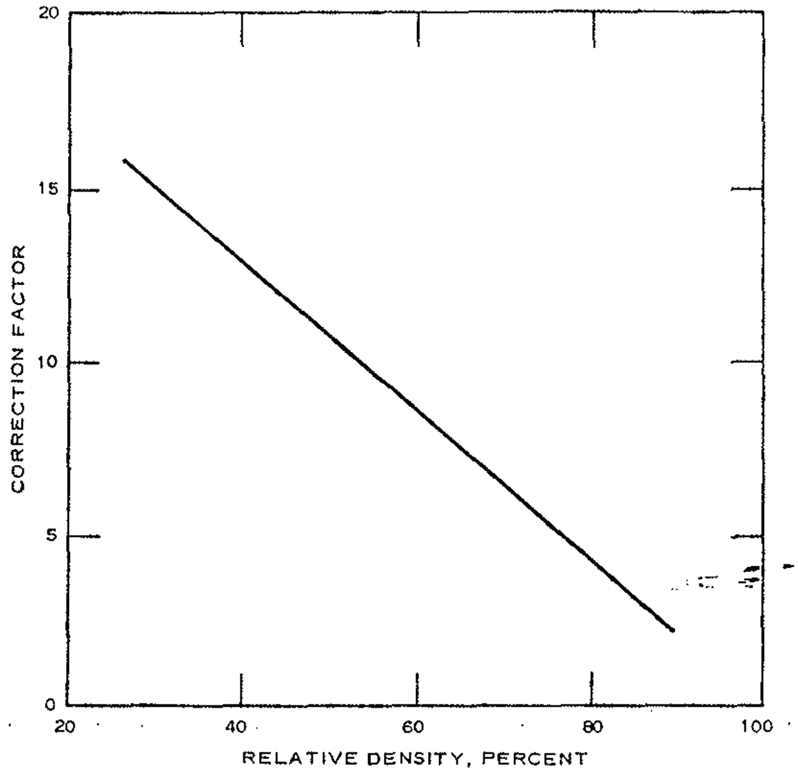


Figure 31. Correction factor for maximum load transfer

D_t = tension skin friction coefficient which is a function of the earth pressure coefficient and the friction angle. The value of 4.06 is assumed by Parker and Reese.

The modified correlation coefficient for a compression pile is calculated by dividing the value of Equation 93 with the correction factor (step 2).

$$U_c = \frac{D_c}{\tan^2(45° + \phi/2) - 1} \tag{93}$$

where

U_c = correlation coefficient for compression loading

D_c = compression skin friction coefficient which is a function of the earth pressure coefficient and the friction angle. Parker and Reese assume the value 5.3 or the value computed from 7.0 - 0.04x.

Step 4. Compute a load transfer curve from a stress-strain curve. Multiply the deviator stresses with the modified correlation coefficient (step 3) to obtain the values of load transfer. Then calculate the displacement of the pile by multiplying the axial strain in the triaxial test with the value obtained from Equation 94 or 95.

$$B_t = 0.15 + 0.012x \tag{94}$$

$$B_c = 0.4 + 0.016x \tag{95}$$

where

B_t = factor correlating upward pile movement to axial strain

B_c = factor correlating downward pile movement to axial strain

Step 5. Repeat steps 1 through 4 for depths up to 15 times the pile diameter. The curve for a depth of 15 pile diameters is used for the remainder of the pile.

Step 6. Construct a point resistance curve by combining any one of the bearing capacity formulas with the theory of elasticity solution for the settlement of a rigid footing on an elastic material (Skempton[18]).

<u>Meyerhof's criteria for tip resistance</u>

106. After Skempton, Yassin, and Gibson,[27] Meyerhof[28] proposed a simple criterion (Equation 96) for generating a point resistance of a pile in sands.

$$z = \frac{dq_b}{30q_{bu}} \tag{96}$$

where q_{bu} = unit ultimate bearing capacity

107. Considering the diversity of values of q_{bu} by various bearing capacity formulas (Vesić,[29] McClelland, Focht, and Emrich[30]), the unit ultimate bearing capacity of a pile point may be readily obtained from the empirical relationship with the standard penetration test (Meyerhof[28]).

$$q_{bu} = 60N \tag{97}$$

where N denotes the number of blows per foot penetration in the standard penetration test.

Summary

108. A set of load transfer curves along a pile in clays can be computed from the criteria by Coyle and Reese.[11] A point resistance curve for a pile in clays can be constructed from Skempton's criteria.

109. The load transfer curves along a pile in sands may be determined by the procedure given by Parker and Reese.[22] A point resistance curve for a pile in sands may be computed either according to the recommendation by Parker and Reese or according to Meyerhof's criteria.

110. Existing soil criteria can only make a rough prediction of the axial behavior of a pile. For a more accurate prediction of axial behavior of a pile, future development is needed of the theory for the mechanism of load transfer and of point resistance. The employment of the finite element method to solve the pile-soil interaction problems can perhaps eliminate the use of semi-empirical criteria to develop load transfer and point resistance curves.

PART VI: DISCRETE ELEMENT THEORY FOR BEAM-COLUMNS

111. The computer code BMCOL51, developed by Matlock and Taylor,[3] utilizes a discrete element mechanical model for describing the load-deformation response of a beam-column. The equations obtained from the discrete element model are similar to those obtained with finite-difference approximations for the differential equations for bending, and the results are approximately the same. However, the equations obtained from the discrete element model can be grouped into a system of equations that allows a variety of boundary conditions to be applied; whereas the system of equations obtained with finite difference theory permit only the application of certain boundary conditions at definite locations along the beam-column, i.e. two at both ends. As a result, BMCOL51 is a more versatile program than COM62 since a wide variety of problems can be solved with BMCOL51 while COM62 is designed exclusively for the analysis of piles or beams on grade. However, BMCOL51 is limited to problems where the reactions can be characterized by linear springs, whereas COM62 can consider nonlinear soil response. It must be noted that BMCOL51 is one of the earlier BMCOL programs written under the guidance of Prof. Matlock. Currently available versions of BMCOL programs are more versatile and can account for nonlinear material and geometric properties.

112. The discrete element model for representing a beam-column will be described in subsequent paragraphs. The development of the model and equations for describing the model are taken directly from Matlock and Haliburton.[31] Likewise, the figures used in the development were extracted from Matlock and Haliburton with changes made to the notation to ensure compatiblity with the remainder of the report. Equations expressing the response of the discrete element model will be derived and used in formulating a set of simultaneous equations for predicting the response of a beam-column. Finally, the procedure utilized in BMCOL51 for solving the simultaneous equations will be presented. This code also has the capability of solving problems with moving loads.

Discrete Element Representation of the Response of a Simple Beam

113. To begin the development, a mechanical model representing a conventional beam will be considered. The model shown in Figure 32 was developed by Matlock and Haliburton.[31] Figure 32(a-c) illustrates how the deformation of a linear elastic beam element under the action of pure bending may be represented. If we consider the overall behavior of the element, then a mechanical analog of the element in Figure 32c may be formed by the rigid plates, hinge, and linear springs in Figure 32d. The stiffness of the springs represent the flexural stiffness of the beam element. To form a beam a number of the mechanical elements can be strung together as shown in Figure 32e. The mechanical model thus formed would truly represent the behavior of the beam if the individual mechanical elements were infinitely small. However, for practical problems, as with any mechanical model or approximate numerical procedure, accuracy must be sacrificed in order that the number of calculations required be kept within practical limits. As it turns out, a cruder model (Figure 32f), where the rigid plates are replaced by rigid bars of length h, may be used to represent the beam without serious loss of accuracy.

114. The equations describing the behavior of the mechanical model may be formulated by considering the deformed segment of a finite-element beam model in Figure 33. The deformable element of the model is represented schematically as a deformable joint with the same behavior as the spring and hinges in Figure 32. If we assume that the effect of a lateral force w distributed along the beam for a distance $h/2$ on either side of a joint) may be represented by a concentrated force

$$W_i = hw_i \tag{98}$$

acting at the deformable joint, then the equation describing the behavior of the beam may be formulated.

115. The change in slope, ϕ_i, between bars A and B may be written as

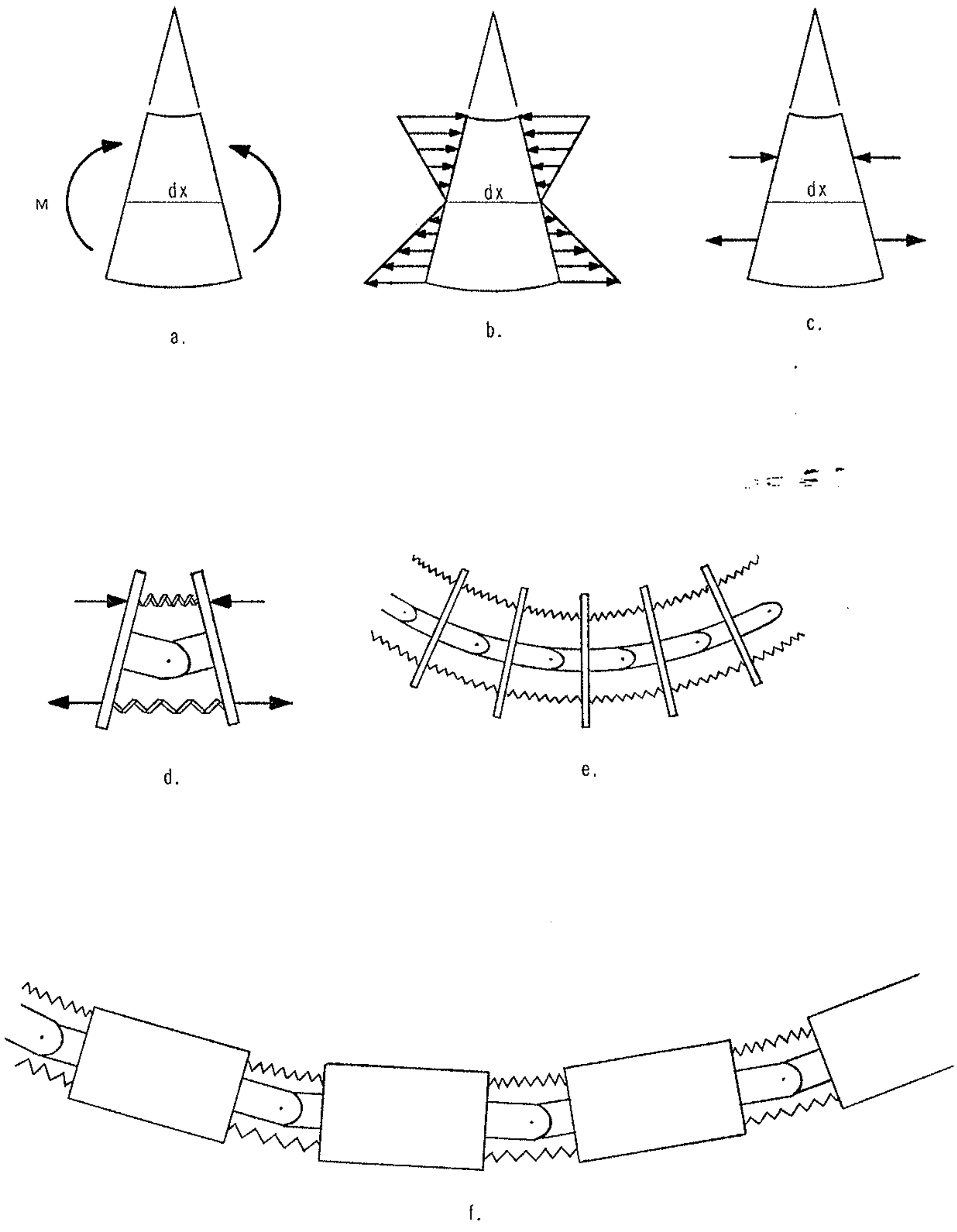


Figure 32. Finite mechanical representation of a conventional beam

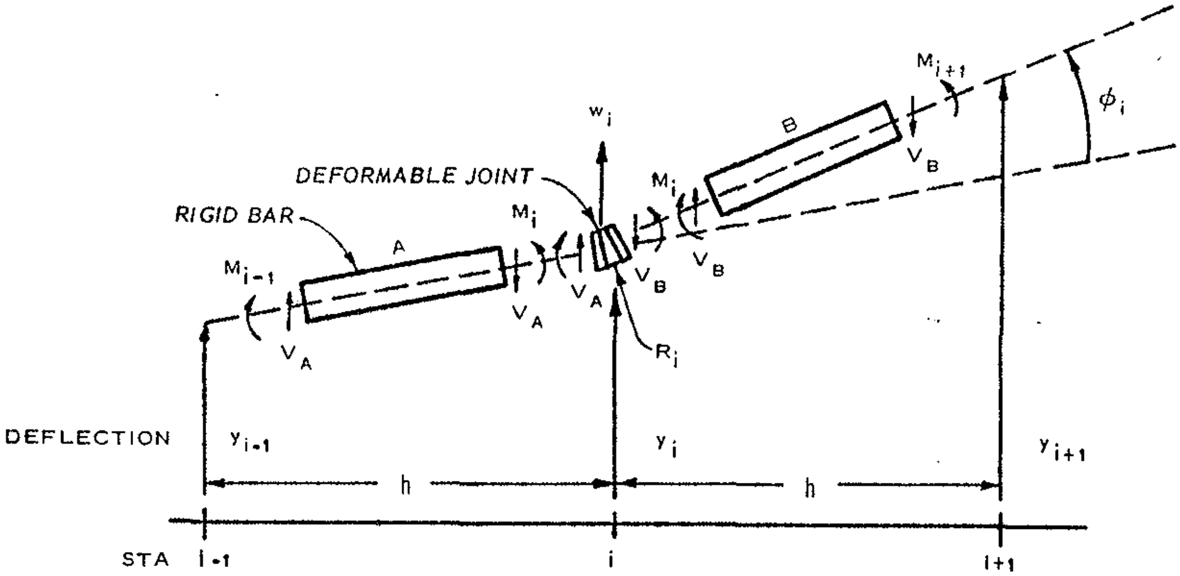


Figure 33. Deformed segment of a finite-element beam model

$$\phi_i = \frac{y_{i-1} - 2y_i + y_{i+1}}{h} \tag{99}$$

In order to establish the similarities between the equations obtained using finite difference approximations and discrete element theory, Equation 99 is written as

$$\phi_i = h\left(\frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}\right) \tag{100}$$

The expression in the parenthesis is equivalent to Equation 2, the expression for the second derivative of deflection. The second derivative represents the curvature of the elastic curve and the finite difference expression represents an approximation of the curvature so that the moment-curvature relationship could be approximated by Equation 12. If we assume that ϕ_i represents the concentration of the beam curvature for one increment, then the moment curvature relationship for an arbitrary joint i may be expressed as

$$M_i = \frac{R_i \phi_i}{h} \tag{101}$$

If we assume that all external forces are applied to the beam as concentrated forces at the deformable joints, then the summation of forces on the deformable joint yields

$$W_i + V_A - V_B = 0 \tag{102}$$

where V_A and V_B are the shear in bars A and B , respectively. Summation of moments about bars A and B yield, respectively,

$$M_{i-1} - M_i + V_A h = 0 \tag{103}$$

and

$$M_i - M_{i+1} + V_B h = 0 \tag{104}$$

Equations 103 and 104 may be substituted into Equation 102 and the shear V_A and V_B eliminated to yield

$$M_{i-1} - 2M_i + M_{i+1} = hW_i \tag{105}$$

The expression for moment in Equation 101 when substituted into Equation 105 yields

$$R_{i-1}\phi_{i-1} - 2R_i\phi_i + R_{i+1}\phi_{i+1} = h^2W_i \tag{106}$$

Substituting the expression for the beam curvature, Equation 99 into Equation 106 yields

$$R_{i-1}\left(\frac{y_{i-2} - 2y_{i-1} + y_i}{h}\right) - 2R_i\left(\frac{y_{i-1} - 2y_i + y_{i+1}}{h}\right) + R_{i+1}\left(\frac{y_i - 2y_{i+1} + y_{i+2}}{h}\right) = h^2W_i \tag{107}$$

Simplifying Equation 107 yields

$$y_{i+2}(R_{i+1}) + y_{i+1}(-2R_{i+1} - 2R_i) + y_i(R_{i+1} + 4R_i + R_{i-1})$$

$$+ y_{i-1}(-2R_i - 2R_{i-1}) + y_{i-2}(R_{i-1}) = h^3W_i \qquad (108)$$

The above equation is identical to Equation 10 if the effects of axial load are omitted in Equation 10 and the expressions for the applied lateral load are equated. The effects of lateral load in Equation 10 are represented by the expression $y_iE_{si}h^4$, where the distributed lateral force that resulted from the soil reaction was given by the expression

$$w_i = E_{si}y_i \qquad (109)$$

Since the expression for the concentrated force at the joint is given by the expression

$$W_i = hw_i \qquad (110)$$

it can be seen that expressions 108 and 10 are identical except for the sign of the term for the lateral force. This difference results simply from the sign convention used in developing the equations and has no physical significance.

Discrete Element Representation of the Response of a Generalized Beam-Column

116. For simple beams, with only lateral forces applied, closed form solutions are available for most cases. However, realistic engineering problems usually involve the application of axial loads and a variety of external loading and restraint conditions. The mechanical model representation of a beam-column and the equations describing the response of the beam-column will be developed below.

117. The external forces and restraints that will be considered are presented in Figure 34. The forces and restraints are shown acting in the positive sense. Lowercase letters represent distributed loads and restraints while corresponding capital letters denote concentrated forces and restraints.

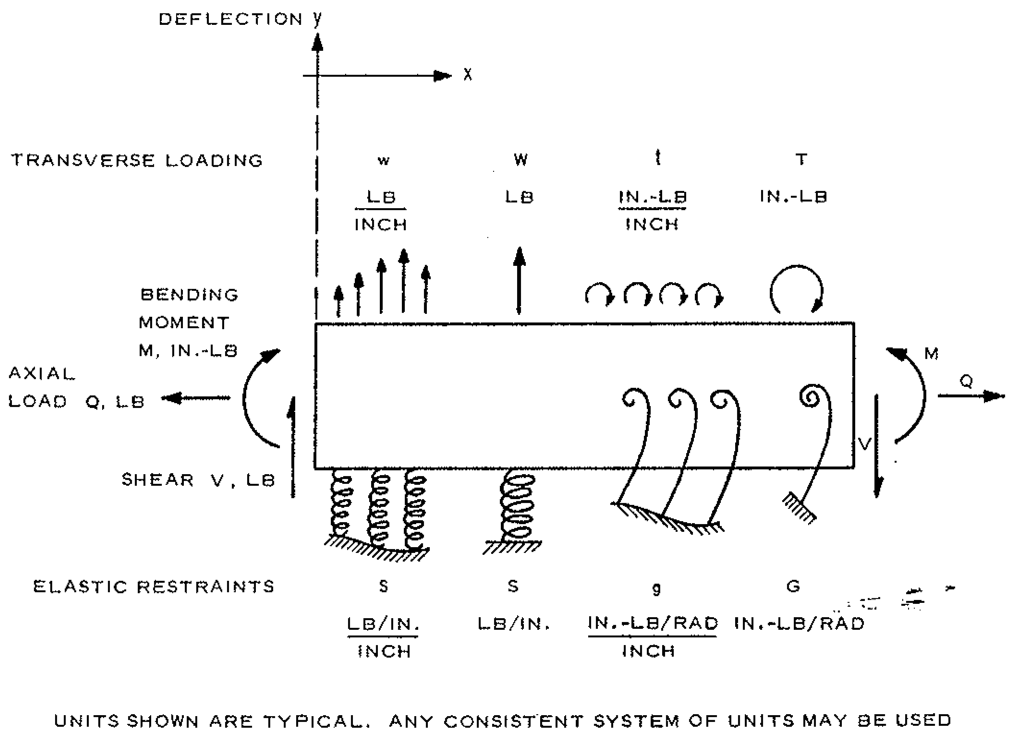


Figure 34. Loads and restraints considered in the generalized beam-column solution. All effects are shown acting in a positive sense in relation to the x-direction

118. Equations describing the behavior of a generalized beam-column may be derived by considering the beam-column element in Figure 35 deflected a distance y and rotated through an angle dy/dx. Summing moments about the right end of the element yields

$$dM - V\,dx - t\,dx - w\frac{(dx)^2}{2} + sy\frac{(dx)^2}{2} - g\,dy - Q\,dy - dQ\frac{dy}{2} = 0 \quad (111)$$

where

t = distributed externally applied moment or torque

w = distributed lateral force

s = distributed lateral restraint

g = distributed rotational restraint

Q = axial force

Neglecting higher order terms and dividing both sides of the equation by dx yields

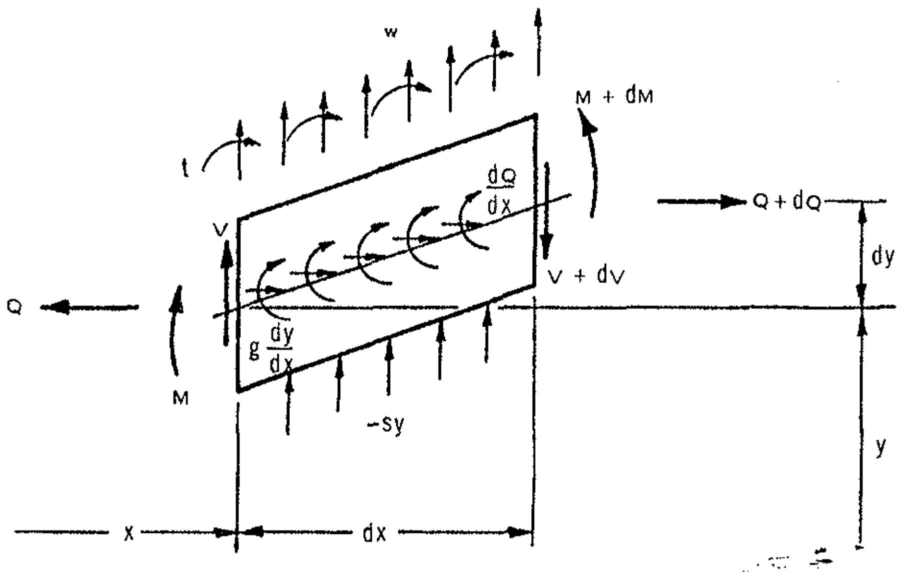


Figure 35. Generalized beam-column element deflected a distance y and tilted through some angle dy/dx

$$\frac{dM}{dx} = V + t + (g + Q)\,\frac{dy}{dx} \tag{112}$$

Summation of vertical forces on the element in Figure 35 yields

$$\frac{dV}{dx} = w - sy \tag{113}$$

Differentiating Equation 112 and substituting Equation 113 into the resulting equation to eliminate the shear yields

$$\frac{d^2M}{dx^2} = w - sy + \frac{d}{dx}\left[t + (g + Q)\,\frac{dy}{dx}\right] \tag{114}$$

The above equation corresponds to Equation 7 but with the addition of the effects of externally applied moments, rotational resistance, an axial load that is variable with x, and externally applied lateral loads. The term $E_s y$ in Equation 7 is analogous to the term sy in the above equation in that they both represent the effects of lateral restraints. However, the variable E_s is a secant modulus value obtained from a nonlinear curve while s symbolizes a linear relationship between load and deflection. The above equation is next converted into finite difference forms and the effects of distributed applied loads and restraints concentrated in order to develop a mechanical model that will represent the response of the beam as predicted by the finite difference equations.

119. The left side of Equation 114 is converted into finite difference form by first writing the expression

$$\left(\frac{d^2M}{dx^2}\right)_i \approx \frac{M_{i-1} - 2M_i + M_{i+1}}{h^2} \tag{115}$$

The equations for the moment at the nodal points given by Equation 101 are substituted into Equation 115 yielding

$$\left(\frac{d^2M}{dx^2}\right)_i \approx \frac{1}{h^4}\Big[y_{i+2}(R_{i+1}) + y_{i+1}(-2R_{i+1} - 2R_i) + y_i(R_{i+1} + 4R_i + R_{i-1}) + Y_{i-1}(-2R_i - 2R_{i-1}) + y_{i-2}(R_{i-1})\Big] \tag{116}$$

The right-hand side of Equation 114 is converted to finite difference form by writing w as w_i and sy as s_iy_i. The remainder of the expression is converted by writing

$$\left(\frac{dt}{dx}\right)_i \approx \frac{t_{i+1} - t_{i-1}}{2h} \tag{117}$$

and

$$\frac{d}{dx}\left[(g + Q)\frac{dy}{dx}\right] = \frac{1}{2h}\left[(g_{i+1} + Q_{i+1})\frac{y_{i+2} - y_i}{2h} - (g_{i-1} + Q_{i-1})\frac{y_i - y_{i-2}}{2h}\right] \tag{118}$$

Writing the right-hand side in its entirety yields

$$\left(\frac{d^2M}{dx^2}\right)_i = w_i - s_i y_i + \frac{1}{2h}\left[t_{i+1} + (g_{i+1} + Q_{i+1})\,\frac{y_{i+2} - y_i}{2h}\right.$$
$$\left. - t_{i-1} - (g_{i-1} + Q_{i-1})\left(\frac{y_i - y_{i-2}}{2h}\right)\right] \qquad (119)$$

Equating Equations 116 and 119 yields for nodal point i the expression

$$\frac{1}{h^4}\left[y_{i+2}(R_{i+1}) + y_{i+1}(-2R_{i+1} - 2R_i) + y_i(R_{i+1} + 4R_i + R_{i-1})\right.$$
$$\left. + y_{i-1}(-2R_{i-1}) + y_{i-2}(R_{i-1})\right]$$
$$= w_i - s_i y_i + \frac{1}{2h}\left[t_{i+1} + (g_{i+1} + Q_{i+1})\left(\frac{y_{i+2} - y_i}{2h}\right)\right.$$
$$\left. - t_{i-1} - (g_{i-1} + Q_{i-1})\left(\frac{y_i - y_{i-2}}{2h}\right)\right] \qquad (120)$$

Combining terms, Equation 120 is written as

$$y_{i+2}\left[R_{i+1} - \frac{h^2}{4}(g_{i+1} + Q_{i+1})\right] + y_{i+1}(-2R_{i+1} - 2R_i)$$
$$+ y_i\left[R_{i+1} + 4R_i + R_{i-1} + h^4 s_i + \frac{h^2}{4}(g_{i+1} + Q_{i-1} + g_{i-1} + Q_{i-1})\right]$$
$$+ y_{i-1}(-2R_i - 2R_{i-1}) + y_{i-2}\left[R_{i-1} - \frac{h^2}{4}(g_{i-1} + Q_{i-1})\right]$$
$$= h^4 w_i + \frac{h^3}{2}(t_{i+1} - t_{i-1}) \qquad (121)$$

The distributed externally applied loads and restraints may be lumped at the nodal points as concentrated forces, W_i , concentrated springs S_i , concentrated moments T_i and concentrated rotational restraints G_i , by the following equations:

$$W_i = hw_i \qquad (122)$$

$$S_i = hs_i \qquad (123)$$

$$T_i = ht_i \tag{124}$$

$$G_i = hg_i \tag{125}$$

Substituting Equations 122-125 into Equation 121 results in an equation describing the behavior of the mechanical model shown in Figure 36. The flexural stiffness R_i is concentrated at the increment point in the

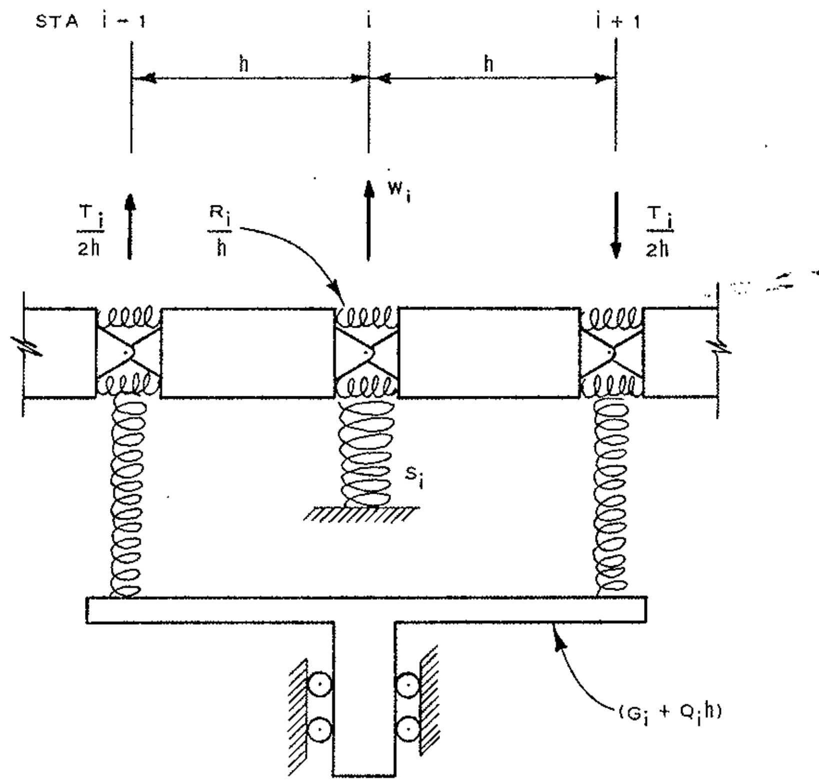


Figure 36. Mechanical model corresponding exactly to beam-column equations

form of a spring-restrained hinge between two rigid segments. All load and support values are ultimately felt by the beam as transverse forces applied at nodal points. This is obvious for the lateral load W_i and for the couple created by forces $T_i/2h$. It is also true for the reaction from the spring S_i as well as for two equal but opposite reactions from the angular restraint mechanism which acts as an exact analog for the combined effect of a rotational spring G_i and the axial tension (or compression) Q_i .

120. The deflections that result from the solution of Equation 121 represent a set of deflections for the nodal points of the mechanical model for the beam-column that will satisfy compatibility and equilibrium at each nodal point and of each nondeformable bar in the model. In the subsequent paragraphs the procedure for solving the equations will be described.

Recursive Solution Technique

121. Equation 121 may be written in the following form:

$$a_i y_{i+2} + b_i y_{i+1} + c_i y_i + d_i y_{i-1} + e_i y_{i-2} = f_i \tag{126}$$

where

$$a_i = R_{i+1} - \frac{h}{4}\left(G_{i+1} + Q_{i+1}h\right) \tag{127}$$

$$b_i = -2\left(R_{i+1} + R_i\right) \tag{128}$$

$$c_i = R_{i+1} + 4R_i + R_{i-1} + h^3 S_i + \frac{h}{4}\left(G_{i+1} + hQ_{i+1} + G_{i-1} + hQ_{i-1}\right) \tag{129}$$

$$d_i = -2\left(R_i + R_{i-1}\right) \tag{130}$$

$$e_i = R_{i-1} - \frac{h}{4}\left(G_{i-1} + hQ_{i-1}\right) \tag{131}$$

$$f_i = h^3 W_i + \frac{h^2}{2}\left(T_{i+1} - T_{i-1}\right) \tag{132}$$

For a beam-column represented by a mechanical model as illustrated in Figure 37a, a set of simultaneous equations composed of Equation 126 written for each nodal point may be formulated. The simultaneous equations when written in matrix form result in the matrix equation

$$[K]\{y\} = \{f\} \tag{133}$$

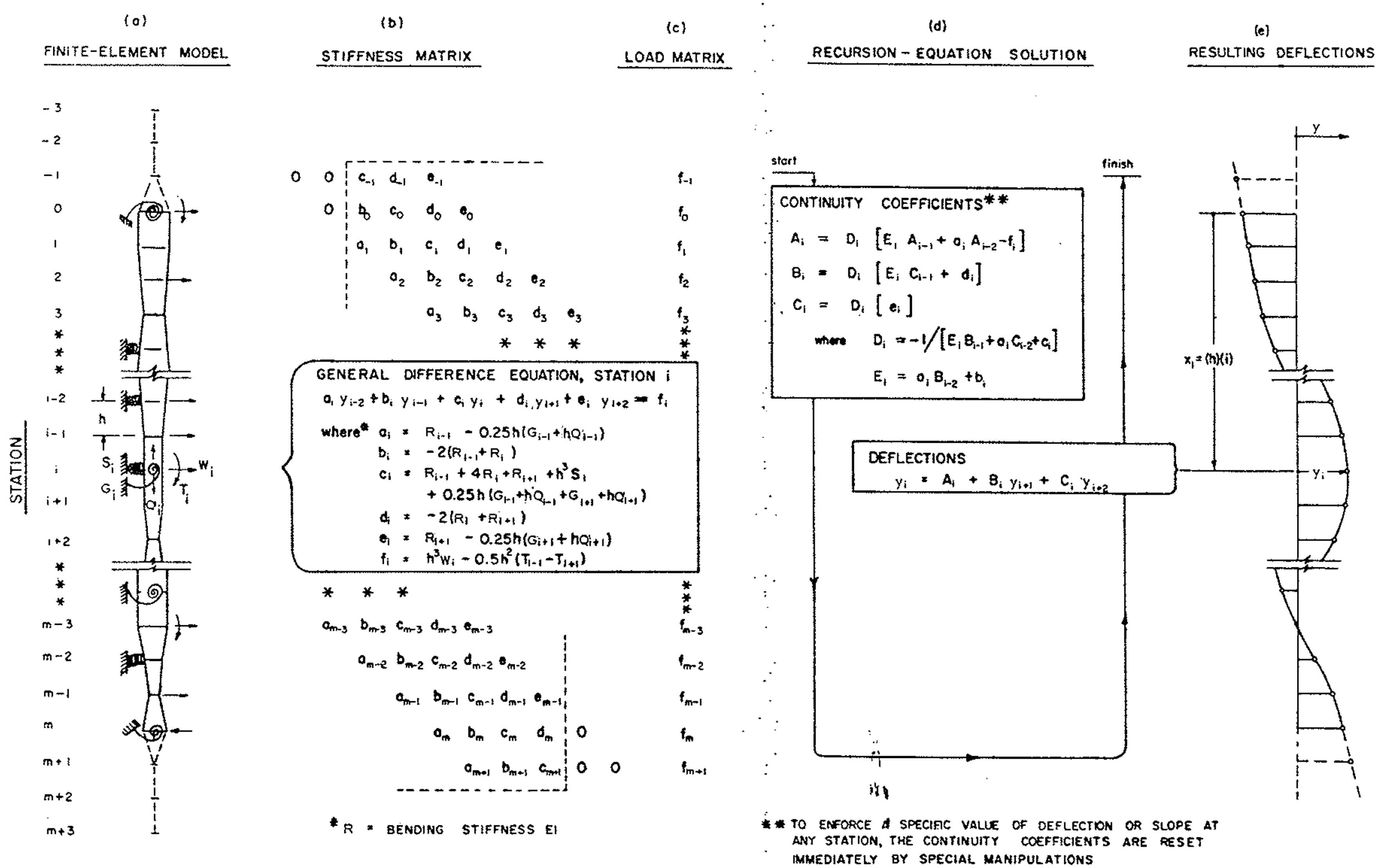


Figure 37. The system of m + 3 simultaneous equations and the method of direct, once-through solution

The stiffness matrix [K] is a diagonally banded matrix containing terms a through e. The deflection matrix {y} is a single column matrix as is the load matrix {f}. The unknown matrix in the matrix equation is the deflection matrix. A recursive technique is utilized to solve for the deflections. Once the deflections are found, the moments, shear, or any force exerted by a restraint spring may be computed by substituting the appropriate deflections into the appropriate equation.

122. In order to begin and end the recursive process, it is necessary to establish three fictitious stations at each end of the beam-column. The fictitious stations have no flexural stiffness and thus act as multiple hinges. If such a system were added to the mechanical model, the response of the remainder of the model would not be affected; therefore, the solution obtained for the equations is not affected by this addition. These fictitious stations are added for computational purposes only and are generated automatically by the computer code BMCOL51.

123. The recursive solution technique is illustrated in Figure 37d. Each equation contains five unknown deflections. In the first pass, two unknown deflections are eliminated from each equation. Starting from the top, the deflections y_{i-2} and y_{i-1} are eliminated. The resulting equations form a diagonally banded matrix in which each equation contains only three unknown deflections. During the reverse pass the solution for the deflection at sta i is computed. The deflection y_i can be determined only when y_{i+1} and y_{i+2} are known. During the forward pass the application of boundary and specified conditions will establish values or relationships between deflections such that y_{i+1} and y_{i+2} are known after completion of the forward pass. This permits the computation of y_i during the reverse pass. After the reverse pass is completed, the deflections at each nodal point are known. With these values, moments, shear slope, or reaction may be obtained by applying the appropriate finite difference equation.

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APPENDIX A: NOTATION

a_i, b_i, c_i	Recursive coefficients computed (in solution of finite difference equations
A	Cross-sectional area of pile
A_b	Area of base of pile
B_c	Factor correlating downward pile movement to axial strain
B_t	Factor correlating upward pile movement to axial strain
c	Cohesion of clay
C	Pile circumference
d	Diameter of pile for circular piles or equivalent diameter of other shapes
D_c	Compression skin friction coefficient
D_t	Tension skin friction coefficient
E	Modulus of elasticity of pile material
E_s	Soil modulus (lateral soil reaction divided by lateral deflection)
f	Shear force per unit area (as a function of the location along a pile)
f_u	Maximum shear transfer in psi
F	Shear force per unit length
F_u, F_v, M_s	Forces and moment exerted by each pile
g	Distributed rotational restraint
G	Concentrated rotational restraint
h	Increment length
H	Constant depending on relative density of sand
I	Moment of inertia of pile section
J_x, J_y, J_m	Secant modulus values
K	Horizontal earth pressure coefficient at pile-soil interface
K_A	Coefficient of active earth pressure
K_o	Coefficient of earth pressure at rest
L	Length of pile
m	Coefficient
M	Bending moment

M_e, P_u, P_v	External moment, horizontal force, and vertical force (applied at origin of u-v coordinate system)
M_i, M_{i+1}, M_{i-1}	Moment at the respective joints
M_t	Bending moment applied to top of pile
N	Number of blows per foot penetration
p	Lateral soil reaction per unit length
p_u	Ultimate lateral soil reaction
P_1	Resultant force per unit length of pile
P_t	Lateral load applied to top of pile
q_b	Normal pressure on base of pile
q_{bu}	Unit ultimate bearing capacity in psi
q_u	Unconfined compressive strength
Q	Axial load; axial force
Q_b	Load due to the normal pressure on the base of a pile
Q_c	Constant axial load in pile
Q_t	Axial load applied to top of pile
R	EI (flexural rigidity)
s	Distributed lateral restraint
S	Concentrated lateral restraint
SF	Force representing concentration of shear transfer at a joint
t	Distributed externally applied moment or torque
T	Concentrated externally applied moment or torque
u-v	Coordinate system (for describing geometry of the foundation)
U_c	Correlation coefficient for compression loading
U_t	Correlation coefficient for uplift loading
V	Shear
V_A, V_B	Shears in bars A and B
w	Distributed lateral force
W	Concentrated lateral force
x	Distance along axis of pile
X	Depth from soil surface
y	Lateral deflection
z	Axial movement of pile

z_b	Axial movement of base of pile
α	Constant (values for loose and dense sand)
β	Constant = $45^o + \emptyset/2$
γ	Effective unit weight of soil
δ	Friction angle (between the pile and the surrounding sand)
$\Delta_v, \Delta_u, \Gamma$	Components of foundation movement
ϵ	Axial soil strain (from triaxial compression test)
ϵ_{50}	Fifty percent of the maximum elastic axial soil strain (from triaxial compression test)
θ	Angular measure of pile batter
λ_1, ν_1	Boundary condition coefficients (computed in solution of finite difference equations)
σ_Δ	Soil deviator stress $(\sigma_1 - \sigma_3)$
$\sigma_{\Delta 50}$	Deviator stress corresponding to 50 percent maximum elastic axial strain
σ_1, σ_3	Axial and confining stress (in triaxial compression test)
$\emptyset$	Angle of internal friction of sand
ψ	A function relating axial load to the relative axial movement between the pile and soil
Ω_t	Slope at top of pile

APPENDIX B: USER'S GUIDE FOR PROGRAM COM62

General Introducton

1. Documentation for the computer program COM62 - to analyze laterally loaded piles in nonlinear soil media - is presented in this appendix and includes a general introduction, program listing, flow charts, guide for data input, and input-output data for two example problems.

2. COM62 is a finite difference computer code (developed by Dr. L. C. Reese, University of Texas (UT), Austin, Texas) that can solve for deflection, shear, moment, and reactions in a single pile under a variety of boundary conditions specified at the top of the pile. The quantities input at the top of the pile can be one of the following combinations: lateral load and a moment; lateral load and a specified slope; lateral load and a specified moment/slope value. The force-deformation characteristics of the soil are represented by a series of nonlinear springs.

3. Typical curves that relate the soil resistance to the lateral movement of the pile are shown in Figure B1. Procedures for obtaining

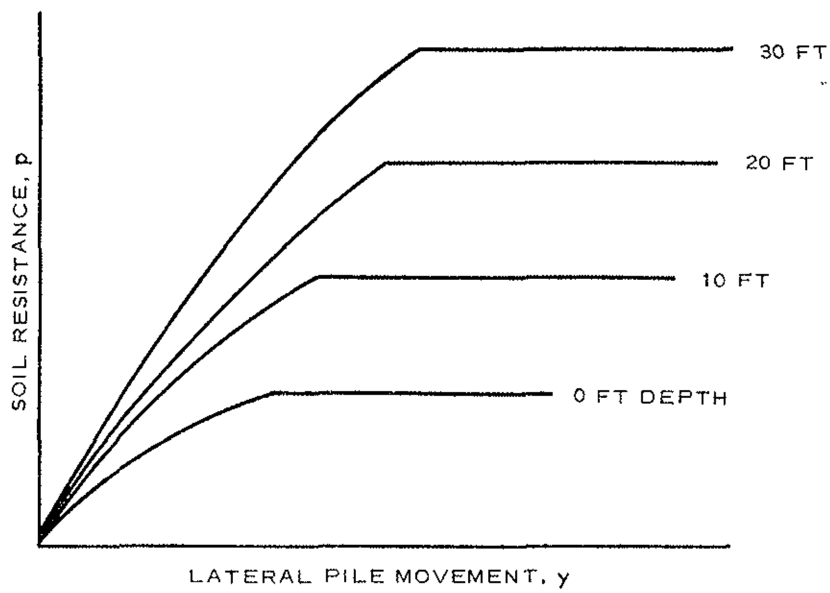

Figure B1. Examples of p-y curves at various depths in soil such curves from laboratory soil test data are described in the text (Part V). The computer program MAKE that can automatically generate such curves from laboratory soil data is documented in Appendix D. COM62 can handle variable flexural rigidity (EI) of the pile and layered soil media. If an axial load is specified at the top of the pile, it is assumed to be constant throughout the length of the pile.

4. In the analysis used in COM62, compatibility is achieved between the inelastic soil and the elastic pile (which is elastically restrained by the superstructure) by repeated application of the elastic theory. The soil stiffness constants are adjusted for each trial in accordance with the specified force-deformation relations for the soil. Thus, the iterative analysis consists of a conventional beam on elastic foundation analysis coupled with the proper prediction of force-deformation characteristics of the soil.

5. Input may be input interactively at execute time, or input may be in a prepared data file. Output will be directed to an output file.

Flow Charts

6. A flow chart for the program is shown in Figure B2. The sequence of operations for subroutine soil is diagramed in Figure B3.

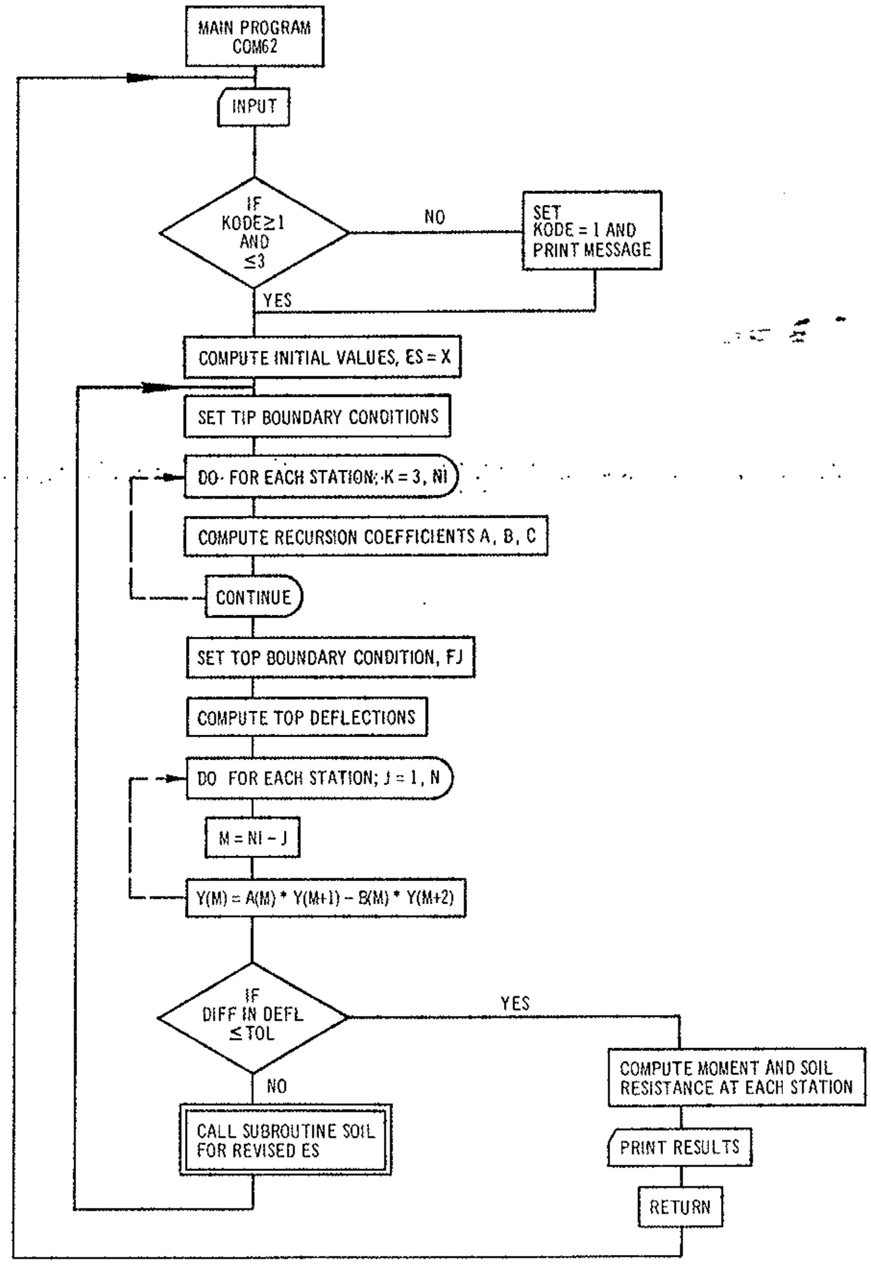

Figure B2. Flow chart for COM62

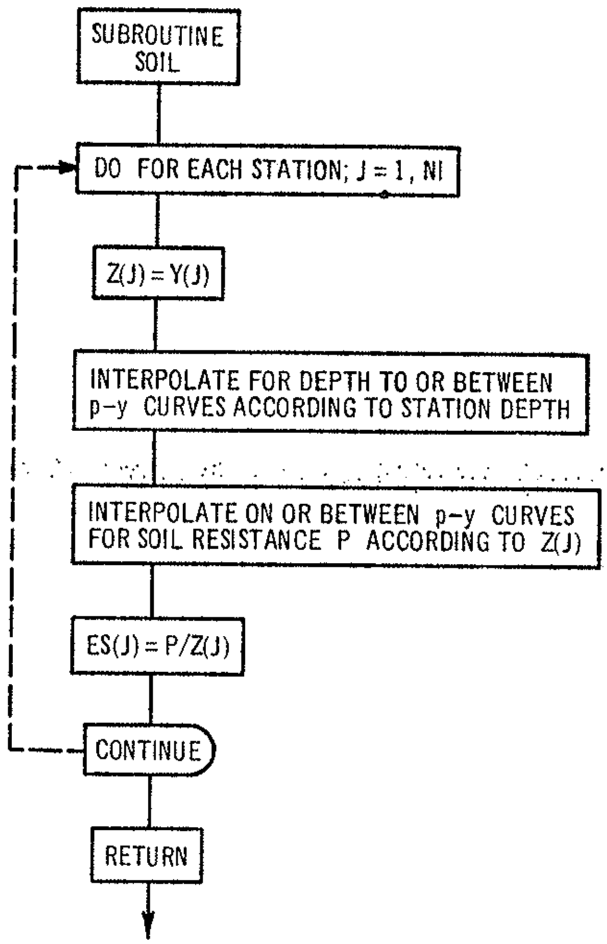

Figure B3. Flow chart of subroutine soil for COM62

Guide for Data Input

7. Data should be input to program COM62 according to the following guide. All input is in free-field format.

Group 1 - Title

RUN

RUN = 60 character problem heading

Group 2 - Problem Parameters

PT, BC2, D, H, TOL, N, KODE, NEWPY

PT = Lateral load at top of pile, lb

BC2 = Secondary boundary condition value (i.e., value of MT (in.-lb), ST, or MT/ST - see KODE below)

D = Pile diameter, in.

H = Increment length, in.

TOL = Increment tolerance for deflections, in.

N = Number of increments
(Product of N and H equals length of pile.)

KODE = Boundary condition control parameter
1 --- use lateral load (PT) and moment (MT).
2 --- use lateral load (PT) and slope (ST).
3 --- use lateral load (PT) and MT/ST.

NEWPY = Control parameter to specify if a new p-y curve will be read in.
0 --- Program will not read p-y curves (will use old p-y data)(Do not specify Group 3).
1 --- A new set of p-y curves will be read.

Group 3 - Soil Resistance-Pile Movement Data

A.

NX, NUM

NX = Number of p-y curves

NUM = Number of points on each p-y curve

B. (i) | X(K) |

(ii) | YM(J,K), PP(J,K) |

X(K) = Distance from top of pile to the K^{th} p-y curve, ft

YM(J,K) = Deflection at J^{th} point on K^{th} p-y curve. J goes from 1 to NUM and K goes from 1 to NX, in.

PP(J,K) = Soil resistance at J^{th} point on K^{th} p-y curve. J goes from 1 to NUM and K goes from 1 to NX, lb/in.

Note: Set B is repeated until NX number of p-y curves have been supplied.
Line (ii) of Set B is repeated within each set until NUM deflections and soil resistances have been supplied for that set.
A p-y curve must always be specified at the top of the pile.

Group 4 - Flexural Rigidity Data

A. | I |

B. | RR(J), XX(J) |

I = Number of different flexural rigidity values for a pile.

RR(J) = The J^{th} flexural rigidity value, lb x in^2. J goes from 1 to I.

XX(J) = The distance from top of pile to point where J^{th} flexural rigidity value occurs. J goes from 1 to I.

Note: Set B should be repeated until I number of flexural rigidity and their location values have been supplied.

Group 5 - Axial Load Data

| PX |

PX = Axial load at the top of pile, lb

Example Problems

8. To illustrate the preparation of input data for program COM62, two example problems (in one run) will be solved. Figures B4 and B5 show the physical problems; Figure B6, the input p-y curve for both examples; and Table B1, the input data to the program. The computer outputs for the first and second examples are given in Tables B2 and B3, respectively. The results are also plotted in Figures B7-B9 for Example Problem 1 and in Figures B10-B12 for Example Problem 2.

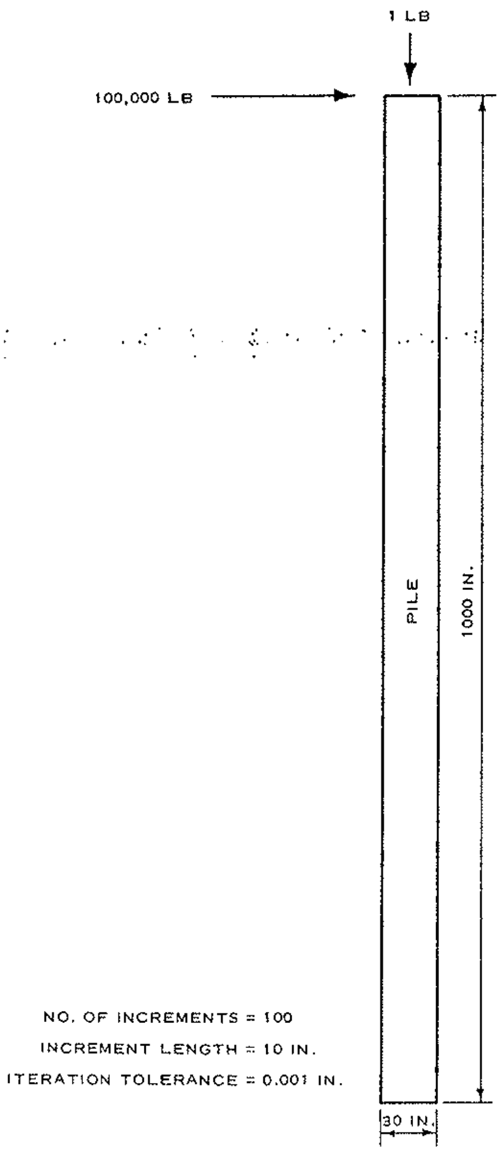

Figure B4. Physical problem for Example Problem 1

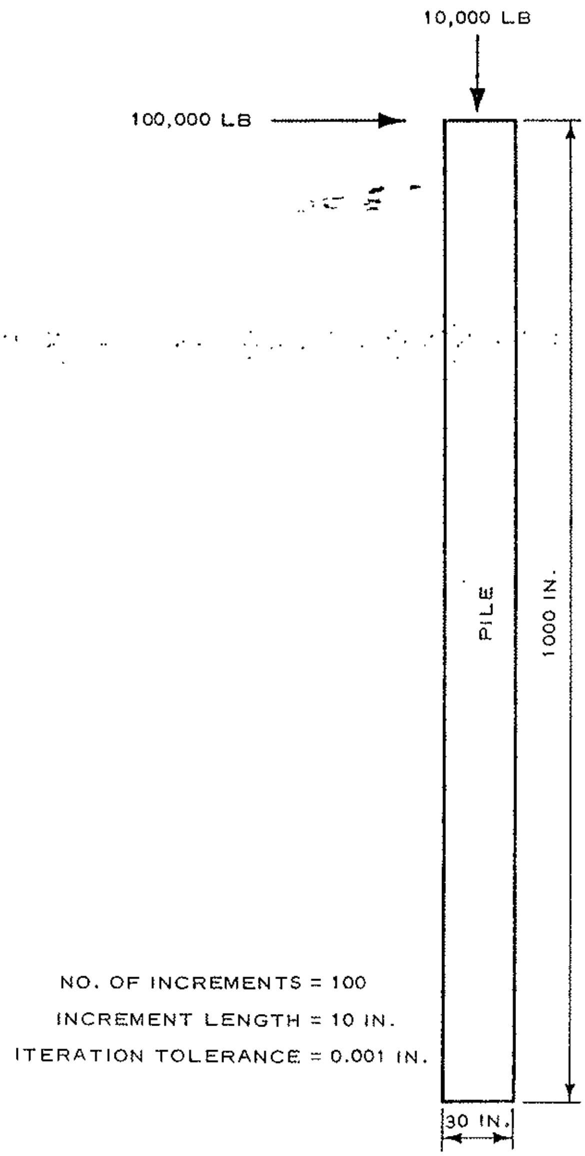

Figure B5. Physical problem for Example Problem 2

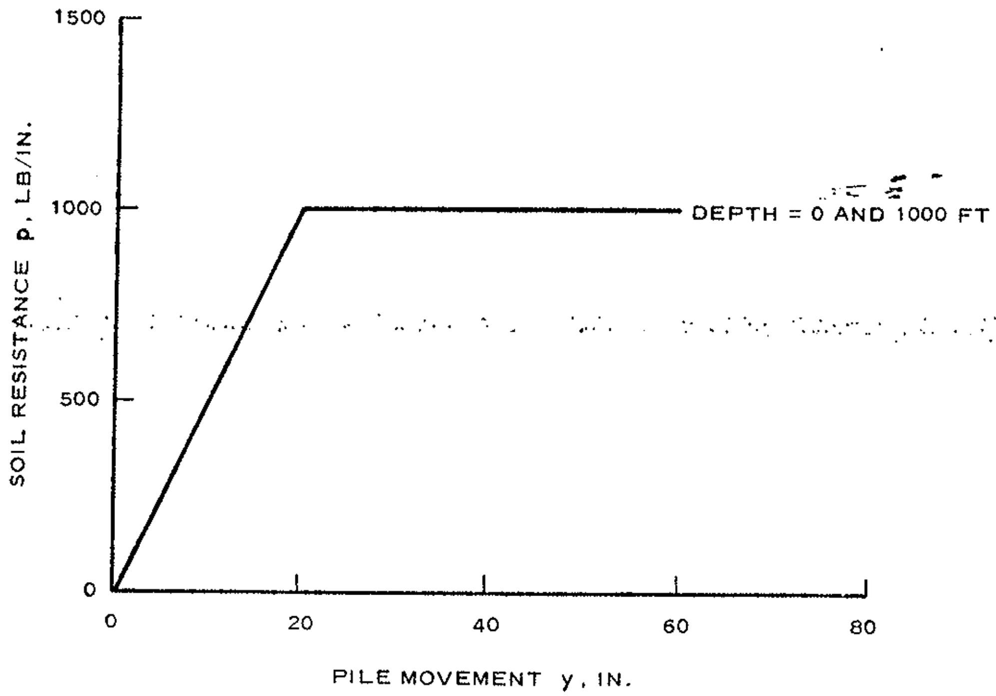

Figure B6. Input p-y curves for Example Problems 1 and 2

Table B1

Input for Example Problems 1 and 2

```
*LIST DAT62

10 RUN 1 COM62 - LATERAL LOAD=100000 AXIAL LOAD=1
20 100000,0,30,10,0.001,100,1,1
30 2,2
40 0.0
50 0,0,0,0
60 20.,1000.
70 1000.
80 0.0,0.0
90 20.,1000.
100 1
110 2.1E11,1000.
120 1.
130 RUN 2  COM62 - LATERAL LOAD=100000 AXIAL LOAD=10000
140 100000.,0.0,30.,10.,0.001,100,1,0
150 1
160 2.1E11,1000.
170 10000.
```

```
DO YOU WANT TO RUN AN EXISTING DATA FILE?
=YES
ENTER NAME OF INPUT DATA FILE
=DAT62
ENTER NAME OF OUTPUT DATA FILE
FILE DESCRIPTION (47 CHARACTERS MAX), TYPE ? FOR INFO ON FORM
=OUT62

*
```

Table B2

Output for Example Problem 1

```
LIST OUT62
 RUN 1 COM62 - LATERAL LOAD=100000 AXIAL LOAD=1


  ITERATION INFORMATION
    ITER. NO.          YT,IN.
        1           0.72159E 01
        2           0.11287E 02
        3           0.11287E 02



                      LATERALLY LOADED PILE PROGRAM

 INPUT INFORMATION
    PT,LB          BC2      BC CASE   DIAMETER,IN
 0.10000E 06   0.               1      0.30000E 02
   INCREMENT LENGTH,IN   NUMBER OF INCREMENTS
       0.10000E 02                100


 AXIAL COMPRESSION AT PILE TOP =      0.10000E 01

 LENGTH OF PILE,FT                  ITERATION TOLERANCE,IN
   0.83333E 02                           0.10000E-02

   DEPTH TO P-Y CURVE,IN.          Y,IN.            P,LB/IN.
        0.                     0.               0.
                               0.20000E 02      0.10000E 04

 OUTPUT INFORMATION
        0.10000E 04            0.               0.
                               0.20000E 02      0.10000E 04

 OUTPUT INFORMATION
   X,FT.        Y,IN.        M,IN-LB   ES,LB/IN2  P,LB/IN.   EI,LB/IN2
 0.         0.1129E 02 0.          0.5000E 02-0.5644E 03 0.2100E 12
 0.8333E 00 0.1098E 02 0.9721E 06 0.5000E 02-0.5489E 03 0.2100E 12
 0.1667E 01 0.1067E 02 0.1889E 07 0.5000E 02-0.5334E 03 0.2100E 12
 0.2500E 01 0.1036E 02 0.2752E 07 0.5000E 02-0.5180E 03 0.2100E 12
 0.3333E 01 0.1005E 02 0.3564E 07 0.5000E 02-0.5026E 03 0.2100E 12
 0.4167E 01 0.9747E 01 0.4324E 07 0.5000E 02-0.4874E 03 0.2100E 12
```

(Continued)

(Sheet 1 of 3)

0.5000E 01	0.9444E 01	0.5035E 07	0.5000E 02	-0.4722E 03	0.2100E 12
0.5833E 01	0.9143E 01	0.5699E 07	0.5000E 02	-0.4571E 03	0.2100E 12
0.6667E 01	0.8844E 01	0.6317E 07	0.5000E 02	-0.4422E 03	0.2100E 12
0.7500E 01	0.8549E 01	0.6892E 07	0.5000E 02	-0.4275E 03	0.2100E 12
0.8333E 01	0.8257E 01	0.7423E 07	0.5000E 02	-0.4129E 03	0.2100E 12
0.9167E 01	0.7969E 01	0.7914E 07	0.5000E 02	-0.3984E 03	0.2100E 12
0.1000E 02	0.7684E 01	0.8366E 07	0.5000E 02	-0.3842E 03	0.2100E 12
0.1083E 02	0.7403E 01	0.8781E 07	0.5000E 02	-0.3702E 03	0.2100E 12
0.1167E 02	0.7127E 01	0.9159E 07	0.5000E 02	-0.3564E 03	0.2100E 12
0.1250E 02	0.6855E 01	0.9501E 07	0.5000E 02	-0.3427E 03	0.2100E 12
0.1333E 02	0.6587E 01	0.9809E 07	0.5000E 02	-0.3294E 03	0.2100E 12
0.1417E 02	0.6324E 01	0.1008E 08	0.5000E 02	-0.3162E 03	0.2100E 12
0.1500E 02	0.6066E 01	0.1033E 08	0.5000E 02	-0.3033E 03	0.2100E 12
0.1583E 02	0.5813E 01	0.1054E 08	0.5000E 02	-0.2907E 03	0.2100E 12
0.1667E 02	0.5565E 01	0.1072E 08	0.5000E 02	-0.2782E 03	0.2100E 12
0.1750E 02	0.5322E 01	0.1088E 08	0.5000E 02	-0.2661E 03	0.2100E 12
0.1833E 02	0.5084E 01	0.1101E 08	0.5000E 02	-0.2542E 03	0.2100E 12
0.1917E 02	0.4851E 01	0.1111E 08	0.5000E 02	-0.2426E 03	0.2100E 12
0.2000E 02	0.4624E 01	0.1120E 08	0.5000E 02	-0.2312E 03	0.2100E 12
0.2083E 02	0.4402E 01	0.1125E 08	0.5000E 02	-0.2201E 03	0.2100E 12
0.2167E 02	0.4185E 01	0.1129E 08	0.5000E 02	-0.2093E 03	0.2100E 12
0.2250E 02	0.3974E 01	0.1130E 08	0.5000E 02	-0.1987E 03	0.2100E 12
0.2333E 02	0.3768E 01	0.1130E 08	0.5000E 02	-0.1884E 03	0.2100E 12
0.2417E 02	0.3567E 01	0.1127E 08	0.5000E 02	-0.1784E 03	0.2100E 12
0.2500E 02	0.3372E 01	0.1123E 08	0.5000E 02	-0.1686E 03	0.2100E 12
0.2583E 02	0.3182E 01	0.1117E 08	0.5000E 02	-0.1591E 03	0.2100E 12
0.2667E 02	0.2998E 01	0.1109E 08	0.5000E 02	-0.1499E 03	0.2100E 12
0.2750E 02	0.2819E 01	0.1100E 08	0.5000E 02	-0.1409E 03	0.2100E 12
0.2833E 02	0.2645E 01	0.1090E 08	0.5000E 02	-0.1322E 03	0.2100E 12
0.2917E 02	0.2476E 01	0.1078E 08	0.5000E 02	-0.1238E 03	0.2100E 12
0.3000E 02	0.2312E 01	0.1065E 08	0.5000E 02	-0.1156E 03	0.2100E 12
0.3083E 02	0.2153E 01	0.1051E 08	0.5000E 02	-0.1077E 03	0.2100E 12
0.3167E 02	0.2000E 01	0.1036E 08	0.5000E 02	-0.9999E 02	0.2100E 12
0.3250E 02	0.1851E 01	0.1020E 08	0.5000E 02	-0.9256E 02	0.2100E 12
0.3333E 02	0.1707E 01	0.1003E 08	0.5000E 02	-0.8537E 02	0.2100E 12
0.3417E 02	0.1568E 01	0.9849E 07	0.5000E 02	-0.7842E 02	0.2100E 12
0.3500E 02	0.1434E 01	0.9662E 07	0.5000E 02	-0.7170E 02	0.2100E 12
0.3583E 02	0.1304E 01	0.9468E 07	0.5000E 02	-0.6522E 02	0.2100E 12
0.3667E 02	0.1179E 01	0.9268E 07	0.5000E 02	-0.5896E 02	0.2100E 12
0.3750E 02	0.1058E 01	0.9061E 07	0.5000E 02	-0.5292E 02	0.2100E 12
0.3833E 02	0.9419E 00	0.8849E 07	0.5000E 02	-0.4709E 02	0.2100E 12
0.3917E 02	0.8296E 00	0.8633E 07	0.5000E 02	-0.4148E 02	0.2100E 12
0.4000E 02	0.7214E 00	0.8412E 07	0.5000E 02	-0.3607E 02	0.2100E 12
0.4083E 02	0.6172E 00	0.8187E 07	0.5000E 02	-0.3086E 02	0.2100E 12
0.4167E 02	0.5170E 00	0.7960E 07	0.5000E 02	-0.2585E 02	0.2100E 12
0.4250E 02	0.4205E 00	0.7730E 07	0.5000E 02	-0.2103E 02	0.2100E 12
0.4333E 02	0.3277E 00	0.7497E 07	0.5000E 02	-0.1639E 02	0.2100E 12
0.4417E 02	0.2385E 00	0.7264E 07	0.5000E 02	-0.1192E 02	0.2100E 12
0.4500E 02	0.1527E 00	0.7028E 07	0.5000E 02	-0.7637E 01	0.2100E 12

(Continued)

Table B2 (Concluded)

0.4583E 02	0.7032E-01	0.6793E 07	0.5000E 02	-0.3516E 01	0.2100E 12
0.4667E 02	-0.8859E-02	0.6557E 07	0.5000E 02	0.4429E 00	0.2100E 12
0.4750E 02	-0.8492E-01	0.6320E 07	0.5000E 02	0.4246E 01	0.2100E 12
0.4833E 02	-0.1580E 00	0.6085E 07	0.5000E 02	0.7898E 01	0.2100E 12
0.4917E 02	-0.2281E 00	0.5850E 07	0.5000E 02	0.1141E 02	0.2100E 12
0.5000E 02	-0.2955E 00	0.5616E 07	0.5000E 02	0.1477E 02	0.2100E 12
0.5083E 02	-0.3602E 00	0.5384E 07	0.5000E 02	0.1801E 02	0.2100E 12
0.5167E 02	-0.4223E 00	0.5153E 07	0.5000E 02	0.2112E 02	0.2100E 12
0.5250E 02	-0.4820E 00	0.4925E 07	0.5000E 02	0.2410E 02	0.2100E 12
0.5333E 02	-0.5393E 00	0.4699E 07	0.5000E 02	0.2697E 02	0.2100E 12
0.5417E 02	-0.5944E 00	0.4476E 07	0.5000E 02	0.2972E 02	0.2100E 12
0.5500E 02	-0.6474E 00	0.4256E 07	0.5000E 02	0.3237E 02	0.2100E 12
0.5583E 02	-0.6983E 00	0.4039E 07	0.5000E 02	0.3491E 02	0.2100E 12
0.5667E 02	-0.7473E 00	0.3825E 07	0.5000E 02	0.3736E 02	0.2100E 12
0.5750E 02	-0.7945E 00	0.3615E 07	0.5000E 02	0.3972E 02	0.2100E 12
0.5833E 02	-0.8400E 00	0.3409E 07	0.5000E 02	0.4200E 02	0.2100E 12
0.5917E 02	-0.8838E 00	0.3208E 07	0.5000E 02	0.4419E 02	0.2100E 12
0.6000E 02	-0.9261E 00	0.3011E 07	0.5000E 02	0.4631E 02	0.2100E 12
0.6083E 02	-0.9670E 00	0.2818E 07	0.5000E 02	0.4835E 02	0.2100E 12
0.6167E 02	-0.1007E 01	0.2630E 07	0.5000E 02	0.5033E 02	0.2100E 12
0.6250E 02	-0.1045E 01	0.2447E 07	0.5000E 02	0.5224E 02	0.2100E 12
0.6333E 02	-0.1082E 01	0.2270E 07	0.5000E 02	0.5410E 02	0.2100E 12
0.6417E 02	-0.1118E 01	0.2098E 07	0.5000E 02	0.5590E 02	0.2100E 12
0.6500E 02	-0.1153E 01	0.1931E 07	0.5000E 02	0.5765E 02	0.2100E 12
0.6583E 02	-0.1187E 01	0.1770E 07	0.5000E 02	0.5936E 02	0.2100E 12
0.6667E 02	-0.1220E 01	0.1616E 07	0.5000E 02	0.6102E 02	0.2100E 12
0.6750E 02	-0.1253E 01	0.1467E 07	0.5000E 02	0.6265E 02	0.2100E 12
0.6833E 02	-0.1285E 01	0.1324E 07	0.5000E 02	0.6424E 02	0.2100E 12
0.6917E 02	-0.1316E 01	0.1188E 07	0.5000E 02	0.6580E 02	0.2100E 12
0.7000E 02	-0.1347E 01	0.1059E 07	0.5000E 02	0.6733E 02	0.2100E 12
0.7083E 02	-0.1377E 01	0.9362E 06	0.5000E 02	0.6883E 02	0.2100E 12
0.7167E 02	-0.1406E 01	0.8203E 06	0.5000E 02	0.7032E 02	0.2100E 12
0.7250E 02	-0.1436E 01	0.7113E 06	0.5000E 02	0.7178E 02	0.2100E 12
0.7333E 02	-0.1465E 01	0.6095E 06	0.5000E 02	0.7323E 02	0.2100E 12
0.7417E 02	-0.1493E 01	0.5150E 06	0.5000E 02	0.7466E 02	0.2100E 12
0.7500E 02	-0.1522E 01	0.4280E 06	0.5000E 02	0.7608E 02	0.2100E 12
0.7583E 02	-0.1550E 01	0.3485E 06	0.5000E 02	0.7749E 02	0.2100E 12
0.7667E 02	-0.1578E 01	0.2767E 06	0.5000E 02	0.7889E 02	0.2100E 12
0.7750E 02	-0.1606E 01	0.2129E 06	0.5000E 02	0.8029E 02	0.2100E 12
0.7833E 02	-0.1633E 01	0.1572E 06	0.5000E 02	0.8167E 02	0.2100E 12
0.7917E 02	-0.1661E 01	0.1097E 06	0.5000E 02	0.8306E 02	0.2100E 12
0.8000E 02	-0.1689E 01	0.7063E 05	0.5000E 02	0.8444E 02	0.2100E 12
0.8083E 02	-0.1717E 01	0.4005E 05	0.5000E 02	0.8583E 02	0.2100E 12
0.8167E 02	-0.1744E 01	0.1793E 05	0.5000E 02	0.8721E 02	0.2100E 12
0.8250E 02	-0.1772E 01	0.4506E 04	0.5000E 02	0.8859E 02	0.2100E 12
0.8333E 02	-0.1799E 01	0.	0.5000E 02	0.8997E 02	0.2100E 12

(Sheet 3 of 3)

Table B3

Output for Example Problem 2

```
RUN 2  COM62 - LATERAL LOAD=100000 AXIAL LOAD=10000


 ITERATION INFORMATION
   ITER. NO.          YT,IN.
       1           0.72177E 01
       2           0.11324E 02
       3           0.11324E 02



                 LATERALLY LOADED PILE PROGRAM

INPUT INFORMATION
   PT,LB          BC2      BC CASE   DIAMETER,IN
0.10000E 06    0.             1      0.30000E 02
  INCREMENT LENGTH,IN   NUMBER OF INCREMENTS
      0.10000E 02                 100

AXIAL COMPRESSION AT PILE TOP =      0.10000E 05

LENGTH OF PILE,FT                  ITERATION TOLERANCE,IN
  0.83333E 02                          0.10000E-02

  DEPTH TO P-Y CURVE,IN.          Y,IN.          P,LB/IN.
       0.                     0.              0.
                              0.20000E 02     0.10000E 04

OUTPUT INFORMATION
       0.10000E 04            0.              0.
                              0.20000E 02     0.10000E 04

OUTPUT INFORMATION
  X,FT.       Y,IN.       M,IN-LB    ES,LB/IN2  P,LB/IN.    EI,LB/IN2
0.          0.1132E 02 0.            0.5000E 02-0.5662E 03 0.2100E 12
0.8333E 00 0.1101E 02 0.9748E 06 0.5000E 02-0.5507E 03 0.2100E 12
0.1667E 01 0.1070E 02 0.1895E 07 0.5000E 02-0.5351E 03 0.2100E 12
0.2500E 01 0.1039E 02 0.2761E 07 0.5000E 02-0.5196E 03 0.2100E 12
0.3333E 01 0.1008E 02 0.3575E 07 0.5000E 02-0.5042E 03 0.2100E 12
0.4167E 01 0.9777E 01 0.4340E 07 0.5000E 02-0.4889E 03 0.2100E 12
0.5000E 01 0.9473E 01 0.5056E 07 0.5000E 02-0.4736E 03 0.2100E 12
0.5833E 01 0.9170E 01 0.5726E 07 0.5000E 02-0.4585E 03 0.2100E 12
0.6667E 01 0.8871E 01 0.6350E 07 0.5000E 02-0.4435E 03 0.2100E 12
0.7500E 01 0.8574E 01 0.6929E 07 0.5000E 02-0.4287E 03 0.2100E 12
```

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(Sheet 1 of 3)

0.8333E 01	0.8281E 01	0.7465E 07	0.5000E 02	-0.4140E 03	0.2100E 12
0.9167E 01	0.7991E 01	0.7960E 07	0.5000E 02	-0.3996E 03	0.2100E 12
0.1000E 02	0.7705E 01	0.8416E 07	0.5000E 02	-0.3853E 03	0.2100E 12
0.1083E 02	0.7423E 01	0.8833E 07	0.5000E 02	-0.3712E 03	0.2100E 12
0.1167E 02	0.7145E 01	0.9213E 07	0.5000E 02	-0.3573E 03	0.2100E 12
0.1250E 02	0.6872E 01	0.9556E 07	0.5000E 02	-0.3436E 03	0.2100E 12
0.1333E 02	0.6603E 01	0.9866E 07	0.5000E 02	-0.3302E 03	0.2100E 12
0.1417E 02	0.6339E 01	0.1014E 08	0.5000E 02	-0.3170E 03	0.2100E 12
0.1500E 02	0.6080E 01	0.1039E 08	0.5000E 02	-0.3040E 03	0.2100E 12
0.1583E 02	0.5826E 01	0.1060E 08	0.5000E 02	-0.2913E 03	0.2100E 12
0.1667E 02	0.5576E 01	0.1078E 08	0.5000E 02	-0.2788E 03	0.2100E 12
0.1750E 02	0.5332E 01	0.1094E 08	0.5000E 02	-0.2666E 03	0.2100E 12
0.1833E 02	0.5093E 01	0.1107E 08	0.5000E 02	-0.2547E 03	0.2100E 12
0.1917E 02	0.4860E 01	0.1118E 08	0.5000E 02	-0.2430E 03	0.2100E 12
0.2000E 02	0.4631E 01	0.1126E 08	0.5000E 02	-0.2316E 03	0.2100E 12
0.2083E 02	0.4408E 01	0.1131E 08	0.5000E 02	-0.2204E 03	0.2100E 12
0.2167E 02	0.4191E 01	0.1135E 08	0.5000E 02	-0.2095E 03	0.2100E 12
0.2250E 02	0.3979E 01	0.1136E 08	0.5000E 02	-0.1989E 03	0.2100E 12
0.2333E 02	0.3772E 01	0.1136E 08	0.5000E 02	-0.1886E 03	0.2100E 12
0.2417E 02	0.3570E 01	0.1133E 08	0.5000E 02	-0.1785E 03	0.2100E 12
0.2500E 02	0.3374E 01	0.1129E 08	0.5000E 02	-0.1687E 03	0.2100E 12
0.2583E 02	0.3184E 01	0.1123E 08	0.5000E 02	-0.1592E 03	0.2100E 12
0.2667E 02	0.2999E 01	0.1116E 08	0.5000E 02	-0.1499E 03	0.2100E 12
0.2750E 02	0.2819E 01	0.1107E 08	0.5000E 02	-0.1409E 03	0.2100E 12
0.2833E 02	0.2644E 01	0.1096E 08	0.5000E 02	-0.1322E 03	0.2100E 12
0.2917E 02	0.2475E 01	0.1084E 08	0.5000E 02	-0.1237E 03	0.2100E 12
0.3000E 02	0.2310E 01	0.1071E 08	0.5000E 02	-0.1155E 03	0.2100E 12
0.3083E 02	0.2151E 01	0.1057E 08	0.5000E 02	-0.1076E 03	0.2100E 12
0.3167E 02	0.1997E 01	0.1042E 08	0.5000E 02	-0.9985E 02	0.2100E 12
0.3250E 02	0.1848E 01	0.1026E 08	0.5000E 02	-0.9239E 02	0.2100E 12
0.3333E 02	0.1704E 01	0.1008E 08	0.5000E 02	-0.8518E 02	0.2100E 12
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0.3583E 02	0.1299E 01	0.9519E 07	0.5000E 02	-0.6496E 02	0.2100E 12
0.3667E 02	0.1174E 01	0.9317E 07	0.5000E 02	-0.5868E 02	0.2100E 12
0.3750E 02	0.1052E 01	0.9109E 07	0.5000E 02	-0.5262E 02	0.2100E 12
0.3833E 02	0.9355E 00	0.8896E 07	0.5000E 02	-0.4678E 02	0.2100E 12
0.3917E 02	0.8229E 00	0.8677E 07	0.5000E 02	-0.4115E 02	0.2100E 12
0.4000E 02	0.7145E 00	0.8455E 07	0.5000E 02	-0.3572E 02	0.2100E 12
0.4083E 02	0.6100E 00	0.8229E 07	0.5000E 02	-0.3050E 02	0.2100E 12
0.4167E 02	0.5095E 00	0.8000E 07	0.5000E 02	-0.2547E 02	0.2100E 12
0.4250E 02	0.4128E 00	0.7768E 07	0.5000E 02	-0.2064E 02	0.2100E 12
0.4333E 02	0.3197E 00	0.7534E 07	0.5000E 02	-0.1599E 02	0.2100E 12
0.4417E 02	0.2303E 00	0.7299E 07	0.5000E 02	-0.1152E 02	0.2100E 12
0.4500E 02	0.1443E 00	0.7062E 07	0.5000E 02	-0.7217E 01	0.2100E 12
0.4583E 02	0.6176E-01	0.6825E 07	0.5000E 02	-0.3088E 01	0.2100E 12
0.4667E 02	-0.1759E-01	0.6587E 07	0.5000E 02	0.8793E 00	0.2100E 12
0.4750E 02	-0.9379E-01	0.6350E 07	0.5000E 02	0.4690E 01	0.2100E 12
0.4833E 02	-0.1670E 00	0.6112E 07	0.5000E 02	0.8349E 01	0.2100E 12

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(Sheet 2 of 3)

0.8333E 01	0.8281E 01	0.7465E 07	0.5000E 02	-0.4140E 03	0.2100E 12
0.9167E 01	0.7991E 01	0.7960E 07	0.5000E 02	-0.3996E 03	0.2100E 12
0.1000E 02	0.7705E 01	0.8416E 07	0.5000E 02	-0.3853E 03	0.2100E 12
0.1083E 02	0.7423E 01	0.8833E 07	0.5000E 02	-0.3712E 03	0.2100E 12
0.1167E 02	0.7145E 01	0.9213E 07	0.5000E 02	-0.3573E 03	0.2100E 12
0.1250E 02	0.6872E 01	0.9556E 07	0.5000E 02	-0.3436E 03	0.2100E 12
0.1333E 02	0.6603E 01	0.9866E 07	0.5000E 02	-0.3302E 03	0.2100E 12
0.1417E 02	0.6339E 01	0.1014E 08	0.5000E 02	-0.3170E 03	0.2100E 12
0.1500E 02	0.6080E 01	0.1039E 08	0.5000E 02	-0.3040E 03	0.2100E 12
0.1583E 02	0.5826E 01	0.1060E 08	0.5000E 02	-0.2913E 03	0.2100E 12
0.1667E 02	0.5576E 01	0.1078E 08	0.5000E 02	-0.2788E 03	0.2100E 12
0.1750E 02	0.5332E 01	0.1094E 08	0.5000E 02	-0.2666E 03	0.2100E 12
0.1833E 02	0.5093E 01	0.1107E 08	0.5000E 02	-0.2547E 03	0.2100E 12
0.1917E 02	0.4860E 01	0.1118E 08	0.5000E 02	-0.2430E 03	0.2100E 12
0.2000E 02	0.4631E 01	0.1126E 08	0.5000E 02	-0.2316E 03	0.2100E 12
0.2083E 02	0.4408E 01	0.1131E 08	0.5000E 02	-0.2204E 03	0.2100E 12
0.2167E 02	0.4191E 01	0.1135E 08	0.5000E 02	-0.2095E 03	0.2100E 12
0.2250E 02	0.3979E 01	0.1136E 08	0.5000E 02	-0.1989E 03	0.2100E 12
0.2333E 02	0.3772E 01	0.1136E 08	0.5000E 02	-0.1886E 03	0.2100E 12
0.2417E 02	0.3570E 01	0.1133E 08	0.5000E 02	-0.1785E 03	0.2100E 12
0.2500E 02	0.3374E 01	0.1129E 08	0.5000E 02	-0.1687E 03	0.2100E 12
0.2583E 02	0.3184E 01	0.1123E 08	0.5000E 02	-0.1592E 03	0.2100E 12
0.2667E 02	0.2999E 01	0.1116E 08	0.5000E 02	-0.1499E 03	0.2100E 12
0.2750E 02	0.2819E 01	0.1107E 08	0.5000E 02	-0.1409E 03	0.2100E 12
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0.2917E 02	0.2475E 01	0.1084E 08	0.5000E 02	-0.1237E 03	0.2100E 12
0.3000E 02	0.2310E 01	0.1071E 08	0.5000E 02	-0.1155E 03	0.2100E 12
0.3083E 02	0.2151E 01	0.1057E 08	0.5000E 02	-0.1076E 03	0.2100E 12
0.3167E 02	0.1997E 01	0.1042E 08	0.5000E 02	-0.9985E 02	0.2100E 12
0.3250E 02	0.1848E 01	0.1026E 08	0.5000E 02	-0.9239E 02	0.2100E 12
0.3333E 02	0.1704E 01	0.1008E 08	0.5000E 02	-0.8518E 02	0.2100E 12
0.3417E 02	0.1564E 01	0.9903E 07	0.5000E 02	-0.7820E 02	0.2100E 12
0.3500E 02	0.1429E 01	0.9715E 07	0.5000E 02	-0.7147E 02	0.2100E 12
0.3583E 02	0.1299E 01	0.9519E 07	0.5000E 02	-0.6496E 02	0.2100E 12
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0.3750E 02	0.1052E 01	0.9109E 07	0.5000E 02	-0.5262E 02	0.2100E 12
0.3833E 02	0.9355E 00	0.8896E 07	0.5000E 02	-0.4678E 02	0.2100E 12
0.3917E 02	0.8229E 00	0.8677E 07	0.5000E 02	-0.4115E 02	0.2100E 12
0.4000E 02	0.7145E 00	0.8455E 07	0.5000E 02	-0.3572E 02	0.2100E 12
0.4083E 02	0.6100E 00	0.8229E 07	0.5000E 02	-0.3050E 02	0.2100E 12
0.4167E 02	0.5095E 00	0.8000E 07	0.5000E 02	-0.2547E 02	0.2100E 12
0.4250E 02	0.4128E 00	0.7768E 07	0.5000E 02	-0.2064E 02	0.2100E 12
0.4333E 02	0.3197E 00	0.7534E 07	0.5000E 02	-0.1599E 02	0.2100E 12
0.4417E 02	0.2303E 00	0.7299E 07	0.5000E 02	-0.1152E 02	0.2100E 12
0.4500E 02	0.1443E 00	0.7062E 07	0.5000E 02	-0.7217E 01	0.2100E 12
0.4583E 02	0.6176E-01	0.6825E 07	0.5000E 02	-0.3088E 01	0.2100E 12
0.4667E 02	-0.1759E-01	0.6587E 07	0.5000E 02	0.8793E 00	0.2100E 12
0.4750E 02	-0.9379E-01	0.6350E 07	0.5000E 02	0.4690E 01	0.2100E 12
0.4833E 02	-0.1670E 00	0.6112E 07	0.5000E 02	0.8349E 01	0.2100E 12

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Table B3 (Concluded)

0.4917E 02	-0.2372E 00	0.5876E 07	0.5000E 02	0.1186E 02	0.2100E 12
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0.5083E 02	-0.3695E 00	0.5407E 07	0.5000E 02	0.1848E 02	0.2100E 12
0.5167E 02	-0.4317E 00	0.5175E 07	0.5000E 02	0.2159E 02	0.2100E 12
0.5250E 02	-0.4915E 00	0.4945E 07	0.5000E 02	0.2457E 02	0.2100E 12
0.5333E 02	-0.5489E 00	0.4718E 07	0.5000E 02	0.2744E 02	0.2100E 12
0.5417E 02	-0.6040E 00	0.4493E 07	0.5000E 02	0.3020E 02	0.2100E 12
0.5500E 02	-0.6570E 00	0.4272E 07	0.5000E 02	0.3285E 02	0.2100E 12
0.5583E 02	-0.7080E 00	0.4053E 07	0.5000E 02	0.3540E 02	0.2100E 12
0.5667E 02	-0.7570E 00	0.3838E 07	0.5000E 02	0.3785E 02	0.2100E 12
0.5750E 02	-0.8043E 00	0.3627E 07	0.5000E 02	0.4021E 02	0.2100E 12
0.5833E 02	-0.8497E 00	0.3420E 07	0.5000E 02	0.4249E 02	0.2100E 12
0.5917E 02	-0.8936E 00	0.3218E 07	0.5000E 02	0.4468E 02	0.2100E 12
0.6000E 02	-0.9359E 00	0.3020E 07	0.5000E 02	0.4680E 02	0.2100E 12
0.6083E 02	-0.9768E 00	0.2826E 07	0.5000E 02	0.4884E 02	0.2100E 12
0.6167E 02	-0.1016E 01	0.2637E 07	0.5000E 02	0.5082E 02	0.2100E 12
0.6250E 02	-0.1055E 01	0.2453E 07	0.5000E 02	0.5273E 02	0.2100E 12
0.6333E 02	-0.1092E 01	0.2275E 07	0.5000E 02	0.5459E 02	0.2100E 12
0.6417E 02	-0.1128E 01	0.2102E 07	0.5000E 02	0.5639E 02	0.2100E 12
0.6500E 02	-0.1163E 01	0.1935E 07	0.5000E 02	0.5814E 02	0.2100E 12
0.6583E 02	-0.1197E 01	0.1773E 07	0.5000E 02	0.5985E 02	0.2100E 12
0.6667E 02	-0.1230E 01	0.1618E 07	0.5000E 02	0.6151E 02	0.2100E 12
0.6750E 02	-0.1263E 01	0.1468E 07	0.5000E 02	0.6314E 02	0.2100E 12
0.6833E 02	-0.1295E 01	0.1325E 07	0.5000E 02	0.6473E 02	0.2100E 12
0.6917E 02	-0.1326E 01	0.1188E 07	0.5000E 02	0.6629E 02	0.2100E 12
0.7000E 02	-0.1356E 01	0.1059E 07	0.5000E 02	0.6782E 02	0.2100E 12
0.7083E 02	-0.1386E 01	0.9356E 06	0.5000E 02	0.6932E 02	0.2100E 12
0.7167E 02	-0.1416E 01	0.8195E 06	0.5000E 02	0.7080E 02	0.2100E 12
0.7250E 02	-0.1445E 01	0.7106E 06	0.5000E 02	0.7227E 02	0.2100E 12
0.7333E 02	-0.1474E 01	0.6088E 06	0.5000E 02	0.7371E 02	0.2100E 12
0.7417E 02	-0.1503E 01	0.5144E 06	0.5000E 02	0.7514E 02	0.2100E 12
0.7500E 02	-0.1531E 01	0.4275E 06	0.5000E 02	0.7656E 02	0.2100E 12
0.7583E 02	-0.1559E 01	0.3480E 06	0.5000E 02	0.7797E 02	0.2100E 12
0.7667E 02	-0.1587E 01	0.2762E 06	0.5000E 02	0.7937E 02	0.2100E 12
0.7750E 02	-0.1615E 01	0.2124E 06	0.5000E 02	0.8077E 02	0.2100E 12
0.7833E 02	-0.1643E 01	0.1567E 06	0.5000E 02	0.8216E 02	0.2100E 12
0.7917E 02	-0.1671E 01	0.1091E 06	0.5000E 02	0.8354E 02	0.2100E 12
0.8000E 02	-0.1699E 01	0.6991E 05	0.5000E 02	0.8493E 02	0.2100E 12
0.8083E 02	-0.1726E 01	0.3933E 05	0.5000E 02	0.8631E 02	0.2100E 12
0.8167E 02	-0.1754E 01	0.1740E 05	0.5000E 02	0.8769E 02	0.2100E 12
0.8250E 02	-0.1781E 01	0.4224E 04	0.5000E 02	0.8907E 02	0.2100E 12
0.8333E 02	-0.1809E 01	0.	0.5000E 02	0.9045E 02	0.2100E 12

(Sheet 3 of 3)

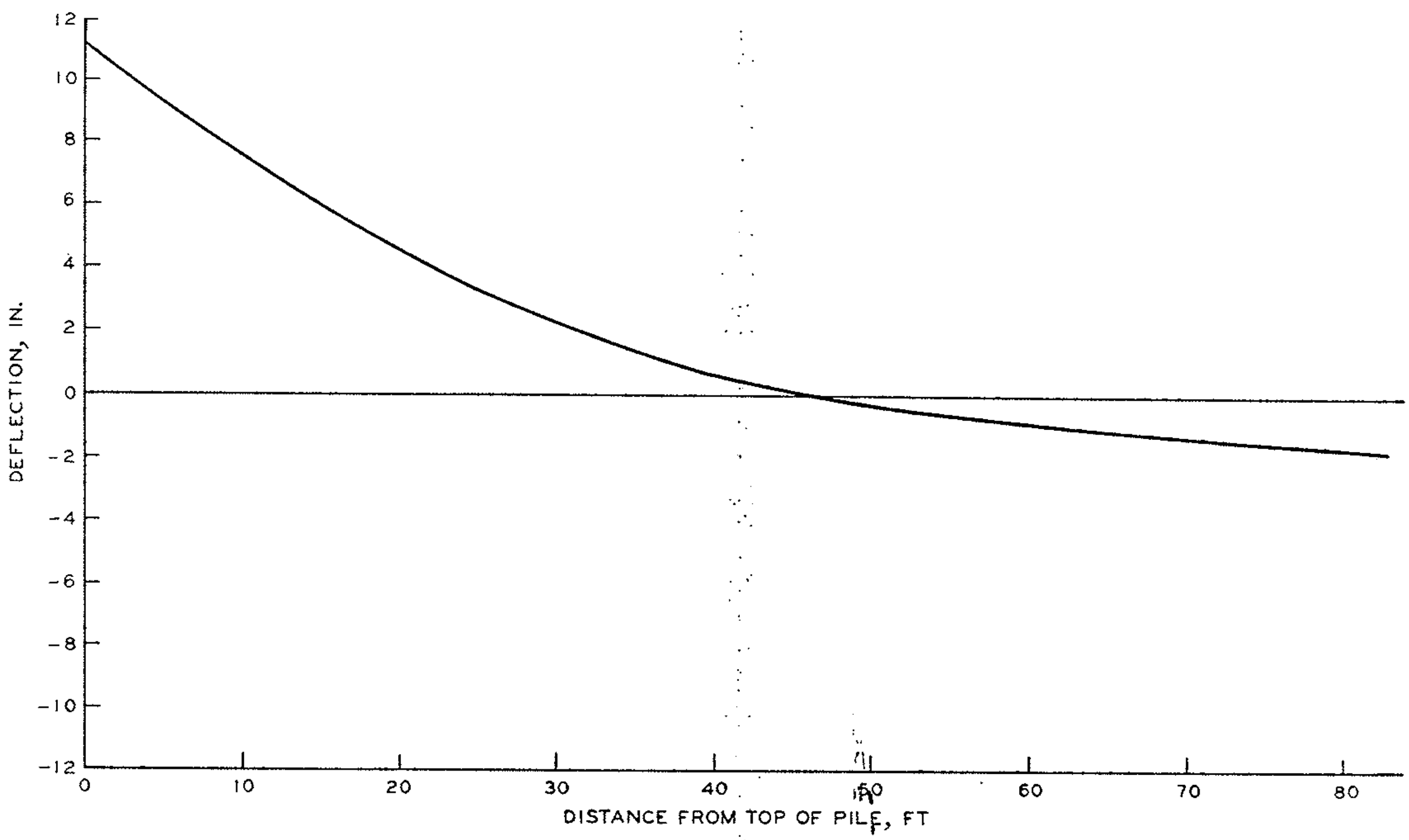

Figure B7. Variation of deflection along pile for Example Problem 1

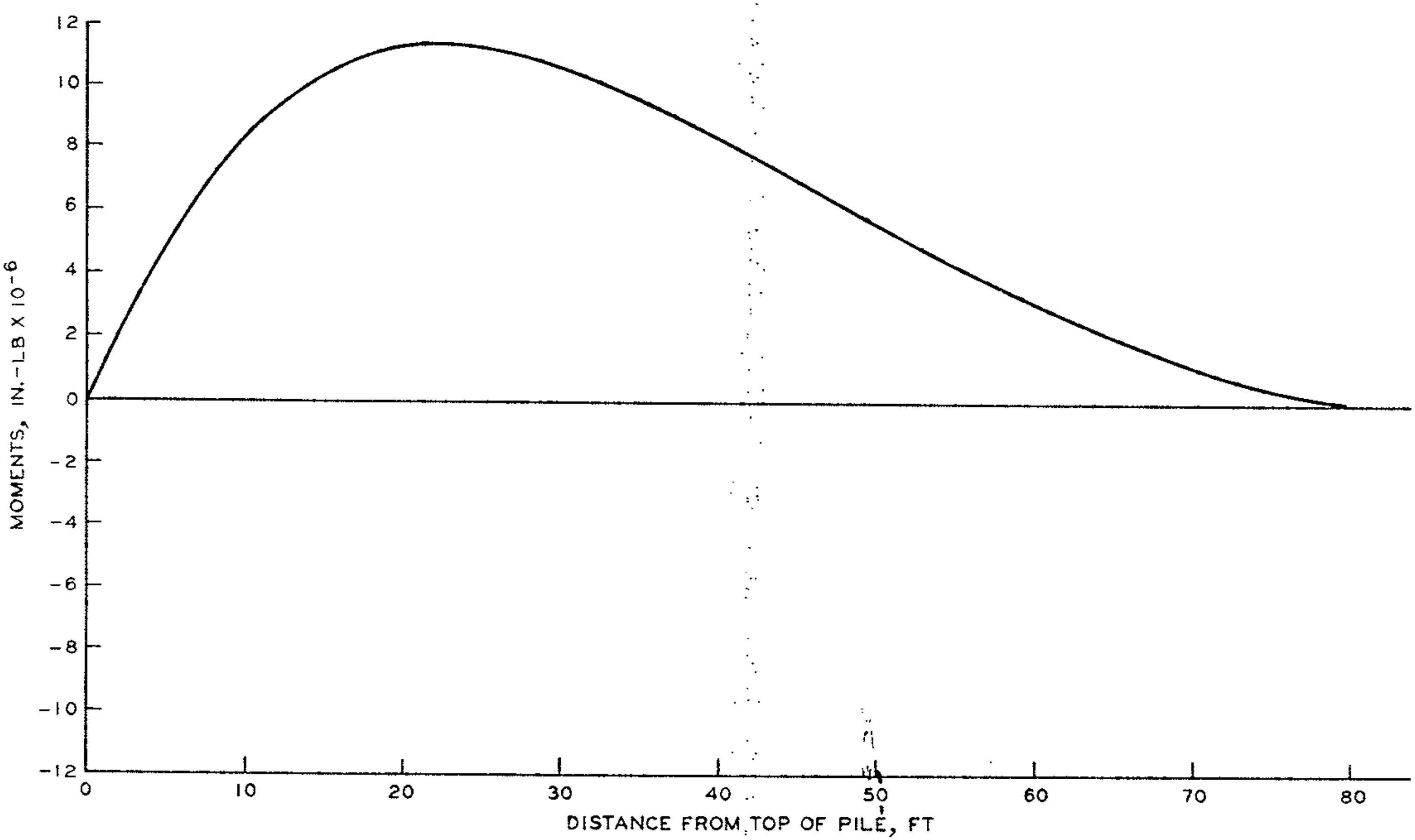

Figure B8. Variation of moment along pile for Example Problem 1

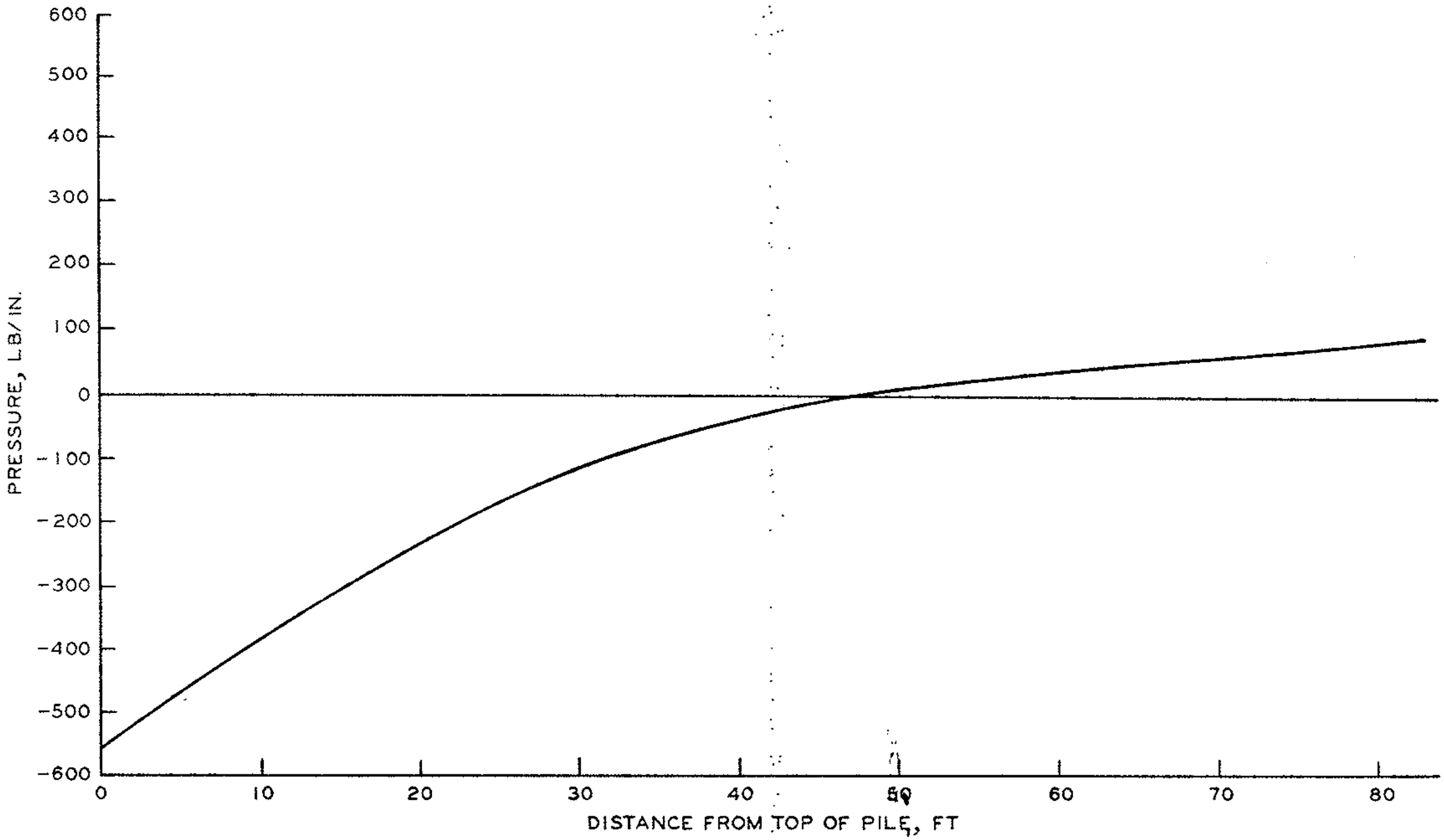

Figure B9. Variation of pressure along pile for Example Problem 1

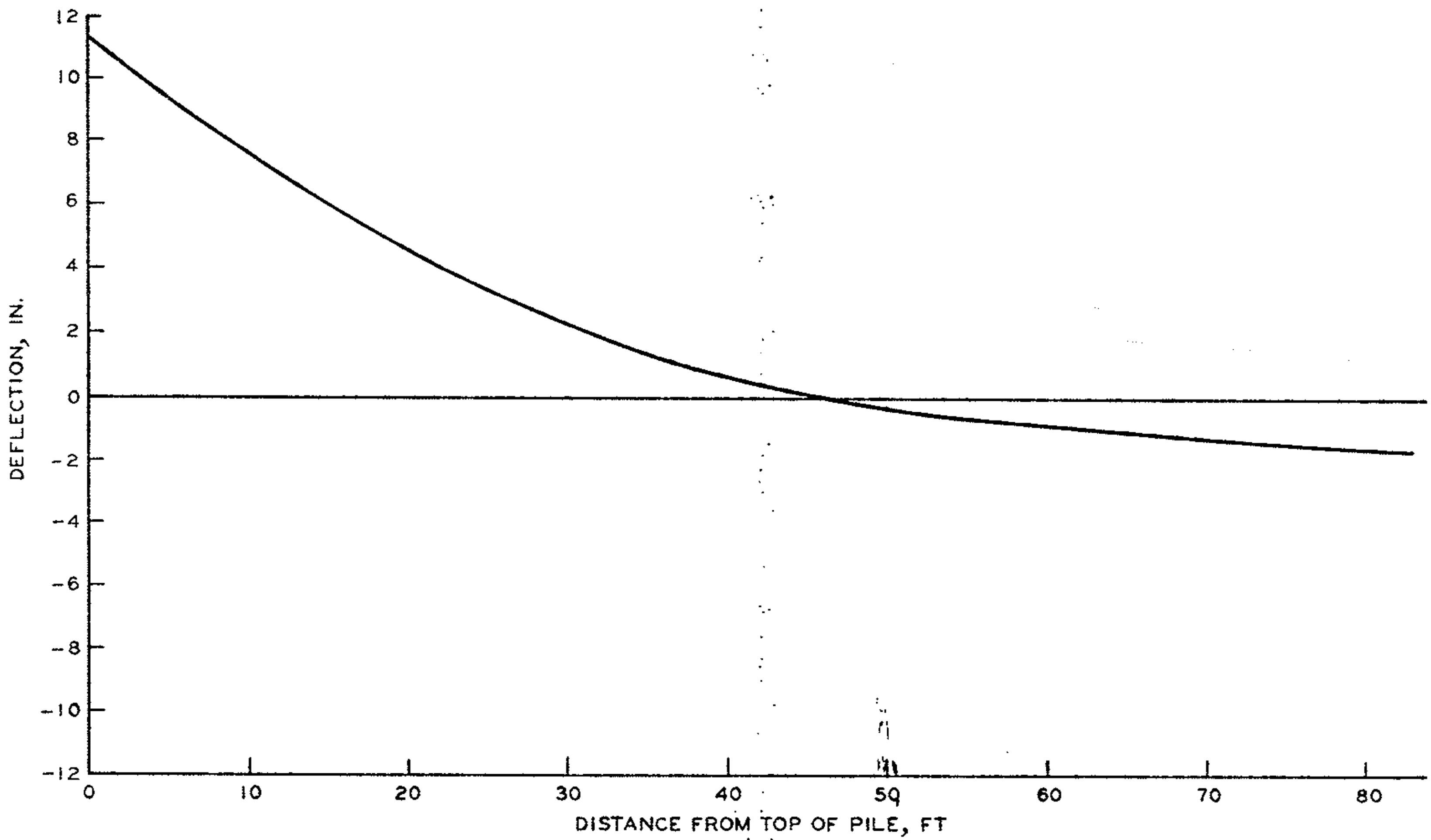

Figure B10. Variation of deflection along pile for Example Problem 2

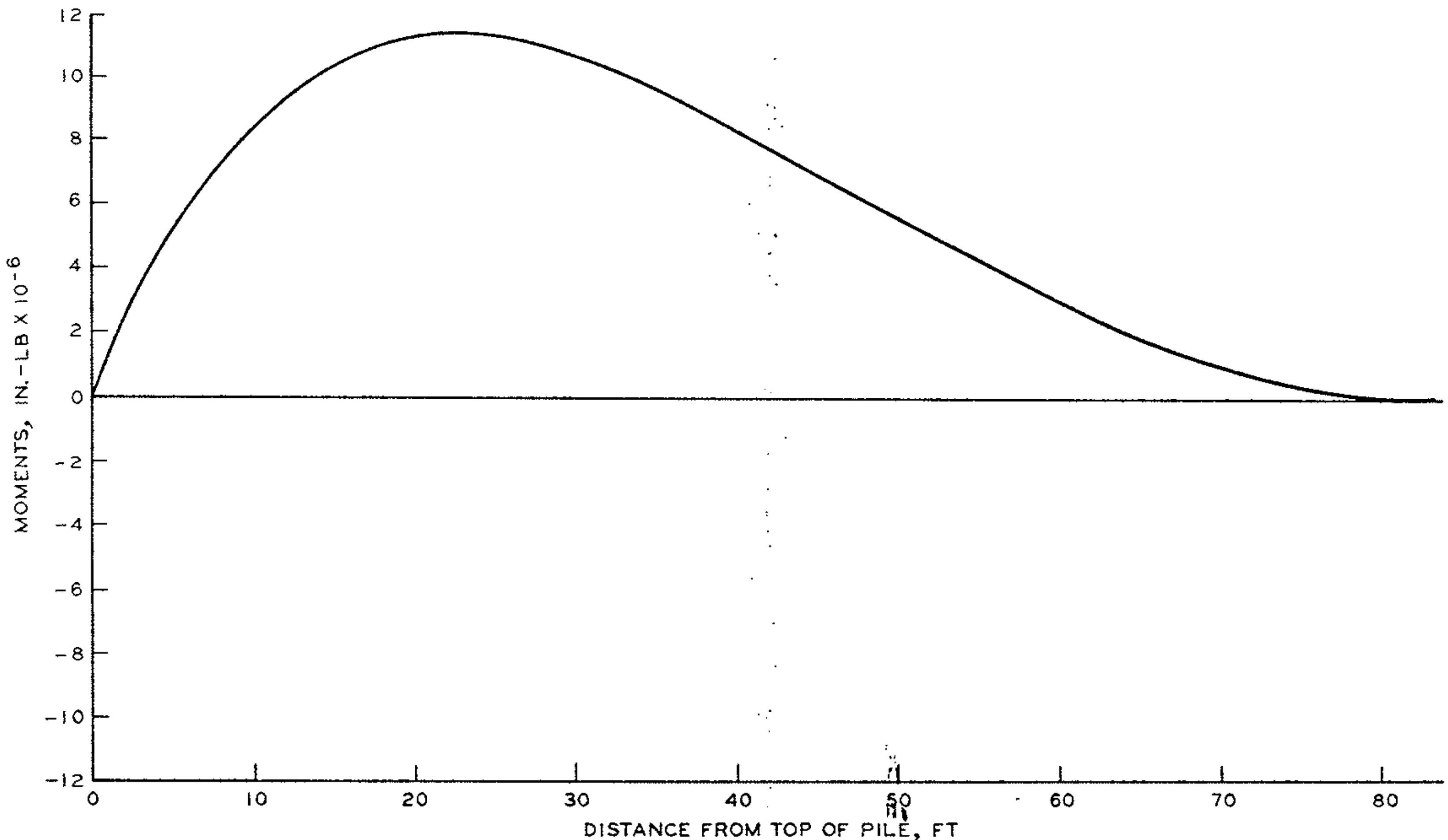

Figure B11. Variation of moment along pile for Example Problem 2

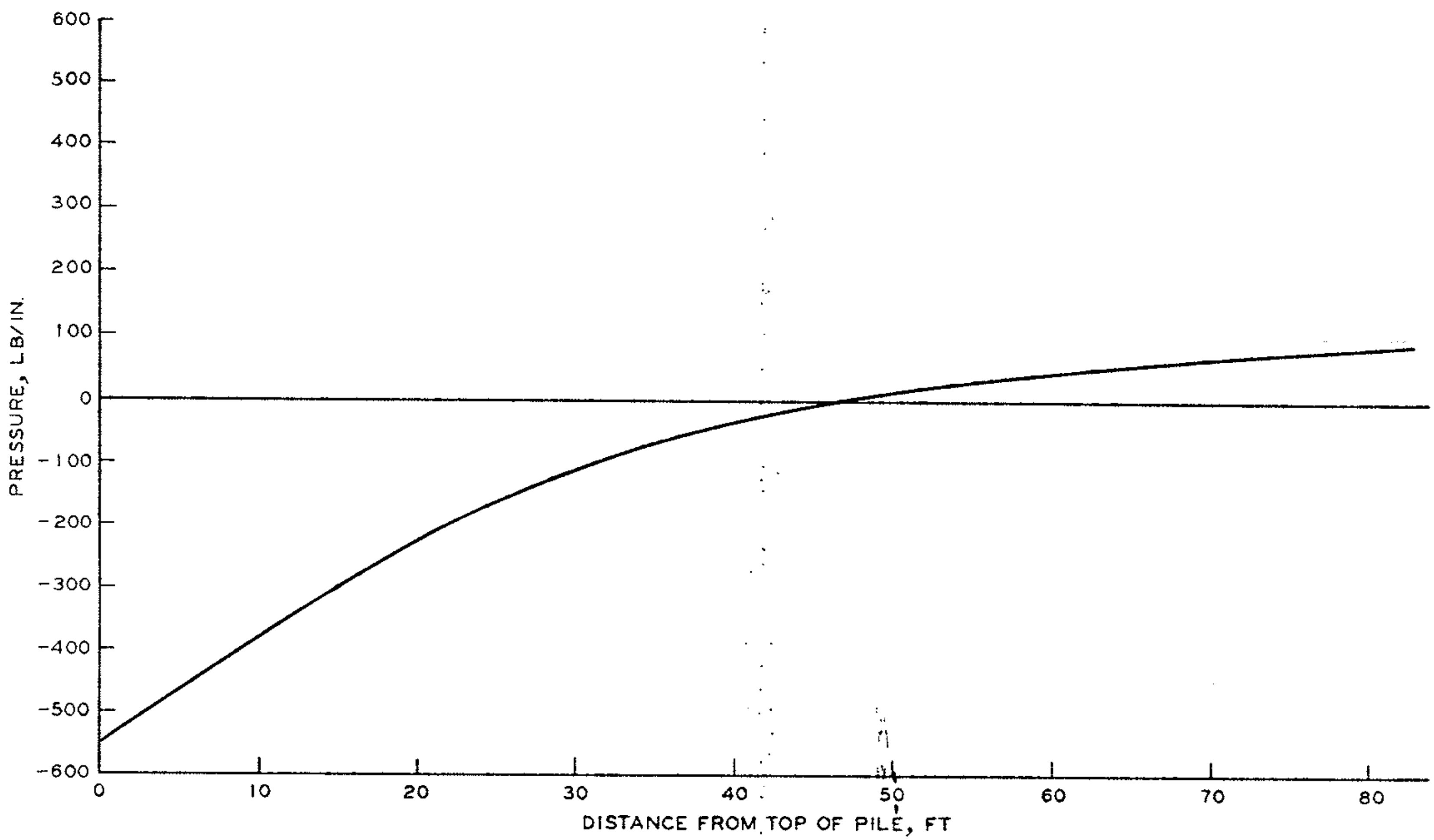

Figure B12. Variation of pressure along pile for Example Problem 2

APPENDIX C: USER'S GUIDE FOR PROGRAM PX4C3

General Introduction

1. Documentation for the computer program PX4C3 - to analyze axially loaded piles in nonlinear soil media - is presented in this appendix and includes a general introduction, program listing, guide for data input, and input-output data for two example problems.

2. PX4C3 is a finite difference computer program (developed by Drs. L. C. Reese, UT at Austin, and H. M. Coyle, Texas A&M University)* that may be used to compute load-displacement relationships for axially loaded piles, where the pile has a constant outside diameter. The program employs a set of load transfer curves along the pile and a point resistance curve at the tip of the pile. Piles of different outside diameters can be solved using PX4C3 by adjusting the load transfer curves for a constant diameter.

3. A load transfer curve relates the skin friction developed on the side of a pile to the absolute axial displacement of a pile section. The point resistance curve refers to a relationship between the total axial soil resistance on the base of the pile tip and the pile tip movement. PX4C3 can handle nonlinear curves for both the above relationships. Some procedures for obtaining these nonlinear soil relations are described in the text (Part V).

4. PX4C3 employs finite difference equations to achieve compatibility between pile displacement and load transfer along the pile and between soil resistance and displacement at the tip of the pile. The method has been found to give good prediction for piles in clays. The program needs to be used with caution as load transfer curves used in the program are presently derived from semi-empirical criteria.

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* H. M. Coyle and L. C. Reese, "Load Transfer for Axially Loaded Piles in Clay," Journal, Soil Mechanics and Foundation Division, American Society of Civil Engineers, Mar 1966.

5. Input may be input interactively at execute time, or input may be in a prepared data file. Output will come directly back to the terminal, or output will be directed to an output file.

Guide for Data Input

6. Data should be input to program PX4C3 according to the following guide. All input is in free-field format.

Group I - Title

RUN

RUN = 60 character problem heading

Group 2 - Problem Parameters

IQ, IJ, NA, IT, LSO

IQ = Number of increments on pile

IJ = Number of assumed tip movements (see Group 7)

NA = Number of depths at which AE values are specified (see Group 4) (AE = cross-sectional area of pile X modulus of elasticity of pile)

IT = Number of points on the point bearing vs tip movement curve (see Group 5)

LSO = Option control to print out load-settlement data only
1 --- if only load settlement results are desired
F1 --- all results are printed

Group 3 - Load Transfer Curve Data

A.

NX, NUM

NX = Number of T-Z curves along the depth of the pile

NUM = Number of points on a T-Z curve. A zero point on a T-Z curve is required to be input.

B. (i)

X(K)

(ii)

PP, ZM

X = Distance from top of pile to K^{th} T-Z curve. K goes from 1 to NX

PP = Load transfer lb/sq ft in T-Z curve

ZM = Pile movement (in.) in T-Z curve

Note: Set B should be repeated until NX T-Z curves have been specified.
Line (ii) of Set B should be repeated with each set until NUM number of PP and ZM values have been specified for that set.

Group 4 - AE Data

AE(M), ZAE(M)

AE = Cross-sectional area times modulus of elasticity, lb

ZAE = Depth to AE value, ft

Note: Repeat until NA sets of AE and ZAE values have been specified by putting one set per line.

Group 5 - Tip Load-Movement Data

TIPLD(M), TIPMV(M)

TIPLD(M) = Point bearing in the point bearing (lb) vs tip movement (in.) curve

TIPMV(M) = Tip movement in the point bearing vs tip movement (in.) curve

Note: Repeat until IT sets of TIPLD and TIPMV values have been specified by putting one set per line.

Group 6 - Pile Data

U, ALGTH, OD

U = Tolerance for convergence on displacements (in.)

ALGTH = Length of pile, ft

OD = Outside diameter of pile, ft

Group 7 - Desired Tip Movements

P

P = Assumed tip displacements, in.

Note: This line is repeated IJ number of times.

Example Problems

Example Problem 1

7. The first example problem illustrating the use of program PX4C3 shows a pile being subjected to a pullout test (Figure C1). The input T-Z curves and the input data for this example are shown in Figure C2 and Table C1, respectively. The computer output is presented in Table C2, and the load-displacement curve is plotted in Figure C3.

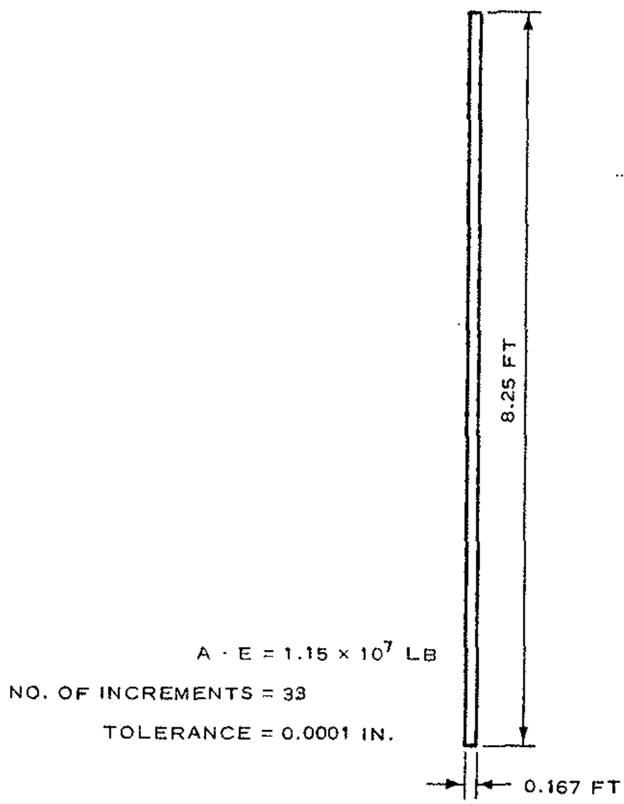

Figure C1. Physical problem (prediction of pullout curve for pile) for Example Problem 1

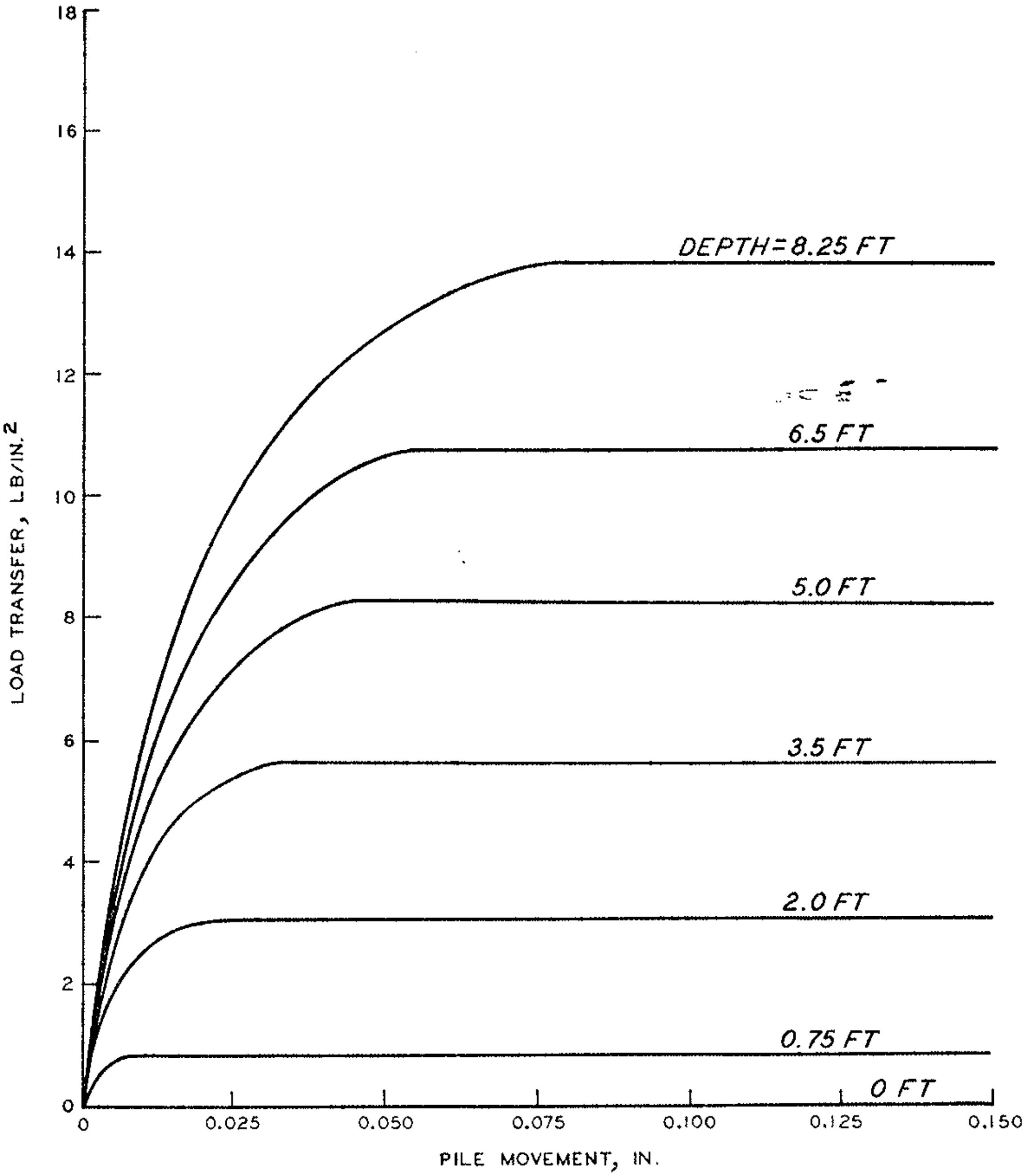

Figure C2. Input load transfer (T) versus pile movement (Z) curves for Example Problem 1

Table C1

Input Data for Example Problem 1

```
10 PREDICTION OF PULLOUT CURVE FOR A PILE
20 33,13,2,2,1
30 7,9
40 0
50 0,0
60 0,0,1
70 0,0,2
80 0,0,3
90 0,0,4
100 0,0.5
110 0,0.6
120 0,10
130 0,10
140 0.75
150 0,0
160 41.52,0.0009
170 81.36,0.0027
180 100.8,0.0045
190 112,0.0062
200 120.3,0.008
210 126,0.0098
220 126.6,0.01
230 126.6,10
240 2
250 0,0
260 132.2,0.0016
270 268.8,0.0048
280 337.2,0.008
290 381.6,0.011
300 410.4,0.0145
310 432,0.0177
320 441.6,0.0192
330 441.6,10
340 3.5
350 0,0
360 237.6,0.0025
370 486,0.0074
380 618,0.0124
390 697.2,0.0174
400 750,0.0222
410 789.6,0.0272
420 805.2,0.0296
430 811.2,10
440 5
450 0,0
460 338.4,0.0033
470 700.8,0.01
```

(Continued)

Table C1 (Concluded)

```
480 890.4,0.0167
490 1008,0.0233
500 1086,0.03
510 1146,0.0367
520 1188,0.0433
530 1188,10
540 6.5
550 0,0
560 436.8,0.004
570 906,0.0126
580 1158,0.021
590 1317,0.0294
600 1440,0.0378
610 1488,0.046
620 1554,0.0545
630 1560,10
640 8.25
650 0,0
660 549.6,0.00535
670 1152,0.0165
680 1476,0.02676
690 1674,0.0374
700 1806,0.0482
710 1908,0.0588
720 1986,0.0695
730 1992,10
740 11500000,0
750 11500000,8.25
760 0,0
770 0,10
780 0.0001,8.25,0.167
790 0.0001
800 0.0007
810 0.0015
820 0.003
830 0.005
840 0.01
850 0.016
860 0.02
870 0.03
880 0.05
890 0.1
900 0.11
910 0.12
```

Table C2

Output Data for Example Problem 1

PREDICTION OF PULLOUT CURVE FOR A PILE

AXIALLY LOADED PILE, CONSTANT OD

P-Z CURVE NO. 1 NO. OF POINTS 9 DEPTH TO CURVE,FT 0.

LOAD TRANSFER LB/SQ FT	PILE MOVEMENT INCHES
0.	0.
0.	0.1000E 00
0.	0.2000E 00
0.	0.3000E 00
0.	0.4000E 00
0.	0.5000E 00
0.	0.6000E 00
0.	0.1000E 02
0.	0.1000E 02

P-Z CURVE NO. 2 NO. OF POINTS 9 DEPTH TO CURVE,FT 0.750E 00

LOAD TRANSFER LB/SQ FT	PILE MOVEMENT INCHES
0.	0.
0.4152E 02	0.9000E-03
0.8136E 02	0.2700E-02
0.1008E 03	0.4500E-02
0.1128E 03	0.6200E-02
0.1203E 03	0.8000E-02
0.1260E 03	0.9800E-02
0.1266E 03	0.1000E-01
0.1266E 03	0.1000E 02

P-Z CURVE NO. 3 NO. OF POINTS 9 DEPTH TO CURVE,FT 0.200E 01

LOAD TRANSFER LB/SQ FT	PILE MOVEMENT INCHES
0.	0.
0.1322E 03	0.1600E-02
0.2688E 03	0.4800E-02
0.3372E 03	0.8000E-02
0.3816E 03	0.1100E-01
0.4104E 03	0.1450E-01
0.4320E 03	0.1770E-01
0.4416E 03	0.1920E-01

(Continued)

(Sheet 1 of 4)

```
     0.4416E 03      0.1000E 02

P-Z CURVE NO.  4     NO. OF POINTS  9     DEPTH TO CURVE,FT  0.350E 01

       LOAD            PILE
       TRANSFER        MOVEMENT
       LB/SQ FT        INCHES
     0.              0.
     0.2376E 03      0.2500E-02
     0.4860E 03      0.7400E-02
     0.6180E 03      0.1240E-01
     0.6972E 03      0.1740E-01
     0.7500E 03      0.2220E-01
     0.7896E 03      0.2720E-01
     0.8052E 03      0.2960E-01
     0.8112E 03      0.1000E 02

 -Z CURVE NO.  5     NO. OF POINTS  9     DEPTH TO CURVE,FT  0.500E 01

       LOAD            PILE
       TRANSFER        MOVEMENT
       LB/SQ FT        INCHES
     0.              0.
     0.3384E 03      0.3300E-02
     0.7008E 03      0.1000E-01
     0.8904E 03      0.1670E-01
     0.1008E 04      0.2330E-01
     0.1086E 04      0.3000E-01
     0.1146E 04      0.3670E-01
     0.1188E 04      0.4330E-01
     0.1188E 04      0.1000E 02

 -Z CURVE NO.  6     NO. OF POINTS  9     DEPTH TO CURVE,FT  0.650E 01

       LOAD            PILE
       TRANSFER        MOVEMENT
       LB/SQ FT        INCHES
     0.              0.
     0.4368E 03      0.4000E-02
     0.9060E 03      0.1260E-01
     0.1158E 04      0.2100E-01
     0.1317E 04      0.2940E-01
     0.1440E 04      0.3780E-01
     0.1488E 04      0.4600E-01
     0.1554E 04      0.5450E-01
     0.1560E 04      0.1000E 02

 -Z CURVE NO.  7     NO. OF POINTS  9     DEPTH TO CURVE,FT  0.825E 01

       LOAD            PILE
       TRANSFER        MOVEMENT
       LB/SQ FT        INCHES
```

(Continued)

(Sheet 2 of 4)

0.	0.
0.5496E 03	0.5358E-02
0.1152E 04	0.1658E-01
0.1476E 04	0.2676E-01
0.1674E 04	0.3748E-01
0.1806E 04	0.4828E-01
0.1908E 04	0.5888E-01
0.1986E 04	0.6958E-01
0.1992E 04	0.1008E 02

AE PILE LBS.	DEPTH FT.
0.11500E 08	0.
0.11500E 08	0.82500E 01

POINT BEARING LOAD	TIP MOVEMENT
0.	0.
0.	0.10000E 02

TOLERANCE	PILE LENGTH FT.	OUTER DIA FT.
0.1000E-03	0.8250E 01	0.1670E 00

ASSUMED TIP MOVEMENT IN	POINT BEARING LB
0.1000E-03	0.
0.7000E-03	0.
0.1500E-02	0.
0.3000E-02	0.
0.5000E-02	0.
0.1000E-01	0.
0.1600E-01	0.
0.2000E-01	0.
0.3000E-01	0.
0.5000E-01	0.
0.1000E 00	0.
0.1100E 00	0.
0.1200E 00	0.

AXIALLY LOADED PILE, CONSTANT OD

(Continued)

Table C2 (Concluded)

TOP LOAD LBS	TOP MOVEMENT INCHES
0.5811E 02	0.3409E-03
0.3918E 03	0.2372E-02
0.7652E 03	0.4950E-02
0.1315E 04	0.9311E-02
0.1809E 04	0.1406E-01
0.2552E 04	0.2338E-01
0.3113E 04	0.3294E-01
0.3366E 04	0.3855E-01
0.3756E 04	0.5119E-01
0.4096E 04	0.7361E-01
0.4190E 04	0.1243E 00
0.4190E 04	0.1343E 00
0.4190E 04	0.1443E 00

(Sheet 4 of 4)

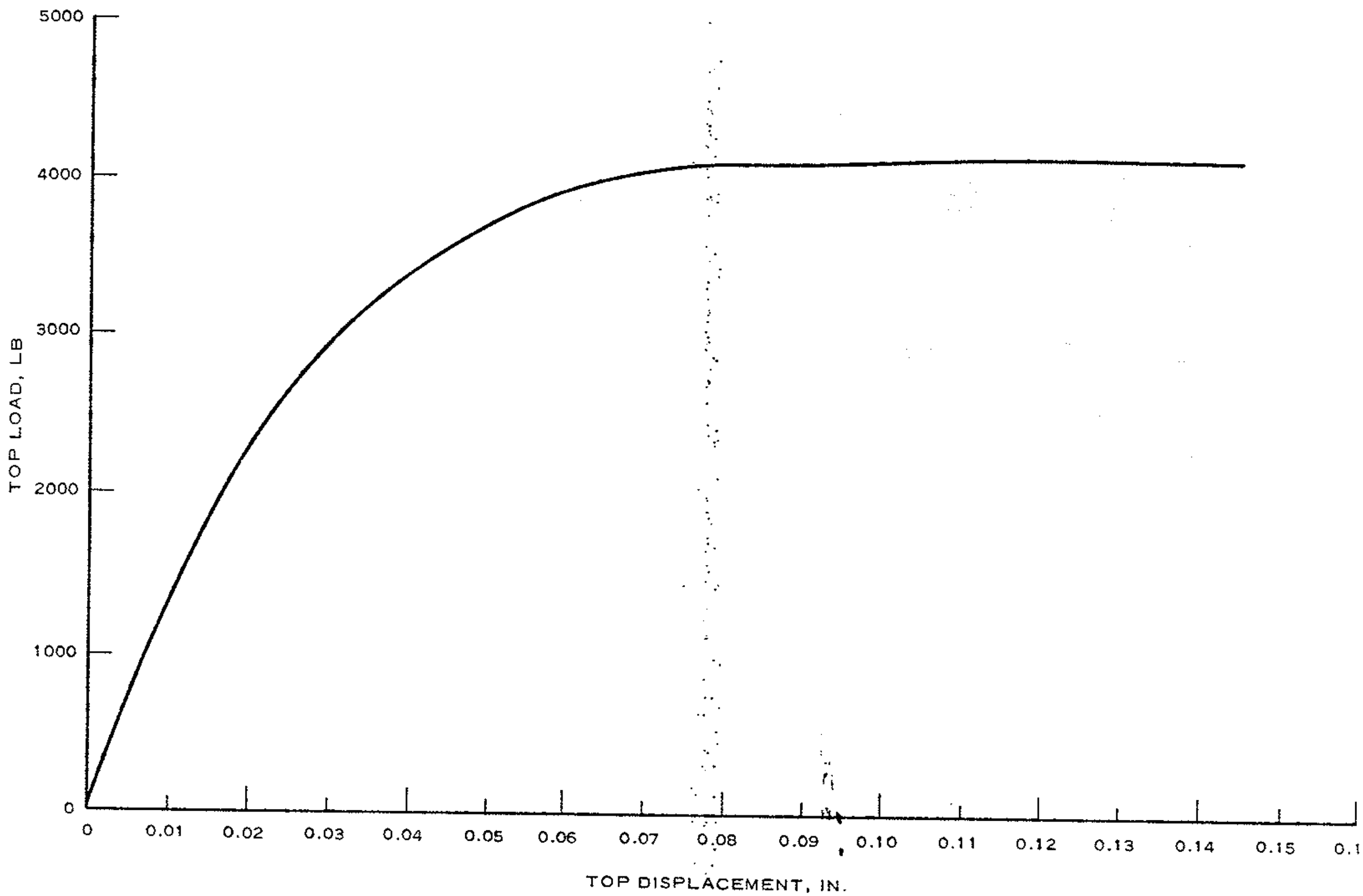

Figure C3. Top load vs top displacement for Example Problem 1

Example Problem 2

8. To demonstrate further the use of program PX4C3, a second example problem is given. Figure C4 shows the physical problem for this example. The input T-Z curves and the input data for this problem are shown in Figure C5 and Table C3, respectively. The computer output is presented in Table C4, and the load-deflection curve for the pile is plotted in Figure C6.

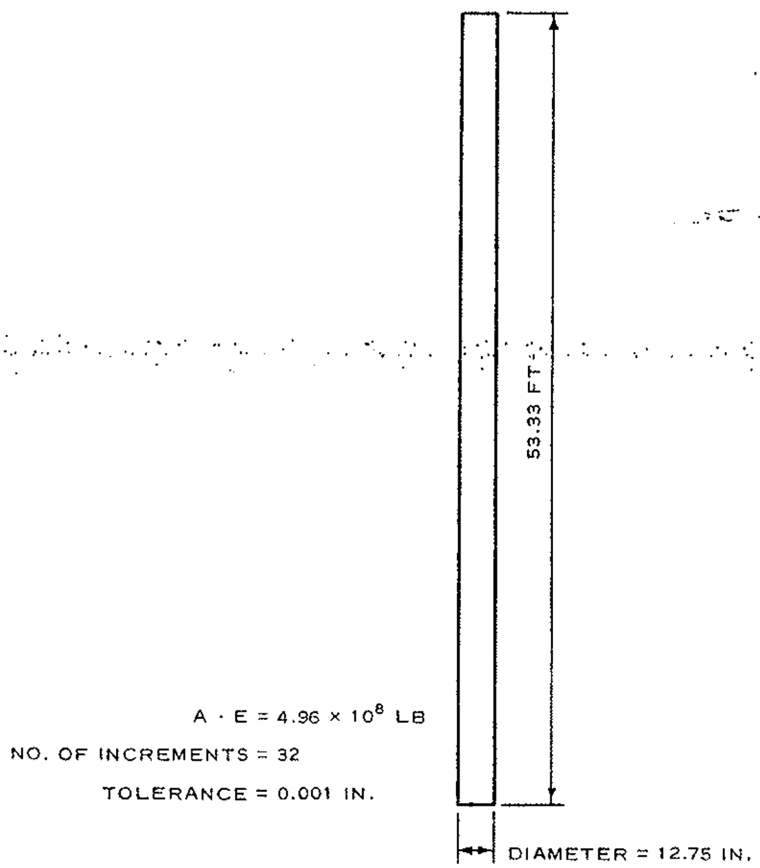

Figure C4. Physical problem (compression testing) for Example Problem 2

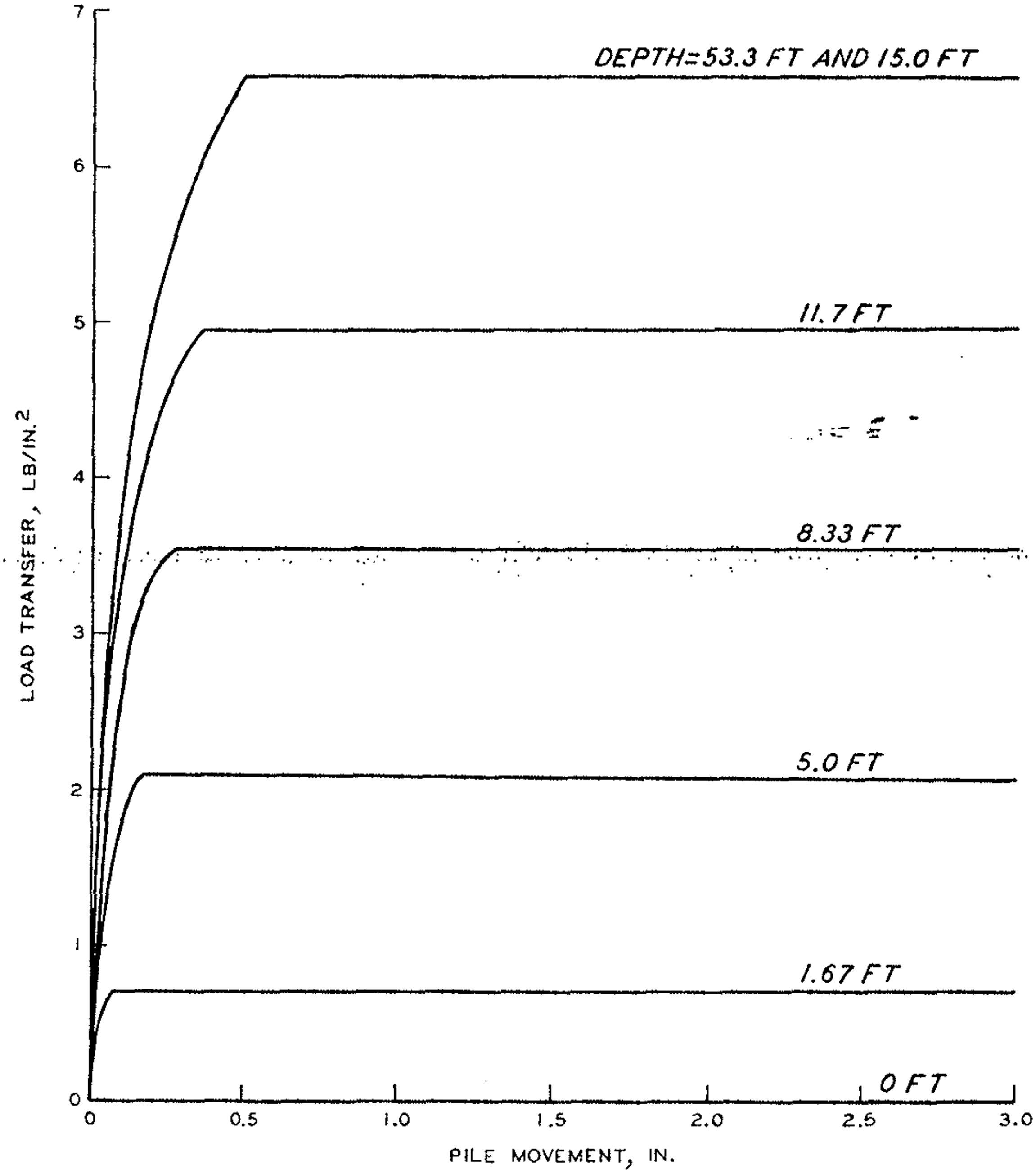

Figure C5. Input load transfer (T) versus pile movement (Z) curves for Example Problem 2

Table C3

Input Data for Example Problem 2

```
TEST 201 12.75 DIA PILE LOADED IN COMPRESSION FROM ARKANSAS RIVER
32,10,2,8,1
7,8
0
0,0
0,5
0,5
0,5
0,5
0,5
0,5
0,5
1.6667
0,0
59.5,0.0183
81.5,0.0366
91,0.055
100,0.0735
100,1
100,2
100,5
5
0,0
169,0.0346
238,0.0691
272,0.1035
295,0.138
310,0.173
310,2
310,5
8.3333
0,0
273,0.051
387,0.102
448,0.153
486,0.204
510,0.255
510,2
510,5
11.6667
0,0
379,0.0674
535,0.1450
615,0.202
675,0.27
```

(Continued)

Table C3 (Concluded)

```
0460 713,0.336
0470 713,2
0480 713,5
0490 15
0500 0,0
0510 480,0.0835
0520 682,0.167
0530 793,0.25
0540 864,0.334
0550 910,0.418
0560 950,0.5
0570 950,5
0580 53.3333
0590 0,0
0600 480,0.0835
0610 682,0.167
0620 793,0.25
0630 864,0.334
0640 910,0.418
0650 950,0.5
0660 950,5
0670 49600000,0
0680 49600000,53.3333
0690 0,0
0700 20000,0.01
0710 40000,0.02
0720 50000,0.04
0730 70000,0.1
0740 80000,0.32
0750 90000,1
0760 90000,10
0770 0.0001,53.3333,1.06
0780 0.02
0790 0.04
0800 0.08
0810 0.1
0820 0.15
0830 0.2
0840 0.3
0850 0.6
0860 1.2
0870 2
```

Table C4

Output Data for Example Problem 2

```
TEST 201 12.75 DIA PILE LOADED IN COMPRESSION FROM ARKANSAS RIVER

       AXIALLY LOADED PILE, CONSTANT OD

P-Z CURVE NO.  1      NO. OF POINTS  8     DEPTH TO CURVE,FT  0.

        LOAD           PILE
        TRANSFER       MOVEMENT
        LB/SQ FT       INCHES
     0.             0.
     0.             0.5000E 01
     0.             0.5000E 01
     0.             0.5000E 01
     0.             0.5000E 01
     0.             0.5000E 01
     0.             0.5000E 01
     0.             0.5000E 01

P-Z CURVE NO.  2      NO. OF POINTS  8     DEPTH TO CURVE,FT  0.167E 01

        LOAD           PILE
        TRANSFER       MOVEMENT
        LB/SQ FT       INCHES
     0.             0.
     0.5950E 02     0.1830E-01
     0.8150E 02     0.3660E-01
     0.9100E 02     0.5500E-01
     0.1000E 03     0.7350E-01
     0.1000E 03     0.1000E 01
     0.1000E 03     0.2000E 01
     0.1000E 03     0.5000E 01

P-Z CURVE NO.  3      NO. OF POINTS  8     DEPTH TO CURVE,FT  0.500E 01

        LOAD           PILE
        TRANSFER       MOVEMENT
        LB/SQ FT       INCHES
     0.             0.
     0.1690E 03     0.3460E-01
     0.2380E 03     0.6910E-01
     0.2720E 03     0.1035E 00
     0.2950E 03     0.1380E 00
     0.3100E 03     0.1730E 00
     0.3100E 03     0.2000E 01
     0.3100E 03     0.5000E 01

P-Z CURVE NO.  4      NO. OF POINTS  8     DEPTH TO CURVE,FT  0.833E 01
```

(Continued)

(Sheet 1 of 3)

LOAD TRANSFER LB/SQ FT	PILE MOVEMENT INCHES
0.	0.
0.2730E 03	0.5100E-01
0.3870E 03	0.1020E 00
0.4480E 03	0.1530E 00
0.4860E 03	0.2040E 00
0.5100E 03	0.2550E 00
0.5100E 03	0.2000E 01
0.5100E 03	0.5000E 01

P-Z CURVE NO. 5 NO. OF POINTS 8 DEPTH TO CURVE,FT 0.117E 02

LOAD TRANSFER LB/SQ FT	PILE MOVEMENT INCHES
0.	0.
0.3790E 03	0.4740E-01
0.5350E 03	0.1450E 00
0.6150E 03	0.2020E 00
0.6750E 03	0.2700E 00
0.7130E 03	0.3360E 00
0.7130E 03	0.2000E 01
0.7130E 03	0.5000E 01

P-Z CURVE NO. 6 NO. OF POINTS 8 DEPTH TO CURVE,FT 0.150E 02

LOAD TRANSFER LB/SQ FT	PILE MOVEMENT INCHES
0.	0.
0.4800E 03	0.8350E-01
0.6820E 03	0.1670E 00
0.7930E 03	0.2500E 00
0.8640E 03	0.3340E 00
0.9100E 03	0.4180E 00
0.9500E 03	0.5000E 00
0.9500E 03	0.5000E 01

P-Z CURVE NO. 7 NO. OF POINTS 8 DEPTH TO CURVE,FT 0.533E 02

LOAD TRANSFER LB/SQ FT	PILE MOVEMENT INCHES
0.	0.
0.4800E 03	0.6350E-01
0.6820E 03	0.1670E 00
0.7930E 03	0.2500E 00
0.8640E 03	0.3340E 00
0.9100E 03	0.4180E 00

(Continued)

(Sheet 2 of 3)

Table C4 (Concluded)

0.9500E 03	0.5000E 00
0.9500E 03	0.5000E 01

AE PILE LBS.	DEPTH FT.
0.49600E 09	0.
0.49600E 09	0.53333E 02

POINT BEARING LOAD	TIP MOVEMENT
0.	0.
0.20000E 05	0.10000E-01
0.40000E 05	0.20000E-01
0.50000E 05	0.40000E-01
0.70000E 05	0.10000E 00
0.80000E 05	0.32000E 00
0.90000E 05	0.10000E 01
0.90000E 05	0.10000E 02

TOLERANCE	PILE LENGTH FT.	OUTER DIA FT.
0.1000E-03	0.5333E 02	0.1060E 01

ASSUMED TIP MOVEMENT IN	POINT BEARING LB
0.2000E-01	0.4000E 05
0.4000E-01	0.5000E 05
0.8000E-01	0.6333E 05
0.1000E 00	0.7000E 05
0.1500E 00	0.7227E 05
0.2000E 00	0.7455E 05
0.3000E 00	0.7909E 05
0.6000E 00	0.8412E 05
0.1200E 01	0.9000E 05
0.2000E 01	0.9000E 05

AXIALLY LOADED PILE, CONSTANT OD

TOP LOAD LBS	TOP MOVEMENT INCHES
0.8487E 05	0.9748E-01
0.1164E 06	0.1461E 00
0.1556E 06	0.2240E 00
0.1701E 06	0.2586E 00
0.1861E 06	0.3229E 00
0.1973E 06	0.3832E 00
0.2184E 06	0.4989E 00
0.2285E 06	0.8144E 00
0.2344E 06	0.1422E 01
0.2344E 06	0.2222E 01

(Sheet 3 of 3)

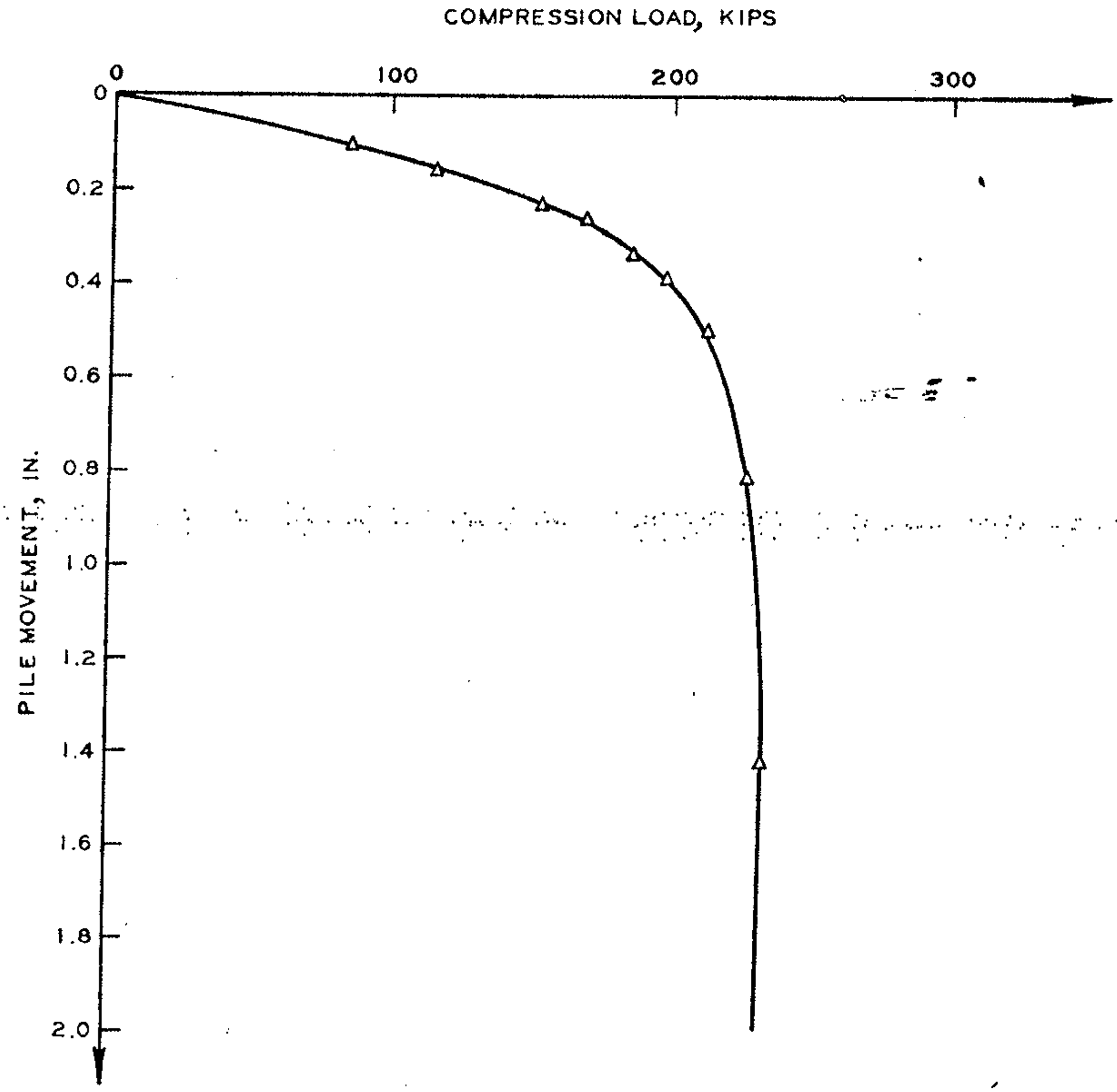

Figure C6. Load-deflection curve for compression loading of a 12.75-in.-diam pile for Example Problem 2

APPENDIX D: USER'S GUIDE FOR PROGRAM MAKE

General Introduction

1. Documentation for the computer program MAKE - to generate soil resistance versus lateral pile movement curves based on laboratory triaxial test results - is presented in this appendix and includes a general introduction, program listing, guide for data input, and input-output data for two example problems.

2. MAKE is a computer program (developed by Dr. Frazier Parker, WES) that can generate soil resistance (p) versus pile movement (y) curves for soils surrounding a laterally loaded pile based on certain laboratory soil test results. The program uses different criteria for clays and sands (as explained in Part V of this report) and can be used (with some minor modifications) as a subroutine to the laterally loaded pile program (COM62) or to the BENT1 program. MAKE can handle any number of stratums of clay or sand and can also account for various pile diameters.

Input may be input interactively at execute time, or input may be in a prepared data file. Output will be directed to an output file.

4. Data should be input to program MAKE according to the following guide. All input is in free-field format and should be in units of pounds, inches and radians. A flow chart for data input is shown in Figure D1.

Group 1 - Profile Data

NSOILP

NSOILP = Number of soil profiles (one value per run)

Group 2 - Soil Data

A.

NSTYPE

NSTYPE = Number of soil stratums (one value per profile)

B.

TSOIL

TSOIL = Alphanumeric designation of type of soil in stratum (sand or clay)(one value per stratum)

= Sand --- input line set C and omit line set D for this stratum

= Clay --- input line set D and omit line set C for this stratum

Note: A space (blank) must be left between the file line number and the parameter TSOIL.

C. Sand Properties

GAMMA, PHI, DIS1, DIS2, KDENSE

GAMMA = Unit weight of soil

PHI = Angle of internal friction

DIS1 = Distance from ground line to top of stratum

DIS2 = Distance from ground line to bottom of stratum

KDENSE = Alphanumeric designation for relative density of sand

= DENSE, MEDIUM, OR LOOSE

D. Clay Properties

(i) | GAMMA, SHEARS, DIS1, DIS2, INFO, ICON |

GAMMA = Unit weight of clay

SHEARS = Cohesion of clay

DIS1 = Distance from ground line to top of stratum

DIS2 = Distance from ground line to bottom of stratum

INFO = Control for input of stress-strain curve
0 --- Omit data for curves by omitting cards that follow in this group
1 --- Input data for curves by specifying cards that follow in this group

ICON = Alphanumeric designation for consistency of clay (SOFT or STIF)

(ii) | NCURVS |

NCURVS = Number of curves per stratum

(iii) | DIST, NPOINT |

DIST = Distance from ground line to curve

NPOINT = Number of points on curve

(iv) | SIGD, EP |

SIGD = Principal stress difference $(\sigma_1 - \sigma_3)$

EP = Axial strain

Note: Repeat (iv) until NPOINT number of points have been specified for that curve.

Note: Repeat (iii) until NCURVS number of curves have been specified for that stratum.

Note: Repeat Group 2B and 2C or Group 2B and 2D NSTYPE times.

Group 3 - Pile Data

A. | NPISP |
|---|

NPISP = Number of different piles in this soil profile (one value per soil profile)

Note: Repeat set B until NPISP sets have been specified.

B. (i) | KS, NOC, NDD |
|---|

KS = Numeric identifier for set of p-y curves

NOC = Number of curves in set

NDD = Number of different diameters used for p-y curves

(ii) | D, DISD1, DISD2 |
|---|

D = Pile diameter

DISD1 = Distance from top of pile to top of section

DISD2 = Distance from top of pile to bottom of section

Note: Repeat (ii) until NDD sets of values have been specified.

(iii) | DTC |
|---|

DTC = Distance from top of pile to p-y curve

Note: Repeat (iii) until NOC values have been specified.

Note: Repeat Groups 2 and 3 NSOILP number of times.

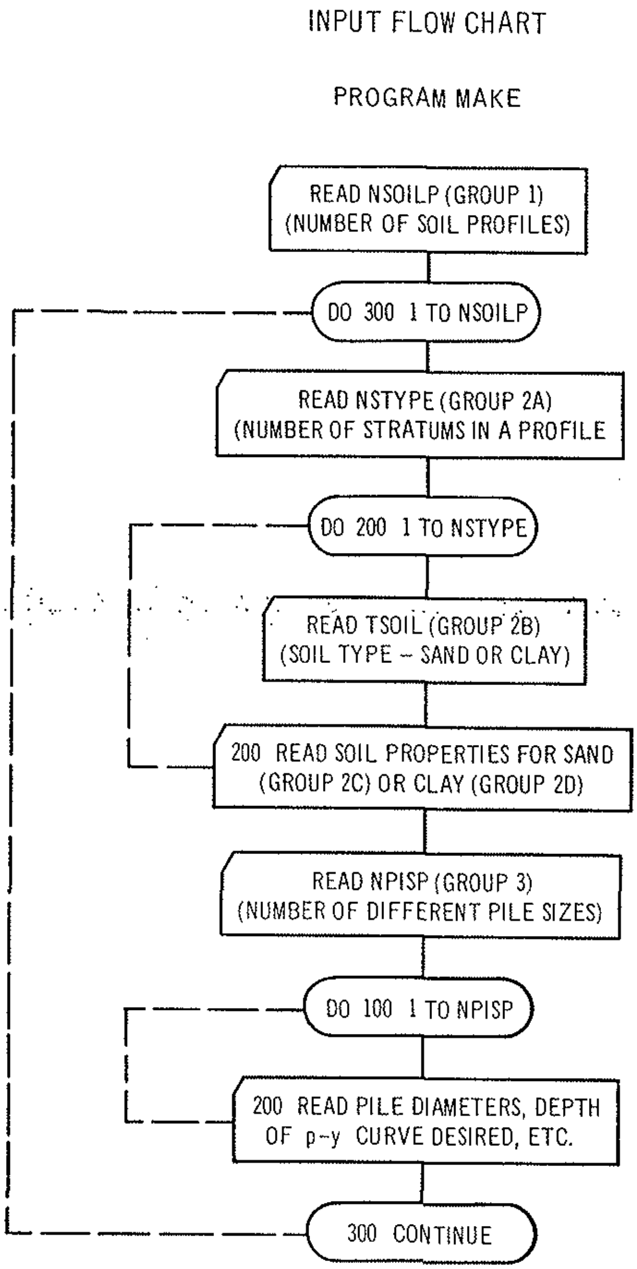

Figure D1. Input flow chart for MAKE

Example Problems

5. To illustrate the use of program MAKE two example problems will be demonstrated. The soil profiles and the pile dimensions used in the two examples are shown in Figure D2. The soil profile for the first example consists of two layers of clay and for the second example,

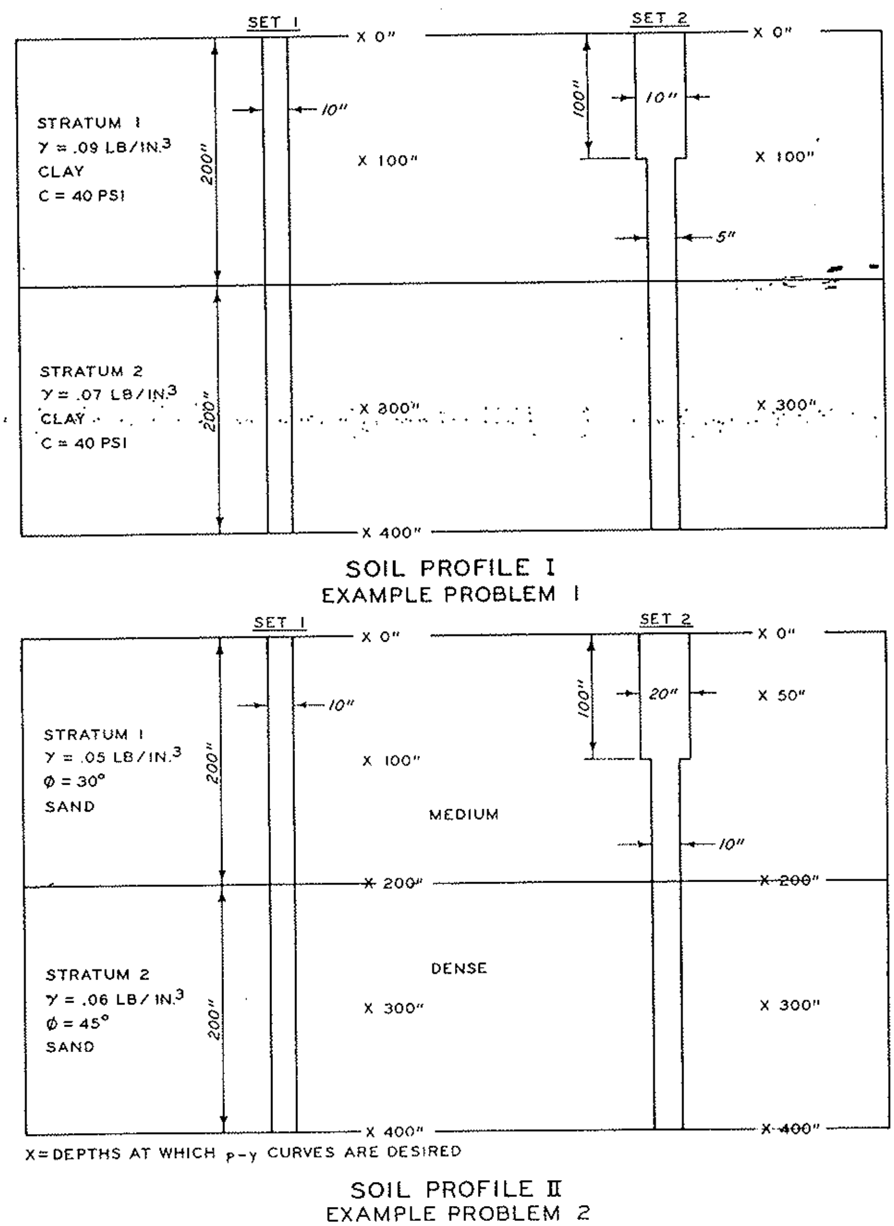

Figure D2. Soil profiles for Example Problems 1 and 2 two layers of sand. Each example requires the generation of two sets of p-y curves corresponding to two different pile configurations. The input and output data for both examples are given in Tables D1 and D2. The generated p-y curves are shown plotted in Figures D3 and D4 for Example Problem 1 and Figures D5 and D6 for Example Problem 2.

Table D1

MAKE Input Data for Example Problems 1 and 2

```
2
2
CLAY
0.09,40,0,200,0,8
CLAY
0.07,60,2,400,0,8
2
1,4,1
10,0,400
0
100
300
400
2,3,2
10,0,100
5,100,400
0
100
300
2
SAND
0.05,0.524,0,200,MEDUM
SAND
0.06,0.7855,200,400,DENSE
2
4,5,1
10,0,400
0
100
200
300
400
5,5,2
20,0,100
10,100,400
0
50
200
300
400
```

Table D2

Output Data for Example Problems 1 and 2

```
     INPUT OF SOIL PARAMETERS


     SOIL PROFILE NO.      1     STRATUM NO.,     1 TYPE SOIL CLAY

    UNIT    COHESION       TOP       BOTTOM           CONSISTENCY
   WEIGTH                 DEPTH      DEPTH
0.9000E-01       0.4000E 02       0.                0.2000E 03       0
     SOIL PROFILE NO.      1     STRATUM NO.,     2 TYPE SOIL CLAY

    UNIT    COHESION       TOP       BOTTOM           CONSISTENCY
   WEIGTH                 DEPTH      DEPTH
0.7000E-01       0.6000E 02       0.2000E 01        0.4000E 03       0


     PROPERTIES OF PILE USED FOR GENERATION OF PY-CURVES

SET IDENTYFIER NO.      1     NUMBER OF CURVES IN SET       4

DIAMETER DISTRIBUTION FOR PILE


   DIAMETER         TOP DIS         BOT DIS
 0.1000E 02     0.              0.4000E 03


CURVE NO.      1 DEPTH TO CURVE 0.
SOIL REACTION          DEFLECTION
   0.                     0.
   0.8000E 03             0.1200E-01
   0.8000E 03             0.1000E 03


CURVE NO.      2 DEPTH TO CURVE 0.1000E 03
SOIL REACTION          DEFLECTION
   0.                     0.
   0.1391E 04             0.4000E-01
   0.1968E 04             0.8000E-01
   0.2410E 04             0.1200E 00
   0.2783E 04             0.1600E 00
   0.3111E 04             0.2000E 00
   0.3408E 04             0.2400E 00
   0.3681E 04             0.2800E 00
   0.3935E 04             0.3200E 00
   0.4174E 04             0.3600E 00
   0.4400E 04             0.4000E 00
   0.4400E 04             0.1000E 03
```

(Continued)　　　　(Sheet 1 of 5)

Table D2 (Continued)

```
CURVE NO.      3 DEPTH TO CURVE 0.3000E 03
SOIL REACTION          DEFLECTION
   0.                  0.
   0.2087E 04          0.4000E-01
   0.2952E 04          0.8000E-01
   0.3615E 04          0.1200E 00
   0.4174E 04          0.1600E 00
   0.4667E 04          0.2000E 00
   0.5112E 04          0.2400E 00
   0.5522E 04          0.2800E 00
   0.5903E 04          0.3200E 00
   0.6261E 04          0.3600E 00
   0.6600E 04          0.4000E 00
   0.6600E 04          0.1000E 03

CURVE NO.      4 DEPTH TO CURVE 0.4000E 03
SOIL REACTION          DEFLECTION
   0.                  0.
   0.2087E 04          0.4000E-01
   0.2952E 04          0.8000E-01
   0.3615E 04          0.1200E 00
   0.4174E 04          0.1600E 00
   0.4667E 04          0.2000E 00
   0.5112E 04          0.2400E 00
   0.5522E 04          0.2800E 00
   0.5903E 04          0.3200E 00
   0.6261E 04          0.3600E 00
   0.6600E 04          0.4000E 00
   0.6600E 04          0.1000E 03
SET IDENTIFIER NO.     2      NUMBER OF CURVES IN SET       3

DIAMETER DISTRIBUTION FOR PILE

   DIAMETER          TOP DIS         BOT DIS
 0.1000E 02      0.              0.1000E 03

 0.5000E 01      0.1000E 03      0.4000E 03


CURVE NO.      1 DEPTH TO CURVE 0.
SOIL REACTION          DEFLECTION
   0.                  0.
   0.8000E 03          0.1200E-01
   0.8000E 03          0.1000E 03
```

(Continued) (Sheet 2 of 5)

CURVE NO. 2 DEPTH TO CURVE 0.1000E 03

SOIL REACTION	DEFLECTION
0.	0.
0.1391E 04	0.4000E-01
0.1968E 04	0.8000E-01
0.2410E 04	0.1200E 00
0.2783E 04	0.1600E 00
0.3111E 04	0.2000E 00
0.3408E 04	0.2400E 00
0.3681E 04	0.2800E 00
0.3935E 04	0.3200E 00
0.4174E 04	0.3600E 00
0.4400E 04	0.4000E 00
0.4400E 04	0.1000E 03

CURVE NO. 3 DEPTH TO CURVE 0.3000E 03

SOIL REACTION	DEFLECTION
0.	0.
0.1044E 04	0.2000E-01
0.1476E 04	0.4000E-01
0.1807E 04	0.6000E-01
0.2087E 04	0.8000E-01
0.2333E 04	0.1000E 00
0.2556E 04	0.1200E 00
0.2761E 04	0.1400E 00
0.2952E 04	0.1600E 00
0.3131E 04	0.1800E 00
0.3300E 04	0.2000E 00
0.3300E 04	0.5000E 02

INPUT OF SOIL PARAMETERS

SOIL PROFILE NO. 2 STRATUM NO., 1 TYPE SOIL SAND

UNIT WEIGHT	ANGLE OF FRIC.	TOP DEPTH	BOTTOM DEPTH	DENSITY
0.5000E-01	0.5240E 00	0.	0.2000E 03	MEDUM

SOIL PROFILE NO. 2 STRATUM NO., 2 TYPE SOIL SAND

UNIT WEIGHT	ANGLE OF FRIC.	TOP DEPTH	BOTTOM DEPTH	DENSITY
0.6000E-01	0.7855E 00	0.2000E 03	0.4000E 03	DENSE

PROPERTIES OF PILE USED FOR GENERATION OF PY-CURVES

SET IDENTIFIER NO. 4 NUMBER OF CURVES IN SET 5

(Continued) (Sheet 3 of 5)

```
DIAMETER DISTRIBUTION FOR PILE

   DIAMETER          TOP DIS          BOT DIS
 0.1000E 02       0.               0.4000E 03


CURVE NO.      1 DEPTH TO CURVE 0.
SOIL REACTION           DEFLECTION
   0.                   0.
   0.                   0.1000E 01
   0.                   0.1000E 03

CURVE NO.      2 DEPTH TO CURVE 0.1000E 03
SOIL REACTION           DEFLECTION
   0.                   0.
   0.1124E 04           0.5057E 00
   0.1124E 04           0.1000E 03

CURVE NO.      3 DEPTH TO CURVE 0.2000E 03
SOIL REACTION           DEFLECTION
   0.                   0.
   0.2908E 04           0.6543E 00
   0.2908E 04           0.1000E 03

CURVE NO.      4 DEPTH TO CURVE 0.3000E 03
SOIL REACTION           DEFLECTION
   0.                   0.
   0.3491E 05           0.1904E 01
   0.3491E 05           0.1000E 03

CURVE NO.      5 DEPTH TO CURVE 0.4000E 03
SOIL REACTION           DEFLECTION
   0.                   0.
   0.4654E 05           0.1904E 01
   0.4654E 05           0.1000E 03
SET IDENTIFIER NO.      5      NUMBER OF CURVES IN SET       5

DIAMETER DISTRIBUTION FOR PILE

   DIAMETER          TOP DIS          BOT DIS
 0.2000E 02       0.               0.1000E 03

 0.1000E 02       0.1000E 03       0.4000E 03
```

(Continued) (Sheet 4 of 5)

Table D2 (Concluded)

```
CURVE NO.     1 DEPTH TO CURVE 0.
SOIL REACTION        DEFLECTION
   0.                0.
   0.                0.1000E 01
   0.                0.2000E 03

CURVE NO.     2 DEPTH TO CURVE 0.5000E 02
SOIL REACTION        DEFLECTION
   0.                0.
   0.3811E 03        0.3430E 00
   0.3811E 03        0.2000E 03

CURVE NO.     3 DEPTH TO CURVE 0.2000E 03
SOIL REACTION        DEFLECTION
   0.                0.
   0.2908E 04        0.6543E 00
   0.2908E 04        0.1000E 03

CURVE NO.     4 DEPTH TO CURVE 0.3000E 03
SOIL REACTION        DEFLECTION
   0.                0.
   0.3491E 05        0.1904E 01
   0.3491E 05        0.1000E 03

CURVE NO.     5 DEPTH TO CURVE 0.4000E 03
SOIL REACTION        DEFLECTION
   0.                0.
   0.4654E 05        0.1904E 01
   0.4654E 05        0.1000E 03
```

(Sheet 5 of 5)

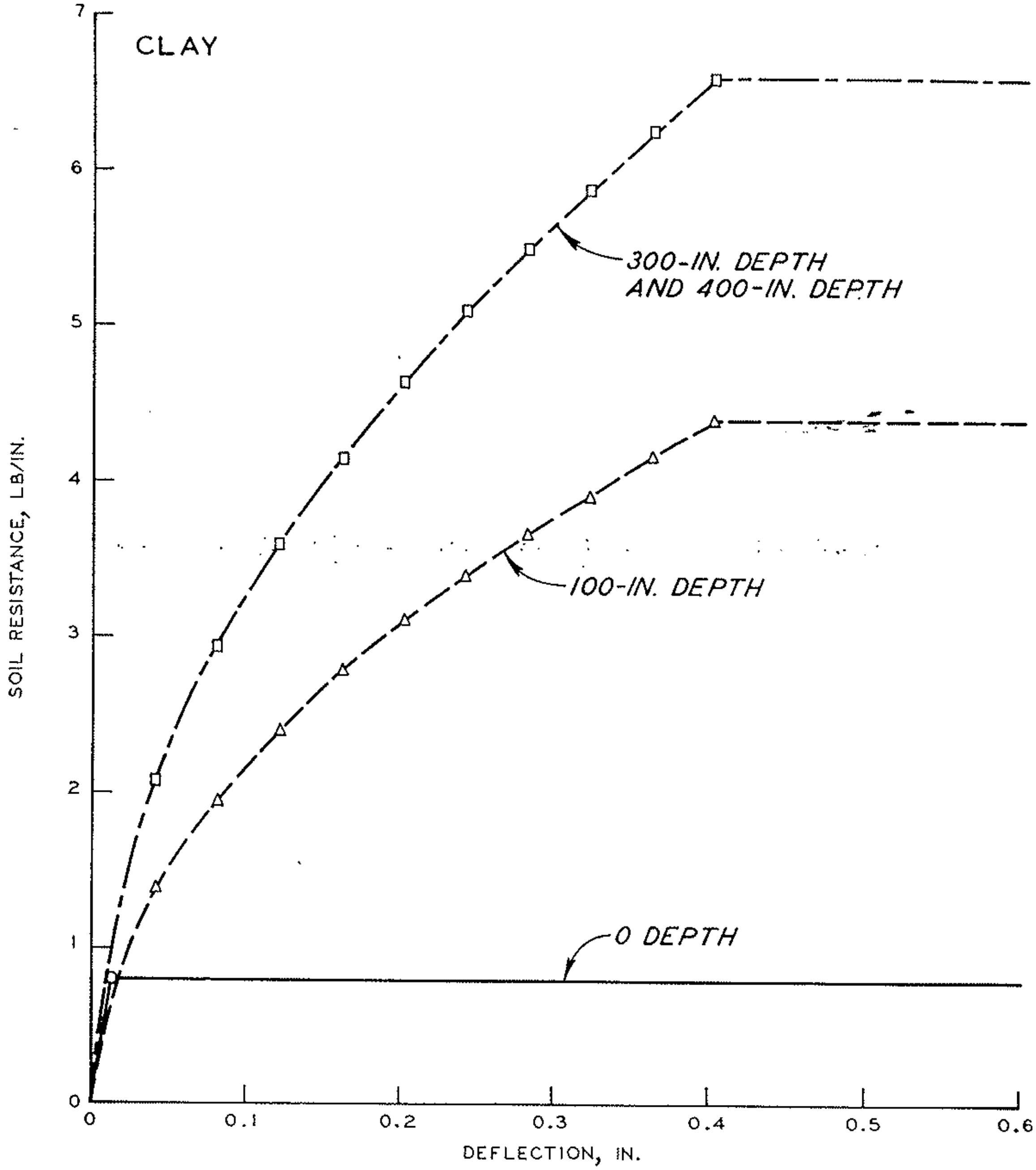

Figure D3. p-y curves for Example Problem 1 - Set 1

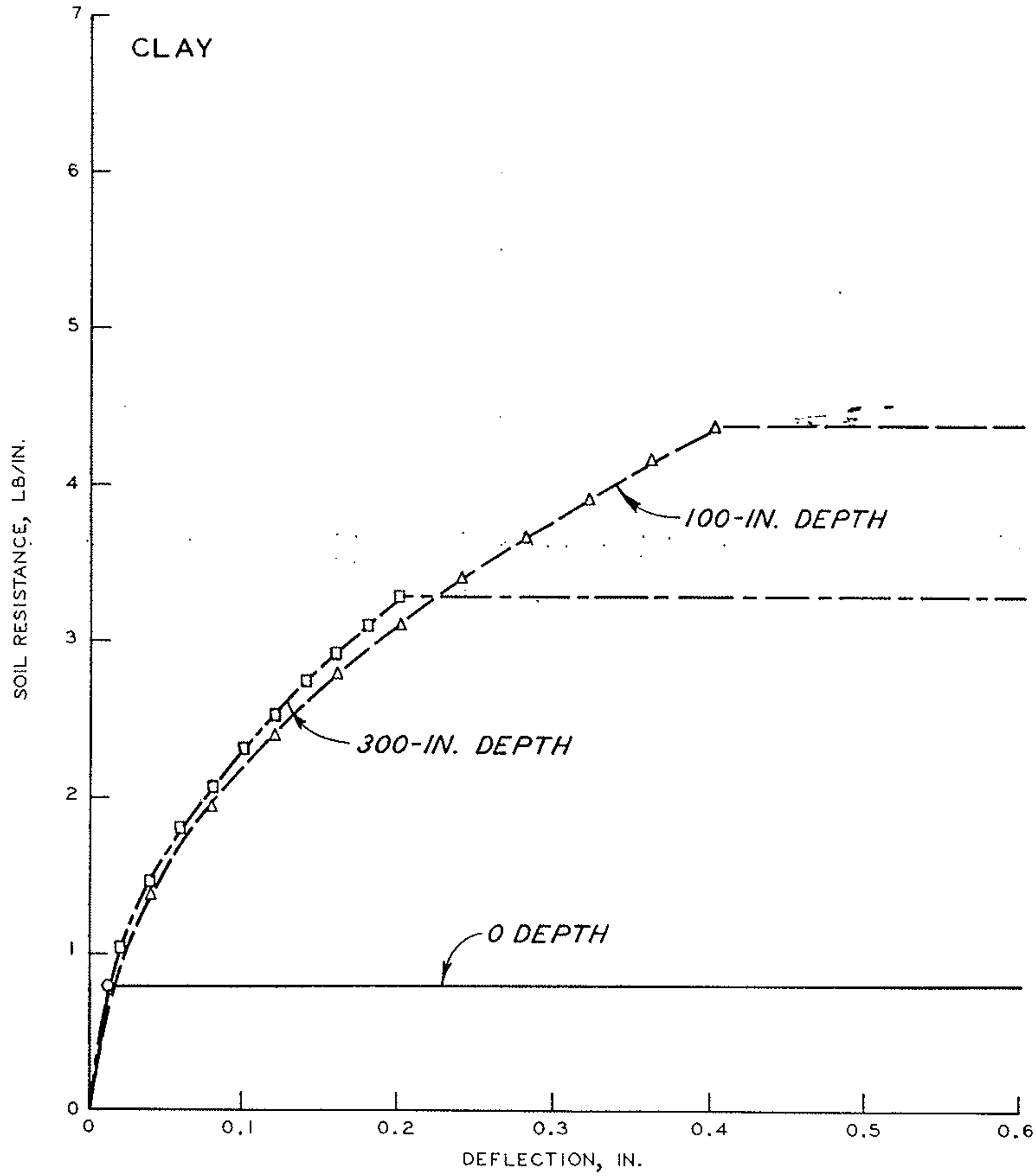

Figure D4. p-y curves for Example Problem 1 - Set 2

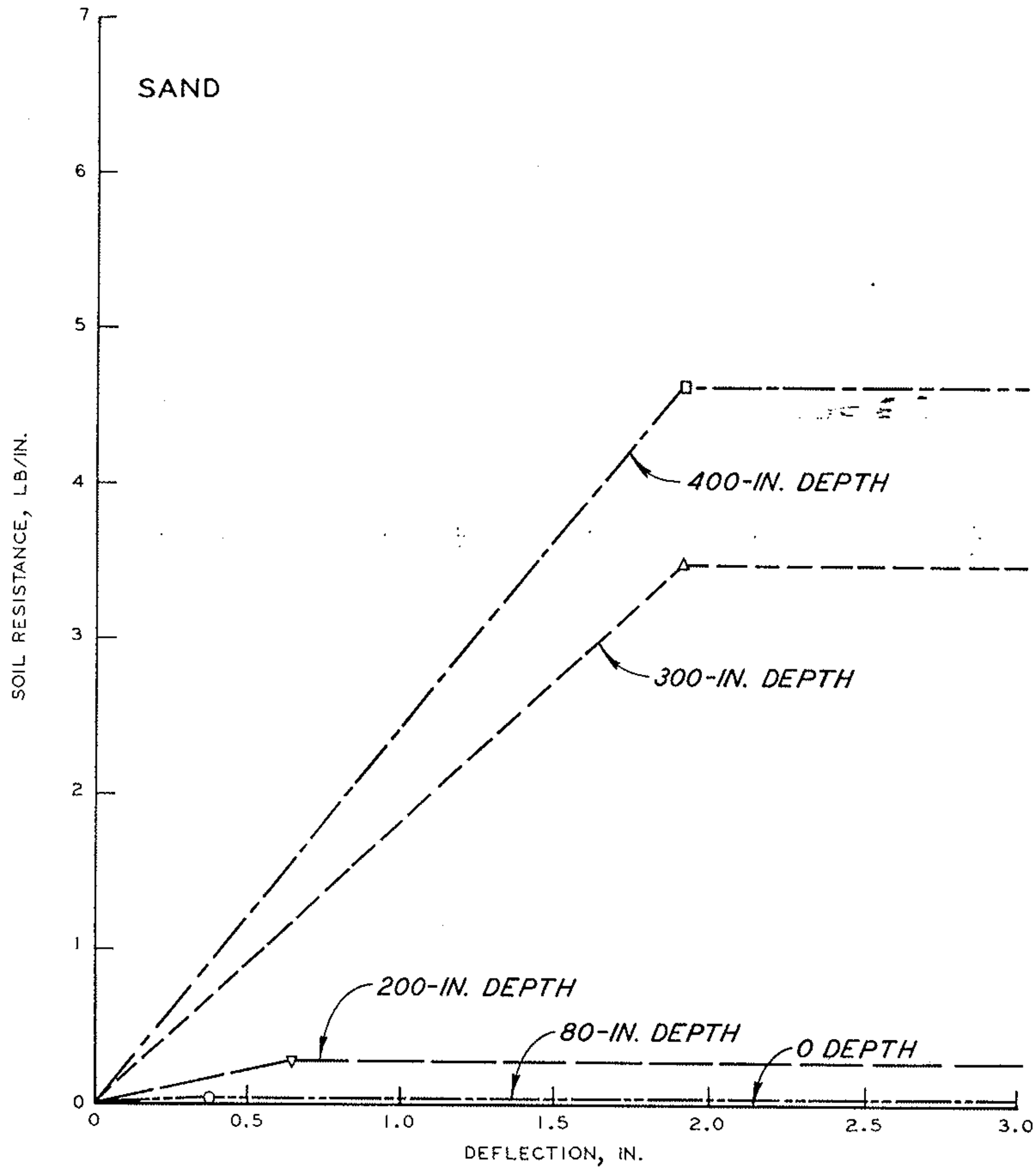

Figure D5. p-y curves for Example Problem 2 - Set 1

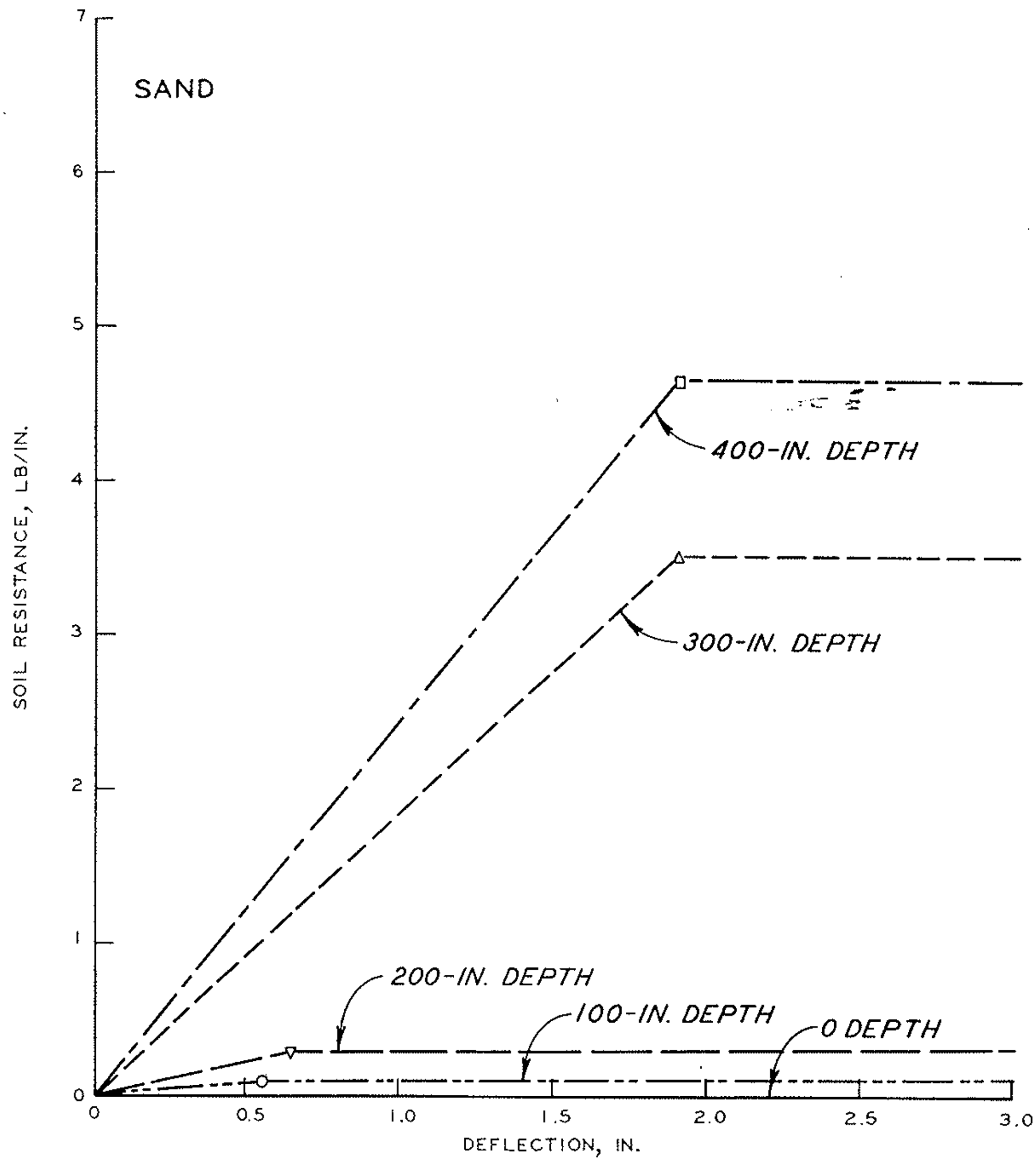

Figure D6. p-y curves for Example Problem 2 - Set 2

APPENDIX E: USER'S GUIDE FOR PROGRAM BENT1

General Introduction

1. Documentation for computer program BENT1 - to analyze two-dimensional group pile problems - is presented in this appendix and includes a general introduction, a computational flow chart, a glossary of notation, program listing, guide for data input, and two example problems with input-output data.

2. BENT1 is a computer program (developed by Drs. L. C. Reese, UT at Austin, and F. Parker,* WES) written to solve two-dimensional problems involving pile-supported foundations subjected to inclined and eccentric loadings. It is a modification of programs developed previously at UT, Austin. It consists of an iterative solution for the three equilibrium equations developed in Part IV using methods described in Part V to handle the nonlinear behavior of individual piles. The purpose of the iterative procedure is to find the deflected position of the structure so that equilibrium and compatibility are satisfied. The pile cap is assumed to be rigid in the analysis. BENT1 uses COM62 and MAKE as subroutines in the program.

3. Input for BENT1 consists essentially of the geometry of the foundation and the axial load-settlement curve. The lateral behavior of individual piles may be either described by inputting a table of p-y curve values or by inputting soil properties and activating subroutine MAKE to generate the p-y curves. Subroutine COM62 is used to compute response of individual piles in the group to lateral loads.

4. The program outputs the lateral load, bending moment, and axial load sustained by each pile in a group pile foundation besides providing other supplementary information. Successive applications of the program can be made to determine the optimum design of pile

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* F. Parker, Jr., and W. R. Cox, "A Method for Analysis of Pile Supported Foundations Considering Nonlinear Soil Behavior," Research Report 1171, 1969, Center for Highway Research, University of Texas, Austin, Tex.

foundations including pile sizes and arrangement of pile in the foundation.

5. Input may be input interactively at execute time, or input may be in a prepared data file. Output will be directed to an output file.

6. A group pile program called GROUP developed by Dr. Katsuyuki Awoshika* under the guidance of Prof. L. C. Reese is presently available. GROUP can perform the same type of analysis as BENT1 but is considered more efficient.

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* K. Awoshika and L. C. Reese, "Analysis of Foundation with Widely Spaced Batter Piles," Research Report 117-3F, 1971, Center for Highway Research, University of Texas, Austin, Tex.

Flow Chart

7. A flow chart for the iterative solution used in the program is shown in Figure E1.

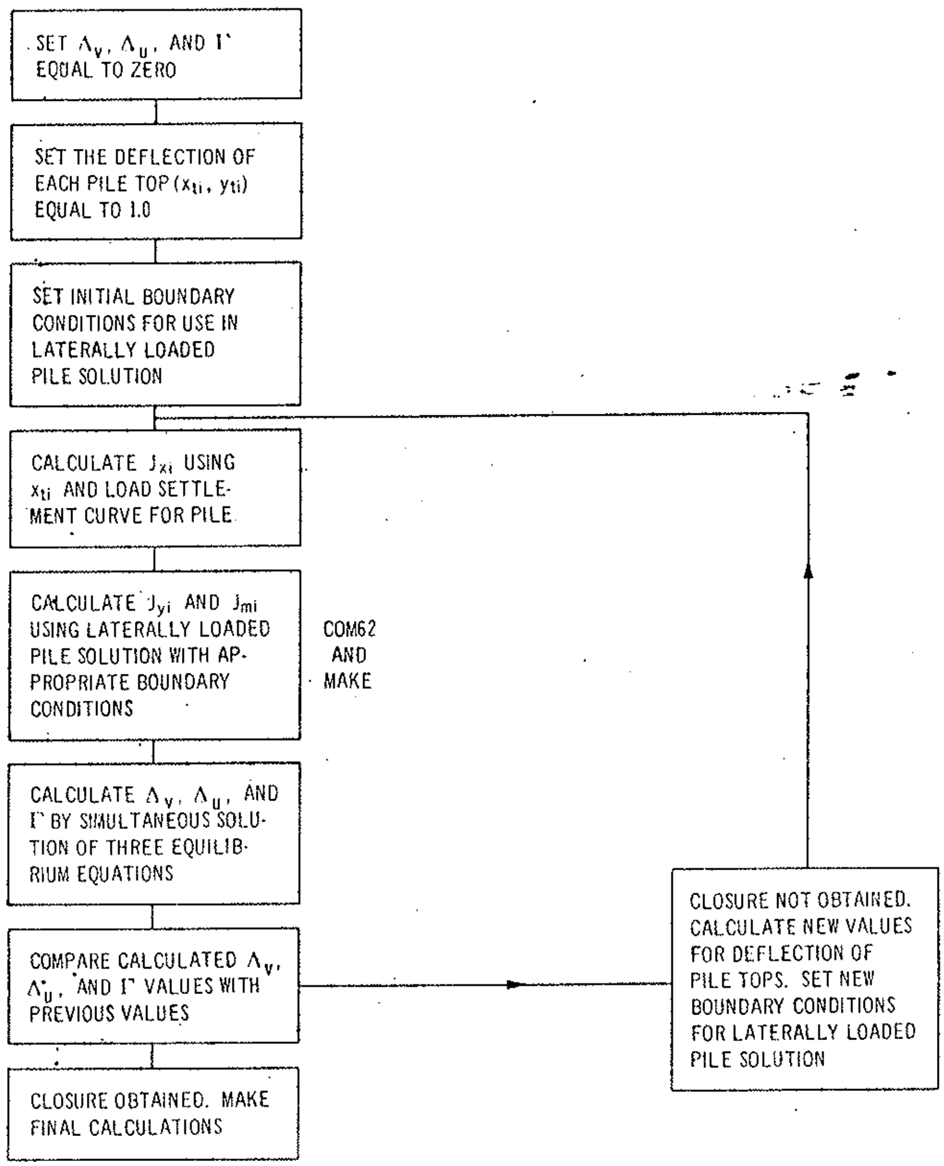

Figure E1. Flow chart for iterative solution in BENT1

Guide for Data Input

8. Data should be input to program BENT1 according to the following guide. All input is in free-field format and should be in units of pounds, inches, and radians. The data input for subroutine MAKE with its input flow chart (Figure E2) is also included.

Group 1 - Title

ANUM

ANUM = 60 character variable to identify problem

Group 2 - Foundation Load and Control Data

PV, PH, TM, TOL, KNPL, KOSC

PV = Vertical load on foundation

PH = Horizontal load on foundation

TM = Moment on foundation

TOL = Iteration tolerance (tolerable deflection difference)

KNPL = Number of pile locations

KOSC = Switch to control oscillating solution
0 --- for normal use
1 --- if solution oscillates

Group 3 - Control Data for Pile Locations

DISTA, DISTB, THETA, POTT, KS, KA

DISTA = Horizontal coordinate of pile top

DISTB = Vertical coordinate of pile top

THETA = Pile batter

POTT = Number of piles at a location

KS = Identifier to relate to p-y curve

KA = Identifier to relate to axial settlement curve

Note: Repeat Group 3 until KNPL sets have been specified

Group 4 - Control Data for Piles

A. | NN, HH, DPS, NDEI, TC, FDBET, E |

NN = Number of increments

HH = Increment length

DPS = Distance from pile top to soil surface

NDEI = Number of different flexural stiffness values specified

TC = Alphanumeric designation for top connection of pile (FIX; PIN; or RES)

FDBET = Rotational restraint value (not needed unless TC=RES)

E = Pile diameter or width

B. | RRI, XX1, XX2 |

RRI = Flexural stiffness (EI) of a section

XX1 = Distance from pile top to top of section

XX2 = Distance from pile top to bottom of section

Note: Repeat Set B until NDEI sets have been specified.

Note: Repeat Group 4 until KNPL sets have been specified.

Group 5 - Control Data for Soil Properties

| NKA, NKS, KOK |

NKA = Number of load settlement curves

NKS = Number of sets of p-y curves

KOK = Switch for input of p-y curves
(KOK = 0 p-y curves input
KOK = 1 p-y curves generated)

Group 6 - Control and Data for Axial Load Settlement Curves

A. | IDEN, IO |

IDEN = Identifier for axial load settlement curve (corresponds to KA)

IO = Number of points on curve

B. | ZZZ, SSS |

ZZZ = Axial settlement

SSS = Axial load

Note: Repeat Set B until IO sets have been supplied.

Note: Repeat Group 6 until NKA sets have been supplied.

Group 7 - Control Data for p-y Curves
(Necessary only if KOK = 0, NKS sets per problem)

A. | IDPY, KNC |

IDPY = Identifier for set of p-y curves (correspond to KS)

KNC = Number of curves in set

B. | NP, XS |

NP = Number of points on curve

XS = Distance from ground line to curve

Note: Repeat Set B until KNC values have been supplied.

C. | YC, PC |

YC = Deflection on curve

PC = Soil reaction on curve

Note: Repeat Set C within each Set B until NP sets are specified.

Note: Repeat Group 7 until NKS sets have been supplied.

If p-y curves are to be generated (i.e., KOK = 1), the data for subroutine MAKE will follow.

Subroutine MAKE

Group 8 - Profile Data

NSOILP

NSOILP = Number of soil profiles (one value per run)

Group 9 - Soil Data

A.

NSTYPE

NSTYPE = Number of soil stratums (one value per profile)

B.

TSOIL

TSOIL = Alphanumeric designation of type of soil in stratum (sand or clay) - one value per stratum

= Sand --- input line Set C and omit line Set D for this stratum

= Clay --- input line Set D and omit line Set C for this stratum

Note: A space (blank) must be left between the file line number and the parameter TSOIL.

C. Sand Properties

GAMMA, PHI, DIS1, DIS2, KDENSE

GAMMA = Unit weight of soil

PHI = Angle of internal friction

DIS1 = Distance from ground line to top of stratum

DIS2 = Distance from ground line to bottom of stratum

KDENSE = Alphanumeric designation for relative density of sand

= DENSE, MEDIUM, or LOOSE

D. Clay Properties

(i) | GAMMA, SHEARS, DIS1, DIS2, INFO, ICON |

GAMMA = Unit weight of clay

SHEARS = Cohesion of clay

DIS1 = Distance from ground line to top of stratum

DIS2 = Distance from ground line to bottom of stratum

INFO = Control for input of stress-strain curve
0 --- omit data for curves by omitting cards that follow in this group
1 --- input data for curves by specifying cards that follow in this group

ICON = Alphanumeric designation for consistency of clay (SOFT or STIF)

(ii) | NCURVS |

NCURVS = Number of curves per stratum

(iii) | DIST, NPOINT |

DIST = Distance from ground line to curve

NPOINT = Number of points on curve

(iv) | SIGD, EP |

SIGD = Principal stress difference ($\sigma_1 - \sigma_3$)

EP = Axial strain

Note: Repeat (iv) until NPOINT number of points have been specified for that curve.

Note: Repeat (iii) until NCURVS number of curves have been specified for that stratum.

Note: Repeat Group 9B and 9C or Group 9B and 9D NSTYPE times.

Group 10 - Pile Data

A. | NPISP |

NPISP = Number of different piles in this soil profile - one value per soil profile

Note: Repeat Set B until NPISP sets have been specified.

B. (i) | KS, NOC, NDD |

KS = Numeric identifier for set of p-y curves

NOC = Number of curves in set

NDD = Number of different diameters used for p-y curves

(ii) | D, DISD1, DISD2 |

D = Pile diameter

DISD1 = Distance from top of pile to top of section

DISD2 = Distance from top of pile to bottom of section

Note: Repeat (ii) until NDD sets of values have been specified.

(iii) | DTC |

DTC = Distance from top of pile to p-y curve

Note: Repeat (iii) until NOC values have been specified.

Note: Repeat Groups 9 and 10 NSOILP number of times.

INPUT FLOW CHART

PROGRAM MAKE

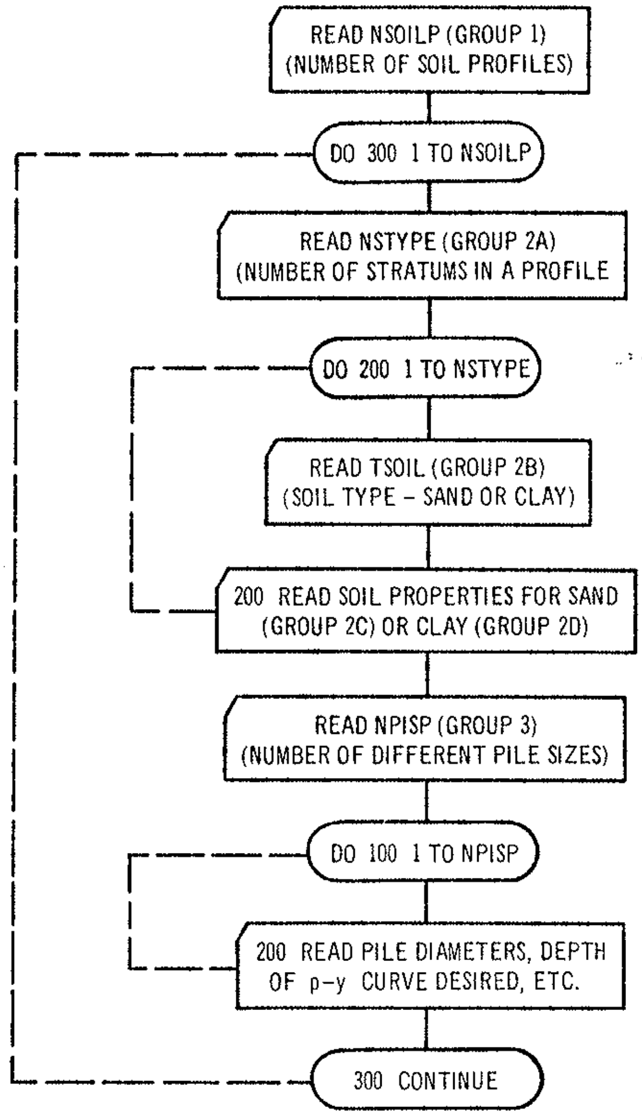

Figure E2. Input flow chart for subroutine MAKE

Example Problems

9. The two example problems (extracted from Parker and Cox*) are associated with actual bents, used by the Texas Highway Department for supporting bridges on the Gulf Coast of Texas. The geometry of the bents, properties of the piles and soil, and loads on the bents were obtained from highway department files.

10. The bents considered in the example problems are used in bridges located on the Gulf Coast of Texas. There are two basic reasons why bents of this type were selected for analysis by the proposed method. The first reason is that soil conditions in this area are consistently bad which makes piles necessary for bridge foundations. The second reason is that high lateral loads are common. These are due primarily to wind and wave action. During hurricanes the lateral loads may be quite high. The use of long piles and high lateral loads makes the proposed method of analysis seem very attractive for these bents.

Copano Bay Causeway

11. The first example problem considered will be one of the bents used in the Copano Bay Causeway. The bridge is located in Aransas County on State Highway 35 between Port Lavaca and Rockport. The bridge is 920 ft in length and provides 50 ft vertical clearance at the center of the span. The roadway is supported by precast-prestressed concrete girders. The bent caps, columns, and footings are reinforced concrete. The bent heights vary from 20 to 50 ft. The bent analyzed is shown in Figure E3. The piles used are battered in four directions to resist horizontal forces perpendicular and parallel to the roadway. Only the case where the horizontal load is perpendicular to the roadway will be considered. For this case, the two interior piles in each footing, which are battered parallel to the roadway, will be treated as vertical piles. The bottom tie beam is considered to provide sufficient rigidity so that the assumption that the pile heads remain in the same plane after movement is valid.

12. The geometry necessary for describing the foundation for the

---

* Ibid p. E1

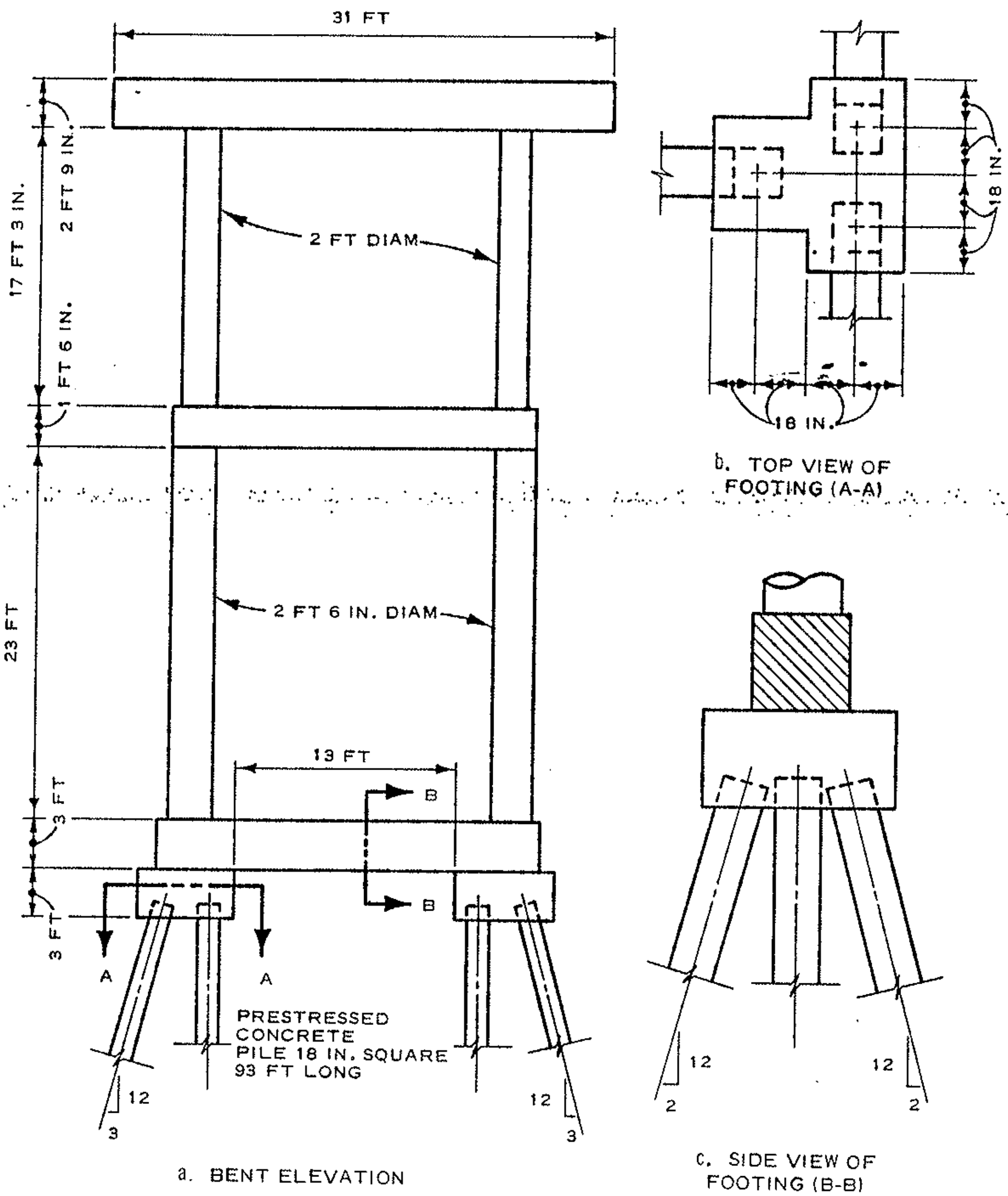

Figure E3. Copano Bay Causeway bent - Example Problem 1 computer solution is presented in Figure E4 and the following tabulation.

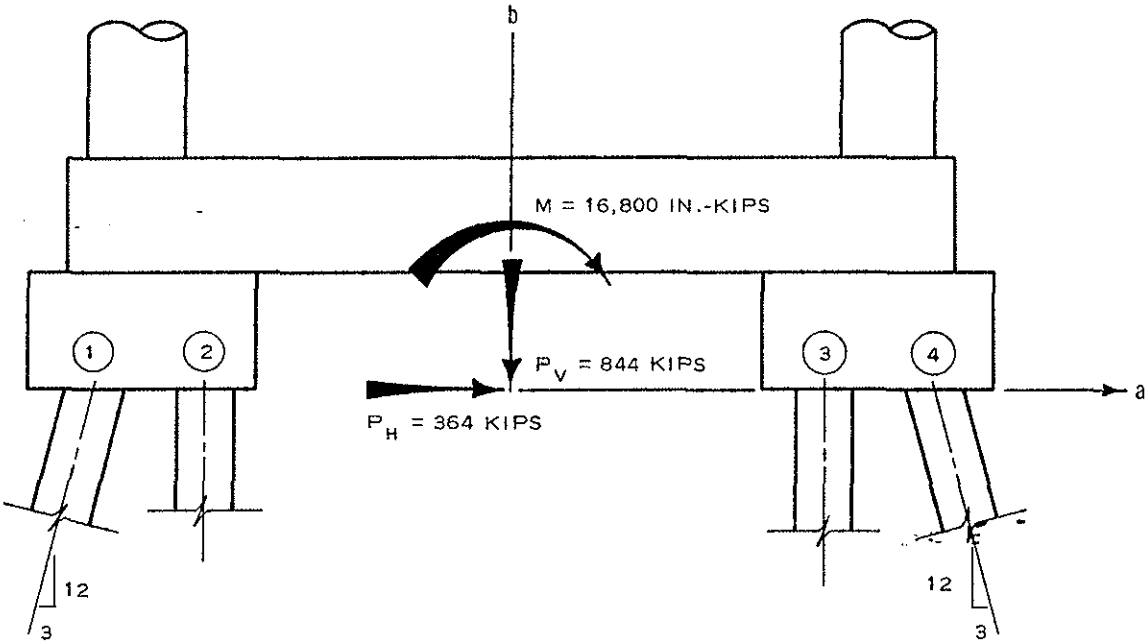

Figure E4. Foundation representation for Example Problem 1

Pile Location	a Coordinate in.	b Coordinate in.	No. Piles at Location	Batter radians
1	-126	0	1	-0.244
2	-90	0	2	0.0
3	+90	0	2	0.0
4	+126	0	1	+0.244

The coordinate system and the resulting forces on the bent are also shown in this figure. The piles are 18-in.-square prestressed concrete piles. They have an effective flexural rigidity of 4.374×10^{10} lb-in.^{2} (assuming a modulus of elasticity for concrete of 5×10^6 psi) and a length of 93 ft.

13. A pile similar to the ones used in the bent was driven near the site of the bent. A load test was performed on this pile. The load settlement curves obtained and used in the computer solution are shown in Figure E5.

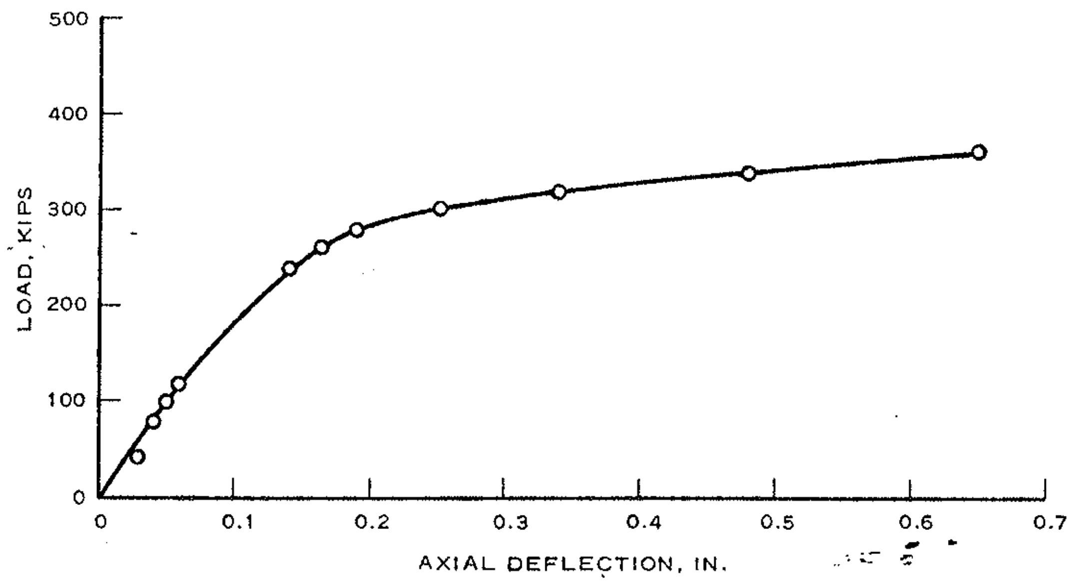

Figure E5. Load deflection curve for Example Problem 1

14. The piles are driven through what is classified as muck or very soft clay to bearing on a dense sand or firm sandy clay. The location of the stiffer strata is variable, and so the length of pile and length of embedment in the stiffer strata will be variable. For this analysis, the piles are assumed to be 93 ft in length with an embedment length of 83 ft.

15. For generation of p-y curves, the soil is treated as a clay. That is, the soil is treated as a frictionless material with the shear strength composed entirely of cohesion. Some thin sand layers are encountered, but their effect is considered insignificant. The tip of the pile may also be buried to several feet in a sand or sandy clay, but the effect on the lateral behavior will be insignificant and will be ignored.

16. After considering boring logs from the vicinity of the bent and after a review of triaxial data, a variation of cohesion with depth was assumed and used for predicting lateral pile-soil interaction. This assumed distribution of cohesion along the pile length is shown in Figure E6. The depth given is the distance from the soil surface. The top of the piles is located at the water surface which is 10 ft above

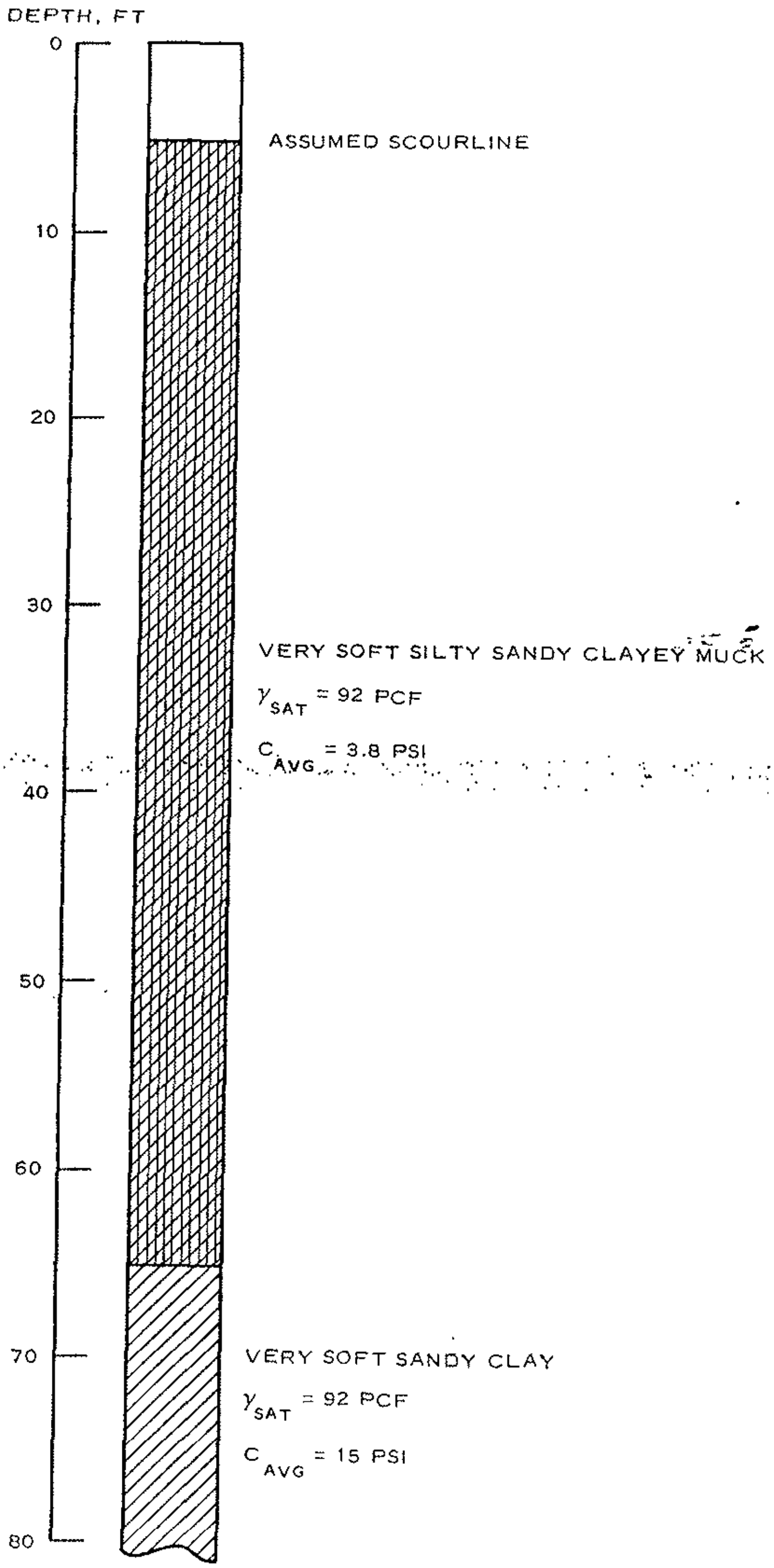

Figure E6. Soil properties for generation of p-y curves for Example Problem 1 the soil surface. The scourline is assumed to be 5 ft below the soil surface. The saturated unit weight of the soil is taken as 92 pcf, and the consistency is soft.

17. A solution was obtained for this problem by using the program BENT1. The movement of the bent is described by the following movements of the origin of the a-b coordinate system:

$$\Delta_v = 7.664 \times 10^{-2} \text{ in.}$$

$$\Delta_u = 1.004 \times 10^{-1} \text{ in.}$$

$$\Gamma = 8.536 \times 10^{-5} \text{ radians}$$

The loads transferred to each pile and the movements of each pile top are given in the following tabulation:

Pile Location	Axial Load per Pile kips	Lateral Load per Pile kips	Moment per Pile in.-kips	Axial Movement in.	Lateral Movement in.
1	78.7	1.7	-253.3	0.0397	0.1134
2	133.4	1.5	-218.9	0.0689	0.1004
3	156.5	1.5	-218.8	0.0843	0.1004
4	193.6	1.1	-155.2	0.1091	0.0763

The forces and movements at the pile tops are related to the x-y coordinate system set up for each pile.

18. The deflection of the a-b coordinate system defines the equilibrium position for the structure. When the foundation is in this position, the piles exert on the foundation the given forces and moments which satisfy the three equilibrium equations. A complete listing of the coded input and output is presented in Tables E1 and E2 beginning on page E38.

19. If the movement of the structure and the loads carried by each pile are considered, it would appear that the design is conservative. This is probably true, but it should be pointed out that factors such as settlement caused by consolidation and cyclic loading have not been considered.

Table E1

Input Data for Example Problem 1

```
100  EX 1 COPANO BAY CAUSEWAY,ARANSAS COUNTY TEXAS,US HIGHWAY 35
110 844000.0,36400.0,16817000.0,0.001,4,0
120 -126.0,0.0,-0.244,1.0,1,1
130 -90.0,0.0,0.,2.0,1,1
140 90.0,0,0,2.0,1,1
150 126.0,0.0,0.244,1.0,1,1
160 31,36.0,120.0,1, FIX ,0,18.0
170 43740000000.0,0.0,1116.0
180 31,36.0,120.0,1, FIX ,0,18.0
190 43740000000.0,0.0,1116.0
200 31,36.0,120.0,1, FIX ,0,18.0
210 43740000000.0,0.0,1116.0
220 31,36.0,120.0,1, FIX ,0,18.0
230 43740000000.0,0.0,1116.0
240 1,1,1
250 1,15
060 -10.0,-360000.0
270 -65.0,-360000.0
280 -0.19,-280000.0
290 -0.16,-260000.0
300 -0.14,-240000.0
310 0.0,0.0
320 0.03,40000.0
330 0.04,80000.0
340 0.05,100000.0
350 0.06,120000.0
360 0.14,240000.0
370 0.16,260000.0
380 0.19,280000.0
390 0.65,360000.0
400 10.0,360000.0
410 1
420 3
430  CLAY
440 0.0,0.001,0.0,60.0,0, SOFT
450  CLAY
460 0.0174,3.8,60.0,894.0,0, SOFT
470  CLAY
480 0.0174,15.0,894.0,1000.0,0, SOFT
490 1
500 1,9,1
510 18.0,0.0,1000.0
520 0.0
530 60.0
540 61.0
550 96.0
560 132.0
570 168.0
580 204.0
590 240.0
600 996.0
```

Table E2

Output Data for Example Problem 1

1EX 1 COPANO BAY CAUSEWAY,ARANSAS COUNTY TEXAS,US HIGHWAY 35

LIST OF INPUT DATA ---

PV	PH	TM	TOL	KNPL	KOSC
0.8440E+06	0.3640E+05	0.1682E+08	0.1000E-02	4	0

CONTROL DATA FOR PILES AT EACH LOCATION

PILE NO	DISTA	DISTB	BATTER	POTT	KS	KA
1	-.1260E+03	0.0000E+00	-.2440E+00	0.1000E+01	1	1
2	-.9000E+02	0.0000E+00	0.0000E+00	0.2000E+01	1	1
3	0.9000E+02	0.0000E+00	0.0000E+00	0.2000E+01	1	1
4	0.1260E+03	0.0000E+00	0.2440E+00	0.1000E+01	1	1

PILE NO.	NN	HH	DPS	NDEI	CONNECTION	FDBET
1	31	0.36000E+02	0.12000E+03	1	FIX	0.0000E+00
2	31	0.36000E+02	0.12000E+03	1	FIX	0.0000E+00
3	31	0.36000E+02	0.12000E+03	1	FIX	0.0000E+00
4	31	0.36000E+02	0.12000E+03	1	FIX	0.0000E+00

	RRI	XX1	XX2
PILE NO 1	0.43740E+11	0.00000E+00	0.11160E+04
PILE NO 2	0.43740E+11	0.00000E+00	0.11160E+04
PILE NO 3	0.43740E+11	0.00000E+00	0.11160E+04
PILE NO 4	0.43740E+11	0.00000E+00	0.11160E+04

AXIAL LOAD SETTLEMENT DATA

IDENTIFIER 1	ZZZ	SSS
	-.10000E+02	-.36000E+06
	-.65000E+02	-.36000E+06
	-.19000E+00	-.28000E+06
	-.16000E+00	-.26000E+06
	-.14000E+00	-.24000E+06
	0.00000E+00	0.00000E+00
	0.30000E-01	0.40000E+05
	0.40000E-01	0.80000E+05
	0.50000E-01	0.10000E+06
	0.60000E-01	0.12000E+06
	0.14000E+00	0.24000E+06

(Continued)

(Sheet 1 of 9)

0.16000E+00	0.26000E+06
0.19000E+00	0.28000E+06
0.65000E+00	0.36000E+06
0.10000E+02	0.36000E+06

INPUT OF SOIL PARAMETERS

SOIL PROFILE NO. 1 STRATUM NO. 1 TYPE SOILCLAY

GAMMA	COHESION	TOP DEPTH	BOTTEM DEPTH	CONSISTENCY
0.0000E+00	0.1000E-02	0.0000E+00	0.6000E+02	SOFT

SOIL PROFILE NO. 1 STRATUM NO. 2 TYPE SOILCLAY

GAMMA	COHESION	TOP DEPTH	BOTTEM DEPTH	CONSISTENCY
0.1740E-01	0.3800E+01	0.6000E+02	0.8940E+03	SOFT

SOIL PROFILE NO. 1 STRATUM NO. 3 TYPE SOILCLAY

GAMMA	COHESION	TOP DEPTH	BOTTEM DEPTH	CONSISTENCY
0.1740E-01	0.1500E+02	0.8940E+03	0.1000E+04	SOFT

DIAMETER DISTRIBUTION FOR PILE

DIAMETER	TOP DIS	BOT DIS
0.1800E+02	0.0000E+00	0.1000E+04

1

P-Y CURVES

SET IDENTIFIER NO. 1 NUMBER OF CURVES IN SET 9

CURVE NO. 1 DEPTH TO CURVE 0.0000E+00

SOIL REACTION	DEFLECTION
0.0000E+00	0.0000E+00
0.3600E-01	0.4320E-01
0.3600E-01	0.1800E+03

CURVE NO. 2 DEPTH TO CURVE 0.6000E+02

(Continued)

(Sheet 2 of 9)

SOIL REACTION	DEFLECTION
0.0000E+00	0.0000E+00
0.6261E-01	0.1440E+00
0.8855E-01	0.2880E+00
0.1084E+00	0.4320E+00
0.1252E+00	0.5760E+00
0.1400E+00	0.7200E+00
0.1534E+00	0.8640E+00
0.1657E+00	0.1008E+01
0.1771E+00	0.1152E+01
0.1878E+00	0.1296E+01
0.1980E+00	0.1440E+01
0.1980E+00	0.1800E+03

CURVE NO. 3 DEPTH TO CURVE 0.6100E+02

SOIL REACTION	DEFLECTION
0.0000E+00	0.0000E+00
0.2379E+03	0.1440E+00
0.3365E+03	0.2880E+00
0.4121E+03	0.4320E+00
0.4759E+03	0.5760E+00
0.5320E+03	0.7200E+00
0.5828E+03	0.8640E+00
0.6295E+03	0.1008E+01
0.6730E+03	0.1152E+01
0.7138E+03	0.1296E+01
0.7524E+03	0.1440E+01
0.7524E+03	0.1800E+03

CURVE NO. 4 DEPTH TO CURVE 0.9600E+02

SOIL REACTION	DEFLECTION
0.0000E+00	0.0000E+00
0.2379E+03	0.1440E+00
0.3365E+03	0.2880E+00
0.4121E+03	0.4320E+00
0.4759E+03	0.5760E+00
0.5320E+03	0.7200E+00
0.5828E+03	0.8640E+00
0.6295E+03	0.1008E+01
0.6730E+03	0.1152E+01
0.7138E+03	0.1296E+01
0.7524E+03	0.1440E+01
0.7524E+03	0.1800E+03

CURVE NO. 5 DEPTH TO CURVE 0.1320E+03

SOIL REACTION	DEFLECTION
0.0000E+00	0.0000E+00
0.2379E+03	0.1440E+00

(Continued)

(Sheet 3 of 9)

0.3365E+03	0.2880E+00
0.4121E+03	0.4320E+00
0.4759E+03	0.5760E+00
0.5320E+03	0.7200E+00
0.5828E+03	0.8640E+00
0.6295E+03	0.1008E+01
0.6730E+03	0.1152E+01
0.7138E+03	0.1296E+01
0.7524E+03	0.1440E+01
0.7524E+03	0.1800E+03

CURVE NO. 6 DEPTH TO CURVE 0.1680E+03

SOIL REACTION	DEFLECTION
0.0000E+00	0.0000E+00
0.2379E+03	0.1440E+00
0.3365E+03	0.2880E+00
0.4121E+03	0.4320E+00
0.4759E+03	0.5760E+00
0.5320E+03	0.7200E+00
0.5828E+03	0.8640E+00
0.6295E+03	0.1008E+01
0.6730E+03	0.1152E+01
0.7138E+03	0.1296E+01
0.7524E+03	0.1440E+01
0.7524E+03	0.1800E+03

CURVE NO. 7 DEPTH TO CURVE 0.2040E+03

SOIL REACTION	DEFLECTION
0.0000E+00	0.0000E+00
0.2379E+03	0.1440E+00
0.3365E+03	0.2880E+00
0.4121E+03	0.4320E+00
0.4759E+03	0.5760E+00
0.5320E+03	0.7200E+00
0.5828E+03	0.8640E+00
0.6295E+03	0.1008E+01
0.6730E+03	0.1152E+01
0.7138E+03	0.1296E+01
0.7524E+03	0.1440E+01
0.7524E+03	0.1800E+03

CURVE NO. 8 DEPTH TO CURVE 0.2400E+03

SOIL REACTION	DEFLECTION
0.0000E+00	0.0000E+00
0.2379E+03	0.1440E+00
0.3365E+03	0.2880E+00
0.4121E+03	0.4320E+00
0.4759E+03	0.5760E+00

(Continued)

0.5320E+03	0.7200E+00
0.5828E+03	0.8640E+00
0.6295E+03	0.1008E+01
0.6730E+03	0.1152E+01
0.7138E+03	0.1296E+01
0.7524E+03	0.1440E+01
0.7524E+03	0.1800E+03

CURVE NO. 9 DEPTH TO CURVE 0.9960E+03

SOIL REACTION	DEFLECTION
0.0000E+00	0.0000E+00
0.9392E+03	0.1440E+00
0.1328E+04	0.2880E+00
0.1627E+04	0.4320E+00
0.1878E+04	0.5760E+00
0.2100E+04	0.7200E+00
0.2301E+04	0.8640E+00
0.2485E+04	0.1008E+01
0.2656E+04	0.1152E+01
0.2818E+04	0.1296E+01
0.2970E+04	0.1440E+01
0.2970E+04	0.1800E+03

1 ITERATION DATA

DV	DH	ALPHA
0.3985E+00	0.1261E+01	-.4492E-03
0.1824E+00	0.3865E+00	0.2338E-03
0.1038E+00	0.1578E+00	0.2297E-03
0.8262E-01	0.1080E+00	0.9721E-04
0.7757E-01	0.1032E+00	0.8648E-04
0.7678E-01	0.1008E+00	0.8561E-04
0.7664E-01	0.1004E+00	0.8536E-04

1EX 1 COPANO BAY CAUSEWAY,ARANSAS COUNTY TEXAS,US HIGHWAY 35

PILE NUM	DISTA,IN	DISTB,IN	THETA, RAD
1	-.12600E+03	0.00000E+00	-.24400E+00

PX,LB	XT,IN	PT,LB	M	T,IN-LB	YT,IN
0.78721E+05	0.39680E-01	0.17341E+04	-.25328E+06	0.11335E+00	

INPUT INFORMATION

TC	TOP DIA,IN	INC. LENGTH,IN	NO. OF INC	KS	KA
FIX	0.1800E+02	0.3600E+02	31	1	1

(Continued)

(Sheet 5 of 9)

PILE LENGTH,IN	DEPTH TO SOIL	ITERATION TOL,BO	UNDRY COND,2
0.1116E+04	0.1200E+03	0.1000E-02	0.0000E+00

OUTPUT INFORMATION

X,IN	Y,IN	MOMENT,IN-LB	ES,LB/	IN P LB/IN
0.00000E+00	0.11335E+00	-.25329E+06	0.00000E+00	0.00000E+00
0.36000E+02	0.10653E+00	-.19032E+06	0.00000E+00	0.00000E+00
0.72000E+02	0.94066E-01	-.12691E+06	0.00000E+00	0.00000E+00
0.10800E+03	0.77842E-01	-.63202E+05	0.00000E+00	0.00000E+00
0.14400E+03	0.59745E-01	0.65153E+03	0.53540E+00	-.31987E-01
0.18000E+03	0.41667E-01	0.64462E+05	0.43481E+00	-.18117E-01
0.21600E+03	0.25499E-01	0.12810E+06	0.16523E+04	-.42132E+02
0.25200E+03	0.13127E-01	0.13684E+06	0.16523E+04	-.21689E+02
0.28800E+03	0.48086E-02	0.11714E+06	0.16523E+04	-.79452E+01
0.32400E+03	-.38435E-04	0.86880E+05	0.16523E+04	0.63506E-01
0.36000E+03	-.23113E-02	0.56498E+05	0.16523E+04	0.38189E+01
0.39600E+03	-.29101E-02	0.30932E+05	0.18842E+04	0.54831E+01
0.43200E+03	-.25924E-02	0.12400E+05	0.21161E+04	0.54857E+01
0.46800E+03	-.19072E-02	0.94991E+03	0.23480E+04	0.44782E+01
0.50400E+03	-.11939E-02	-.46995E+04	0.25799E+04	0.30803E+01
0.54000E+03	-.61993E-03	-.63459E+04	0.28118E+04	0.17431E+01
0.57600E+03	-.23392E-03	-.57184E+04	0.30437E+04	0.71200E+00
0.61200E+03	-.17354E-04	-.41548E+04	0.32756E+04	0.56846E-01
0.64800E+03	0.76110E-04	-.25079E+04	0.35075E+04	-.26695E+00
0.68400E+03	0.95266E-04	-.12011E+04	0.37394E+04	-.35624E+00
0.72000E+03	0.78834E-04	-.35317E+03	0.39713E+04	-.31307E+00
0.75600E+03	0.51938E-04	0.89837E+02	0.42032E+04	-.21830E+00
0.79200E+03	0.27703E-04	0.24971E+03	0.44351E+04	-.12286E+00
0.82800E+03	0.10867E-04	0.24977E+03	0.46670E+04	-.50719E-01
0.86400E+03	0.14327E-05	0.18351E+03	0.48989E+04	-.70189E-02
0.90000E+03	-.25647E-05	0.10773E+03	0.51308E+04	0.13159E-01
0.93600E+03	-.33701E-05	0.48755E+02	0.53627E+04	0.18073E-01
0.97200E+03	-.27309E-05	0.13085E+02	0.55946E+04	0.15278E-01
0.10080E+04	-.17040E-05	-.28149E+01	0.58265E+04	0.99281E-02
0.10440E+04	-.76045E-06	-.58414E+01	0.60584E+04	0.46071E-02
0.10800E+04	0.99826E-08	-.28833E+01	0.62909E+04	-.62799E-04
0.11160E+04	0.69498E-06	-.46574E+02	0.65222E+04	-.45328E-02

1EX 1 COPANO BAY CAUSEWAY,ARANSAS COUNTY TEXAS,US HIGHWAY 35

PILE NUM	DISTA,IN	DISTB,IN	THE TA, RAD
2	-.90000E+02	0.00000E+00	0.00000E+00

PX,LB	XT,IN	PT,LB	M T,IN-LB	YT,IN
0.13344E+06	0.68963E-01	0.14908E+04	-.21892E+06	0.10041E+00

INPUT INFORMATION

TC	TOP DIA,IN	INC. LENGTH,IN	NO. OF INC	KS	KA
FIX	0.1800E+02	0.3600E+02	31	1	1

(Continued)

(Sheet 6 of 9)

PILE LENGTH,IN DEPTH TO SOIL ITERATION TOL.BO UNDRY COND.2

0.1116E+04 0.1200E+03 0.1000E-02 0.0000E+00

OUTPUT INFORMATION

X,IN	Y,IN	MOMENT,IN-LB	ES,LB/ IN	P LB/IN
0.00000E+00	0.10040E+00	-.21891E+06	0.00000E+00	0.00000E+00
0.36000E+02	0.94089E-01	-.16440E+06	0.00000E+00	0.00000E+00
0.72000E+02	0.82902E-01	-.10923E+06	0.00000E+00	0.00000E+00
0.10800E+03	0.68478E-01	-.53645E+05	0.00000E+00	0.00000E+00
0.14400E+03	0.52465E-01	0.21595E+04	0.58554E+00	-.30720E-01
0.18000E+03	0.36516E-01	0.57916E+05	0.43481E+00	-.15878E-01
0.21600E+03	0.22283E-01	0.11342E+06	0.16523E+04	-.36817E+02
0.25200E+03	0.11410E-01	0.12076E+06	0.16523E+04	-.18853E+02
0.28800E+03	0.41159E-02	0.10319E+06	0.16523E+04	-.68007E+01
0.32400E+03	-.12051E-03	0.76410E+05	0.16523E+04	0.15913E+00
0.36000E+03	-.20929E-02	0.49577E+05	0.16523E+04	0.34582E+01
0.39600E+03	-.25964E-02	0.27030E+05	0.18842E+04	0.48921E+01
0.43200E+03	-.22990E-02	0.10715E+05	0.21161E+04	0.48649E+01
0.46800E+03	-.16841E-02	0.66426E+03	0.23480E+04	0.39543E+01
0.50400E+03	-.10495E-02	-.42651E+04	0.25799E+04	0.27076E+01
0.54000E+03	-.54130E-03	-.56685E+04	0.28118E+04	0.15220E+01
0.57600E+03	-.20104E-03	-.50769E+04	0.30437E+04	0.61190E+00
0.61200E+03	-.11204E-04	-.36723E+04	0.32756E+04	0.36702E-01
0.64800E+03	0.69821E-04	-.22055E+04	0.35075E+04	-.24490E+00
0.68400E+03	0.85498E-04	-.10474E+04	0.37394E+04	-.31971E+00
0.72000E+03	0.70138E-04	-.29958E+03	0.39713E+04	-.27854E+00
0.75600E+03	0.45903E-04	0.88494E+02	0.42032E+04	-.19294E+00
0.79200E+03	0.24289E-04	0.22617E+03	0.44351E+04	-.10772E+00
0.82800E+03	0.93773E-05	0.22333E+03	0.46670E+04	-.43764E-01
0.86400E+03	0.10823E-05	0.16290E+03	0.48989E+04	-.53025E-02
0.90000E+03	-.23859E-05	0.94947E+02	0.51308E+04	0.12241E-01
0.93600E+03	-.30409E-05	0.42486E+02	0.53627E+04	0.16308E-01
0.97200E+03	-.24372E-05	0.10991E+02	0.55946E+04	0.13635E-01
0.10080E+04	-.15077E-05	-.28761E+01	0.58265E+04	0.87846E-02
0.10440E+04	-.66346E-06	-.53471E+01	0.60584E+04	0.40195E-02
0.10800E+04	0.22341E-07	-.25876E+01	0.62905E+04	-.14054E-03
0.11160E+04	0.63148E-06	-.41871E+02	0.65222E+04	-.41186E-02

1EX 1 COPANO BAY CAUSEWAY,ARANSAS COUNTY TEXAS,US HIGHWAY 35

PILE NUM	DISTA,IN	DISTB,IN	THETA, RAD
3	0.90000E+02	0.00000E+00	0.00000E+00

PX,LB	XT,IN	PT,LB	M	T,IN-LB	YT,IN
0.15649E+06	0.84327E-01	0.14825E+04		-.21883E+06	0.10041E+00

INPUT INFORMATION

TC	TOP DIA,IN	INC, LENGTH,IN	NO, OF INC	KS	KA
FIX	0.1800E+02	0.3600E+02	31	1	1

PILE LENGTH,IN DEPTH TO SOIL ITERATION TOL.BO UNDRY COND.2

(Continued)

0.1116E+04 0.1200E+03 0.1000E-02 0.0000E+00

OUTPUT INFORMATION

X,IN	Y,IN	MOMENT,IN-LB	ES,LB/ IN	P LB/IN
0.00000E+00	0.10039E+00	-.21882E+06	0.00000E+00	0.00000E+00
0.36000E+02	0.94085E-01	-.16447E+06	0.00000E+00	0.00000E+00
0.72000E+02	0.82897E-01	-.10934E+06	0.00000E+00	0.00000E+00
0.10800E+03	0.68469E-01	-.53722E+05	0.00000E+00	0.00000E+00
0.14400E+03	0.52450E-01	0.21539E+04	0.58566E+00	-.30718E-01
0.18000E+03	0.36494E-01	0.57980E+05	0.43481E+00	-.15868E-01
0.21600E+03	0.22257E-01	0.11351E+06	0.16523E+04	-.36775E+02
0.25200E+03	0.11382E-01	0.12086E+06	0.16523E+04	-.18807E+02
0.28800E+03	0.40895E-02	0.10328E+06	0.16523E+04	-.67571E+01
0.32400E+03	-.14334E-03	0.76460E+05	0.16523E+04	0.23683E+00
0.36000E+03	-.21107E-02	0.49592E+05	0.16523E+04	0.34875E+01
0.39600E+03	-.26087E-02	0.27013E+05	0.18842E+04	0.49153E+01
0.43200E+03	-.23063E-02	0.10679E+05	0.21161E+04	0.48804E+01
0.46800E+03	-.16875E-02	0.62135E+03	0.23480E+04	0.39623E+01
0.50400E+03	-.10503E-02	-.43044E+04	0.25799E+04	0.27097E+01
0.54000E+03	-.54064E-03	-.56985E+04	0.28118E+04	0.15202E+01
0.57600E+03	-.19980E-03	-.50960E+04	0.30437E+04	0.60814E+00
0.61200E+03	-.99577E-05	-.36817E+04	0.32756E+04	0.32617E-01
0.64800E+03	0.70800E-04	-.22081E+04	0.35075E+04	-.24833E+00
0.68400E+03	0.86133E-04	-.10460E+04	0.37394E+04	-.32209E+00
0.72000E+03	0.70474E-04	-.29657E+03	0.39713E+04	-.27987E+00
0.75600E+03	0.46027E-04	0.91541E+02	0.42032E+04	-.19346E+00
0.79200E+03	0.24292E-04	0.22850E+03	0.44351E+04	-.10773E+00
0.82800E+03	0.93285E-05	0.22478E+03	0.46670E+04	-.43536E-01
0.86400E+03	0.10245E-05	0.16359E+03	0.48989E+04	-.50189E-02
0.90000E+03	-.24324E-05	0.95133E+02	0.51308E+04	0.12480E-01
0.93600E+03	-.30706E-05	0.42412E+02	0.53627E+04	0.16467E-01
0.97200E+03	-.24522E-05	0.10835E+02	0.55946E+04	0.13719E-01
0.10080E+04	-.15127E-05	-.30120E+01	0.58265E+04	0.88137E-02
0.10440E+04	-.66244E-06	-.54226E+01	0.60584E+04	0.40134E-02
0.10800E+04	0.27127E-07	-.26068E+01	0.62906E+04	-.17064E-03
0.11160E+04	0.63946E-06	-.42248E+02	0.65222E+04	-.41707E-02

1EX 1 COPANO BAY CAUSEWAY,ARANSAS COUNTY TEXAS,US HIGHWAY 35

PILE NUM	DISTA,IN	DISTB,IN	THETA, RAD
4	0.12600E+03	0.00000E+00	0.24400E+00

PX,LB	XT,IN	PT,LB	M	T,IN-LB	YT,IN
0.19360E+06	0.10906E+00	0.10624E+04	-.15520E+06		0.76322E-01

INPUT INFORMATION

TC	TOP DIA,IN	INC. LENGTH,IN	NO. OF INC	KS	KA
FIX	0.1800E+02	0.3600E+02	31	1	1

PILE LENGTH,IN DEPTH TO SOIL ITERATION TOL.BOUNDRY COND.2

(Continued)

(Sheet 8 of 9)

0.1116E+04	0.1200E+03	0.1000E-02	0.0000E+00

OUTPUT INFORMATION

-X,IN	Y,IN	MOMENT,IN-LB	ES,LB/ IN	P LB/IN
0.00000E+00	0.76289E-01	-.15521E+06	0.00000E+00	0.00000E+00
0.36000E+02	0.70917E-01	-.11591E+06	0.00000E+00	0.00000E+00
0.72000E+02	0.62110E-01	-.75965E+05	0.00000E+00	0.00000E+00
0.10800E+03	0.51053E-01	-.35575E+05	0.00000E+00	0.00000E+00
0.14400E+03	0.38941E-01	0.50177E+04	0.67393E+00	-.26243E-01
0.18000E+03	0.26978E-01	0.45548E+05	0.43481E+00	-.11730E-01
0.21600E+03	0.16365E-01	0.85802E+05	0.16523E+04	-.27039E+02
0.25200E+03	0.82936E-02	0.90521E+05	0.16523E+04	-.13704E+02
0.28800E+03	0.29046E-02	0.76960E+05	0.16523E+04	-.47993E+01
0.32400E+03	-.20405E-03	0.56739E+05	0.16523E+04	0.33716E+00
0.36000E+03	-.16316E-02	0.36629E+05	0.16523E+04	0.26959E+01
0.39600E+03	-.19739E-02	0.19802E+05	0.18842E+04	0.37191E+01
0.43200E+03	-.17294E-02	0.76821E+04	0.21161E+04	0.36596E+01
0.46800E+03	-.12573E-02	0.26080E+03	0.23480E+04	0.29522E+01
0.50400E+03	-.77750E-03	-.33360E+04	0.25799E+04	0.20059E+01
0.54000E+03	-.39653E-03	-.43140E+04	0.28116E+04	0.11149E+01
0.57600E+03	-.14338E-03	-.38224E+04	0.30437E+04	0.43641E+00
0.61200E+03	-.34921E-05	-.27432E+04	0.32756E+04	0.11438E-01
0.64800E+03	0.55119E-04	-.16334E+04	0.35075E+04	-.19333E+00
0.68400E+03	0.65332E-04	-.76487E+03	0.37394E+04	-.24430E+00
0.72000E+03	0.52882E-04	-.20853E+03	0.39713E+04	-.21001E+00
0.75600E+03	0.34253E-04	0.76829E+02	0.42032E+04	-.14397E+00
0.79200E+03	0.17901E-04	0.17516E+03	0.44351E+04	-.79392E-01
0.82800E+03	0.67385E-05	0.16960E+03	0.46670E+04	-.31449E-01
0.86400E+03	0.60140E-06	0.12230E+03	0.48989E+04	-.29462E-02
0.90000E+03	-.19118E-05	0.70493E+02	0.51308E+04	0.98092E-02
0.93600E+03	-.23364E-05	0.30988E+02	0.53627E+04	0.12529E-01
0.97200E+03	-.18427E-05	0.75431E+01	0.55946E+04	0.10309E-01
0.10080E+04	-.11256E-05	-.25841E+01	0.58265E+04	0.65584E-02
0.10440E+04	-.48505E-06	-.41969E+01	0.60584E+04	0.29386E-02
0.10800E+04	0.31152E-07	-.19772E+01	0.62906E+04	-.19596E-03
0.11160E+04	0.48877E-06	-.31941E+02	0.65222E+04	-.31879E-02

Houston Ship Channel

20. The second example problem considered will be one of the bents used in a bridge across the Houston Ship Channel. The bridge is located in Harris County on Interstate Highway 610. Details of the bent analyzed are shown in Figure E7. The bent is reinforced concrete and is supported by 142 eighteen-inch-square, precast-prestressed concrete piles. The piles in this example are battered parallel to the roadway to resist horizontal loads from the superstructure. It is assumed that the 7-ft-thick pile cap provides sufficient rigidity so that the assumption of plane movement is valid.

21. The geometry necessary for describing the foundation for the computer solution is shown in Figure E8 and the following tabulation:

Pile Location	a Coordinate in.	b Coordinate in.	No. Piles at Location	Batter radians
1	-150	0	24	-0.166
2	-90	0	23	-0.083
3	-30	0	24	-0.042
4	30	0	24	0.042
5	90	0	23	0.083
6	150	0	24	0.166

The coordinate system and the loads on the structure are also designated in the figure. The piles have an effective flexural rigidity of 4.374×10^{10} lb-in.^{2} (assuming a modulus of elasticity of concrete of 5×10^6 psi) and a length of 44 ft.

22. No axial load-deflection curves obtained from load tests are available for the piles used in the bent. As a result, it was necessary to estimate the axial behavior of the piles. The ultimate bearing capacity of the piles was estimated as 650 kips in compression and 600 kips in tension. The ultimate deflection is estimated as 0.5 in. The load-deflection relationship is assumed to be linear resulting in a curve as shown in Figure E9.

23. The properties of the soil used for predicting the lateral pile-soil interaction were obtained from the highway department borings.

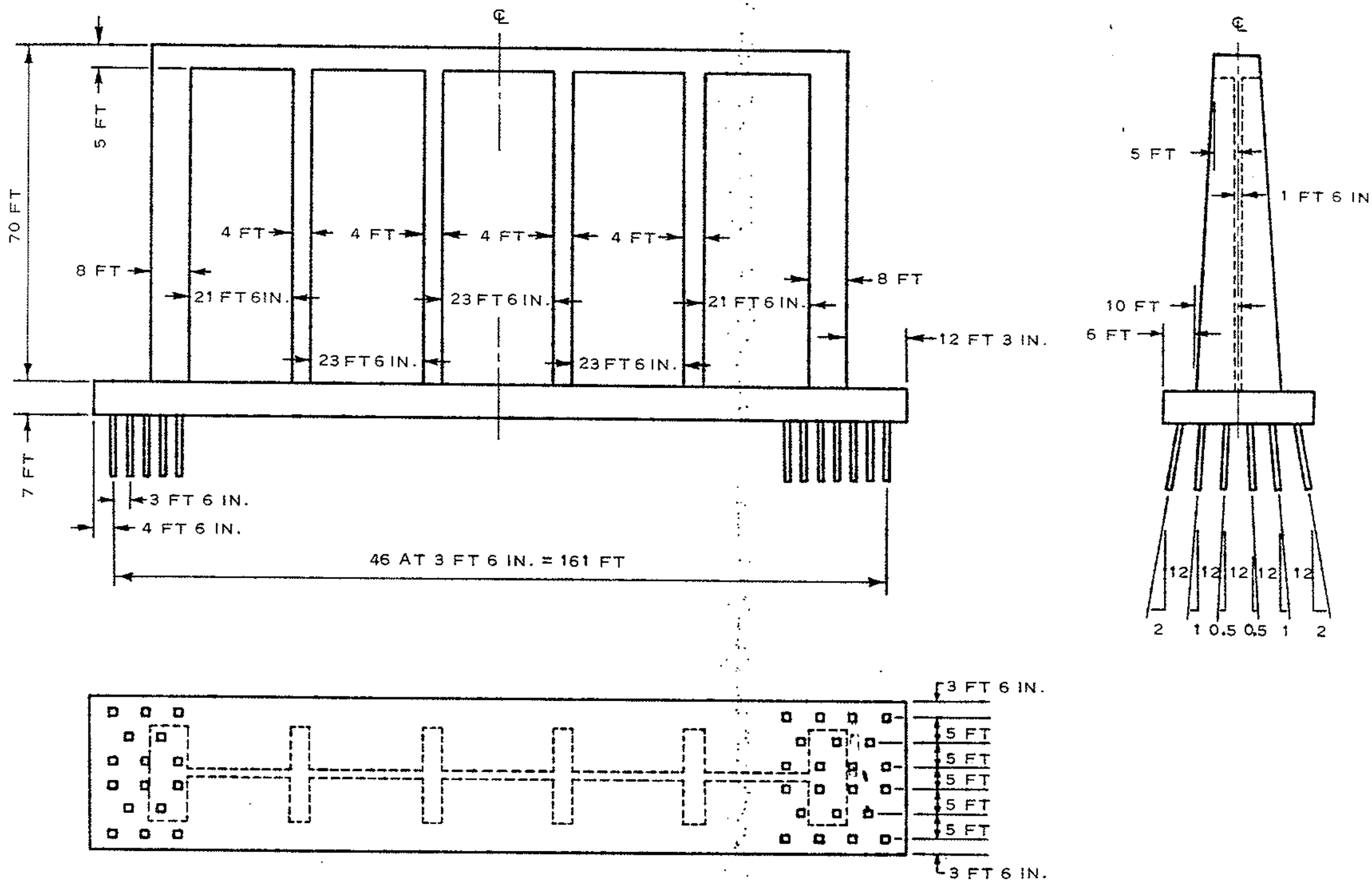

Figure E7. Houston Ship Channel bent - Example Problem 2

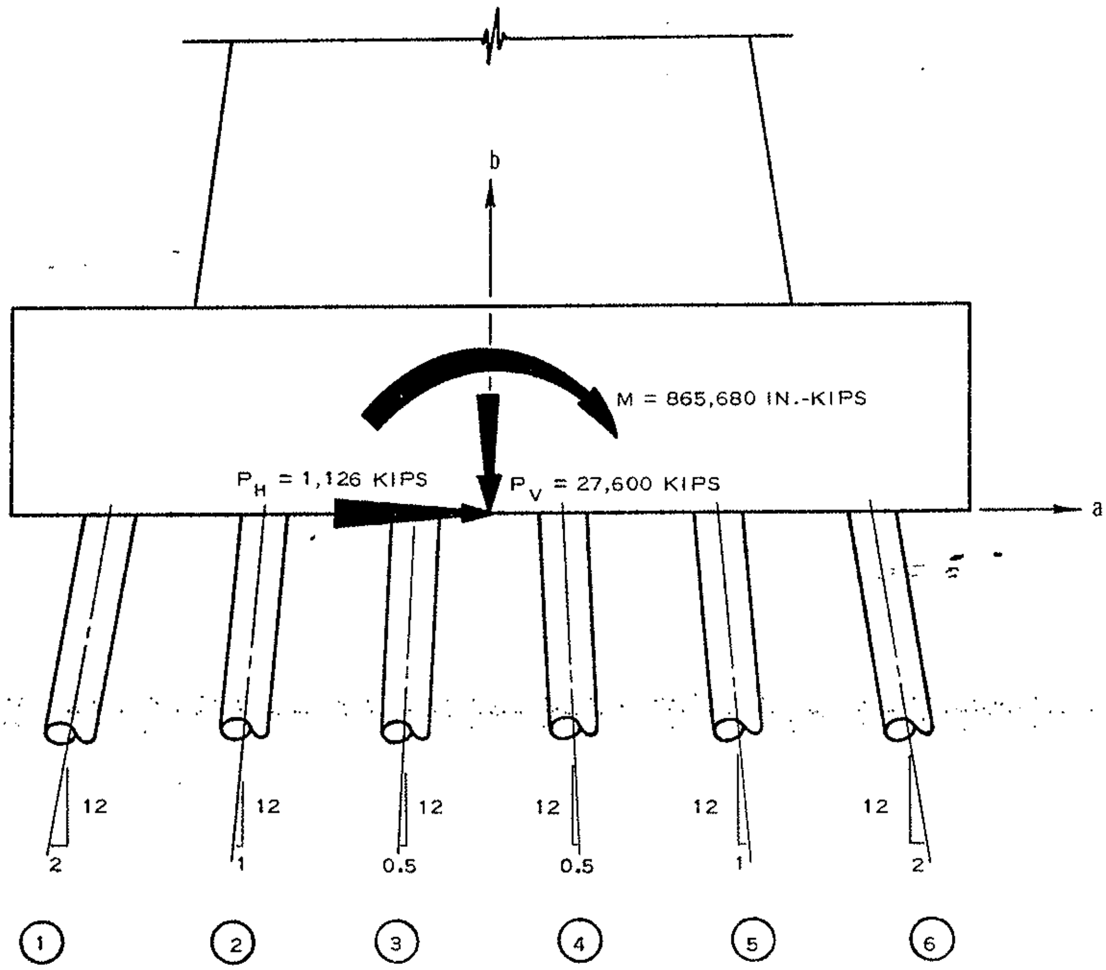

Figure E8. Foundation representation for Example Problem 2

The properties used for generation of p-y curves are illustrated in Figure E10. It should be pointed out that the profile shown is a simplification of the actual profile. The top 13 ft of soil, defined as very dense sandy silt, will be treated as a sand when p-y curves are generated. That is, it will be treated as a cohesionless material. The bottom 31 ft, defined as very stiff silty clay, will be treated as a clay. That is, it will be treated as a frictionless material. Depths given are measured from the top of the pile. From the given soil properties, p-y curves are generated.

24. A solution was obtained for the Ship Channel problem by using the program BENT1. The movement of the bent, when loaded, is described

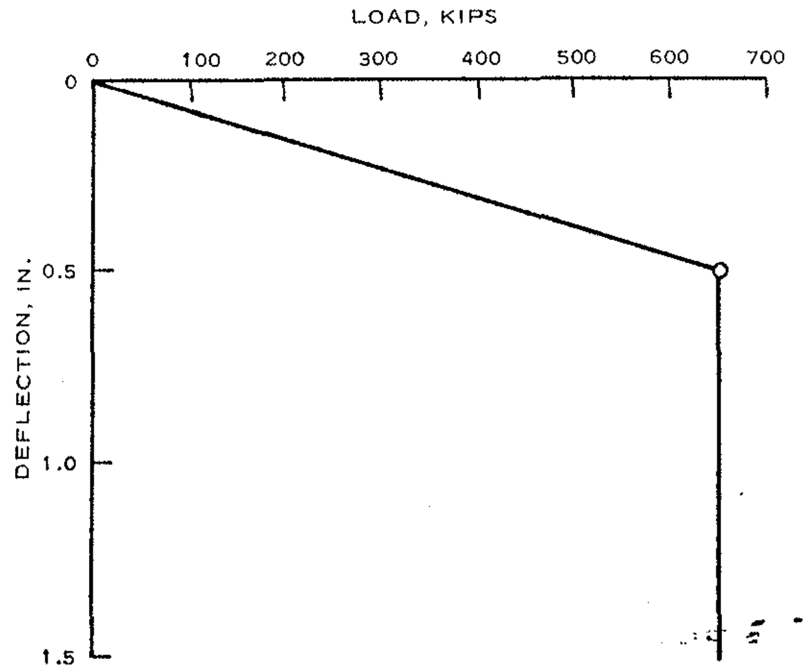

Figure E9. Estimated axial load deformation for Example Problem 2

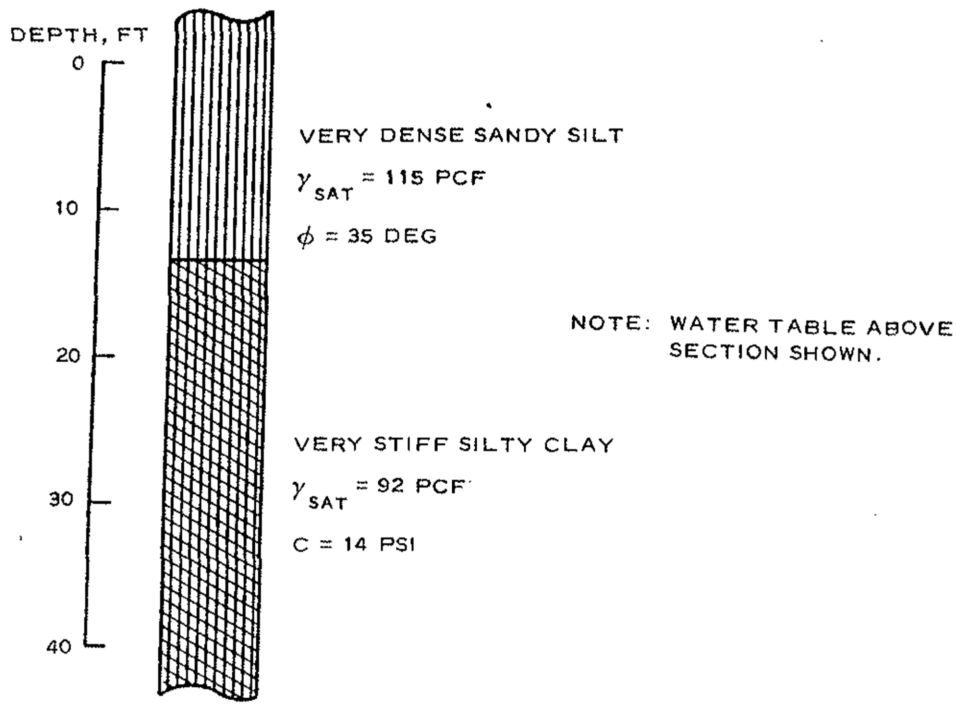

Figure E10. Soil properties for p-y curves for Example Problem 2 by the following movements of the origin of the a-b coordinate system:

$$\Delta_v = 1.512 \times 10^{-1} \text{ in.}$$

$$\Delta_u = 3.321 \times 10^{-2} \text{ in.}$$

$$\Gamma = 4.183 \times 10^{-4} \text{ radians}$$

The loads transferred to each pile and the movements of each pile top are given in this tabulation:

Pile Location	Axial Load per Pile kips	Lateral Load per Pile kips	Moment per Pile in.-kips	Axial Movement in.	Lateral Movement in.
1	106.3	3.3	-46.0	0.0818	0.0474
2	143.6	2.5	0.4	0.1104	0.0425
3	178.3	2.0	32.8	0.1372	0.0390
4	214.5	0.3	122.1	0.1650	0.0263
5	248.3	0.2	83.8	0.1910	0.0174
6	281.5	0.0	-15.2	0.2165	-0.0026

The forces and movements at the pile tops are related to the x-y coordinate systems set up for each pile. A complete listing of the coded input and output for the program is given in Tables E3 and E4 beginning on page E53.

25. The small deflections and loads obtained for the piles would tend to indicate that the design is conservative. This is probably true and is to be expected. However, it should be pointed out that a number of factors, such as consolidation and cyclic loading, have not been considered and that the load deflection curve used is only a rough approximation. The value used for ultimate load is probably fairly reliable, but the deflection at which the load stops increasing is only an educated guess. Because of this, a linear variation of load with movement was considered to provide sufficient refinement. The effect will be disclosed in the loads and deflections obtained for the piles. The loads obtained will probably be fairly accurate, but the accuracy of the movements obtained will depend on the accuracy of the value which was assumed for the deflection at which the load stops increasing.

Table E3

Input Data for Example Problem 2

```
100  EX 2 HOUQTON SHIP CHANNEL BRIDGE,HARRIS CO.,HIGHWAY I-610
110 2.7600E+07,1.1260E+06,8.6568E+08,0.001,6,0
120 -150.0,0.0,-0.166,24.0,1,1
130 -90.0,0.0,-0.083,23.0,1,1
140 -30.0,0.0,-0.042,24.0,1,1
150 30.0,0.0,0.042,24.0,1,1
160 90.0,0.0,0.083,23.0,1,1
170 150.0,0,0.166,24.0,1,1
180 33,16.0,0.0,1, FIX ,0,18.0
190 4.3740E+10,0.0,528.0
200 33,16.0,0.0,1, FIX ,0,18.0
210 4.3740E+10,0.0,528.0
220 33,16.0,0.0,1, FIX ,0,18.0
230 4.3740E+10,0.0,528.0
240 33,16.0,0.0,1, FIX ,0,18.0
250 4.3540E+10,0.0,528.0
260 33,16,0,1, FIX ,0,18
270 4.3740E+10,0,528.0
280 33,16.0,0,1, FIX ,0,18.0
290 4.3740E+10,0,5.2800E)02
300 1,1,1
310 1,5
320 -10.0,-6.000E+05
330 -0.5,-6.000E+05
340 0,0
350 0.5,6.5000E+05
360 10.0,6.5000E+05
370 1
380 2
390  SAND
400 0.03,0.6,0,156.0, DENSE
410  CLAY
420 0.017,14.0,156.0,528.0,0, STIF
430 1
440 1,10,1
450 18.0,0,528.0
460 0
470 12.0
480 24.0
490 48.0
500 96.0
510 144.0
520 228.0
530 229.0
540 240.0
550 528.0

READY
```

Table E4

Output Data for Example Problem 2

1EX 2 HOUSTON SHIP CHANNEL BRIDGE,HARRIS CO.,HIGHWAY I-610

LIST OF INPUT DATA ---

PV	PH	TM	TOL	KNPL	KOSC
0.2760E+08	0.1126E+07	0.8657E+09	0.1000E-02	6	0

CONTROL DATA FOR PILES AT EACH LOCATION

PILE NO	DISTA	DISTB	BATTER	POTT	KS	KA
1	-.1500E+03	0.0000E+00	-.1660E+00	0.2400E+02	1	1
2	-.9000E+02	0.0000E+00	-.8300E-01	0.2300E+02	1	1
3	-.3000E+02	0.0000E+00	-.4200E-01	0.2400E+02	1	1
4	0.3000E+02	0.0000E+00	0.4200E-01	0.2400E+02	1	1
5	0.9000E+02	0.0000E+00	0.8300E-01	0.2300E+02	1	1
6	0.1500E+03	0.0000E+00	0.1660E+00	0.2400E+02	1	1

PILE NO.	NN	HH	DPS	NDEI	CONNECTION	FDBET
1	33	0.16000E+02	0.00000E+00	1	FIX	0.0000E+00
2	33	0.16000E+02	0.00000E+00	1	FIX	0.0000E+00
3	33	0.16000E+02	0.00000E+00	1	FIX	0.0000E+00
4	33	0.16000E+02	0.00000E+00	1	FIX	0.0000E+00
5	33	0.16000E+02	0.00000E+00	1	FIX	0.0000E+00
6	33	0.16000E+02	0.00000E+00	1	FIX	0.0000E+00

	RRI	XX1	XX2
PILE NO 1	0.43740E+11	0.00000E+00	0.52800E+03
PILE NO 2	0.43740E+11	0.00000E+00	0.52800E+03
PILE NO 3	0.43740E+11	0.00000E+00	0.52800E+03
PILE NO 4	0.43740E+11	0.00000E+00	0.52800E+03
PILE NO 5	0.43740E+11	0.00000E+00	0.52800E+03
PILE NO 6	0.43740E+11	0.00000E+00	0.52800E+03

AXIAL LOAD SETTLEMENT DATA

	ZZZ	SSS
IDENTIFIER 1	-.10000E+02	-.60000E+06
	-.50000E+00	-.60000E+06
	0.00000E+00	0.00000E+00

(Continued)

(Sheet 1 of 10)

```
                         0.50000E+00    0.65000E+06
                         0.10000E+02    0.65000E+06

   INPUT OF SOIL PARAMETERS

   SOIL PROFILE NO.    1    STRATUM NO.    1      TYPE SOILSAND

     GAMMA     ANGLE OF FRIC.  TOP DEPTH    BOTTEM DEPTH    DENSITY
   0.3000E-01    0.6000E+00    0.0000E+00     0.1560E+03    DENSE

   SOIL PROFILE NO.    1    STRATUM NO.    2      TYPE SOILCLAY

     GAMMA        COHESION      TOP DEPTH    BOTTEM    DEPTH  CONSISTENCY
   0.1700E-01    0.1400E+02    0.1560E+03    0.5280E+03       STIF

                DIAMETER DISTRIBUTION FOR PILE

             DIAMETER       TOP DIS        BOT DIS
             0.1800E+02     0.0000E+00     0.5280E+03

1
                             P-Y CURVES
SET IDENTIFIER NO.     1     NUMBER OF CURVES IN SET     10

CURVE NO.     1 DEPTH TO CURVE 0.0000E+00

SOIL REACTION          DEFLECTION
    0.0000E+00           0.0000E+00
    0.0000E+00           0.1000E+01
    0.0000E+00           0.1800E+03

CURVE NO.     2 DEPTH TO CURVE 0.1200E+02

SOIL REACTION          DEFLECTION
    0.0000E+00           0.0000E+00
    0.3363E+02           0.8409E-01
    0.3363E+02           0.1800E+03

CURVE NO.     3 DEPTH TO CURVE 0.2400E+02

SOIL REACTION          DEFLECTION
```

(Continued)

(Sheet 2 of 10)

0.0000E+00	0.0000E+00
0.9157E+02	0.1144E+00
0.9157E+02	0.1800E+03

CURVE NO. 4 DEPTH TO CURVE 0.4800E+02

SOIL REACTION	DEFLECTION
0.0000E+00	0.0000E+00
0.2803E+03	0.1752E+00
0.2803E+03	0.1800E+03

CURVE NO. 5 DEPTH TO CURVE 0.9600E+02

SOIL REACTION	DEFLECTION
0.0000E+00	0.0000E+00
0.9494E+03	0.2967E+00
0.9494E+03	0.1800E+03

CURVE NO. 6 DEPTH TO CURVE 0.1440E+03

SOIL REACTION	DEFLECTION
0.0000E+00	0.0000E+00
0.2007E+04	0.4182E+00
0.2007E+04	0.1800E+03

CURVE NO. 7 DEPTH TO CURVE 0.2280E+03

SOIL REACTION	DEFLECTION
0.0000E+00	0.0000E+00
0.8766E+03	0.3600E-01
0.1239E+04	0.7200E-01
0.1518E+04	0.1080E+00
0.1753E+04	0.1440E+00
0.1960E+04	0.1800E+00
0.2147E+04	0.2160E+00
0.2319E+04	0.2520E+00
0.2479E+04	0.2880E+00
0.2630E+04	0.3240E+00
0.2772E+04	0.3600E+00
0.2772E+04	0.1800E+03

CURVE NO. 8 DEPTH TO CURVE 0.2290E+03

SOIL REACTION	DEFLECTION
0.0000E+00	0.0000E+00
0.8766E+03	0.3600E-01
0.1239E+04	0.7200E-01
0.1518E+04	0.1080E+00
0.1753E+04	0.1440E+00

(Continued)

0,1960E+04	0.1800E+00
0,2147E+04	0.2160E+00
0,2319E+04	0.2520E+00
0,2479E+04	0.2880E+00
0,2630E+04	0.3240E+00
0,2772E+04	0,3600E+00
0,2772E+04	0.1800E+03

CURVE NO. 9 DEPTH TO CURVE 0.2400E+03

SOIL REACTION	DEFLECTION
0,0000E+00	0.0000E+00
0,8766E+03	0.3600E-01
0,1239E+04	0.7200E-01
0,1518E+04	0.1080E+00
0,1753E+04	0.1440E+00
0,1960E+04	0.1800E+00
0,2147E+04	0.2160E+00
0,2319E+04	0.2520E+00
0,2479E+04	0.2880E+00
0,2630E+04	0.3240E+00
0,2772E+04	0.3600E+00
0,2772E+04	0.1800E+03

CURVE NO. 10 DEPTH TO CURVE 0,5280E+03

SOIL REACTION	DEFLECTION
0,0000E+00	0.0000E+00
0,8766E+03	0.3600E-01
0,1239E+04	0.7200E-01
0,1518E+04	0.1080E+00
0,1753E+04	0.1440E+00
0,1960E+04	0.1800E+00
0,2147E+04	0.2160E+00
0,2319E+04	0.2520E+00
0,2479E+04	0.2880E+00
0,2630E+04	0.3240E+00
0,2772E+04	0.3600E+00
0,2772E+04	0.1800E+03

1 ITERATION DATA

DV	DH	ALPHA
0.3026E+00	0.7467E-01	0,8315E-03
0.1513E+00	0.6979E-01	0,3648E-03
0.1512E+00	0.2654E-01	0,4280E-03
0.1512E+00	0.3435E-01	0,4165E-03
0,1512E+00	0.3261E-01	0,4191E-03

(Continued)

(Sheet 4 of 10)

0.1512E+00 0.3310E-01 0.4185E-03

1EX 2 HOUSTON SHIP CHANNEL BRIDGE,HARRIS CO.,HIGHWAY I-610

PILE NUM	DISTA,IN	DISTB,IN	THETA, RAD
1	-.15000E+03	0.00000E+00	-.16600E+00

PX,LB	XT,IN	PT,LB	M T,IN-LB	YT,IN
0.10631E+06	0.81780E-01	0.32327E+04	-.41201E+05	0.47258E-01

INPUT INFORMATION

TC	TOP DIA,IN	INC. LENGTH,IN	NO, OF INC	KS	KA
FIX	0.1800E+02	0.1600E+02	33	1	1

PILE LENGTH,IN	DEPTH TO SOIL	ITERATION TOL,BO	UNDRY COND,2
0.5280E+03	0.0000E+00	0.1000E-02	0.0000E+00

OUTPUT INFORMATION

X,IN	Y,IN	MOMENT,IN-LB	ES,LB/IN	P LB/IN
0.00000E+00	0.46555E-01	-.45409E+05	0.00000E+00	0.00000E+00
0.16000E+02	0.39727E-01	0.70409E+04	0.53333E+03	-.21188E+02
0.32000E+02	0.32940E-01	0.54062E+05	0.10666E+04	-.35136E+02
0.48000E+02	0.26470E-01	0.92054E+05	0.16000E+04	-.42352E+02
0.64000E+02	0.20538E-01	0.11914E+06	0.21333E+04	-.43815E+02
0.80000E+02	0.15304E-01	0.13495E+06	0.26667E+04	-.40810E+02
0.96000E+02	0.10859E-01	0.14022E+06	0.32000E+04	-.34750E+02
0.11200E+03	0.72354E-02	0.13651E+06	0.37333E+04	-.27012E+02
0.12800E+03	0.44104E-02	0.12580E+06	0.42667E+04	-.18818E+02
0.14400E+03	0.23218E-02	0.11018E+06	0.48000E+04	-.11144E+02
0.16000E+03	0.87804E-03	0.91660E+05	0.85237E+04	-.74842E+01
0.17600E+03	-.29238E-04	0.71158E+05	0.12247E+05	0.35809E+00
0.19200E+03	-.52005E-03	0.50703E+05	0.15971E+05	0.83058E+01
0.20800E+03	-.71411E-03	0.32343E+05	0.19695E+05	0.14064E+02
0.22400E+03	-.71887E-03	0.17563E+05	0.23419E+05	0.16835E+02
0.24000E+03	-.62084E-03	0.70818E+04	0.24350E+05	0.15117E+02
0.25600E+03	-.48137E-03	0.46640E+03	0.24350E+05	0.11721E+02
0.27200E+03	-.33916E-03	-.31487E+04	0.24350E+05	0.82584E+01
0.28800E+03	-.21538E-03	-.46477E+04	0.24350E+05	0.52445E+01
0.30400E+03	-.11881E-03	-.48012E+04	0.24350E+05	0.28930E+01
0.32000E+03	-.50336E-04	-.42112E+04	0.24350E+05	0.12256E+01
0.33600E+03	-.65086E-05	-.33047E+04	0.24350E+05	0.15848E+00
0.35200E+03	0.17977E-04	-.23556E+04	0.24350E+05	-.43773E+00
0.36800E+03	0.28676E-04	-.15171E+04	0.24350E+05	-.69824E+00
0.38400E+03	0.30495E-04	-.85643E+03	0.24350E+05	-.74254E+00
0.40000E+03	0.27302E-04	-.38530E+03	0.24350E+05	-.66479E+00
0.41600E+03	0.21854E-04	-.84113E+02	0.24350E+05	-.53212E+00
0.43200E+03	0.15913E-04	0.80902E+02	0.24350E+05	-.38747E+00
0.44800E+03	0.10446E-04	0.14667E+03	0.24350E+05	-.25436E+00
0.46400E+03	0.58374E-05	0.14724E+03	0.24350E+05	-.14214E+00

(Continued)

(Sheet 5 of 10)

0.48000E+03	0.20906E-05	0.11132E+03	0.24350E+05	-.50904E-01
0.49600E+03	-.10047E-05	0.62309E+02	0.24350E+05	0.24465E-01
0.51200E+03	-.37353E-05	0.19518E+02	0.24350E+05	0.90954E-01
0.52800E+03	-.63517E-05	0.15323E+04	0.24350E+05	0.15466E+00

1EX 2 HOUSTON SHIP CHANNEL BRIDGE,HARRIS CO.,HIGHWAY I-610

PILE NUM	DISTA,IN	DISTB,IN	THETA, RAD
2	-.90000E+02	0.00000E+00	-.83000E-01

PX,LB	XT,IN	PT,LB	M T,IN-LB	YT,IN
0.14357E+06	0.11043E+00	0.24860E+04	0.33676E+04	0.42397E-01

INPUT INFORMATION

TC	TOP DIA,IN	INC, LENGTH,IN	NO. OF INC	KS	KA
FIX	0.1800E+02	0.1600E+02	33	1	1

PILE LENGTH,IN	DEPTH TO SOIL	ITERATION TOL,BO	UNDRY COND,2
0.5280E+03	0.0000E+00	0.1000E-02	0.0000E+00

OUTPUT INFORMATION

X,IN	Y,IN	MOMENT,IN-LB	ES,LB/IN	P LB/IN
0.00000E+00	0.41881E-01	0.29538E+03	0.00000E+00	0.00000E+00
0.16000E+02	0.35187E-01	0.41033E+05	0.53333E+03	-.18766E+02
0.32000E+02	0.28733E-01	0.76932E+05	0.10666E+04	-.30648E+02
0.48000E+02	0.22729E-01	0.10492E+06	0.16000E+04	-.36366E+02
0.64000E+02	0.17339E-01	0.12351E+06	0.21333E+04	-.36989E+02
0.80000E+02	0.12672E-01	0.13253E+06	0.26667E+04	-.33791E+02
0.96000E+02	0.87804E-02	0.13278E+06	0.32000E+04	-.28097E+02
0.11200E+03	0.56663E-02	0.12574E+06	0.37333E+04	-.21154E+02
0.12800E+03	0.32880E-02	0.11316E+06	0.42667E+04	-.14029E+02
0.14400E+03	0.15721E-02	0.96910E+05	0.48000E+04	-.75460E+01
0.16000E+03	0.42335E-03	0.78640E+05	0.85237E+04	-.36085E+01
0.17600E+03	-.26512E-03	0.59381E+05	0.12247E+05	0.32471E+01
0.19200E+03	-.60605E-03	0.40903E+05	0.15971E+05	0.96793E+01
0.20800E+03	-.70758E-03	0.24869E+05	0.19695E+05	0.13936E+02
0.22400E+03	-.66356E-03	0.12380E+05	0.23419E+05	0.15540E+02
0.24000E+03	-.54708E-03	0.38604E+04	0.24350E+05	0.13321E+02
0.25600E+03	-.40800E-03	-.12528E+04	0.24350E+05	0.99346E+01
0.27200E+03	-.27626E-03	-.38217E+04	0.24350E+05	0.67268E+01
0.28800E+03	-.16688E-03	-.46653E+04	0.24350E+05	0.40635E+01
0.30400E+03	-.84813E-04	-.44648E+04	0.24350E+05	0.20651E+01
0.32000E+03	-.28874E-04	-.37319E+04	0.24350E+05	0.70307E+00
0.33600E+03	0.52227E-05	-.28158E+04	0.24350E+05	-.12717E+00
0.35200E+03	0.22839E-04	-.19299E+04	0.24350E+05	-.55613E+00
0.36800E+03	0.29161E-04	-.11847E+04	0.24350E+05	-.71005E+00
0.38400E+03	0.28548E-04	-.62040E+03	0.24350E+05	-.69513E+00
0.40000E+03	0.24304E-04	-.23347E+03	0.24350E+05	-.59179E+00
0.41600E+03	0.18694E-04	0.21639E+01	0.24350E+05	-.45519E+00
0.43200E+03	0.13096E-04	0.12126E+03	0.24350E+05	-.31889E+00
0.44800E+03	0.82083E-05	0.15863E+03	0.24350E+05	-.19987E+00

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(Sheet 6 of 10)

```
   0.46400E+03  0.42489E-05  0.14469E+03  0,24350E+05  -.10345E+00
   0.48000E+03  0.11362E-05  0.10414E+03  0,24350E+05  -.27668E-01
   0.49600E+03  -.13668E-05  0.56434E+02  0,24350E+05  0.33280E-01
   0,51200E+03  -.35395E-05  0.17193E+02  0,24350E+05  0.86186E-01
   0.52800E+03  -.56116E-05  0.13128E+04  0,24350E+05  0.13664E+00
1EX 2 HOUSTON SHIP CHANNEL BRIDGE,HARRIS CO.,HIGHWAY I-610

   PILE NUM    DISTA,IN       DISTB,IN      THE        TA, RAD
     3        -.30000E+02    0.00000E+00   -.42000E-01

     PX,LB       XT,IN          PT,LB         M        T,IN-LB       YT,IN
   0.17832E+06  0.13717E+00   0.19724E+04   0.32899E+05 0.38889E-01
   INPUT INFORMATION

      TC     TOP DIA,IN    INC, LENGTH,IN NO. OF       INC    KS KA

      FIX     0.1800E+02     0.1600E+02      33          1

   PILE LENGTH,IN DEPTH TO SOIL  ITERATION TOL,BO      UNDRY COND.2

      0.5280E+03      0.0000E+00      0.1000E-02      0.0000E+00

                                OUTPUT INFORMATION

       X,IN          Y,IN      MOMENT,IN-LB     ES,LB/       IN         P LB/IN

   0,00000E+00  0,38671E-01  0.31607E+05  0,18626E-08  -.72030E-10
   0,16000E+02  0.32068E-01  0.64343E+05  0,53333E+03  -.17103E+02
   0.32000E+02  0.25842E-01  0.92634E+05  0,10666E+04  -.27565E+02
   0,48000E+02  0,20158E-01  0.11377E+06  0,16000E+04  -.32253E+02
   0.64000E+02  0,15140E-01  0.12653E+06  0,21333E+04  -.32298E+02
   0,80000E+02  0,10862E-01  0.13089E+06  0,26667E+04  -.28966E+02
   0,96000E+02  0,73509E-02  0.12770E+06  0,32000E+04  -.23523E+02
   0,11200E+03  0,45868E-02  0.11835E+06  0,37333E+04  -.17124E+02
   0,12800E+03  0,25155E-02  0.10450E+06  0,42667E+04  -.10732E+02
   0,14400E+03  0.10558E-02  0.87796E+05  0,48000E+04  -.50682E+01
   0,16000E+03  0.11006E-03  0.69698E+05  0,85237E+04  -.93816E+00
   0,17600E+03  -.42781E-03  0,51286E+05  0,12247E+05  0.52396E+01
   0,19200E+03  -.66552E-03  0.34163E+05  0,15971E+05  0.10629E+02
   0,20800E+03  -.70328E-03  0.19724E+05  0.19695E+05  0.13851E+02
   0.22400E+03  -.62560E-03  0.88114E+04  0,23419E+05  0.14651E+02
   0,24000E+03  -.49635E-03  0.16399E+04  0,24350E+05  0.12086E+02
   0,25600E+03  -.35750E-03  -.24394E+04  0,24350E+05  0.87051E+01
   0,27200E+03  -.23293E-03  -.42877E+04  0,24350E+05  0.56718E+01
   0,28800E+03  -.13346E-03  -.46795E+04  0,24350E+05  0.32496E+01
   0,30400E+03  -.61368E-04  -.42345E+04  0,24350E+05  0.14943E+01
   0.32000E+03  -.14063E-04  -.34026E+04  0,24350E+05  0.34244E+00
   0.33600E+03  0,13327E-04  -.24794E+04  0,24350E+05  -.32451E+00
   0.35200E+03  0,26206E-04  -.16367E+04  0,24350E+05  -.63811E+00
   0.36800E+03  0,29506E-04  -.95569E+03  0,24350E+05  -.71846E+00
   0.38400E+03  0,27213E-04  -.45759E+03  0,24350E+05  -.66261E+00
   0,40000E+03  0,22241E-04  -.12864E+03  0,24350E+05  -.54155E+00
   0.41600E+03  0,16516E-04  0,61805E+02  0,24350E+05  -.40216E+00
   0.43200E+03  0.11153E-04  0.14923E+03  0,24350E+05  -.27158E+00
```

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(Sheet 7 of 10)

0.44800E+03	0.66639E-05	0.16698E+03	0.24350E+05	-.16226E+00
0.46400E+03	0.31518E-05	0.14301E+03	0.24350E+05	-.76744E-01
0.48000E+03	0.47659E-06	0.99245E+02	0.24350E+05	-.11604E-01
0.49600E+03	-.16177E-05	0.52407E+02	0.24350E+05	0.39391E-01
0.51200E+03	-.34053E-05	0.15598E+02	0.24350E+05	0.82918E-01
0.52800E+03	-.51016E-05	0.11614E+04	0.24350E+05	0.12422E+00

1EX 2 HOUSTON SHIP CHANNEL BRIDGE,HARRIS CO.,HIGHWAY I-610

PILE NUM	DISTA,IN	DISTB,IN	THETA, RAD
4	0.30000E+02	0.00000E+00	0.42000E-01

PX,LB	XT,IN	PT,LB	M	T,IN-LB	YT,IN
0.21454E+06	0.16503E+00	0.52109E+03	0.10217E+06	0.26189E-01	

INPUT INFORMATION

TC	TOP DIA,IN	INC. LENGTH,IN	NO. OF INC	KS	KA
FIX	0.1800E+02	0.1600E+02	33	1	1

PILE LENGTH,IN	DEPTH TO SOIL	ITERATION TOL,BO	UNDRY COND,2
0.5280E+03	0.0000E+00	0.1000E-02	0.0000E+00

OUTPUT INFORMATION

X,IN	Y,IN	MOMENT,IN-LB	ES,LB/IN	P LB/IN
0.00000E+00	0.29491E-01	0.12162E+06	0.00000E+00	0.00000E+00
0.16000E+02	0.23152E-01	0.13132E+06	0.53333E+03	-.12347E+02
0.32000E+02	0.17581E-01	0.13769E+06	0.10666E+04	-.18753E+02
0.48000E+02	0.12816E-01	0.13909E+06	0.16000E+04	-.20506E+02
0.64000E+02	0.88653E-02	0.13507E+06	0.21333E+04	-.18913E+02
0.80000E+02	0.57050E-02	0.12603E+06	0.26667E+04	-.15213E+02
0.96000E+02	0.32824E-02	0.11293E+06	0.32000E+04	-.10503E+02
0.11200E+03	0.15208E-02	0.97017E+05	0.37333E+04	-.56776E+01
0.12800E+03	0.32697E-03	0.79521E+05	0.42667E+04	-.13951E+01
0.14400E+03	-.40142E-03	0.61567E+05	0.48000E+04	0.19268E+01
0.16000E+03	-.76948E-03	0.44030E+05	0.85237E+04	0.65588E+01
0.17600E+03	-.87984E-03	0.28116E+05	0.12247E+05	0.10775E+02
0.19200E+03	-.82564E-03	0.14926E+05	0.15971E+05	0.13186E+02
0.20800E+03	-.68408E-03	0.50928E+04	0.19695E+05	0.13473E+02
0.22400E+03	-.51272E-03	-.12979E+04	0.23419E+05	0.12007E+02
0.24000E+03	-.34895E-03	-.46131E+04	0.24350E+05	0.84967E+01
0.25600E+03	-.21218E-03	-.57473E+04	0.24350E+05	0.51664E+01
0.27200E+03	-.10904E-03	-.55518E+04	0.24350E+05	0.26551E+01
0.28800E+03	-.38404E-04	-.46696E+04	0.24350E+05	0.93512E+00
0.30400E+03	0.49050E-05	-.35421E+04	0.24350E+05	-.11943E+00
0.32000E+03	0.27482E-04	-.24408E+04	0.24350E+05	-.66919E+00
0.33600E+03	0.35775E-04	-.15077E+04	0.24350E+05	-.87110E+00
0.35200E+03	0.35243E-04	-.79571E+03	0.24350E+05	-.85814E+00
0.36800E+03	0.30053E-04	-.30241E+03	0.24350E+05	-.73178E+00
0.38400E+03	0.23094E-04	0.39369E+01	0.24350E+05	-.56233E+00
0.40000E+03	0.16158E-04	0.16632E+03	0.24350E+05	-.39344E+00
0.41600E+03	0.10195E-04	0.22777E+03	0.24350E+05	-.24825E+00
0.43200E+03	0.55659E-05	0.22539E+03	0.24350E+05	-.13553E+00
0.44800E+03	0.22555E-05	0.18803E+03	0.24350E+05	-.54921E-01
0.46400E+03	0.45585E-07	0.13637E+03	0.24350E+05	-.11099E-02

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(Sheet 8 of 10)

0.48000E+03	-.13662E-05	0.84251E+02	0.24350E+05	0.33267E-01
0.49600E+03	-.22849E-05	0.40547E+02	0.24350E+05	0.55637E-01
0.51200E+03	-.29663E-05	0.11035E+02	0.24350E+05	0.72228E-01
0.52800E+03	-.35831E-05	0.71760E+03	0.24350E+05	0.87247E-01

1EX 2 HOUSTON SHIP CHANNEL BRIDGE,HARRIS CO.,HIGHWAY I-610

PILE NUM	DISTA,IN	DISTB,IN	THETA, RAD
5	0.90000E+02	0.00000E+00	0.83000E-01

PX,LB	XT,IN	PT,LB	M	T,IN-LB	YT,IN
0.24828E+06	0.19099E+00	0.80773E+02	0.93473E+05	0.17321E-01	

INPUT INFORMATION

TC	TOP DIA,IN	INC, LENGTH,IN	NO, OF INC	KS	KA
FIX	0.1800E+02	0.1600E+02	33	1	1

PILE LENGTH,IN	DEPTH TO SOIL	ITERATION TOL,BO	UNDRY COND.2
0.5280E+03	0.0000E+00	0.1000E-02	0.0000E+00

OUTPUT INFORMATION

X,IN	Y,IN	MOMENT,IN-LB	ES,LB/IN	P LB/IN
0.00000E+00	0.26722E-01	0.14859E+06	0.00000E+00	0.00000E+00
0.16000E+02	0.20462E-01	0.15143E+06	0.53333E+03	-.10912E+02
0.32000E+02	0.15087E-01	0.15127E+06	0.10666E+04	-.16093E+02
0.48000E+02	0.10598E-01	0.14676E+06	0.16000E+04	-.16958E+02
0.64000E+02	0.69687E-02	0.13770E+06	0.21333E+04	-.14867E+02
0.80000E+02	0.41448E-02	0.12463E+06	0.26667E+04	-.11052E+02
0.96000E+02	0.20503E-02	0.10855E+06	0.32000E+04	-.65609E+01
0.11200E+03	0.59111E-03	0.90638E+05	0.37333E+04	-.22068E+01
0.12800E+03	-.33759E-03	0.72026E+05	0.42667E+04	0.14404E+01
0.14400E+03	-.84474E-03	0.53678E+05	0.48000E+04	0.40547E+01
0.16000E+03	-.10377E-02	0.36291E+05	0.85237E+04	0.88452E+01
0.17600E+03	-.10183E-02	0.21115E+05	0.12247E+05	0.12471E+02
0.19200E+03	-.87530E-03	0.91007E+04	0.15971E+05	0.13980E+02
0.20800E+03	-.67904E-03	0.65234E+03	0.19695E+05	0.13374E+02
0.22400E+03	-.47895E-03	-.43734E+04	0.23419E+05	0.11216E+02
0.24000E+03	-.30447E-03	-.65214E+04	0.24350E+05	0.74137E+01
0.25600E+03	-.16815E-03	-.67620E+04	0.24350E+05	0.40944E+01
0.27200E+03	-.71410E-04	-.59446E+04	0.24350E+05	0.17388E+01
0.28800E+03	-.94612E-05	-.46734E+04	0.24350E+05	0.23038E+00
0.30400E+03	0.25135E-04	-.33365E+04	0.24350E+05	-.61203E+00
0.32000E+03	0.40204E-04	-.21514E+04	0.24350E+05	-.97895E+00
0.33600E+03	0.42682E-04	-.12137E+04	0.24350E+05	-.10392E+01
0.35200E+03	0.38055E-04	-.54043E+03	0.24350E+05	-.92663E+00
0.36800E+03	0.30266E-04	-.10353E+03	0.24350E+05	-.73696E+00
0.38400E+03	0.21870E-04	0.14485E+03	0.24350E+05	-.53253E+00
0.40000E+03	0.14323E-04	0.25669E+03	0.24350E+05	-.34875E+00
0.41600E+03	0.82775E-05	0.27888E+03	0.24350E+05	-.20155E+00
0.43200E+03	0.38645E-05	0.24907E+03	0.24350E+05	-.94098E-01
0.44800E+03	0.90918E-06	0.19481E+03	0.24350E+05	-.22138E-01
0.46400E+03	-.90596E-06	0.13459E+03	0.24350E+05	0.22060E-01
0.48000E+03	-.19334E-05	0.79830E+02	0.24350E+05	0.47076E-01
0.49600E+03	-.24935E-05	0.37003E+02	0.24350E+05	0.60716E-01

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(Sheet 9 of 10)

0.51200E+03	-.28371E-05	0.96659E+01	0.24350E+05	0.69083E-01
0.52800E+03	-.31242E-05	0.58283E+03	0.24350E+05	0.76072E-01

1EX 2 HOUSTON SHIP CHANNEL BRIDGE,HARRIS CO.,HIGHWAY I-610

PILE NUM	DISTA,IN	DISTB,IN	THETA, RAD
6	0.15000E+03	0.00000E+00	0.16600E+00

PX,LB	XT,IN	PT,LB	M T,IN-LB	YT,IN
0.28149E+06	0.21653E+00	0.11586E-01	-.15874E+05	-.27207E-02

INPUT INFORMATION

TC	TOP DIA,IN	INC, LENGTH,IN	NO, OF INC,	KS	KA
FIX	0.1800E+02	0.1600E+02	33	1	1

PILE LENGTH,IN	DEPTH TO SOIL	ITERATION TOL,BO	UNDRY COND,2
0.5280E+03	0.0000E+00	0.1000E-02	0.0000E+00

OUTPUT INFORMATION

X,IN	Y,IN	MOMENT,IN-LB	ES,LB/IN	P LB/IN
0.00000E+00	0.26238E-01	0.15307E+06	0.18626E-08	-.48872E-10
0.16000E+02	0.19991E-01	0.15483E+06	0.53333E+03	-.10661E+02
0.32000E+02	0.14649E-01	0.15361E+06	0.10666E+04	-.15626E+02
0.48000E+02	0.10207E-01	0.14813E+06	0.16000E+04	-.16332E+02
0.64000E+02	0.66322E-02	0.13822E+06	0.21333E+04	-.14149E+02
0.80000E+02	0.38661E-02	0.12447E+06	0.26667E+04	-.10309E+02
0.96000E+02	0.18285E-02	0.10787E+06	0.32000E+04	-.58512E+01
0.11200E+03	0.42222E-03	0.89598E+05	0.37333E+04	-.15763E+01
0.12800E+03	-.45967E-03	0.70773E+05	0.42667E+04	0.19613E+01
0.14400E+03	-.92734E-03	0.52333E+05	0.48000E+04	0.44512E+01
0.16000E+03	-.10887E-02	0.34947E+05	0.85237E+04	0.92799E+01
0.17600E+03	-.10455E-02	0.19879E+05	0.12247E+05	0.12805E+02
0.19200E+03	-.88605E-03	0.80564E+04	0.15971E+05	0.14151E+02
0.20800E+03	-.67939E-03	-.15682E+03	0.19695E+05	0.13380E+02
0.22400E+03	-.47365E-03	-.49444E+04	0.23419E+05	0.11092E+02
0.24000E+03	-.29684E-03	-.68842E+04	0.24350E+05	0.72279E+01
0.25600E+03	-.16033E-03	-.69623E+04	0.24350E+05	0.39039E+01
0.27200E+03	-.64564E-04	-.60296E+04	0.24350E+05	0.15721E+01
0.28800E+03	-.40895E-05	-.46844E+04	0.24350E+05	0.99576E-01
0.30400E+03	0.28968E-04	-.33061E+04	0.24350E+05	-.70536E+00
0.32000E+03	0.42676E-04	-.21028E+04	0.24350E+05	-.10391E+01
0.33600E+03	0.44077E-04	-.11621E+04	0.24350E+05	-.10732E+01
0.35200E+03	0.38676E-04	-.49435E+03	0.24350E+05	-.94174E+00
0.36800E+03	0.30381E-04	-.66799E+02	0.24350E+05	-.73977E+00
0.38400E+03	0.21696E-04	0.17148E+03	0.24350E+05	-.52828E+00
0.40000E+03	0.14014E-04	0.27424E+03	0.24350E+05	-.34123E+00
0.41600E+03	0.79372E-05	0.28919E+03	0.24350E+05	-.19327E+00
0.43200E+03	0.35529E-05	0.25419E+03	0.24350E+05	-.86512E-01
0.44800E+03	0.65643E-06	0.19662E+03	0.24350E+05	-.15984E-01
0.46400E+03	-.10892E-05	0.13464E+03	0.24350E+05	0.26523E-01
0.48000E+03	-.20470E-05	0.79226E+02	0.24350E+05	0.49842E-01
0.49600E+03	-.25409E-05	0.36441E+02	0.24350E+05	0.61871E-01
0.51200E+03	-.28217E-05	0.94337E+01	0.24350E+05	0.68706E-01
0.52800E+03	-.30471E-05	0.55916E+03	0.24350E+05	0.74197E-01

(Sheet 10 of 10)

APPENDIX F: USER'S GUIDE FOR PROGRAM BMCOL51

General Introduction

1. Documentation for computer program BMCOL51 - to solve a wide variety of beam-column structural problems for moving loads - is presented in this appendix and includes a general introduction, listing of the program, summary flow chart, guide for data input, and input-output for two example problems.

2. BMCOL51 is a finite difference program (developed by Prof. H. Matlock and Dr. T. P. Taylor,* UT at Austin) that can solve a variety of simple and complex beam-column structural problems accounting for movable loads. It is one of the earlier BMCOL programs written under the guidance of Prof. Matlock. Later versions of the BMCOL programs are available and are much more efficient and versatile than BMCOL51. However, BMCOL51 is documented herein principally to show the power and versatility of this family of programs. The documentation is extracted from the report written by Matlock and Taylor.

3. Beam-column equations developed in Part VI are programmed in BMCOL51. Changes in load (including moving loads), flexural stiffness, support conditions, and axial loads can vary in a freely discontinuous manner from joint to joint in the model. The finite difference representation of the fourth order differential equations are consecutively solved starting at one end of the beam in terms of known boundary conditions and adjacent joints. At the other end, the process reverses and the deflections are computed in a back substitution from joint to joint. By numerical differentiation of the deflections, the slope, bending moment, shear, and reaction are determined at each point. Plot routines in the program can be activated to produce plots of deflections, moments, shear, or reactions along the length of the beam-column.

4. BMCOL51 can consider only linear soil supports; however, other

---

* H. Matlock and T. P. Taylor, "A Computer Program to Analyze Beam-Columns Under Movable Loads," Research Report 56-4, 1968, Center for Highway Research, University of Texas, Austin, Tex.

BMCOL programs are currently available that can account for nonlinear behavior. Programs to perform dynamic analyses are also available. A list of available reports describig some of the more versatile programs is included in paragraph 7.

5. Some of the uses of BMCOL51 can be in obtaining general solutions for linear beam-columns, moving load problems, beam on elastic foundation problems, variable beam-size problems, and buckling problems.

6. BMCOL51 runs in the WES G-635 computer in batch/remote batch/ Card-In mode. The program is saved in the system (under a user number) as BMCOL51. To run a batch/remote batch job the user can access the program through proper control cards and read in his data in the form of cards. To run a Card-In job, the user first reads in his data in a file and then runs the program (with the data file created from a terminal or from cards). He can direct his output to any printer at the end of the run by giving proper commands.

7. The beam-column related reports (as of April 1974) of the Center for Highway Research, University of Texas at Austin, are as follows:

Report No. 56-1, "A Finite-Element Method of Solution for Linearly Elastic Beam-Columns" by H. Matlock and T. A. Haliburton, presents a solution for beam-columns that is a basic tool in subsequent reports. September 1966.

Report No. 56-2, "A Computer Program to Analyze Bending of Bent Caps" by H. Matlock and W. B. Ingram, describes the application of the beam-column solution to the particular problem of bridge bent caps. October 1966.

Report No. 56-3, "A Finite-Element Method of Solution for Structural Frames" by H. Matlock and B. R. Grubbs, describes a solution for frames with no sway. May 1967.

Report No. 56-4, "A Computer Program to Analyze Beam-Columns Under Movable Loads" by H. Matlock and T. P. Taylor, describes the application of the beam-column solution to problems with any configuration of movable nondynamic loads. June 1968.

Report No. 56-5, "A Finite-Element Method for Bending Analysis of Layered Structural Systems" by W. B. Ingram and H. Matlock, describes an alternating-direction iteration method for solving two-dimensional systems of layered grids-over-beams and plates-over-beams. June 1967.

Report No. 56-6, "Discontinuous Orthotrophic Plates and Pavement Slabs" by W. R. Hudson and Hudson Matlock, describes an alternating-direction iteration method for solving complex two-dimensional plate and slab problems with emphasis on pavement slabs. May 1966.

Report No. 56-7, "A Finite-Element Analysis of Structural Frames" by T. A. Haliburton and H. Matlock, describes a method of analysis for rectangular plane frames with three degrees of freedom at each joint. July 1967.

Report No. 56-8, "A Finite-Element Method for Transverse Vibrations of Beams and Plates" by H. Salani and H. Matlock, describes an implicit procedure for determining the transient and steady-state vibrations of beams and plates, including pavement slabs. June 1968.

Report No. 56-9, "A Direct Computer Solution for Plates and Pavement Slabs: by C. F. Stelzer, Jr., and W. R. Hudson, describes a direct method for solving complex two-dimensional plate and slab problems. October 1967.

Report No. 56-10, "A Finite-Element Method of Analysis for Composite Beams" by T. P. Taylor and H. Matlock, describes a method of analysis for composite beams with any degree of horizontal shear interaction. January 1968.

Report No. 56-11, "A Discrete-Element Solution of Plates and Pavement Slabs Using a Variable-Increment-Length Model" by C. M. Pearre III and W. R. Hudson, presents a method for solving freely discontinuous plates and pavement slabs subjected to a variety of loads. April 1969.

Report No. 56-12, "A Discrete-Element Method of Analysis for Combined Bending and Shear Deformations of a Beam" by D. F. Tankersley and W. P. Dawkins, presents a method of analysis for the combined effects of bending and shear deformations. December 1969.

Report No. 56-13, "A Discrete-Element Method of Multiple-Loading Analysis for Two-Way Bridge Floor Slabs" by J. J. Panak and H. Matlock, includes a procedure for analysis of two-way bridge floor slabs continuous over many supports. January 1970.

Report No. 56-14, "A Direct Computer Solution for Plane Frames" by W. P. Dawkins and J. R. Ruser, Jr., presents a direct method of solution for the computer analysis of plane frame structures. May 1969.

Report No. 56-15, "Experimental Verification of Discrete-Element Solutions for Plates and Slabs" by S. L. Agarwal and W. R. Hudson, presents a comparison of discrete-element solutions with small-dimension test results for plates and slabs, including some cyclic data. April 1970.

Report No. 56-16, "Experimental Evaluation of Subgrade Modulus and Its Application in Model Slab Studies" by Q. S. Siddiqi and W. R. Hudson, describes a series of experiments to evaluate layered foundation coefficients of subgrade reaction for use in the discrete-element method. January 1970.

Report No. 56-17, "Dynamic Analysis of Discrete-Element Plates on Nonlinear Foundations" by A. E. Kelly and H. Matlock, presents a numerical method for the dynamic analysis of plates on nonlinear foundations. July 1970.

Report No. 56-18, "A Discrete-Element Analysis for Anisotropic Skew Plates and Grids" by M. R. Vora and H. Matlock, describes a tridirectional model and a computer program for the analysis of anisotropic skew plates or slabs with grid-beams. August 1970.

Report No. 56-19, "An Algebraic Equation Solution Process Formulated in Anticipation of Banded Linear Equations" by F. L. Endres and H. Matlock, describes a system of equation-solving routines that may be applied to a wide variety of problems by using them within appropriate programs. January 1971.

Report No. 56-20, "Finite-Element Method of Analysis for Plane Curved Girders" by W. P. Dawkins, presents a method of analysis that may be applied to plane curved highway bridge girders and other structural members composed of straight and curved sections. June 1971.

Report No. 56-21, "Linearly Elastic Analysis of Plane Frames Subjected to Complex Loading Conditions" by C. O. Hays and H. Matlock, presents a design-oriented computer solution for plane frames structures and trusses that can analyze with a large number of loading conditions. June 1971.

Report No. 56-22, "Analysis of Bending Stiffness Variation at Cracks in Continuous Pavements" by A. Abou-Ayyash and W. R. Hudson, describes an evaluation of the effect of transverse cracks on the longitudinal bending rigidity of continuously reinforced concrete pavements. April 1972.

Report No. 56-23, "A Nonlinear Analysis of Statically Loaded Plane Frames Using a Discrete Element Model" by C. O. Hays and H. Matlock, describes a method of analysis which considers support, material, and geometric nonlinearities for plane frames subjected to complex loads and restraints. May 1972.

Report No. 56-24, "A Discrete-Element Method for Transverse Vibrations of Beam-Columns Resting on Linearly Elastic or Inelastic Supports" by J. Hsiao-Chieh Chan and H. Matlock, presents a new approach to predict the hysteretic behavior of inelastic supports in dynamic problems. June 1972.

Report No. 56-25, "A Discrete-Element Method of Analysis for Orthogonal Slab and Grid Bridge Floor Systems" by J. J. Panak and H. Matlock, presents a computer program particularly suited to highway bridge structures composed of slabs with supporting beam-diaphragm systems. May 1972.

Report No. 56-26, "Application of Slab Analysis Methods to Rigid Pavement Problems" by H. J. Treybig, W. R. Hudson, and A. Abou-Ayyash, illustrates how the program of Report No. 56-25 can be specifically applied to a typical continuously reinforced pavement with shoulders. May 1972.

Report No. 56-27, "Final Summary of Discrete-Element Methods of Analysis for Pavement Slabs" by W. Ronald Hudson, H. J. Treybig, and A. Abou-Ayyash, presents a summary of the project developments which can be used for pavement slabs. August 1972.

Report No. 56-28, "Finite-Element Analysis of Bridge Decks" by M. R. Abdelraouf and H. Matlock, presents a finite-element analysis which is compared with a discrete-element analysis of a typical bridge superstructure. August 1972.

Report No. 56-29F, "Final Project Report" by J. J. Panak, summarizes the project history and describes the major developments and findings in concise form. August 1972.

Flow Chart

8. A summary flow chart for this program is shown in Figure F1.

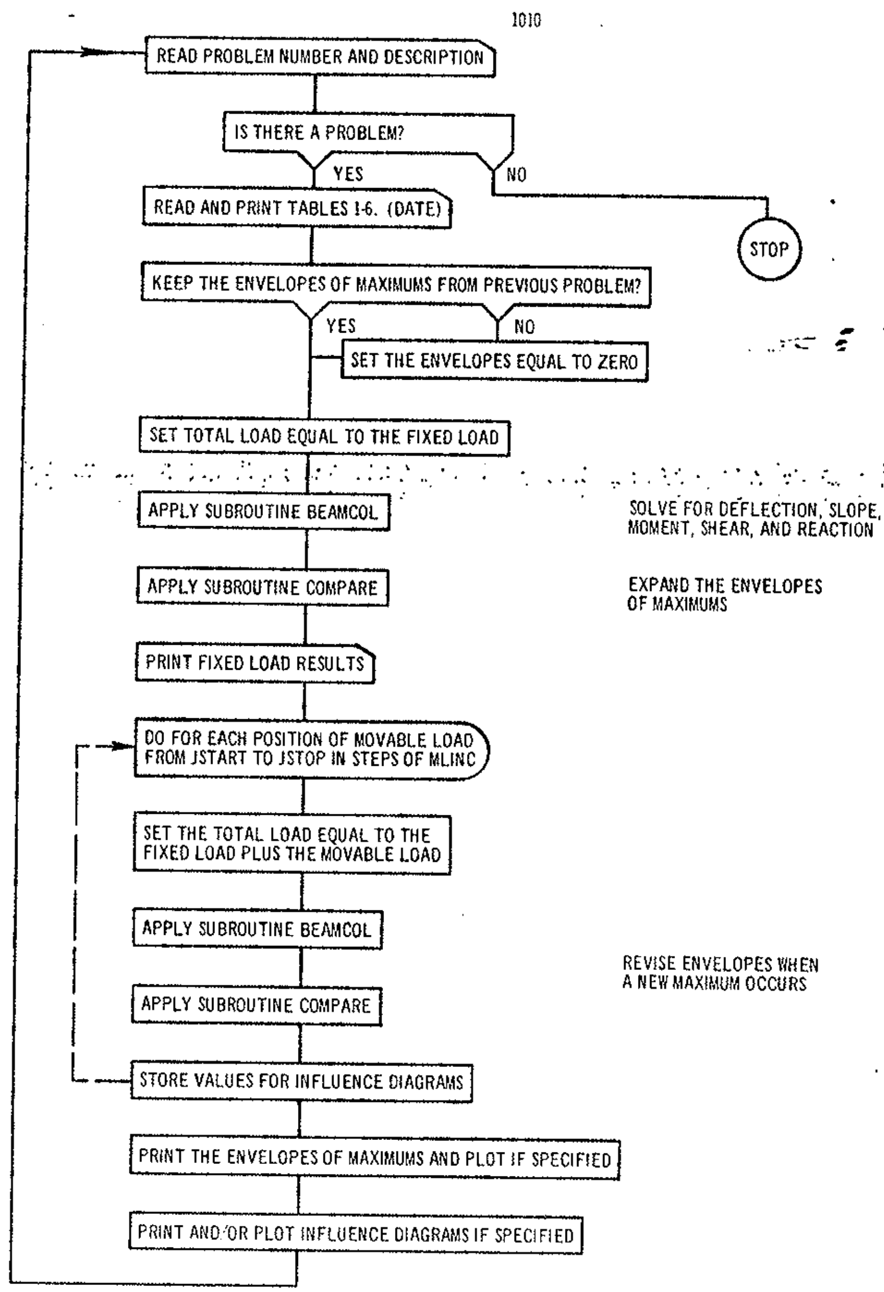


Figure F1. Summary flow diagram for BMCOL51

Guide for Data Input

10. The data is input to the code in the card forms presented in this guide (extracted from a report by H. Matlock and T. P. Taylor*).

IDENTIFICATION OF PROGRAM AND RUN (2 alphanumeric cards per run)

1 80

1 80

IDENTIFICATION OF PROBLEM (one card each problem; program stops if PROB NUM = 0)

PROB NUM

Description of problem (alphanumeric)

1 5 11 80

TABLE 1. PROGRAM CONTROL DATA (one card each problem)

ENTER "1" TO HOLD PRIOR | NUM CARDS ADDED FOR TABLE | ENTER "1" TO PLOT ENVELOPES FOR

ENVELOPFS | TABLE 2 3 4 5 6 | 2 3 4 5 6 | DEFL MOM SHEAR REACT

10 15 20 25 30 35 40 45 50 55 60 65 70 75 80

TABLE 2. CONSTANTS AND MOVABLE-LOAD DATA (one card, or none if Table 2 of preceding problem is held)

MOVABLE - LOAD - DATA

NUM INCRS | INCR LENGTH | NUM INCRS IN PATTERN | START STA | STOP STA | STEP SIZE

10 21 30 45 50 55 60

TABLE 3. SPECIFIED DEFLECTIONS AND SLOPES (number of cards according to Table 1; none if preceding Table 3 is held)

STATION | CASE | DEFLECTION | SLOPE

10 20 30 40

CASE = 1 for deflection only, 2 for slope only, 3 for both

* Ibid p. F1.

(Continued)

Data Input Guide (Continued)

TABLE 4. FIXED LOADS AND RESTRAINTS, (number of cards according to Table 1). Data added to storage as lumped quantities per increment length, linearly interpolated between values input at indicated end stations, with 1/2-values at each end station. Concentrated effects are established as full values at single stations by setting final station = initial station.

STA	TO STA	ENTER 1 IF CONT'D ON NEXT CARD	F BENDING STIFFNESS	Q_f TRANSVERSE FORCE	S SPRING SUPPORT	T TRANSVERSE COUPLE	R ROTATIONAL RESTRAINT	P AXIAL TENSION OR COMPRESSION
6–10	10–15	15–20	20–30	30–40	40–50	50–60	60–70	70–80

TABLE 5. MOVABLE LOADS (Number of cards according to Table 1). Data added to storage just as in Table 4.

STA	TO STA	ENTER 1 IF CONT'D ON NEXT CARD	Q_m TRANSVERSE FORCE
6–10	10–15	15–20	31–40

TABLE 6. SPECIFIED STATIONS FOR INFLUENCE DIAGRAMS (4 cards or none).

*1 = Plotted Output
2 = Tabulated Output
3 = Plotted and Tabulated Output

NUM OF DIAGRAMS	TYPE OF OUTPUT*	SPECIFIED STATIONS (max = 5 per variable)					FOR:
6–10	10–15	20–25	25–30	30–35	35–40	40–45	DEFLECTION
6–10	10–15	20–25	25–30	30–35	35–40	40–45	MOMENT
6–10	10–15	20–25	25–30	30–35	35–40	40–45	SHEAR
6–10	10–15	20–25	25–30	30–35	35–40	40–45	REACTION

STOP CARD (one blank card at end of run)
1–80

(Continued)

GENERAL PROGRAM NOTES

The data cards must be stacked in proper order for the program to run.

A consistent system of units must be used for all input data, for example: kips and inches.

All 5-space or less words are understood to be right justified integers or whole decimal numbers | -43 |

All 10-space words are right justified floating-point decimal numbers . . . . | -4.321E+03 |

TABLE 1. PROGRAM-CONTROL DATA

For each of Tables 2, 3, and 6, a choice must be made between holding all of the data from the preceding problem or entering entirely new data. If the hold option for any of these tables is set equal to 1, the number of cards input for that table must be zero.

For Tables 4 and 5, the data is accumulated in storage by adding to previously stored data. The number of cards input is independent of the hold option, except that the cumulative total of cards can not exceed 100.

Card counts in Table 1 should be rechecked carefully after the coding of each problem is completed.

The plot option for each of the envelopes of maximums is independent of the others. No plots are drawn for those options that are blank or zero. For each plot option that is set equal to 1, a plot is drawn on 4 × 10 in. axes.

TABLE 2. CONSTANTS AND MOVABLE-LOAD DATA

The maximum number of increments into which the beam-column may be divided is 200. Typical units for the value of increment length are inches.

The number of increments in the movable-load pattern may not exceed the number of increments in the beam-column. The start station is the first position at which the zero station of the movable-load pattern is placed.

(Continued)

Data Input Guide (Continued)

Any positive start station for the movable load is permissible. A negative start station is permissible if one step of the load pattern will place some portion of the load pattern on the beam-column.

Any stop station on the beam is allowed. A stop station of no more than one step of movement past the right end of the beam-column is permissible.

The movable-load pattern may be moved across the beam in steps of as many increments as desired.

To plot envelopes of maximum for fixed-load solutions, enter zero increment of pattern length, zero start and stop stations, and 1 for step size. No Table 5 necessary.

TABLE 3. SPECIFIED DEFLECTIONS AND SLOPES

The maximum number of stations at which deflections and slopes may be specified is 20.

A slope may not be specified closer than 3 increments from another specified slope.

A deflection may not be specified closer than 2 increments from a specified slope, except that both a deflection and a slope may be specified at the same station.

TABLE 4. STIFFNESS AND FIXED-LOAD DATA

Typical units,						
variables:	F	Q	S	T	R	P
values per station:	$lb \times in^2$	lb	lb/in	$in \times lb$	$\frac{in \times lb}{rad}$	lb

Axial tension or compression values P must be stated at each station in the same manner as any other distributed data; there is no mechanism in the program to automatically distribute the internal effects of an externally applied axial force.

Data should not be entered in this table (nor held from the preceding problem) which would express effects at fictitious stations beyond the ends of the real beam-column.

The left end of the beam-column must be located at station 0.

(Continued)

For the interpolation and distribution process, there are four variations in the station numbering and the referencing for continuation to succeeding cards. These variations are explained and illustrated on page 6.

There are no restrictions on the order of cards in Table 4, except that within a distribution sequence the stations must be in regular order.

TABLE 5. MOVABLE-LOAD DATA

The data in Table 5 is governed by the same rules as Table 4.

TABLE 6. SPECIFIED STATIONS FOR INFLUENCE DIAGRAMS

The number of cards in Table 6 is either 4 or 0.

A maximum of 5 stations may be specified for each of the four variables, deflection, bending moment, shear, and support reaction.

The data cards must be stacked in the same order as the above list.

If no influence lines are desired for one variable, a blank card must be inserted for that variable.

Shear is computed one-half increment to the left of the designated station.

If 1 is specified for the type of output, the influence lines are plotted on 4 X 10 in. axes.

If 2 is specified for the type of output, the influence lines are tabulated in numerical form in Table 10.

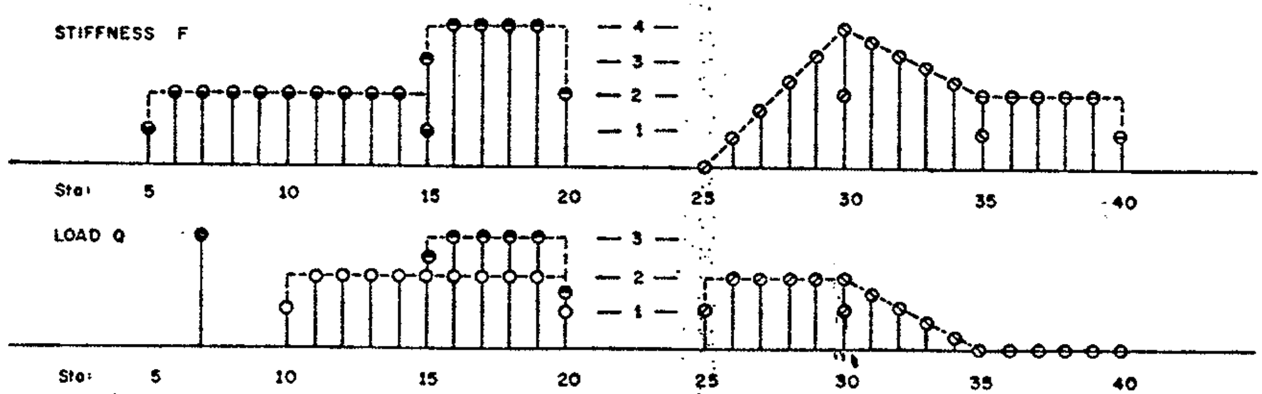
If 3 is specified for the type of output, the influence lines are tabulated and plotted.

(Continued)

Data Input Guide (Concluded)

	FROM STA	TO STA	CONT'D TO NEXT CARD P	F	Q	etc.
Individual — card Input						
Case a.1 Data concentrated at one sta	7	7	0 = NO		3.0	
Case a.2. Data uniformly distributed	5	15	0 = NO	2.0		
	15	20	0 = NO	4.0	1.0	
	10	20	0 = NO		2.0	
Multiple — card Sequence						
Case b. First — of — sequence	25		1 = YES	0.0	2.0	
Case c. Interior — of — sequence		30	1 = YES	4.0	2.0	
		35	1 = YES	2.0	0.0	
Case d. End — of — sequence		40	0 = NO	2.0		

Resulting Distribution of Data



Example Problems

Example Problem 1

9. The first example problem demonstrating the use of program BMCOL51 shows a simply supported beam with the variable cross section loaded (Figure F2). The input and output data for this example are presented in Tables F1 and F2. The results of the variation of deflection and moment along the beam are plotted in Figures F3 and F4.

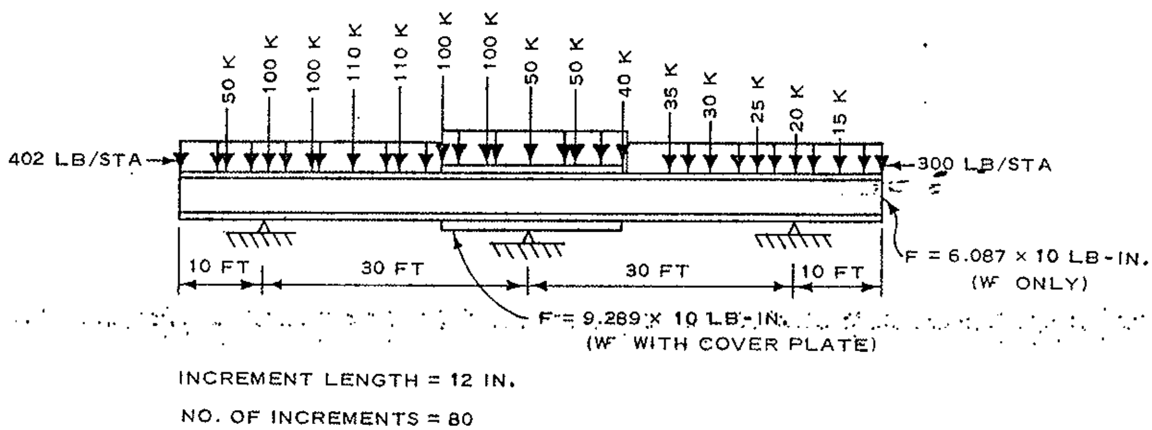


Figure F2. Physical problem for Example Problem 1 (steel bent cap, simply supported, and fixed loads)

Table F1

<u>Input Data for Example Problem 1</u>

PROGRAM						DATE	
REQUESTED BY		PREPARED BY		CHECKED BY		PAGE OF	
1–10	11–20	21–30	31–40	41–50	51–60	61–70	71–80
		BMCOL51 UNIVERSITY OF TEXAS AT AUSTIN					
		EXAMPLE PROBLEMS FOR REPORT					
001	STEEL BENT	CAP, SIMPLY SUPPORTED, FIXED LOADS, WITH PLOTS					
			1	3 17		1 1	
80		1.200E+01			1		
10	1	0.000E+00					
40	1	0.000E+00					
70	1	0.000E+00					
0	80	6.087E+11	-3.000E+02				
30	50	3.202E+11	-1.020E+02				
5	5		-5.000E+04				
10	10		-1.000E+05				
15	15		-1.000E+05				
20	20		-1.100E+05				
25	25		-1.100E+05				
30	30		-1.000E+05				
35	35		-1.000E+05				
40	40		-5.000E+04				
45	45		-5.000E+04				
50	50		-4.000E+04				
55	55		-3.500E+04				
60	60		-3.000E+04				
65	65		-2.500E+04				
70	70		-2.000E+04				
75	75		-1.500E+04				

BES FORM NO. 1233
SEPTEMBER 1962

Table F2

Output Data for Example Problem 1

```
PROGRAM BMCOL 51 - MASTER -  MATLOCK-TAYLOR  -   REVISION DATE = 08 MAR 68
CE394.2  HOMEWORK  PROBLEM 001, DATA CODED FOR EXAMPLE PROBLEM GIVEN IN
REPORT 56-1 (S)  FOR CENTER FOR HIGHWAY RESEARCH. CODED BY F.PARKER


PROB
  001     STEEL BENT CAP, SIMPLY SUPPORTED,FIXED LOADS, NO ENVELOPES OR


TABLE 1 - PROGRAM-CONTROL DATA
                                            ENVELOPES        TABLE NUMBER
                                            OF MAXIMUMS   2    3    4    5    6

     HOLD FROM PRECEDING PROBLEM (1=HOLD)        0        0    0   -0    0    0
     NUM CARDS INPUT THIS PROBLEM                         1    3   17    0    0

                                                       DEFL  MOM  SHR  RCT
     OPTION (IF=1) TO PLOT ENVELOPES OF MAXIMUMS          1    1    0    0


TABLE 2 - CONSTANTS
     NUM INCREMENTS                                                      80
     INCREMENT LENGTH                                              0.120E 02
     NUMBER OF INCREMENTS FOR MOVABLE LOAD                                0
     INITIAL POSITION OF MOVABLE LOAD STA ZERO                            0
     FINAL POSITION OF MOVABLE LOAD STA ZERO                              0
     NUMBER OF INCREMENTS BETWEEN EACH POSITION OF MOVABLE LOAD           1


TABLE 3 - SPECIFIED DEFLECTIONS AND SLOPES

     STA      CASE       DEFLECTION        SLOPE
      10        1          0.               NONE
      40        1          0.               NONE
      70        1          0.               NONE
```

(Continued)

(Sheet 1 of 11)

TABLE 4 - STIFFNESS AND FIXED-LOAD DATA

FROM	TO	CONTD	F	QF	S	T	M	P
0	80	0	0.609E 12	-0.300E 03	0.	0.	0.	0.
30	50	0	0.320E 12	-0.102E 03	0.	0.	0.	0.
5	5	0	0.	-0.500E 05	0.	0.	0.	0.
10	10	0	0.	-0.100E 06	0.	0.	0.	0.
15	15	0	0.	-0.100E 06	0.	0.	0.	0.
20	20	0	0.	-0.110E 06	0.	0.	0.	0.
25	25	0	0.	-0.110E 06	0.	0.	0.	0.
30	30	0	0.	-0.100E 06	0.	0.	0.	0.
35	35	0	0.	-0.100E 06	0.	0.	0.	0.
40	40	0	0.	-0.500E 05	0.	0.	0.	0.
45	45	0	0.	-0.500E 05	0.	0.	0.	0.
50	50	0	0.	-0.400E 05	0.	0.	0.	0.
55	55	0	0.	-0.350E 05	0.	0.	0.	0.
60	60	0	0.	-0.300E 05	0.	0.	0.	0.
65	65	0	0.	-0.250E 05	0.	0.	0.	0.
70	70	0	0.	-0.200E 05	0.	0.	0.	0.
75	75	0	0.	-0.150E 05	0.	0.	0.	0.

TABLE 5 - MOVABLE-LOAD DATA

FROM TO CONTD QM

NONE

TABLE 6 - SPECIFIED STATIONS FOR INFLUENCE DIAGRAMS
(SHEAR IS COMPUTED ONE HALF INCREMENT
TO THE LEFT OF THE DESIGNATED STATION)

NONE

(Continued)

```
PROGRAM BMCOL 51 - MASTER -  MATLOCK-TAYLOR  -   REVISION DATE = 08 MAR 68
CE394.2  HOMEWORK  PROBLEM 001, DATA CODED FOR EXAMPLE PROBLEM GIVEN IN
REPORT 56-1 (S)  FOR CENTER FOR HIGHWAY RESEARCH. CODED BY F.PARKER

PROB (CONTD)
  001      STEEL BENT CAP, SIMPLY SUPPORTED,FIXED LOADS, NO ENVELOPES OR
```

TABLE 7 - FIXED-LOAD RESULTS

STA I	DIST	DEFL	SLOPE	MOM	SHEAR	SUP REACT
-1	-0.120E 02	0.362E 00		0.		0.
			-0.271E-02		0.	
0	0.	0.329E 00		0.		0.
			-0.271E-02		-0.154E 03	
1	0.120E 02	0.296E 00		-0.184E 04		0.
			-0.271E-02		-0.458E 03	
2	0.240E 02	0.264E 00		-0.734E 04		0.
			-0.272E-02		-0.760E 03	
3	0.360E 02	0.231E 00		-0.165E 05		0.
			-0.272E-02		-0.106E 04	
4	0.480E 02	0.199E 00		-0.292E 05		0.
			-0.272E-02		-0.136E 04	
5	0.600E 02	0.166E 00		-0.456E 05		0.
			-0.272E-02		-0.517E 05	
6	0.720E 02	0.133E 00		-0.666E 06		0.
			-0.273E-02		-0.520E 05	
7	0.840E 02	0.101E 00		-0.129E 07		0.
			-0.276E-02		-0.523E 05	
8	0.960E 02	0.676E-01		-0.192E 07		0.
			-0.279E-02		-0.526E 05	
9	0.108E 03	0.341E-01		-0.255E 07		0.
			-0.284E-02		-0.529E 05	
10	0.120E 03	0.		-0.318E 07		0.372E 06
			-0.291E-02		0.219E 06	
11	0.132E 03	-0.349E-01		-0.557E 06		0.
			-0.292E-02		0.218E 06	
12	0.144E 03	-0.699E-01		0.206E 07		0.
			-0.288E-02		0.218E 06	
13	0.156E 03	-0.104E 00		0.468E 07		0.
			-0.278E-02		0.218E 06	
14	0.168E 03	-0.138E 00		0.729E 07		0.
			-0.264E-02		0.218E 06	
15	0.180E 03	-0.169E 00		0.990E 07		0.
			-0.245E-02		0.117E 06	
16	0.192E 03	-0.199E 00		0.113E 08		0.
			-0.222E-02		0.117E 06	
17	0.204E 03	-0.226E 00		0.127E 08		0.
			-0.197E-02		0.117E 06	
18	0.216E 03	-0.249E 00		0.141E 08		0.
			-0.169E-02		0.116E 06	
19	0.228E 03	-0.269E 00		0.155E 08		0.
			-0.139E-02		0.116E 06	
20	0.240E 03	-0.286E 00		0.169E 08		0.
			-0.105E-02		0.572E 04	

(Continued)

(Sheet 3 of 11)

Table F2 (Continued)

21	0.252E 03	-0.299E 00	-0.720E-03	0.170E 08	0.542E 04	0.
22	0.264E 03	-0.307E 00	-0.384E-03	0.170E 08	0.512E 04	0.
23	0.276E 03	-0.312E 00	-0.470E-04	0.171E 08	0.483E 04	0.
24	0.288E 03	-0.313E 00	0.291E-03	0.172E 08	0.453E 04	0.
25	0.300E 03	-0.309E 00	0.630E-03	0.172E 08	-0.106E 06	0.
26	0.312E 03	-0.302E 00	0.945E-03	0.159E 08	-0.106E 06	0.
27	0.324E 03	-0.290E 00	0.123E-02	0.147E 08	-0.106E 06	0.
28	0.336E 03	-0.275E 00	0.150E-02	0.134E 08	-0.107E 06	0.
29	0.348E 03	-0.257E 00	0.174E-02	0.121E 08	-0.107E 06	0.
30	0.360E 03	-0.237E 00	0.191E-02	0.108E 08	-0.207E 06	0.
31	0.372E 03	-0.214E 00	0.201E-02	0.834E 07	-0.208E 06	0.
32	0.384E 03	-0.190E 00	0.209E-02	0.585E 07	-0.208E 06	0.
33	0.396E 03	-0.165E 00	0.213E-02	0.335E 07	-0.208E 06	0.
34	0.408E 03	-0.139E 00	0.214E-02	0.848E 06	-0.209E 06	0.
35	0.420E 03	-0.113E 00	0.212E-02	-0.166E 07	-0.309E 06	0.
36	0.432E 03	-0.877E-01	0.205E-02	-0.537E 07	-0.310E 06	0.
37	0.444E 03	-0.631E-01	0.193E-02	-0.909E 07	-0.310E 06	0.
38	0.456E 03	-0.399E-01	0.177E-02	-0.128E 08	-0.311E 06	0.
39	0.468E 03	-0.187E-01	0.156E-02	-0.165E 08	-0.311E 06	0.
40	0.480E 03	0.	0.129E-02	-0.203E 08	0.158E 06	0.520E 06
41	0.492E 03	0.155E-01	0.106E-02	-0.184E 08	0.158E 06	0.
42	0.504E 03	0.282E-01	0.844E-03	-0.165E 08	0.158E 06	0.
43	0.516E 03	0.383E-01	0.656E-03	-0.146E 08	0.157E 06	0.
44	0.528E 03	0.462E-01	0.492E-03	-0.127E 08	0.157E 06	0.
45	0.540E 03	0.521E-01	0.352E-03	-0.108E 08	0.106E 06	0.
46	0.552E 03	0.563E-01	0.229E-03	-0.953E 07	0.106E 06	0.
47	0.564E 03	0.591E-01	0.122E-03	-0.826E 07	0.106E 06	0.
48	0.576E 03	0.605E-01	0.321E-04	-0.699E 07	0.105E 06	0.
49	0.588E 03	0.609E-01	-0.419E-04	-0.573E 07	0.105E 06	0.
50	0.600E 03	0.604E-01		-0.447E 07		0.

(Continued)

Table F2 (Continued)

51	0.612E 03	0.591E-01	-0.112E-03	-0.369E 07	0.645E 05	0.
52	0.624E 03	0.569E-01	-0.184E-03	-0.292E 07	0.642E 05	0.
53	0.636E 03	0.540E-01	-0.242E-03	-0.216E 07	0.639E 05	0.
54	0.648E 03	0.506E-01	-0.285E-03	-0.140E 07	0.636E 05	0.
55	0.660E 03	0.468E-01	-0.312E-03	-0.636E 06	0.633E 05	0.
56	0.672E 03	0.429E-01	-0.325E-03	-0.300E 06	0.280E 05	0.
57	0.684E 03	0.389E-01	-0.331E-03	0.316E 05	0.277E 05	0.
58	0.696E 03	0.350E-01	-0.330E-03	0.360E 06	0.274E 05	0.
59	0.708E 03	0.311E-01	-0.323E-03	0.685E 06	0.271E 05	0.
60	0.720E 03	0.274E-01	-0.309E-03	0.101E 07	0.268E 05	0.
61	0.732E 03	0.239E-01	-0.290E-03	0.964E 06	-0.353E 04	0.
62	0.744E 03	0.207E-01	-0.271E-03	0.918E 06	-0.383E 04	0.
63	0.756E 03	0.176E-01	-0.252E-03	0.868E 06	-0.413E 04	0.
64	0.768E 03	0.148E-01	-0.235E-03	0.815E 06	-0.443E 04	0.
65	0.780E 03	0.122E-01	-0.219E-03	0.758E 06	-0.473E 04	0.
66	0.792E 03	0.973E-02	-0.204E-03	0.398E 06	-0.300E 05	0.
67	0.804E 03	0.738E-02	-0.197E-03	0.337E 05	-0.303E 05	0.
68	0.816E 03	0.503E-02	-0.196E-03	-0.334E 06	-0.306E 05	0.
69	0.828E 03	0.260E-02	-0.202E-03	-0.705E 06	-0.309E 05	0.
70	0.840E 03	0.	-0.216E-03	-0.108E 07	-0.312E 05	0.694E 05
71	0.852E 03	-0.285E-02	-0.238E-03	-0.866E 06	0.178E 05	0.
72	0.864E 03	-0.591E-02	-0.255E-03	-0.655E 06	0.175E 05	0.
73	0.876E 03	-0.912E-02	-0.268E-03	-0.448E 06	0.172E 05	0.
74	0.888E 03	-0.124E-01	-0.276E-03	-0.245E 06	0.169E 05	0.
75	0.900E 03	-0.158E-01	-0.281E-03	-0.450E 05	0.166E 05	0.
76	0.912E 03	-0.192E-01	-0.282E-03	-0.288E 05	0.135E 04	0.
77	0.924E 03	-0.226E-01	-0.283E-03	-0.162E 05	0.105E 04	0.
78	0.936E 03	-0.260E-01	-0.283E-03	-0.720E 04	0.749E 03	0.
79	0.948E 03	-0.294E-01	-0.283E-03	-0.180E 04	0.450E 03	0.
80	0.960E 03	-0.328E-01	-0.283E-03	0.	0.150E 03	0.
81	0.972E 03	-0.362E-01	-0.283E-03	0.	0.	0.

(Continued)

PROB (CONTD)
001 STEEL BENT CAP, SIMPLY SUPPORTED,FIXED LOADS, NO ENVELOPES OR

TABLE 8A- ENVELOPES OF MAXIMUMS * = HELD FROM PRIOR PROBLEM

STA	MAX +DEFL	LOC	MAX -DEFL	LOC	MAX +MOM	LOC	MAX -MOM	LOC
-1	0.362E 00	-4	0.	999	0.	999	0.	999
0	0.329E 00	-4	0.	999	0.	999	0.	999
1	0.296E 00	-4	0.	999	0.	999	-0.184E 04	-4
2	0.264E 00	-4	0.	999	0.	999	-0.734E 04	-4
3	0.231E 00	-4	0.	999	0.	999	-0.165E 05	-4
4	0.199E 00	-4	0.	999	0.	999	-0.292E 05	-4
5	0.166E 00	-4	0.	999	0.	999	-0.456E 05	-4
6	0.133E 00	-4	0.	999	0.	999	-0.666E 06	-4
7	0.101E 00	-4	0.	999	0.	999	-0.129E 07	-4
8	0.676E-01	-4	0.	999	0.	999	-0.192E 07	-4
9	0.341E-01	-4	0.	999	0.	999	-0.255E 07	-4
10	0.	999	0.	999	0.	999	-0.318E 07	-4
11	0.	999	-0.349E-01	-4	0.	999	-0.557E 06	-4
12	0.	999	-0.699E-01	-4	0.206E 07	-4	0.	999
13	0.	999	-0.104E 00	-4	0.468E 07	-4	0.	999
14	0.	999	-0.138E 00	-4	0.729E 07	-4	0.	999
15	0.	999	-0.169E 00	-4	0.990E 07	-4	0.	999
16	0.	999	-0.199E 00	-4	0.113E 08	-4	0.	999
17	0.	999	-0.226E 00	-4	0.127E 08	-4	0.	999
18	0.	999	-0.249E 00	-4	0.141E 08	-4	0.	999
19	0.	999	-0.269E 00	-4	0.155E 08	-4	0.	999
20	0.	999	-0.286E 00	-4	0.169E 08	-4	0.	999
21	0.	999	-0.299E 00	-4	0.170E 08	-4	0.	999
22	0.	999	-0.307E 00	-4	0.170E 08	-4	0.	999
23	0.	999	-0.312E 00	-4	0.171E 08	-4	0.	999
24	0.	999	-0.313E 00	-4	0.172E 08	-4	0.	999
25	0.	999	-0.309E 00	-4	0.172E 08	-4	0.	999
26	0.	999	-0.302E 00	-4	0.159E 08	-4	0.	999
27	0.	999	-0.290E 00	-4	0.147E 08	-4	0.	999
28	0.	999	-0.275E 00	-4	0.134E 08	-4	0.	999
29	0.	999	-0.257E 00	-4	0.121E 08	-4	0.	999
30	0.	999	-0.237E 00	-4	0.108E 08	-4	0.	999
31	0.	999	-0.214E 00	-4	0.834E 07	-4	0.	999
32	0.	999	-0.190E 00	-4	0.585E 07	-4	0.	999
33	0.	999	-0.165E 00	-4	0.335E 07	-4	0.	999
34	0.	999	-0.139E 00	-4	0.848E 06	-4	0.	999
35	0.	999	-0.113E 00	-4	0.	999	-0.166E 07	-4
36	0.	999	-0.877E-01	-4	0.	999	-0.537E 07	-4
37	0.	999	-0.631E-01	-4	0.	999	-0.909E 07	-4
38	0.	999	-0.399E-01	-4	0.	999	-0.128E 08	-4
39	0.	999	-0.187E-01	-4	0.	999	-0.165E 08	-4
40	0.	999	0.	999	0.	999	-0.203E 08	-4
41	0.155E-01	-4	0.	999	0.	999	-0.184E 08	-4
42	0.282E-01	-4	0.	999	0.	999	-0.165E 08	-4
43	0.383E-01	-4	0.	999	0.	999	-0.146E 08	-4
44	0.462E-01	-4	0.	999	0.	999	-0.127E 08	-4

(Continued)

45	0.521E-01	-4	0.	999	0.	999	-0.108E 08	-4
46	0.563E-01	-4	0.	999	0.	999	-0.953E 07	-4
47	0.591E-01	-4	0.	999	0.	999	-0.826E 07	-4
48	0.605E-01	-4	0.	999	0.	999	-0.699E 07	-4
49	-0.609E-01	-4	0.	999	0.	999	-0.573E 07	-4
50	0.604E-01	-4	0.	999	0.	999	-0.447E 07	-4
51	0.591E-01	-4	0.	999	0.	999	-0.369E 07	-4
52	0.569E-01	-4	0.	999	0.	999	-0.292E 07	-4
53	0.540E-01	-4	0.	999	0.	999	-0.216E 07	-4
54	0.506E-01	-4	0.	999	0.	999	-0.140E 07	-4
55	0.468E-01	-4	0.	999	0.	999	-0.636E 06	-4
56	0.429E-01	-4	0.	999	0.	999	-0.300E 06	-4
57	0.389E-01	-4	0.	999	0.316E 05	-4	0.	999
58	0.350E-01	-4	0.	999	0.360E 06	-4	0.	999
59	0.311E-01	-4	0.	999	0.685E 06	-4	0.	999
60	0.274E-01	-4	0.	999	0.101E 07	-4	0.	999
61	0.239E-01	-4	0.	999	0.964E 06	-4	0.	999
62	0.207E-01	-4	0.	999	0.918E 06	-4	0.	999
63	0.176E-01	-4	0.	999	0.868E 06	-4	0.	999
64	0.148E-01	-4	0.	999	0.815E 06	-4	0.	999
65	0.122E-01	-4	0.	999	0.758E 06	-4	0.	999
66	0.973E-02	-4	0.	999	0.598E 06	-4	0.	999
67	0.738E-02	-4	0.	999	0.337E 05	-4	0.	999
68	0.503E-02	-4	0.	999	0.	999	-0.334E 06	-4
69	0.260E-02	-4	0.	999	0.	999	-0.705E 06	-4
70	0.	999	0.	999	0.	999	-0.108E 07	-4
71	0.	999	-0.285E-02	-4	0.	999	-0.866E 06	-4
72	0.	999	-0.591E-02	-4	0.	999	-0.655E 06	-4
73	0.	999	-0.912E-02	-4	0.	999	-0.448E 06	-4
74	0.	999	-0.124E-01	-4	0.	999	-0.245E 06	-4
75	0.	999	-0.158E-01	-4	0.	999	-0.450E 05	-4
76	0.	999	-0.192E-01	-4	0.	999	-0.288E 05	-4
77	0.	999	-0.226E-01	-4	0.	999	-0.162E 05	-4
78	0.	999	-0.260E-01	-4	0.	999	-0.720E 04	-4
79	0.	999	-0.294E-01	-4	0.	999	-0.180E 04	-4
80	0.	999	-0.328E-01	-4	0.	999	0.	999
81	0.	999	-0.362E-01	-4	0.	999	0.	999

(Continued)

TABLE 8B- ENVELOPES OF MAXIMUMS * = HELD FROM PRIOR PROBLEM

STA	MAX +SHEAR	LOC	MAX -SHEAR	LOC	MAX +REACT	LOC	MAX -REACT	LOC
-1					0.	999	0.	999
	0.	999	0.	999				
0					0.	999	0.	999
	0.	999	-0.154E 03	-4				
1					0.	999	0.	999
	0.	999	-0.458E 03	-4				
2					0.	999	0.	999
	0.	999	-0.760E 03	-4				
3					0.	999	0.	999
	0.	999	-0.106E 04	-4				
4					0.	999	0.	999
	0.	999	-0.136E 04	-4				
5					0.	999	0.	999
	0.	999	-0.517E 05	-4				
6					0.	999	0.	999
	0.	999	-0.520E 05	-4				
7					0.	999	0.	999
	0.	999	-0.523E 05	-4				
8					0.	999	0.	999
	0.	999	-0.526E 05	-4				
9					0.	999	0.	999
	0.	999	-0.529E 05	-4				
10					0.372E 06	-4	0.	999
	0.219E 06	-4	0.	999				
11					0.	999	0.	999
	0.218E 06	-4	0.	999				
12					0.	999	0.	999
	0.218E 06	-4	0.	999				
13					0.	999	0.	999
	0.218E 06	-4	0.	999				
14					0.	999	0.	999
	0.218E 06	-4	0.	999				
15					0.	999	0.	999
	0.117E 06	-4	0.	999				
16					0.	999	0.	999
	0.117E 06	-4	0.	999				
17					0.	999	0.	999
	0.117E 06	-4	0.	999				
18					0.	999	0.	999
	0.116E 06	-4	0.	999				
19					0.	999	0.	999
	0.116E 06	-4	0.	999				
20					0.	999	0.	999
	0.572E 04	-4	0.	999				
21					0.	999	0.	999
	0.542E 04	-4	0.	999				
22					0.	999	0.	999
	0.512E 04	-4	0.	999				
23					0.	999	0.	999
	0.483E 04	-4	0.	999				
24					0.	999	0.	999
	0.453E 04	-4	0.	999				
25					0.	999	0.	999
	0.	999	-0.106E 06	-4				
26					0.	999	0.	999
	0.	999	-0.106E 06	-4				
27					0.	999	0.	999
	0.	999	-0.106E 06	-4				
28					0.	999	0.	999
	0.	999	-0.107E 06	-4				

(Continued)

(Sheet 8 of 11)

Table F2 (Continued)

29	0.	999	-0.107E 06	-4	0.	999	0.	999
30	0.	999	-0.207E 06	-4	0.	999	0.	999
31	0.	999	-0.208E 06	-4	0.	999	0.	999
32	0.	999	-0.208E 06	-4	0.	999	0.	999
33	0.	999	-0.208E 06	-4	0.	999	0.	999
34	0.	999	-0.209E 06	-4	0.	999	0.	999
35	0.	999	-0.309E 06	-4	0.	999	0.	999
36	0.	999	-0.310E 06	-4	0.	999	0.	999
37	0.	999	-0.310E 06	-4	0.	999	0.	999
38	0.	999	-0.311E 06	-4	0.	999	0.	999
39	0.	999	-0.311E 06	-4	0.	999	0.	999
40	0.158E 06	-4	0.	999	0.520E 06	-4	0.	999
41	0.158E 06	-4	0.	999	0.	999	0.	999
42	0.158E 06	-4	0.	999	0.	999	0.	999
43	0.157E 06	-4	0.	999	0.	999	0.	999
44	0.157E 06	-4	0.	999	0.	999	0.	999
45	0.106E 06	-4	0.	999	0.	999	0.	999
46	0.106E 06	-4	0.	999	0.	999	0.	999
47	0.106E 06	-4	0.	999	0.	999	0.	999
48	0.105E 06	-4	0.	999	0.	999	0.	999
49	0.105E 06	-4	0.	999	0.	999	0.	999
50	0.645E 05	-4	0.	999	0.	999	0.	999
51	0.642E 05	-4	0.	999	0.	999	0.	999
52	0.639E 05	-4	0.	999	0.	999	0.	999
53	0.636E 05	-4	0.	999	0.	999	0.	999
54	0.633E 05	-4	0.	999	0.	999	0.	999
55	0.280E 05	-4	0.	999	0.	999	0.	999
56	0.277E 05	-4	0.	999	0.	999	0.	999
57	0.274E 05	-4	0.	999	0.	999	0.	999
58	0.271E 05	-4	0.	999	0.	999	0.	999
59	0.268E 05	-4	0.	999	0.	999	0.	999
60	0.	999	-0.353E 04	-4	0.	999	0.	999

(Continued)

Table F2 (Continued)

61					0.	999	0.	999
	0.	999	-0.383E 04	-4				
62					0.	999	0.	999
	0.	999	-0.413E 04	-4				
63					0.	999	0.	999
	0.	999	-0.443E 04	-4				
64					0.	999	0.	999
	0.	999	-0.473E 04	-4				
65					0.	999	0.	999
	0.	999	-0.300E 05	-4				
66					0.	999	0.	999
	0.	999	-0.303E 05	-4				
67					0.	999	0.	999
	0.	999	-0.306E 05	-4				
68					0.	999	0.	999
	0.	999	-0.309E 05	-4				
69					0.	999	0.	999
	0.	999	-0.312E 05	-4				
70					0.694E 05	-4	0.	999
	0.178E 05	-4	0.	999				
71					0.	999	0.	999
	0.175E 05	-4	0.	999				
72					0.	999	0.	999
	0.172E 05	-4	0.	999				
73					0.	999	0.	999
	0.169E 05	-4	0.	999				
74					0.	999	0.	999
	0.166E 05	-4	0.	999				
75					0.	999	0.	999
	0.135E 04	-4	0.	999				
76					0.	999	0.	999
	0.105E 04	-4	0.	999				
77					0.	999	0.	999
	0.749E 03	-4	0.	999				
78					0.	999	0.	999
	0.450E 03	-4	0.	999				
79					0.	999	0.	999
	0.150E 03	-4	0.	999				
80					0.	999	0.	999
	0.	999	0.	999				
81					0.	999	0.	999

(Continued)

```
TABLE 9 -- SCALES FOR PLOTS OF THE ENVELOPES OF MAXIMUMS
     HORIZONTAL SCALE
        10 INCHES =100. STATIONS

     VERTICAL SCALES
                    LENGTH       MAXIMUM
        VARIABLE   OF AXIS        VALUE
        DEFLECT   2 INCHES = 0.400E 00
        MOMENT    2 INCHES = 0.400E 08


PROB (CONTD)
  001      STEEL BENT CAP, SIMPLY SUPPORTED,FIXED LOADS, NO ENVELOPES OR


TABLE 10A -- INFLUENCE DIAGRAMS FOR DEFLECTION

    LOCATION              DESIGNATED STATIONS FOR INFLUENCE DIAGRAMS
     OF LOAD       STA          STA          STA          STA          STA

              NONE


TABLE 10B -- INFLUENCE DIAGRAMS FOR MOMENT

    LOCATION              DESIGNATED STATIONS FOR INFLUENCE DIAGRAMS
     OF LOAD       STA          STA          STA          STA          STA

              NONE


TABLE 10C -- INFLUENCE DIAGRAMS FOR SHEAR
             ( SHEAR IS COMPUTED ONE HALF INCREMENT
               TO THE LEFT OF THE DESIGNATED STATION )

    LOCATION              DESIGNATED STATIONS FOR INFLUENCE DIAGRAMS
     OF LOAD       STA          STA          STA          STA          STA

              NONE


TABLE 10D -- INFLUENCE DIAGRAMS FOR SUPPORT REACTION

    LOCATION              DESIGNATED STATIONS FOR INFLUENCE DIAGRAMS
     OF LOAD       STA          STA          STA          STA          STA

              NONE
```

(Sheet 11 of 11)

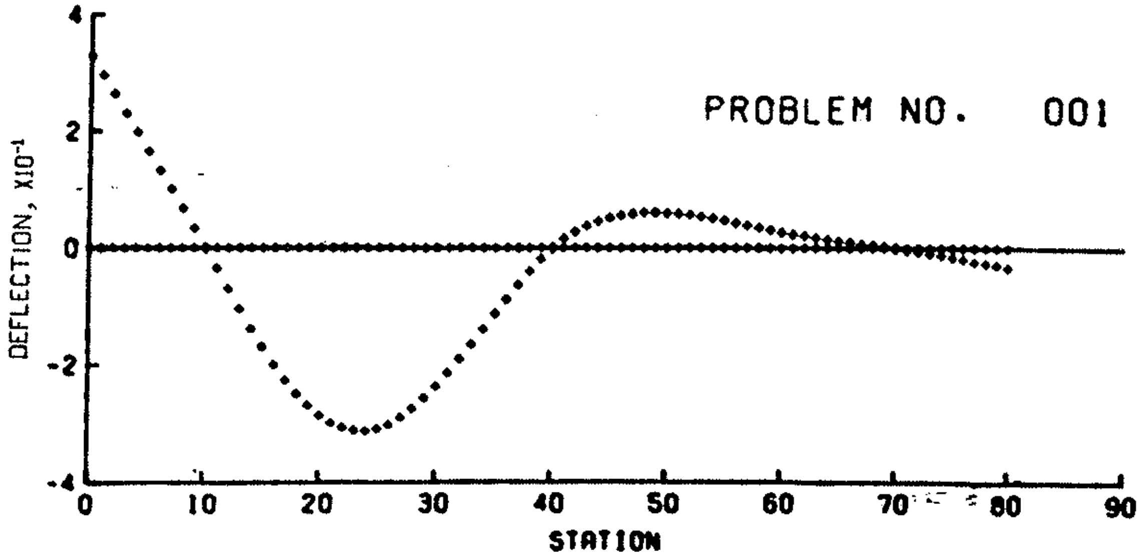


Figure F3. Variation of deflection along beam

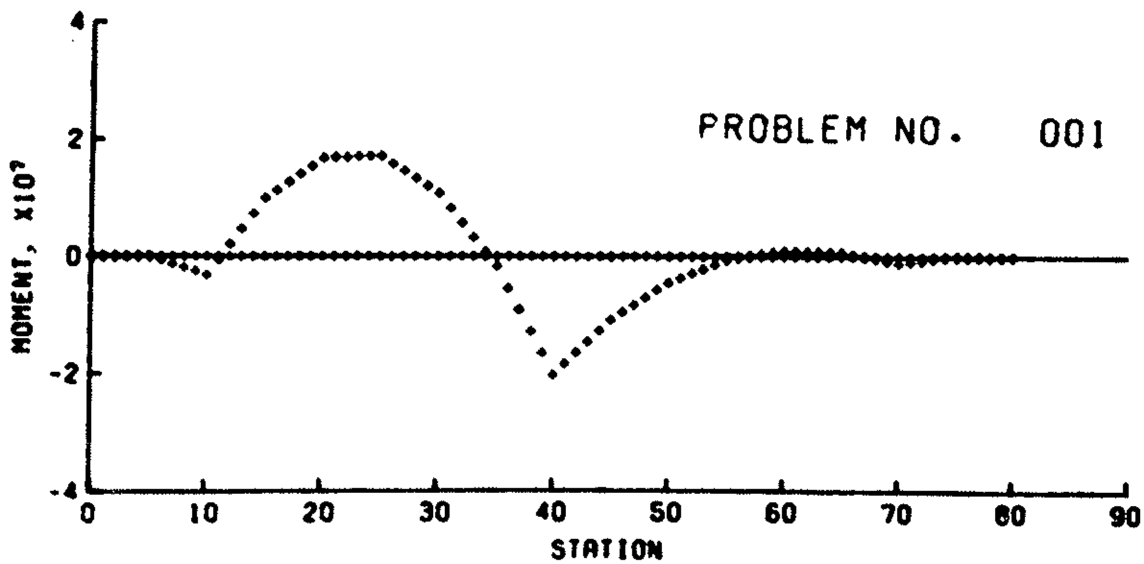


Figure F4. Variation of moment along beam

Example Problem 2

10 To illustrate further the use of program BMCOL51, a second example - a braced trench problem - is given. Figure F5 shows the physical problem for this example. The input and output data are presented in Tables F3 and F4. The results of the variation of deflection and moment along the trench support are plotted in Figures F6 and F7.

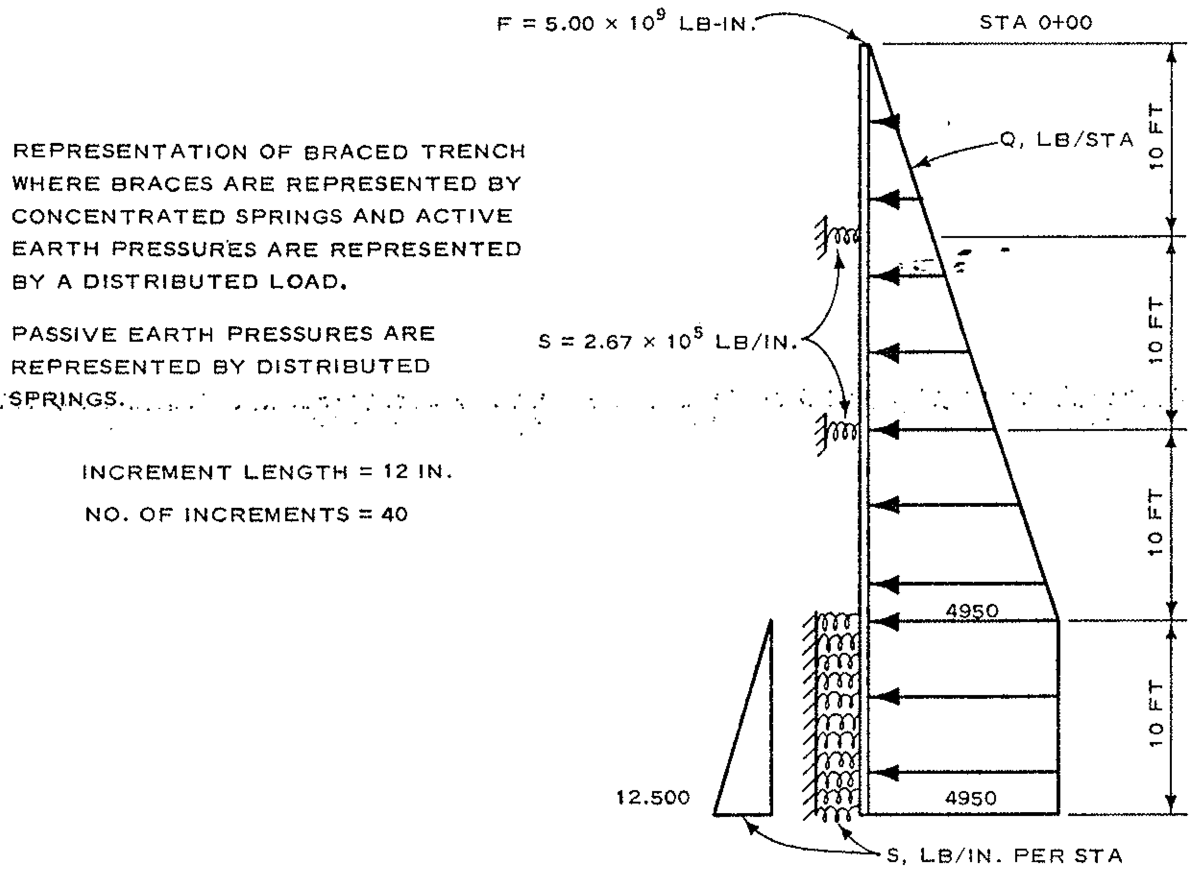


Figure F5. Physical problem for Example Problem 2 (braced trench problem)

Table F3

Input Data for Example Problem 2

```
                        BMCOL51 UNIVERSITY OF TEXAS AT AUSTIN
                     EXAMPLE PROBLEMS FOR REPORT

  002     SHEET PILE WITH AT REST PRESSURE; FIXED LOADS, WITH PLOTS
                                       1    0    6              1    1
        40
         0   40
        10   10                          2.670E+05
        20   20                          2.670E+05
         0         1
             30    1          -4.950E+03
             40               -4.950E+03 1.260E+04
```

WES FORM NO. 1233
SEPTEMBER 1962

Table F4

Output Data for Example Problem 2

PROGRAM BMCOL 51 - MASTER - MATLOCK-TAYLOR - REVISION DATE = 08 MAR 68
CE394.2 HOMEWORK PROBLEM 001, DATA CODED FOR EXAMPLE PROBLEM GIVEN IN
REPORT 56-1 (S) FOR CENTER FOR HIGHWAY RESEARCH. CODED BY F.PARKER

PROB
002 SHEET PILE WITH AT REST PRESSURE,FIXED LOADS, NO ENVELOPES OR

TABLE 1 - PROGRAM-CONTROL DATA

	ENVELOPES OF MAXIMUMS	TABLE NUMBER 2	3	4	5	6
HOLD FROM PRECEDING PROBLEM (1=HOLD)	0	0	0	0	0	0
NUM CARDS INPUT THIS PROBLEM		1	0	6	0	0

	DEFL	MOM	SHR	RCT
OPTION (IF=1) TO PLOT ENVELOPES OF MAXIMUMS	1	1	0	0

TABLE 2 - CONSTANTS

NUM INCREMENTS	40
INCREMENT LENGTH	0.120E 02
NUMBER OF INCREMENTS FOR MOVABLE LOAD	0
INITIAL POSITION OF MOVABLE LOAD STA ZERO	0
FINAL POSITION OF MOVABLE LOAD STA ZERO	0
NUMBER OF INCREMENTS BETWEEN EACH POSITION OF MOVABLE LOAD	1

TABLE 3 - SPECIFIED DEFLECTIONS AND SLOPES

STA	CASE	DEFLECTION	SLOPE
	NONE		

TABLE 4 - STIFFNESS AND FIXED-LOAD DATA

FROM	TO	CONTD	F	QF	S	T	R	P
0	40	0	0.500E 10	0.	0.	0.	0.	0.
10	10	0	0.	0.	0.267E 06	0.	0.	0.
20	20	0	0.	0.	0.267E 06	0.	0.	0.
0		1	0.	0.	0.	0.	0.	0.
	30	1	0.	-0.495E 04	0.	0.	0.	0.
	40	0	0.	-0.495E 04	0.125E 05	0.	0.	0.

TABLE 5 - MOVABLE-LOAD DATA

FROM	TO	CONTD	QM
		NONE	

TABLE 6 - SPECIFIED STATIONS FOR INFLUENCE DIAGRAMS
(SHEAR IS COMPUTED ONE HALF INCREMENT
TO THE LEFT OF THE DESIGNATED STATION)

NONE

(Continued)

(Sheet 1 of 7)

```
PROGRAM BMCOL 51 - MASTER -  MATLOCK-TAYLOR  -   REVISION DATE = 06 MAR 68
CE394.2  HOMEWORK  PROBLEM 001, DATA CODED FOR EXAMPLE PROBLEM GIVEN IN
REPORT 56-1 (S)  FOR CENTER FOR HIGHWAY RESEARCH. CODED BY F.PARKER
```

```
PROB (CONTD)
  002      SHEET PILE WITH AT REST PRESSURE,FIXED LOADS, NO ENVELOPES OR
```

TABLE 7 - FIXED-LOAD RESULTS

STA I	DIST	DEFL	SLOPE	MOM	SHEAR	SUP REACT
-1	-0.120E 02	-0.631E 00		0.		0.
			0.477E-02		0.108E-01	
0	0.	-0.574E 00		0.129E 00		0.
			0.477E-02		0.	
1	0.120E 02	-0.517E 00		0.129E 00		0.
			0.477E-02		-0.165E 03	
2	0.240E 02	-0.460E 00		-0.198E 04		0.
			0.476E-02		-0.495E 03	
3	0.360E 02	-0.402E 00		-0.792E 04		0.
			0.474E-02		-0.990E 03	
4	0.480E 02	-0.346E 00		-0.198E 05		0.
			0.470E-02		-0.165E 04	
5	0.600E 02	-0.289E 00		-0.396E 05		0.
			0.460E-02		-0.247E 04	
6	0.720E 02	-0.234E 00		-0.693E 05		0.
			0.443E-02		-0.346E 04	
7	0.840E 02	-0.181E 00		-0.111E 06		0.
			0.417E-02		-0.462E 04	
8	0.960E 02	-0.131E 00		-0.166E 06		0.
			0.377E-02		-0.594E 04	
9	0.108E 03	-0.855E-01		-0.238E 06		0.
			0.320E-02		-0.742E 04	
10	0.120E 03	-0.471E-01		-0.327E 06		0.126E 05
			0.241E-02		0.351E 04	
11	0.132E 03	-0.161E-01		-0.285E 06		0.
			0.173E-02		0.169E 04	
12	0.144E 03	0.263E-02		-0.264E 06		0.
			0.110E-02		-0.288E 03	
13	0.156E 03	0.158E-01		-0.268E 06		0.
			0.455E-03		-0.243E 04	
14	0.168E 03	0.213E-01		-0.297E 06		0.
			-0.258E-03		-0.474E 04	
15	0.180E 03	0.182E-01		-0.354E 06		0.
			-0.111E-02		-0.722E 04	
16	0.192E 03	0.488E-02		-0.440E 06		0.
			-0.216E-02		-0.986E 04	
17	0.204E 03	-0.211E-01		-0.559E 06		0.
			-0.351E-02		-0.127E 05	
18	0.216E 03	-0.632E-01		-0.711E 06		0.
			-0.521E-02		-0.156E 05	
19	0.228E 03	-0.126E 00		-0.898E 06		0.
			-0.737E-02		-0.186E 05	
20	0.240E 03	-0.214E 00		-0.112E 07		0.572E 05
			-0.101E-01		0.351E 05	

(Continued)

Table F4 (Continued)

21	0.252E 03	-0.335E 00	-0.117E-01	-0.702E 06	0.316E 05	0.
22	0.264E 03	-0.476E 00	-0.125E-01	-0.323E 06	0.280E 05	0.
23	0.276E 03	-0.626E 00	-0.125E-01	0.131E 05	0.242E 05	0.
24	0.288E 03	-0.776E 00	-0.118E-01	0.304E 06	0.202E 05	0.
25	0.300E 03	-0.917E 00	-0.105E-01	0.547E 06	0.161E 05	0.
26	0.312E 03	-0.104E 01	-0.868E-02	0.740E 06	0.118E 05	0.
27	0.324E 03	-0.115E 01	-0.656E-02	0.882E 06	0.737E 04	0.
28	0.336E 03	-0.123E 01	-0.423E-02	0.970E 06	0.275E 04	0.
29	0.348E 03	-0.128E 01	-0.182E-02	0.100E 07	-0.203E 04	0.
30	0.360E 03	-0.130E 01	0.527E-03	0.979E 06	-0.698E 04	0.
31	0.372E 03	-0.129E 01	0.268E-02	0.895E 06	-0.103E 05	0.161E 04
32	0.384E 03	-0.126E 01	0.453E-02	0.771E 06	-0.121E 05	0.315E 04
33	0.396E 03	-0.121E 01	0.603E-02	0.626E 06	-0.125E 05	0.452E 04
34	0.408E 03	-0.113E 01	0.717E-02	0.475E 06	-0.118E 05	0.566E 04
35	0.420E 03	-0.105E 01	0.797E-02	0.333E 06	-0.102E 05	0.654E 04
36	0.432E 03	-0.951E 00	0.848E-02	0.211E 06	-0.806E 04	0.713E 04
37	0.444E 03	-0.850E 00	0.875E-02	0.114E 06	-0.557E 04	0.743E 04
38	0.456E 03	-0.745E 00	0.886E-02	0.471E 05	-0.308E 04	0.745E 04
39	0.468E 03	-0.638E 00	0.889E-02	0.102E 05	-0.847E 03	0.718E 04
40	0.480E 03	-0.532E 00	0.889E-02	-0.647E-01	0.539E-02	0.332E 04
41	0.492E 03	-0.425E 00		0.		0.

(Continued)

(Sheet 3 of 7)

PROB (CONTD)
002 SHEET PILE WITH AT REST PRESSURE,FIXED LOADS, NO ENVELOPES ON

TABLE 8A- ENVELOPES OF MAXIMUMS * = HELD FROM PRIOR PROBLEM

STA	MAX- +DEFL	LOC	MAX -DEFL	LOC	MAX +MOM	LOC	MAX -MOM	LOC
-1	0.	999	-0.631E 00	0	0.	999	0.	999
0	0.	999	-0.574E 00	0	0.129E 00	0	0.	999
1	0.	999	-0.517E 00	0	0.129E 00	0	0.	999
2	0.	999	-0.460E 00	0	0.	999	-0.198E 04	0
3	0.	999	-0.402E 00	0	0.	999	-0.792E 04	0
4	0.	999	-0.346E 00	0	0.	999	-0.198E 05	0
5	0.	999	-0.289E 00	0	0.	999	-0.396E 05	0
6	0.	999	-0.234E 00	0	0.	999	-0.693E 05	0
7	0.	999	-0.181E 00	0	0.	999	-0.111E 06	0
8	0.	999	-0.131E 00	0	0.	999	-0.166E 06	0
9	0.	999	-0.855E-01	0	0.	999	-0.238E 06	0
10	0.	999	-0.471E-01	0	0.	999	-0.327E 06	0
11	0.	999	-0.181E-01	0	0.	999	-0.285E 06	0
12	0.263E-02	0	0.	999	0.	999	-0.264E 06	0
13	0.158E-01	0	0.	999	0.	999	-0.268E 06	0
14	0.213E-01	0	0.	999	0.	999	-0.297E 06	0
15	0.182E-01	0	0.	999	0.	999	-0.354E 06	0
16	0.488E-02	0	0.	999	0.	999	-0.440E 06	0
17	0.	999	-0.211E-01	0	0.	999	-0.559E 06	0
18	0.	999	-0.632E-01	0	0.	999	-0.711E 06	0
19	0.	999	-0.126E 00	0	0.	999	-0.898E 06	0
20	0.	999	-0.214E 00	0	0.	999	-0.112E 07	0
21	0.	999	-0.335E 00	0	0.	999	-0.702E 06	0
22	0.	999	-0.476E 00	0	0.	999	-0.323E 06	0
23	0.	999	-0.626E 00	0	0.131E 05	0	0.	999
24	0.	999	-0.776E 00	0	0.304E 06	0	0.	999
25	0.	999	-0.917E 00	0	0.547E 06	0	0.	999
26	0.	999	-0.104E 01	0	0.740E 06	0	0.	999
27	0.	999	-0.115E 01	0	0.882E 06	0	0.	999
28	0.	999	-0.123E 01	0	0.970E 06	0	0.	999
29	0.	999	-0.128E 01	0	0.100E 07	0	0.	999
30	0.	999	-0.130E 01	0	0.979E 06	0	0.	999
31	0.	999	-0.129E 01	0	0.895E 06	0	0.	999
32	0.	999	-0.126E 01	0	0.771E 06	0	0.	999
33	0.	999	-0.121E 01	0	0.626E 06	0	0.	999
34	0.	999	-0.113E 01	0	0.475E 06	0	0.	999
35	0.	999	-0.105E 01	0	0.333E 06	0	0.	999
36	0.	999	-0.951E 00	0	0.211E 06	0	0.	999
37	0.	999	-0.850E 00	0	0.114E 06	0	0.	999
38	0.	999	-0.745E 00	0	0.471E 05	0	0.	999
39	0.	999	-0.638E 00	0	0.102E 05	0	0.	999
40	0.	999	-0.532E 00	0	0.	999	-0.647E-01	0
41	0.	999	-0.425E 00	0	0.	999	0.	999

(Continued)

TABLE 88- ENVELOPES OF MAXIMUMS * = HELD FROM PRIOR PROBLEM

STA	MAX +SHEAR	LOC	MAX -SHEAR	LOC	MAX +REACT	LOC	MAX -REACT	LOC
-1	0.108E-01	0	0.	999	0.	999	0.	999
0	0.	999	0.	999	0.	999	0.	999
1	0.	999	-0.165E 03	0	0.	999	0.	999
2	0.	999	-0.495E 03	0	0.	999	0.	999
3	0.	999	-0.990E 03	0	0.	999	0.	999
4	0.	999	-0.165E 04	0	0.	999	0.	999
5	0.	999	-0.247E 04	0	0.	999	0.	999
6	0.	999	-0.346E 04	0	0.	999	0.	999
7	0.	999	-0.462E 04	0	0.	999	0.	999
8	0.	999	-0.594E 04	0	0.	999	0.	999
9	0.	999	-0.742E 04	0	0.	999	0.	999
10	0.351E 04	0	0.	999	0.126E 05	0	0.	999
11	0.169E 04	0	0.	999	0.	999	0.	999
12	0.	999	-0.288E 03	0	0.	999	0.	999
13	0.	999	-0.243E 04	0	0.	999	0.	999
14	0.	999	-0.474E 04	0	0.	999	0.	999
15	0.	999	-0.722E 04	0	0.	999	0.	999
16	0.	999	-0.986E 04	0	0.	999	0.	999
17	0.	999	-0.127E 05	0	0.	999	0.	999
18	0.	999	-0.156E 05	0	0.	999	0.	999
19	0.	999	-0.188E 05	0	0.	999	0.	999
20	0.351E 05	0	0.	999	0.572E 05	0	0.	999

(Continued)

21					0.	999	0.	999
	0.316E 05	0	0.	999				
22					0.	999	0.	999
	0.280E 05	0	0.	999				
23					0.	999	0.	999
	0.242E 05	0	0.	999				
24					0.	999	0.	999
	0.202E 05	0	0.	999				
25					0.	999	0.	999
	0.161E 05	0	0.	999				
26					0.	999	0.	999
	0.118E 05	0	0.	999				
27					0.	999	0.	999
	0.737E 04	0	0.	999				
28					0.	999	0.	999
	0.275E 04	0	0.	999				
29					0.	999	0.	999
	0.	999	-0.203E 04	0				
30					0.	999	0.	999
	0.	999	-0.698E 04	0				
31					0.161E 04	0	0.	999
	0.	999	-0.103E 05	0				
32					0.315E 04	0	0.	999
	0.	999	-0.121E 05	0				
33					0.452E 04	0	0.	999
	0.	999	-0.125E 05	0				
34					0.566E 04	0	0.	999
	0.	999	-0.118E 05	0				
35					0.654E 04	0	0.	999
	0.	999	-0.102E 05	0				
36					0.713E 04	0	0.	999
	0.	999	-0.806E 04	0				
37					0.743E 04	0	0.	999
	0.	999	-0.557E 04	0				
38					0.745E 04	0	0.	999
	0.	999	-0.308E 04	0				
39					0.718E 04	0	0.	999
	0.	999	-0.847E 03	0				
40					0.332E 04	0	0.	999
	0.539E-02	0	0.	999				
41					0.	999	0.	999

(Continued)

```
TABLE 9 -- SCALES FOR PLOTS OF THE ENVELOPES OF MAXIMUMS
     HORIZONTAL SCALE
        10 INCHES = 50. STATIONS

     VERTICAL SCALES
                    LENGTH       MAXIMUM
        VARIABLE   OF AXIS        VALUE
      - DEFLECT   2 INCHES = 0.200E 01
        MOMENT    2 INCHES = 0.200E 07


PROB (CONTD)
  002     SHEET PILE WITH AT REST PRESSURE,FIXED LOADS, NO ENVELOPES OR


TABLE 10A -- INFLUENCE DIAGRAMS FOR DEFLECTION

    LOCATION              DESIGNATED STATIONS FOR INFLUENCE DIAGRAMS
     OF LOAD        STA          STA          STA          STA          STA

                NONE


TABLE 10B -- INFLUENCE DIAGRAMS FOR MOMENT

    LOCATION              DESIGNATED STATIONS FOR INFLUENCE DIAGRAMS
     OF LOAD        STA          STA          STA          STA          STA

                NONE


TABLE 10C -- INFLUENCE DIAGRAMS FOR SHEAR
             ( SHEAR IS COMPUTED ONE HALF INCREMENT
               TO THE LEFT OF THE DESIGNATED STATION )

    LOCATION              DESIGNATED STATIONS FOR INFLUENCE DIAGRAMS
     OF LOAD        STA          STA          STA          STA          STA

                NONE


TABLE 10D -- INFLUENCE DIAGRAMS FOR SUPPORT REACTION

    LOCATION              DESIGNATED STATIONS FOR INFLUENCE DIAGRAMS
     OF LOAD        STA          STA          STA          STA          STA

                NONE


PROGRAM BMCOL 51 - MASTER -  MATLOCK-TAYLOR  -   REVISION DATE = 08 MAR 68
CE394.2  HOMEWORK  PROBLEM 001, DATA CODED FOR EXAMPLE PROBLEM GIVEN IN
REPORT 56-1 (S)  FOR CENTER FOR HIGHWAY RESEARCH. CODED BY F.PARKER


RETURN THIS PAGE TO TIME RECORD FILE -- HM
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(Sheet 7 of 7)

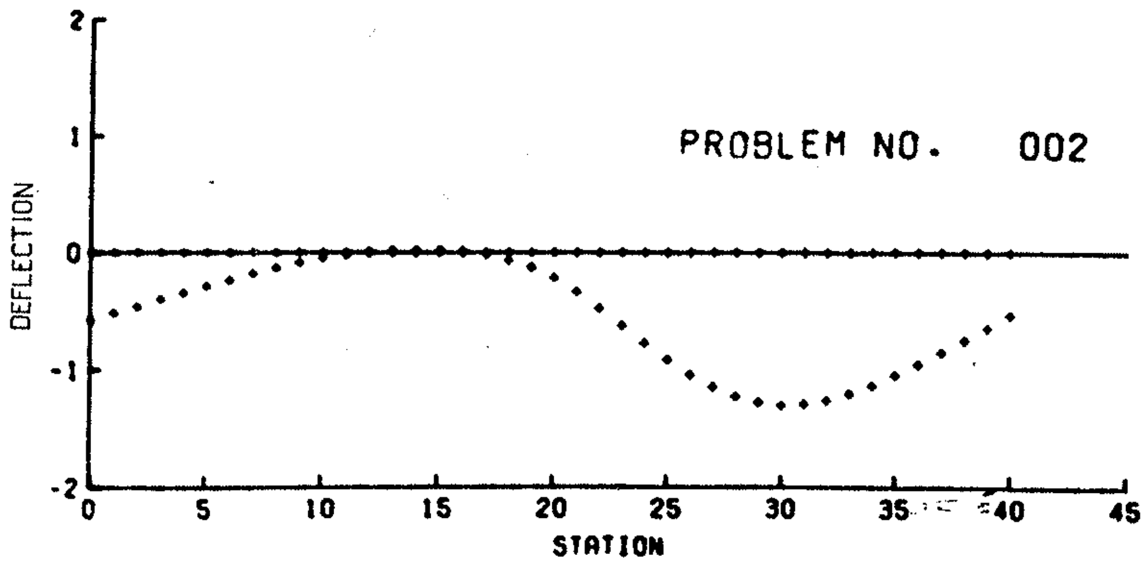

Figure F6. Variation of deflection along trench support

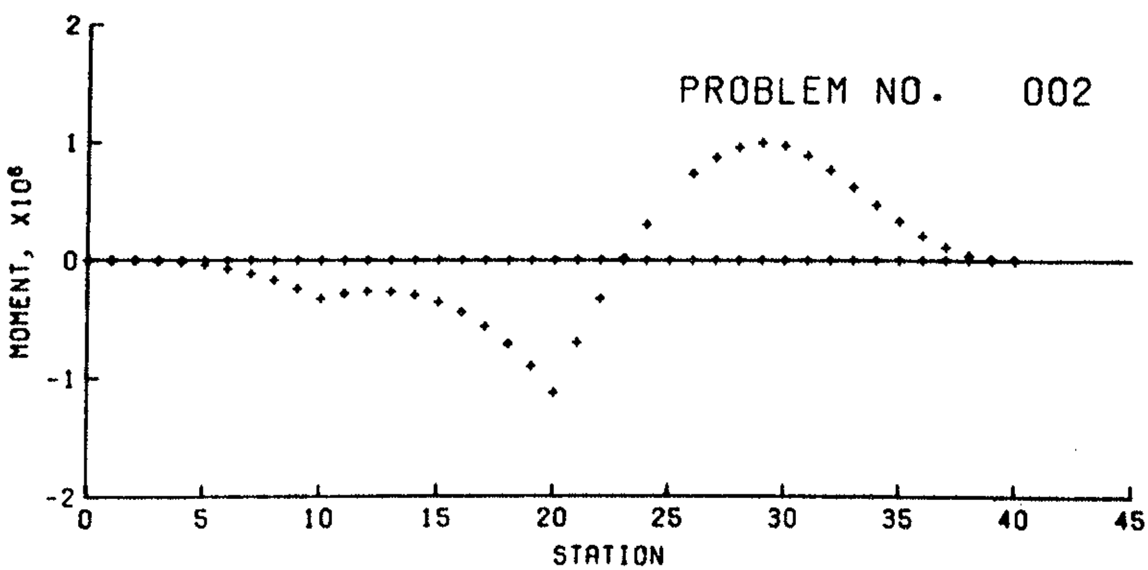

Figure F7. Variation of moment along trench support