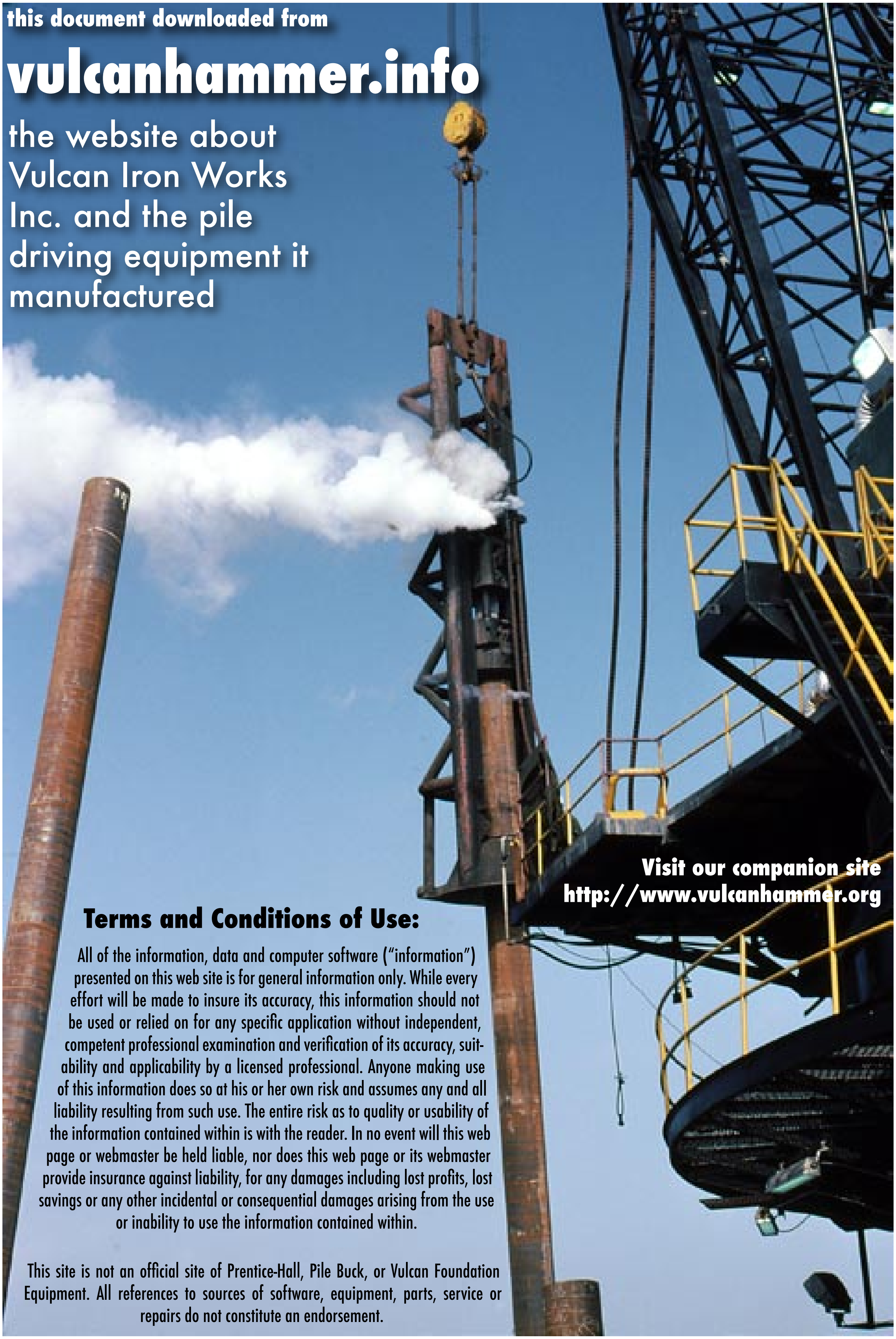

Comments on One-Dimensional Wave Mechanics

Dhanvin Mehta

1. INTRODUCTION:

Good. introduces a wave theory approach i.e., discrete element as continuum

The classical wave theory, which describes the propagation stress waves in the rods, forms the underlying basis of pile driving theory and pile driving computer programs. In this respect, piles may be considered as rods and the same analogy can be extended to the other parts of the driving system. The purpose of this study is to describe essential differences in the development, modelling as well as basic ideas behind the computer program popularly called 'Weap' (Wave Equation Analysis of Piles) which was developed for the Federal Highway Department, and the algorithm based on wave equation theory, extended and refined realistically to include complications due to presence of discontinuities in pile cross section, skin friction as well as internal damping. Only air steam hammers are considered as a part of the latter approach mainly because of the simplified model adopted, and for the ease in the program development. The conflicts associated with any other hammer system during modelling, using wave theory approach are self evident, and needs no further elaboration.

2. MODELLING:

Similarity:

Pile driving is a dynamic process, demanding a way of calculation that assesses as correctly as possible phenomena, such as forces, stresses etc. as a function of time. At moment of impact when a pile is hit by a hammer, stress-waves start to propagate with speed of sound in the pile, as well as in the hammer, the anvil, and the cushion block; upward in the hammer and downward in all other parts.

As soon as a stress-wave encounters a discontinuity (change of cross section or material) or a resistance (from the soil), the wave is (partly) reflected creating a wave in opposite direction and in every point of the system the stresses and particle-velocities vary with time.

In order to analyse the effect of the blow it is necessary that these stresses, velocities and displacements are assessed; this can be done with a pile driving computer program based on wave equation.

The principal part of such a program constitutes a mathematical model of the system dealing with the following functions:

a. It defines in a general way the essential properties of the components of the system:

   - dimensions.
   - weights.
   - material constants.

b. It defines the boundary conditions, i.e. the soil and how the soil resistances act. These relations are based on the laws of dynamics, and they are laid down in a set of formulae.

The mathematical model permits the state of stress and the state of motion of all components of the system to be calculated for a series of sequential (small) timesteps. Timesteps in this context have to be considered with an order of magnitude of about 0,1 millisecond. In this way variation of motion and stressed can be followed.

The mathematical model must be supplied with numerical data; the "input".

The above two programs equal with respect to these principles, their mathematical models, however, differ.

B. <u>Weap or Smith Type Program</u>:

The term Wave Equation is the name that has been attached to computer programs for discrete dynamic pile analysis. The pile and driving system (Figure 1a) is represented by a series of masses and springs as shown in Figure 1b. The soil is modeled by a spring (R, Static) and a dashpot (R, Dynamic) attached to each mass. The soil resistances so represented are shown in Figure 1.1c and are linear elastic plastic for the spring where the maximum forc͡e, R_u, is reached at a displacement q, called the quake, and linearly proportional to the element velocity for the dashpot (commonly know as the damping force).

The analysis proceeds by giving the ram an initial velocity. At each element the displacement can be calculated for a small time increment with element velocities determined from the previous time

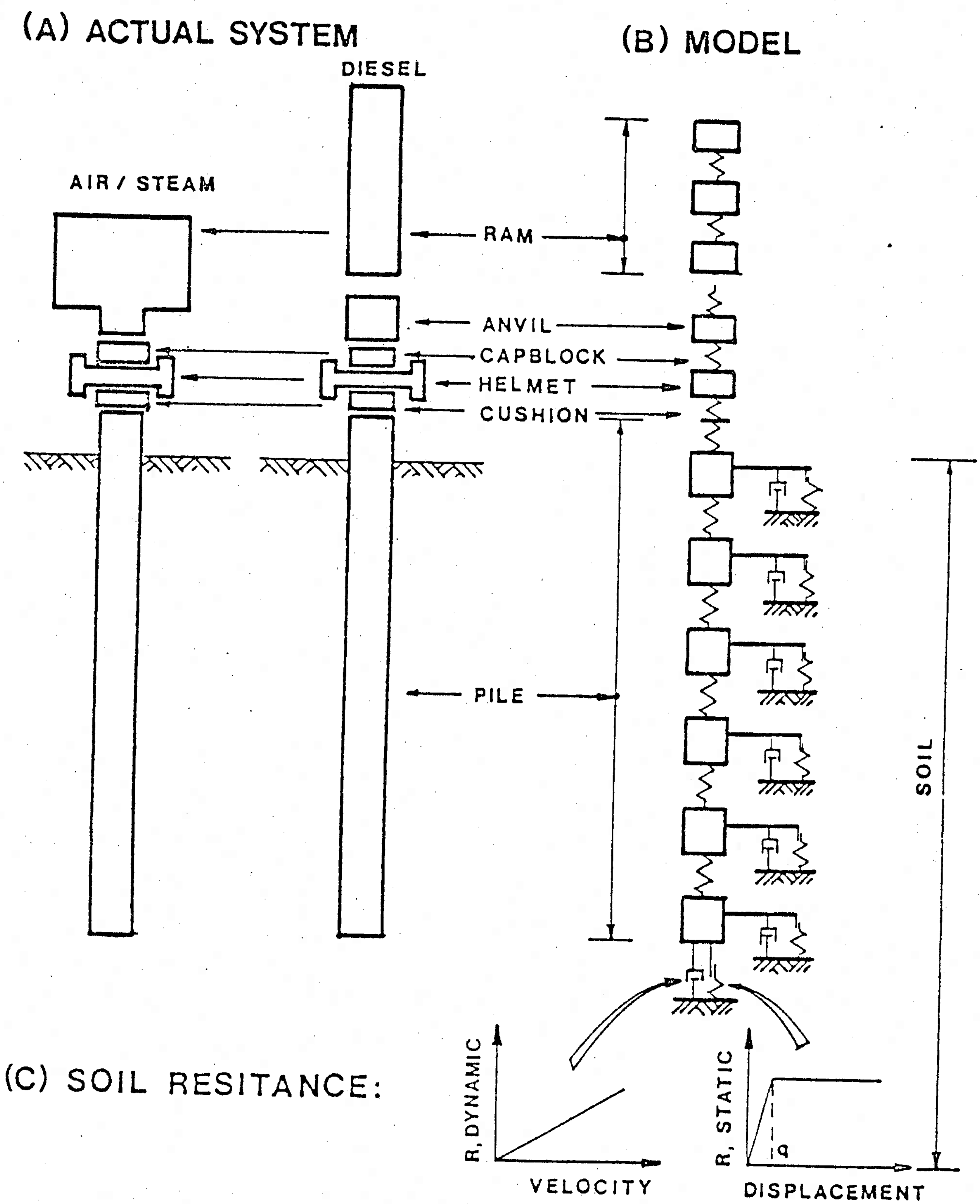

FIGURE 1 (A) THE SYSTEM TO BE ANALYZED;
(B) THE WAVE EQUATION MODEL AND
(C) THE COMPONENTS OF THE SOIL RESISTANCE MODEL

increment. With these displacements and velocities the forces acting on each mass can be determined. They arise from the pile spring deformations, from the soil spring deformation and from the dashpot force. Using Newton's Second Law the mass accelerations can be calculated and by integration also the velocity. The computation then proceeds to the next time increment.

In application a set of soil forces R_u and damping forces are assigned at each element. Then the ram is given it's rated impact velocity and the dynamic computation outlined above is continued through successive time increments until all soil forces are less than R_u. The total permanent displacement will have then been calculated and a point on the bearing graph is known. The capacity value is known as R_{ut} and is equal to the sum of the R_u values at each element. The blow count is obtained from the calculated permanent set. In this procedure the permanent set (or blow count) is determined for a set of assigned resistances. However, the bearing graph is plotted, by tradition, with the blow count as the independent variable. A variety of R_{ut} values can be used to calculate the total shape of the bearing graph.

In addition to the bearing graph the wave equation also gives stresses in the pile and they can also be shown as a function of blow count.

In practice, the wave equation bearing graph can be used in a manner quite similar to the dynamic formula bearing graph. In addition, driving stresses can be rationally limited. While the shape of the two curves are quite similar the differences are substantial. A particular wave equation bearing graph is associated with a single driving system pile type, soil profile and a particular pile penetration. If any one of the above items are changed, the bearing graph changes.

The above description summarizes very briefly the operation of traditional wave equation programs such as the Weap program.

C. Wave Theory Programs:

Different from the Weap or SMITH-type program, the Wave Theory Program is based directly on the properties of waves. As has been stated earlier, there are stress-waves that propagate with the speed of sound in the pile, hammer and anvil.

The Wave Theory Program follows these waves during their propagation. When the waves encounter discontinuities or (soil) resistances, their modifications (reflections and transmissions) are computed.

The state of motion follows from the stress-waves in a straight-forward manner; displacements are calculated from the velocities. The Wave Theory Program automatically establishes its own coordinate system in such a way that it takes exactly one timestep for the waves to travel from any grid-point to the next.

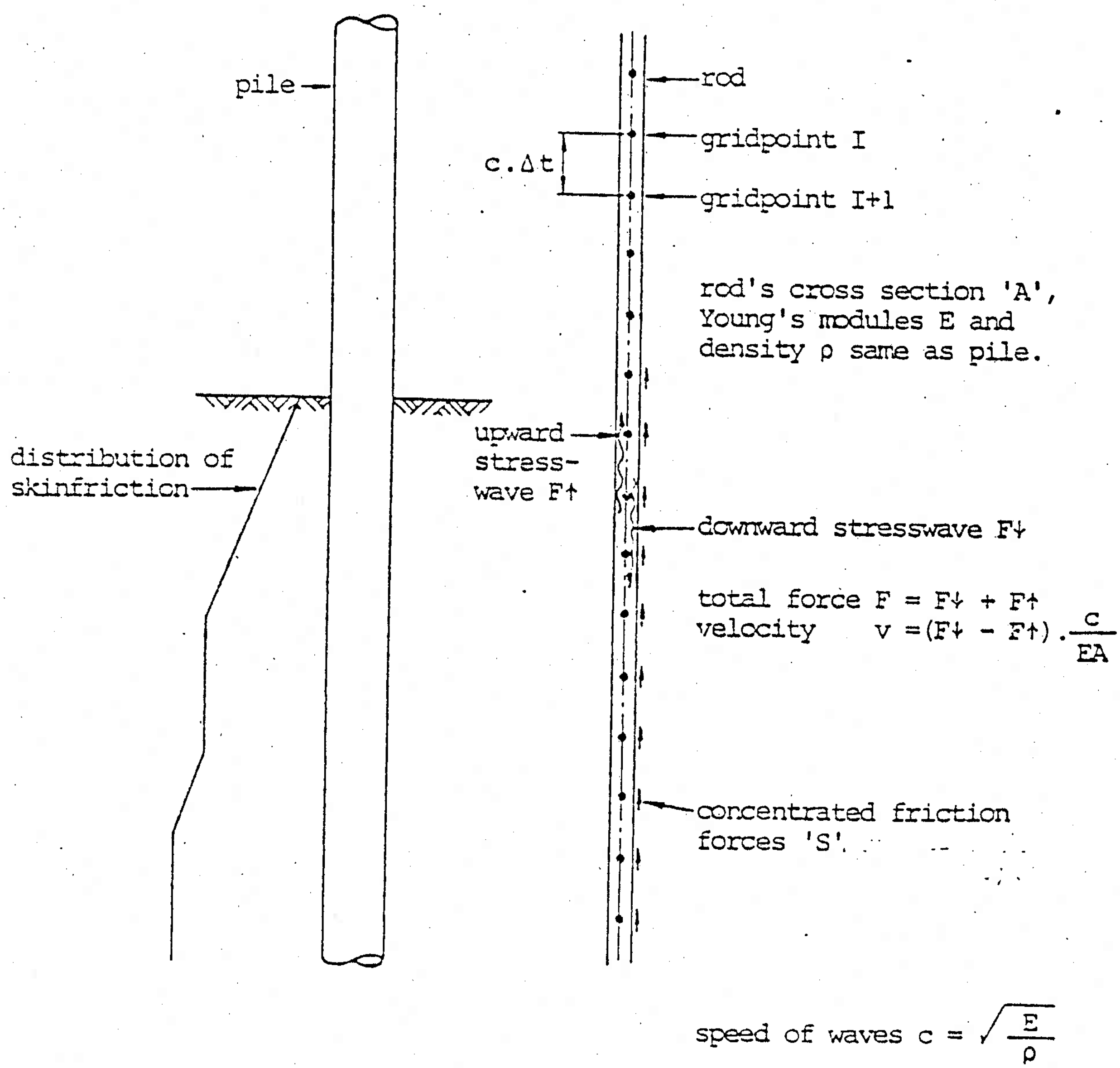
pile
rod
gridpoint I
c.Δt
gridpoint I+1
rod's cross section 'A', Young's modules E and density ρ same as pile.
upward stress-wave F↑
distribution of skinfriction
downward stresswave F↓
total force F = F↓ + F↑
velocity v =(F↓ - F↑) . c/EA
concentrated friction forces 'S'.
speed of waves c = √(E/ρ)

Figure 2

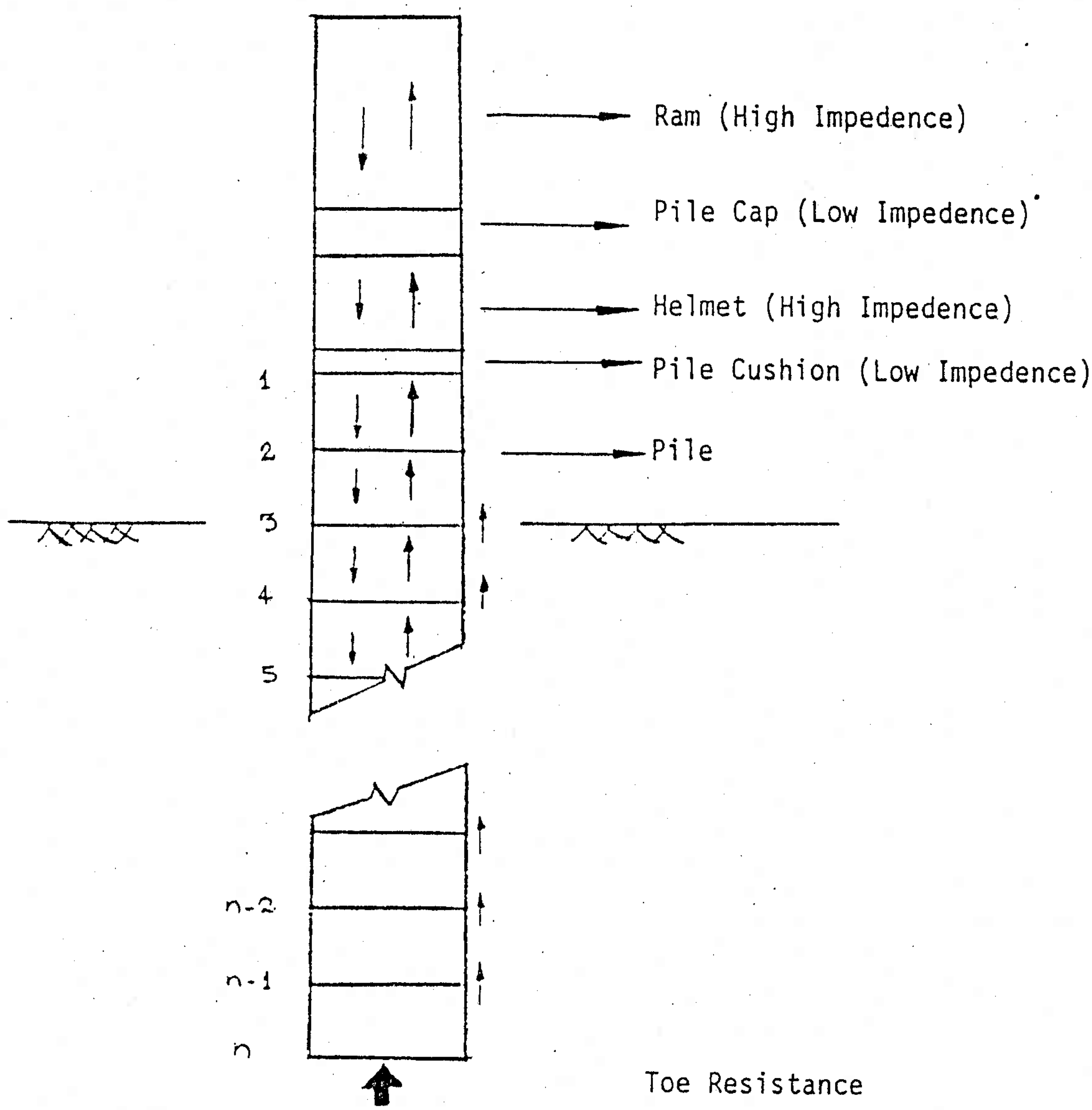

Figure 3. Pile & Hammer Model

Below groundlevel The Wave Theory Program automatically distributes the skinfriction over the grid-points; continuously distributed skinfriction when given so by the input, is replaced by a set of concentrated friction forces at these grid-points having the same resultant.

Opposite to the Weap or SMITH-type program, where mass is concentrated in a number of discrete points, The Wave Theory Program maintains the continuity of mass; also continuity of elasticity is maintained, where Weap uses interspaced springs.

With this model The Wave Theory Program computes motion (velocity and displacement) and forces (stresses) for exactly the same points, the grid-points.

Skinfriction for both programs is assessed in a similar way; it is assumed to increase with velocity (damping).

For displacements below a certain limit, call quake, elastic behavior of the soil is assumed in both the programs.

CONCLUSION:

Because of the difference between the mathematical models on which the two programs are base, differences in output must be expected.

However, with respect to set-per-blow the results of both programs need not differ widely, when applied to identical cases.

The wave theory, extended in a simple way in order to be able to account for skinfriction, provides a good description of the pile driving process.

This theory has lead to an economic pile driving computer program, which in a straight-forward manner provides realistic results, without numerical integration of the differential equations.

In the theory, as well as in the program the interaction of hammer, pile and soil is correctly represented; the impact diagram, often erroneously considered to be a property of the hammer alone, is calculated by the program.

The pile driving program creates possibility to:

- make reliable predictions, on condition that good soil data are available

- estimate the driving resistance and the bearing capacity on the base of blowcounts for all hammers

- interpret pile driving tests.

TABLE 1

	Wave Theory Program	Weap
mass	continuous	concentrated point masses
stiffness	continuous	concentrated springs
method of analysis	calculates the initial wave, propagates for each timestep Δt the waves, both in hammer and pile, and calculates the modifications at discontinuities	solves for each time-step t the set of equations of dynamic equilibrium by numerical integration
skinfriction	the program replaces the given continuous distribution by a large number of concentrated frictional forces	concentrated in the centers of gravity of gravity of the elements; accounted for by spring dashpot systems
toe-resistance	same as Weap	represented by spring and dashpot
interaction of hammer, pile and soil	calculated	calculated