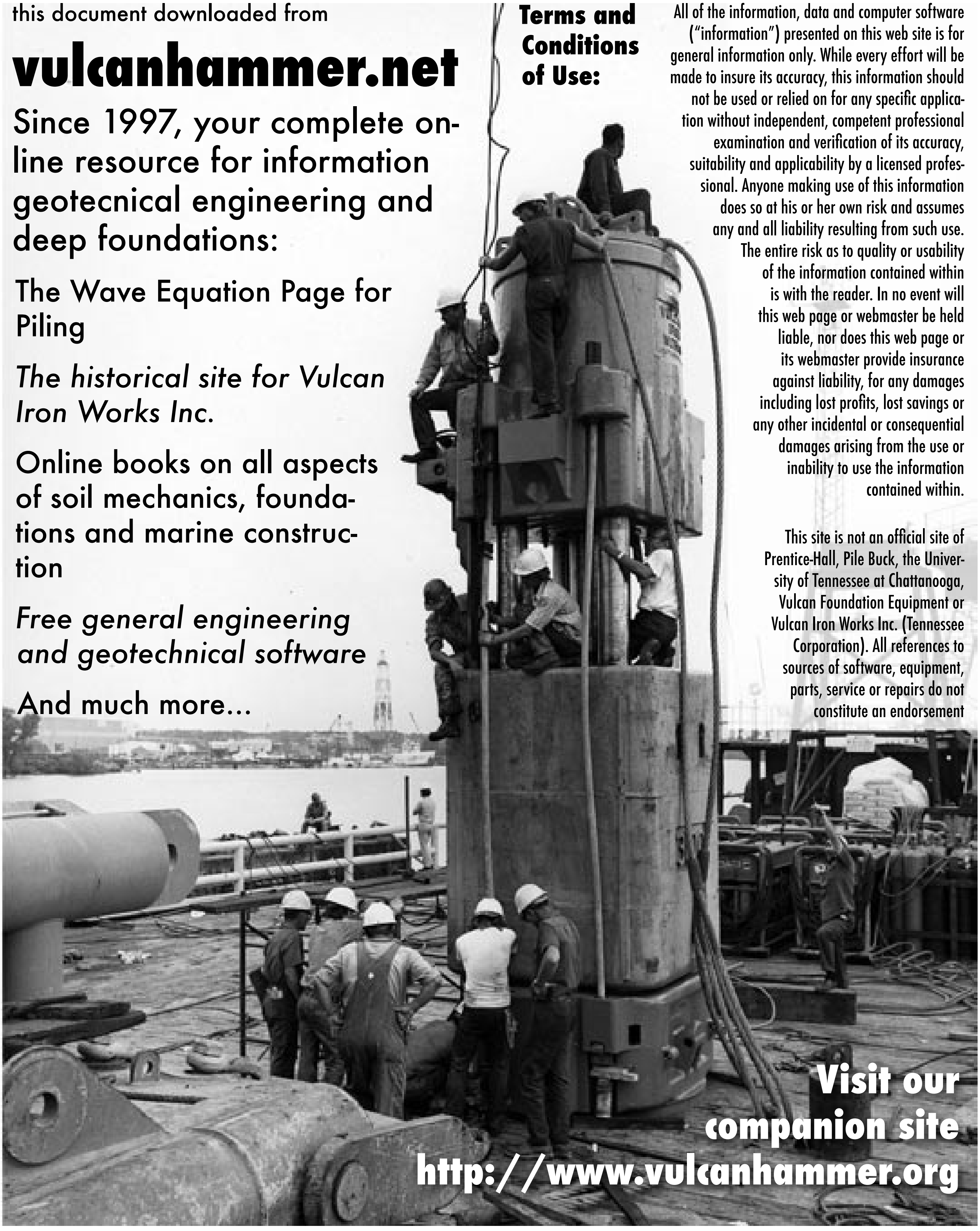

DYNAMIC BEHAVIOR OF PILING

A Dissertation

By

Lee Leon Lowery, Jr.

Submitted to the Graduate College of the
Texas A&M University in
partial fulfillment of the requirements for the degree of

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Chairman of Committee

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CHAPTER I

INTRODUCTION

General Background

The problem of pile-driving analysis has been of great interest to engineers for many years. Ever since the first engineer proposed a method for predicting the load carrying capacity of a pile, the whole subject of pile driving has become a much debated field in engineering. In other areas new methods of analysis for structural elements and systems are constantly being proposed with little or no resulting discussion. However, the proposal of a new piling analysis is sure to stir much interest and often some rather heated discussions.

Since over four-hundred pile-driving formulas have been proposed[1], not including the countless formula modifications which are used[2], many engineers resort to the use of only one or two formulas regardless of the driving conditions encountered[3]. Although many of the erroneous assumptions made in these formulas have been widely discussed[4,5], the fact that they omit many significant parameters which affect the problem seems to have received less attention. However, when the driving formulas omit parameters which change from case to case, the engineer has no means of determining how significant the parameter may be, nor can he tell in which direction or to what extent the change will vary the results. Thus, to obtain an accurate solution obviously requires that fewer erroneous assumptions be made regarding the dynamic behavior of the materials and equipment used in pile driving, and that all significant parameters are included in the analysis.

The first of these problems was solved when it was noted that pile driving is actually a case of longitudinal impact, governed by the wave equation rather than by statics or rigid-body dynamics[6,7]. However, since the exact simulation and solution of the wave equation applied to piling are extremely complex for all but the simplest problems, many significant parameters still had to be neglected.

The second problem was later solved by Smith[8], who proposed a numerical solution to the wave equation, capable of including any of the known parameters involved in pile-driving analysis. This method of analysis was applicable to tapered, stepped, and composite piles, to non-linear soil resistance and soil damping, to piles having several cushions, followers, helmets, etc. In other words, it was a completely general method for simulation and analysis of the complex problem of pile driving.

It should be noted that much of the experimental work used in this report was reported by other investigators. These cases are referenced, and the problem number or name used herein will be the same as used by the original reporter. This will enable the reader to identify the problem being solved and to determine exactly what information was reported by referring to the original paper.

Objectives

The objectives of this research are:

1. To review and summarize Smith's original method of analysis and to derive a more general solution.

2. To determine how the numerical solution is affected by the elasticity of the ram.

3. To compare results given by the wave equation with those determined by laboratory experiments and field tests.

4. To illustrate the significance of the parameters involved, including cushion stiffness and damping, ram velocity, material damping in the pile, soil damping and quake, and to determine the quantitative effect of these parameters where possible.

5. To show how the wave equation can be used to determine the dynamic or impact characteristics of the materials involved.

6. To determine the dynamic properties of the cushion subjected to impact loading.

7. To study the effect of internal damping in the pile and its significance.

Literature Review

The basic purpose of any pile driving formula is to permit the design of a functional yet economical foundation. According to Chellis[9], there are four basic types of driving formulas:

1. Empirical formulas, which are based on statistical investigations of pile load tests,

2. Static formulas, which are based on the side frictional forces and point bearing force on the pile, as determined by soils investigations,

3. Dynamic formulas, which assume that the dynamic soil resistance is equal to the static load capacity of the pile, and

4. The wave equation, which assumes only those parameters for which the behavior has not yet been determined experimentally. Each of the preceeding formulas has advantages and disadvantages which have

been widely noted[10,11] and need not be restated at this time.

Isaacs is thought to have first noted that the wave equation is applicable to the problem of pile driving[12]. However, Fox[13] was probably the first person to propose that an exact solution be used for pile-driving analysis. Shortly thereafter, Glanville, Grime, Fox, and Davies[14] published the first correlations between experimental studies and results determined by the exact solution to the wave equation developed by Fox. Since this exact solution was extremely complex, they were forced to use simplified boundary conditions including zero side frictional resistance, a perfectly elastic cushion block, and an elastic soil spring acting only at the tip of the pile. However, even using these simplified boundary conditions, they obtained reasonably accurate results.

In 1940 Cummings[15] discussed several errors inherent in dynamic pile-driving formulas and reviewed the previous work done using the wave equation. However, he also noted that even for the simplest problems, "the complete solution includes long and complicated mathematical expressions so that its use for a practical problem would involve laborious numerical calculations."

A practical pile-driving problem usually involves side frictional soil resistance, soil damping constants, nonlinear cushion and capblock springs, and other factors which prevent a direct solution of the resulting differential equation. However, in 1950 Smith[16] proposed a mathematical model and a corresponding numerical method of analysis which enabled him to account for the effects of any parameters which might influence the problem. He has since continued to update his method and has published various other works[17,18,19,20,21].

Smith's method of solution did not really become popular until 1960 when he published a summary of his numerical method applied to pile-driving analysis[22]. In this paper he recommended a number of material constants and several material behavior curves to account for the dynamic action of the soil, cushions, and pile material.

Smith's method of analysis received considerable interest[23], and two applications of the wave equation were suggested:

1. The immediate application of the wave equation, using the most probable material properties to predict ultimate driving resistance and driving stresses.

2. To perform extensive parameter studies in order to determine trends and to gain more insight into

the behavior of pile driving, and also to determine the relative significance of these parameters.

Immediately after the appearance of Smith's paper in 1960, the Bridge Division of the Texas Highway Department initiated a research project with the Texas Transportation Institute to perform exhaustive studies of the behavior of piling by the wave equation. Their first report dealt with a computer program based on Smith's numerical solution[24]. This program was immediately used to study the magnitude of stress in prestressed concrete piling which failed during driving[25], and later to check conditions at other sites which might cause pile breakage due to excessive driving stresses[26].

In September, 1962 Hirsch[27] published a major work designed to correlate field data, including driving stresses and pile displacements, with the theoretical computer solution.

Forehand and Reese[28] investigated the possibility of predicting the ultimate bearing capacity of piling using the wave equation, but since complete data was available for relatively few problems, they were unable to draw many firm conclusions. They also studied the dynamic action of the soil during driving and recommended some values for the soil parameters used in the wave equation.

In August, 1963 several extensions of Smith's method were presented by Samson, Hirsch, and Lowery[29]. Two simple cases for which the "exact" solution was known were compared with Smith's numerical solution to indicate the method's accuracy. A third section of the paper presented the results of a short parameter study indicating how certain trends in pile driving might be determined and how to study the significance of various parameters. The results for several theoretical and field test problems were also compared.

In 1963 Hirsch, Samson, and Lowery[30] published a study on the methods employed in measuring dynamic stresses and displacements of piling during driving, and presented further experimental and theoretical comparisons "to demonstrate that the computer solution of the wave equation offers a rational approach to the problems associated with the structural behavior of piling during driving." This report was based on an extensive study by Hirsch[31].

Another major investigation by Hirsch[32] involved a study of the variables which affected the behavior of concrete piles during driving. Over 2100 separate problems were solved and the results were presented in the form of graphs for use by design engineers. The author drew many valuable conclusions from this study, some of which were already known from years of experience, while others were original and had not previously been recognized.

In another paper dealing with stress wave theory, Samson, Bundy, and Hirsch[33] discussed various practical applications of the wave equation related to the design of long prestressed concrete piles driven at several Texas Gulf Coast sites.

In several instances, Hirsch and Samson combined practical pile-driving experience with the use of the wave equation to determine correct driving practices and to design prestressed concrete piles[34,35].

Hirsch's[36] latest publication deals with the dynamic load-deformation properties of various pile cushion materials and other dynamic properties of materials required to simulate as closely as possible the actual behavior of a pile during driving.

CHAPTER II

A NUMERICAL METHOD OF ANALYSIS

The Basic Solution

Since 1931 it has been realized that pile driving involved theories of longitudinal impact rather than statics and that the behavior of piling during driving was actually governed by the wave equation. However, the application of the wave equation to pile driving was restricted to very simple problems because the exact solution was complex, involved much labor, and for most practical cases, required many simplifying assumptions.

However, in 1950 Smith[37] proposed an approximate solution based on concentrating the distributed mass of the pile (shown in Figure 2.1a) into a series of small weights, W(1) thru W(MP), connected by weightless springs K(1) thru K(MP-1), with the addition of soil resistance acting on the masses, as illustrated in Figure 2.1b. Time also was divided into small increments. This numerical solution to the wave equation is then applied by the repeated use of the following equations, developed by Smith[38]:

$$D(m,t) = D(m,t-1) + 12\Delta t V(m,t-1) \qquad \text{Eq. 2.1}$$

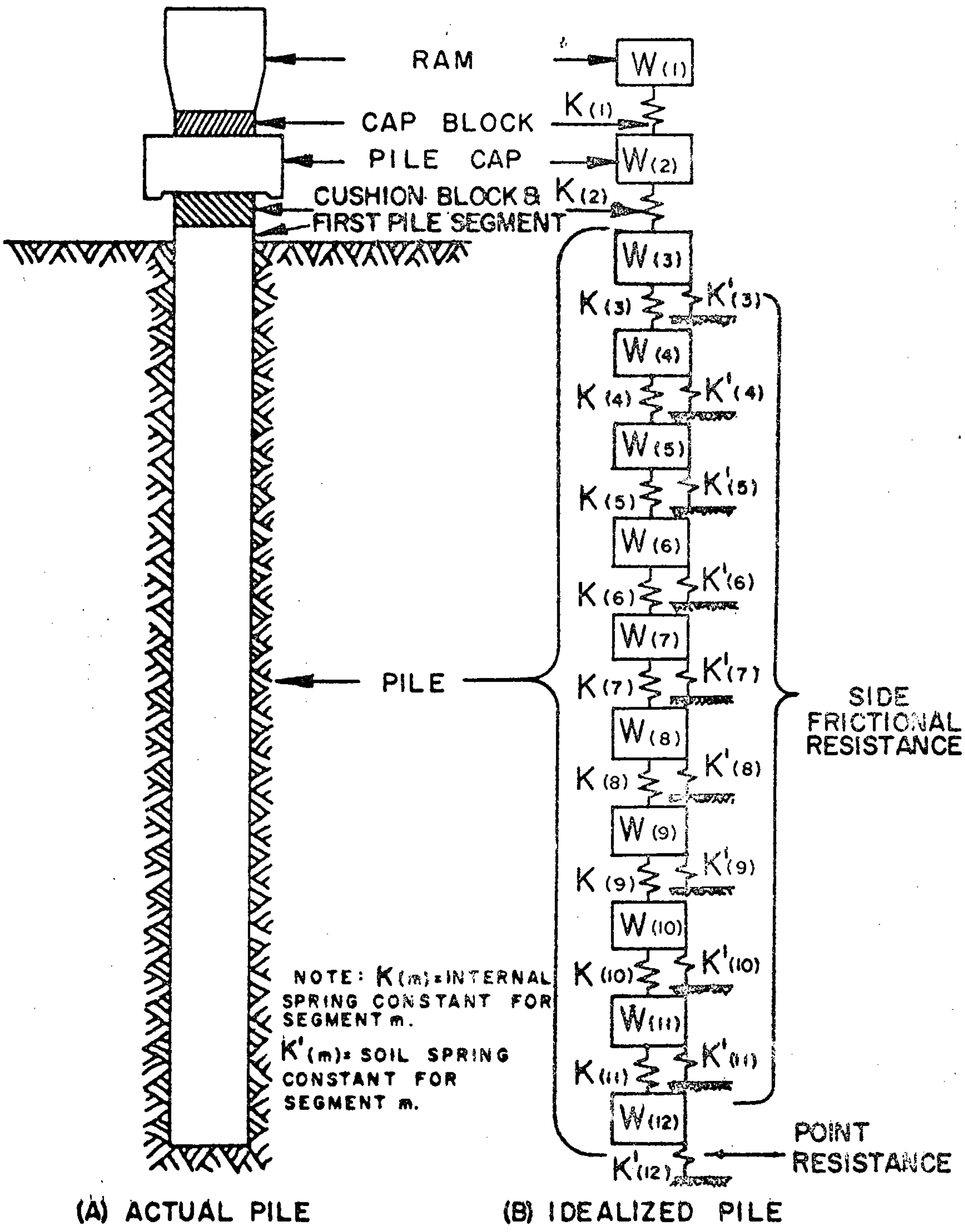


FIGURE 2.1 - IDEALIZATION OF A PILE FOR PURPOSE OF ANALYSIS

$$C(m,t) = D(m,t) - D(m+1,t) \qquad \text{Eq. 2.2}$$

$$F(m,t) = C(m,t)K(m) \qquad \text{Eq. 2.3}$$

$$R(m,t) = [D(m,t)-\underline{D'(m,t)}]\, K'(m)[1+J(m)V(m,t-1)] \qquad \text{Eq. 2.4}$$

$$V(m,t) = V(m,t-1)+[F(m,t)-R(m,t)]\, g\Delta t/W(m) \qquad \text{Eq. 2.5}$$

where $\underline{m}$ is the mass number; $\underline{t}$ denotes the time interval number; $\underline{\Delta t}$ is the size of the time interval (sec); D(m,t) is the total displacement of mass number $\underline{m}$ during time interval number $\underline{t}$(in.); V(m,t) is the velocity of mass $\underline{m}$ during time interval $\underline{t}$(ft/sec); C(m,t) is the compression of spring $\underline{m}$ during time interval $\underline{t}$(in.); F(m,t) is the force exerted by spring number $\underline{m}$ between segment numbers (m) and (m+t) during time interval $\underline{t}$(lb); and K(m) is the spring rate of mass $\underline{m}$ (lb/in.). Note that since certain parameters do not change with time, they are assigned a single subscript.

The quantity R(m,t) is the total soil resistance acting on segment $\underline{m}$(lb/in.); K'(m) is the spring rate of the soil spring causing the external soil resistance force on mass $\underline{m}$(lb/in.); D(m,t) is the total inelastic soil displacement or yielding during the $\underline{t}$ at segment $\underline{m}$(in.); J(m) is a damping constant for the soil acting on segment number ($\underline{m}$)(sec/ft); g is the gravitational acceleration (ft/sec^2); and W(m) is the weight of segment number m(lb).

The solution is begun by initializing the time-dependent parameters to zero and by giving the ram an initial velocity. Then an incremental amount of time Δt elapses during which the ram moves down an amount given by Equation 2.1. The displacements D(m,I) of the other masses are computed in the same manner.

Equation 2.2 is then used to determine the compressions C(m,I), after which the internal spring forces acting between the masses are found from Equation 2.3 and the external soil forces R(m,I) are computed from Equation 2.4.

Finally, a new velocity V(m,I) is determined for each mass using Equation 2.5, after which another time interval elapses. New displacements, compressions, forces, and velocities are again computed using the same equations and the cycle is repeated until the solution is obtained. Smith[39] and others[40,41], give a detailed explanation of this method of solution and the computer programming required. The dynamic behavior of various parameters will be discussed later.

Smith would have probably caused little interest had he simply given a numerical solution for the wave equation. Instead he presented a simple, physical model, easily visualized, using parameters which are readily understood. This and the simplicity of the equations required for a solution doubtlessly account for much of the wave equation's increasing popularity as a means of studying the behavior of piling.

Modifications of the Original Solution

Although the original method of analysis proposed by Smith can be used to solve many of the problems given in this report, it has been greatly extended to include other idealizations. The major additions and changes are summarized here for reference only, and are fully discussed in later chapters.

1. The relationship between soil resistance to penetration of the pile was originally limited to a series of straight lines. The revised program allows the use of any shape for this curve, as noted in Chapter VI.

2. The elastic soil deformation "Q" and the soil damping constant "J" were each limited to one value at the point of the pile and a second value for side resistance. These parameters have been generalized to include different values at each pile segment.

3. A new method by which internal damping in the pile can be accounted for is now included. This method is explained in Chapter V.

4. A second method is included to account for the coefficient of restitution of the capblock or cushion-block.

5. For correlation with experimental data, it is now possible to place forces directly on the head of the pile rather than having to calculate them from the hammer-cushion-anvil properties. This method was used extensively where the force vs time curve at the head of the pile was known; since then the hammer, cushion, and anvil properties did not influence the solution.

6. The linear force vs compression curve for various cushion materials used previously has been generalized as noted in Chapter IV.

7. The effect of gravity on the solution can now be accounted for.

8. A special "parameter study" sub-program was written which was included in the general program. This feature was used to vary specific parameters or groups of parameters between specified limits in order to study their influence on the solution, and to see if trends could be found.

9. For possible later use, several pile-driving formulas were included in the computer program.

10. The soil resistance on the point segment now uses two springs, one for the side friction acting on

the side of the pile and a second spring for point bearing.

CHAPTER III

PILE DRIVING HAMMERS

Ram Idealization

Smith[42] suggests that since the ram is usually short in length, in many cases it can accurately be represented by a single weight having infinite stiffness. The example illustrated in Figure 2.1 makes this assumption since K(1) represents the spring constant of only the cap block, the elasticity of the ram having been neglected. He also notes that where greater accuracy is desired, or when the ram is long and slender, it can also be divided into a series of weights and springs. However, no work has been done to determine how long the ram can be before its elasticity affects the accuracy of the solution, and therefore should also be divided into several segments. The most common hammers in the above class include drop, air, and steam hammers. Figures 3.1 and 3.2 show how the ram may be idealized.

In order to determine how great is the influence of dividing the ram into a number of segments, several ram lengths ranging from 2 to 10 feet were assumed, driving a 100 ft pile with point resistance only. For this parameter study the total weight of the pile varied

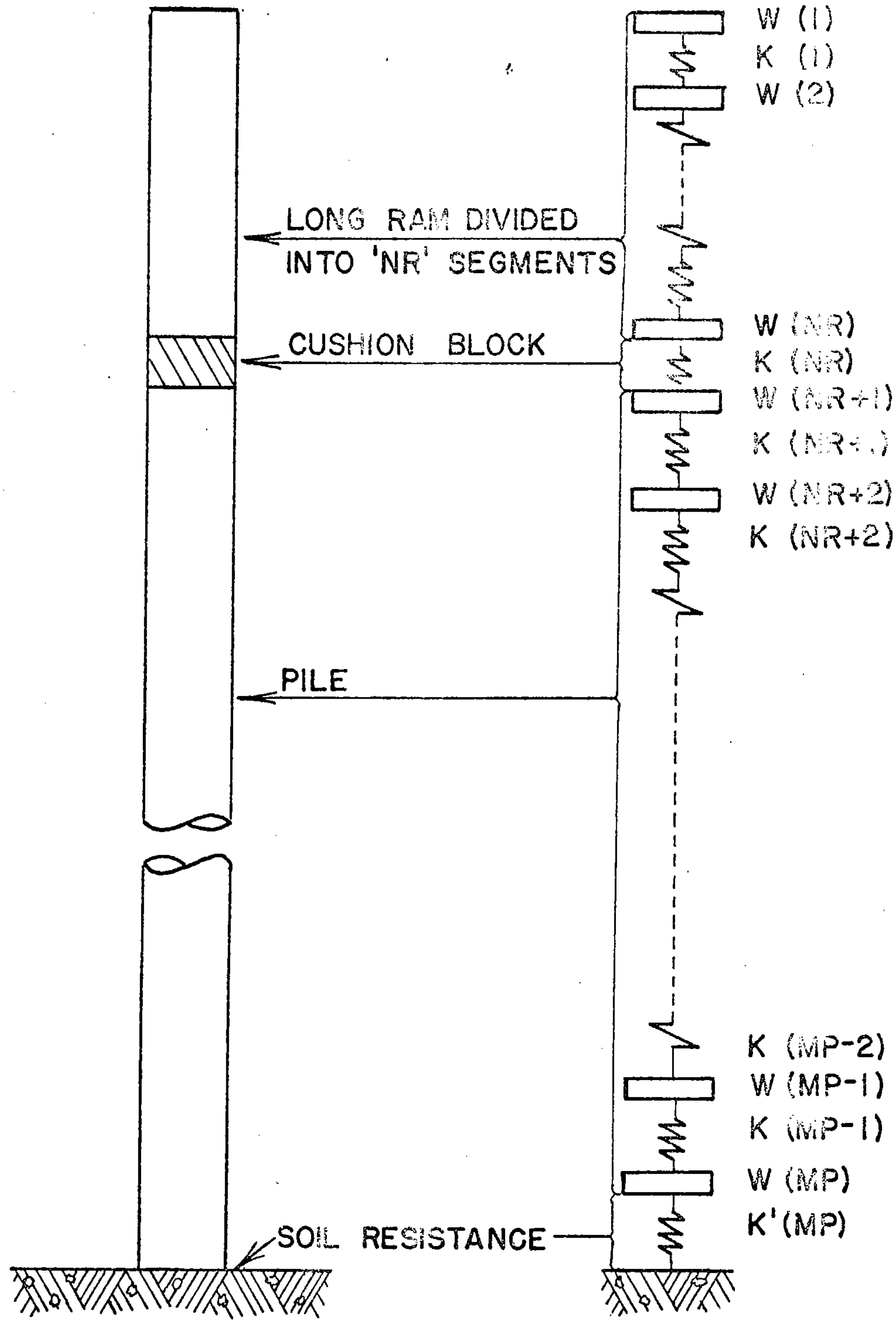


FIGURE 3.1–IDEALIZATION FOR A LONG RAM STRIKING DIRECTLY ON A CUSHION BLOCK

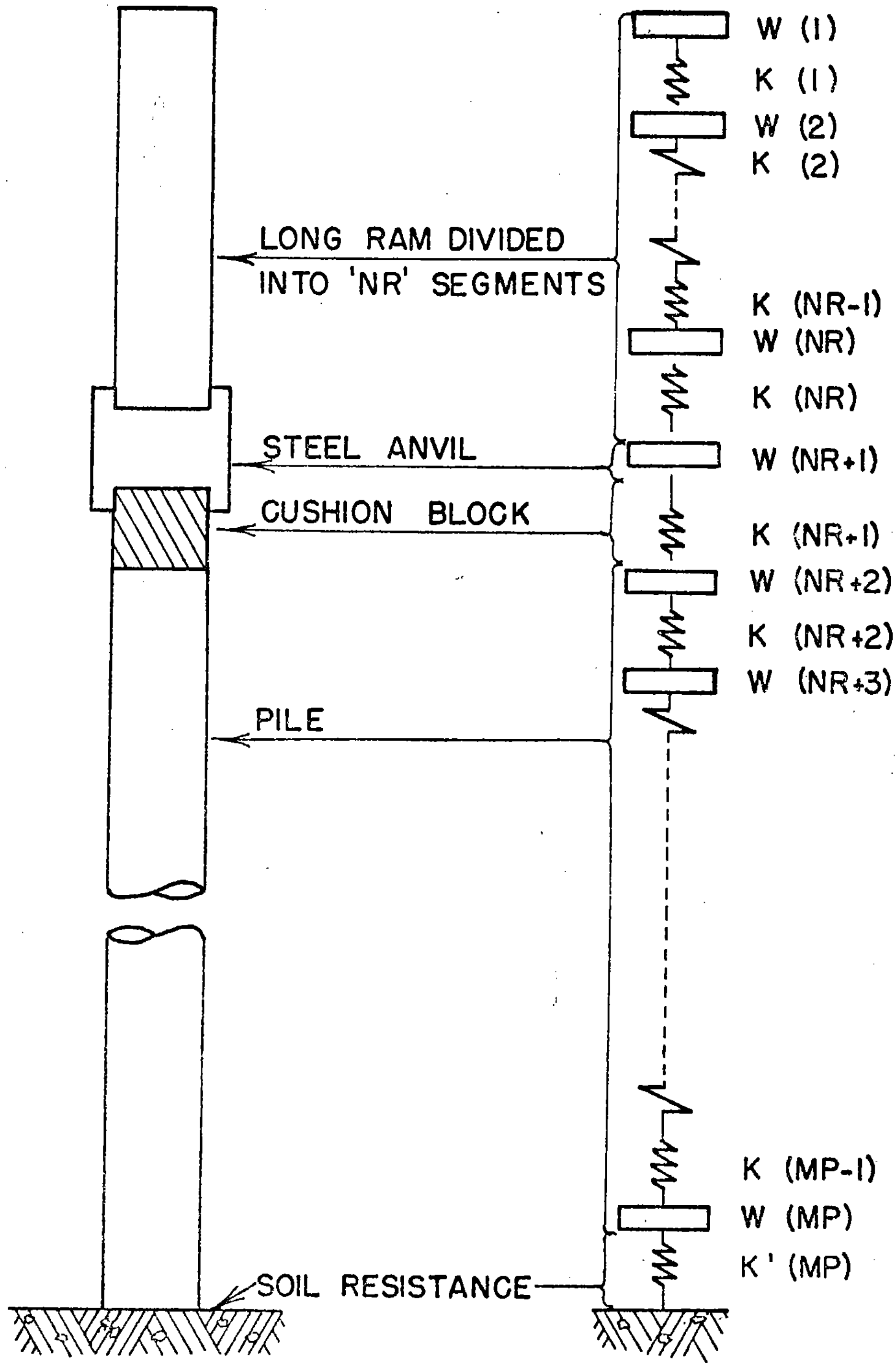


FIGURE 3.2-IDEALIZATION FOR A LONG RAM STRIKING DIRECTLY ON A STEEL ANVIL

from 1,500 lb to 10,000 lb, while the ultimate soil resistance ranged from zero to 10,000 lb. The cushion was assumed to have a stiffness of 2,000 kip/in.

Table 3.1 lists the results found for a typical problem solved in this series, the problem consisting of a 10 ft ram traveling at 20 ft/sec, striking a cushion having a stiffness of 2,000 kip/in. The pile used was a 100 ft 12H53 steel pile, driven by a 5,000 lb ram with an initial velocity of 12.4 ft/sec. No pile cap was included in the solution, the cushion being placed directly between the hammer and the head of the pile. Since the ram was divided into very short lengths, the pile was also divided into short segments.

As shown in Table 3.1, the solution is not changed to any extent, regardless of whether the ram is divided into 1, 2, or 10 segments. The time interval Δt was held constant in each case.

This is further evidenced by noting Table 3.1, which gives the effects of dividing the pile into segments shorter than the normal ten ft lengths. When the pile segment length changes from 10 ft to 1.25 ft the tensile stress changes almost 25 percent whereas changing the number of ram segments affects the solution less than one percent. However, the use of a driving cap is common practice and its addition into the system usually

TABLE 3.1 EFFECT OF BREAKING THE RAM INTO SEGMENTS WHEN RAM STRIKES A CUSHION

Number of Ram Divisions	Length of Pile Segments (ft)	Maximum Compressive Force in Pile (kip)	Maximum Tensile Force in Pile (kip)	Maximum Point Displacement (in)
1	10.0	305.4	273.9	3.019
1	5.0	273.8	245.9	3.042
1	2.5	265.6	224.8	3.053
1	1.25	263.1	219.0	3.057
2	1.25	262.6	218.8	3.058
10	1.25	262.9	218.5	3.059? it will not reach a max
		262.9	219.9 △	
		262.9	217.9 ▽	no soil resistance

reduces these percentages. It has previously been shown that the lengths of the pile segments normally have little effect on the solution[41].

In certain hammers such as a diesel hammer, the ram strikes directly on a steel anvil rather than on a cushion. This makes the choice of a spring rate between the ram and anvil difficult because the impact occurs between two steel elements. One possible solution is to place the spring constant of the entire ram between the weights representing the ram and anvil. Also, the ram can be broken into a series of weights and springs as is the pile.

To determine when the ram in this case should be divided, a parameter study was run in which the ram length varied between 6 and 10 ft and the anvil weight from 1,000 to 2,000 lb. In each case the ram diameter was held constant and the ram was divided equally into segment lengths as noted in Table 3.2. These variables were picked because of their possible influence on the solution.

The pile used was again a 12H53 point bearing pile with a cushion of 2,000 kip/in. spring constant placed between the anvil and head of the pile. The soil parameters used were RU_{total} = 500 kip, Q = 0.1 in.,

TABLE 3.2 EFFECT OF BREAKING RAM INTO SEGMENTS WHEN RAM STRIKES A STEEL ANVIL

Anvil Weight lb	Ram Length ft	Number of Ram Divisions	Length of Each Ram Segment ft	Maximum Compressive Force on Pile: At Head kip	Maximum Compressive Force on Pile: At Center kip	Maximum Compressive Force on Pile: At Tip kip	Maximum Point Displacement in.
2000	10	1	10	513	513	884	0.207
		2	5	437	438	774	0.159
		5	2	373	373	674	0.124
		10	1	375	375	678	0.125
	8	1	8	478	478	833	0.183
		4	2	359	359	648	0.117
		8	1	360	360	651	0.118
	6	1	6	430	430	763	0.155
		3	2	344	344	621	0.110
		6	1	342	342	616	0.109
1000	10	1	10	508	509	878	0.160
		2	5	451	451	789	0.159
		5	2	381	382	691	0.151
		10	1	371	372	681	0.153
	8	1	8	487	488	846	0.151
		4	2	443	444	785	0.144
		8	1	369	370	675	0.134
		10	0.8	337	338	665	0.133
	6	1	6	457	457	798	0.137
		3	2	361	362	666	0.128
		6	1	316	316	562	0.109
		10	0.6	320	320	611	0.113

What about relative length of ram to pile? L_r / L_p

and J = 0.15 sec/ft. These factors were held constant for all problems listed in Table 3.2.

The most obvious result shown by Table 3.2 is that when the steel ram impacts directly on a steel anvil dividing the ram into segments has a marked effect on the solution.

For what conditions?

An unexpected result of this study is that even a short ram should be broken into segments. As seen in Table 3.2, regardless of the total ram length, the solutions for forces and displacements continued to change until a ram segment length of 2 ft for the 2,000 lb anvil, and a segment length of 1 ft for the 1,000 lb anvil was reached.

Energy Output of Hammer

One of the most significant parameters involved in pile driving is the velocity of the ram immediately before impact. This velocity is often used to determine the maximum kinetic energy of the hammer and its energy output rating, and must also be known or assumed before the wave equation can be applied.

Although most manufacturers of the pile-driving equipment specify the output energies of their hammers, these are usually downgraded by foundation experts because of the lack of a consistent method of determining the output and because of the difficulty encountered in verifying the recommended values. A number of possible factors such as poor hammer condition, lack of lubrication, and wear are also known to seriously reduce the energy output of a hammer. However, to determine how much the rated energy of any given hammer should be reduced is not a simple task.

Chellis[43] discusses several reasons for this energy reduction and recommends a number of possible efficiency factors for the commonly used hammers, based on his observations and previous experience.

The Michigan Study of Pile Driving Hammers

In 1965 the Michigan State Highway Commission[44] completed an exhaustive research program designed to obtain a better understanding of the complex problem of pile driving, and though a number of specific objectives were given, one objective was of primary importance. As noted by Housel[45], "Hammer energy actually delivered to the pile, as compared with the manufacturer's rated energy, was the focal point of a major portion of this investigation of pile-driving hammers." In other words, they hoped to determine more accurate energy ratings for the hammers tested, and to compare these values with the manufacturer's ratings.

The energy transmitted to the pile was termed "ENTHRU" by the authors and was determined by the summation

$$ENTHRU = \Sigma F \Delta S$$

where F, the force on the top of the pile, was measured by a specially designed load cell, and ΔS, the resulting movement of the head of the pile, was found using displacement transducers and/or accelerometers.

However, since so many parameters influence the problem, and since these parameters are continually changing during the pile driving operation (e.g. condition of the cushion, length of the pile, soil resistance, etc.), it seems unlikely that a single efficiency factor could be found for any given hammer. More likely a range of efficiencies will result.

As noted in the Michigan report[46]: "Hammer type and the operation; soil conditions; pile type, mass, rigidity, and length; and the type and condition of cap blocks were all factors that affected ENTHRU, but when, how, and how much could not be ascertained with any degree of certainty." However, since the wave equation is able to account for these factors, their effects can be determined.

Before analyzing any of the Michigan cases, further explanation of the reported data is required.

As noted in the Michigan report, ENTHRU was not actually a direct measure of the hammer's efficiency or energy output since the forces and displacements were measured below the capblock, as shown in Figure 3.3. Thus ENTHRU was defined as the amount of work done on the load cell.

The maximum displacement of the head of the pile was also reported and was designated LIMSET. Oscillographic records of force vs time measured in the load cell were also reported. Since force acceleration was measured only at these points, the maximum observed values will be called FMAX and AMAX, respectively.

Problem Information

In selecting which of the Michigan pile problems to solve by the wave equation, it was decided to run at least two problems for each hammer used at each of the three testing sites. As shown in Table 3.3, cases selected from the Belleville site include two pile lengths for each of four different hammers. Similarly, the Detroit and Muskegon site problems are summarized in Tables 3.4 and 3.5. Figures 3.4 and 3.5 illustrate how these problems were idealized for purposes of analysis.

Even though the Michigan study is one of the most

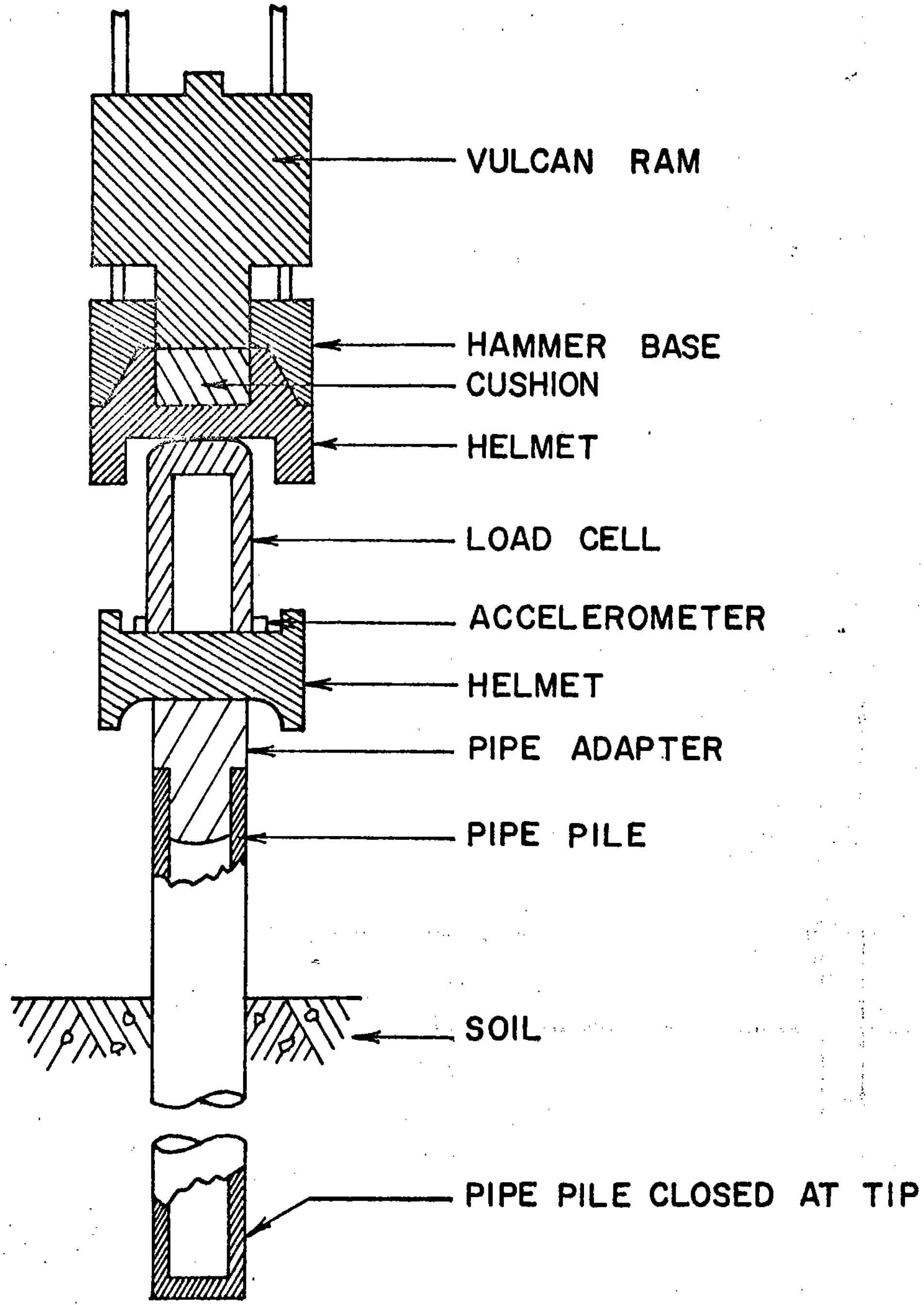


FIGURE 3.3 – TYPICAL PILE DRIVING ASSEMBLY (AFTER REFERENCE 44)

TABLE 3.3 SUMMARY OF BELLEVILLE CASES SOLVED BY WAVE EQUATION

PILE I.D.	CASE	HAMMER*	CUSHION	PILE INFORMATION TYPE	TOTAL LENGTH (ft)	EMBEDDED LENGTH (ft)
BLTP-6	10.0/57.9	V-1	Oak	12H53	32.5/72.5	10.0/57.9
BLTP-4	25.0/66.4	LB-312	Micarta	12 in. Pipe 0.25 in. wall	40.7/81.3	15.0/56.4
BRP-4	20.0/50.0	M-DE30	Oak	12H53	40.0/60.0	20.0/50.0
BLTP-5	15.0/60.0	D-D12	German Oak	12 in. Pipe 0.179 in. wall	40.0/80.0	5.0/50.0

* Hammer designations are as follow:

V-1 = Vulcan 1
V-50C = Vulcan 50C
V-80C = Vulcan 80C
LB-312 = Link Belt 312
LB-520 = Link Belt 520
M-DE30 = McKiernen-Terry DE-30
M-DE40 = McKiernen-Terry DE-40
D-D12 = Delmag D-12
D-D22 = Delmag D=22

TABLE 3.4 SUMMARY OF DETROIT CASES SOLVED BY WAVE EQUATION

PILE I.D.	CASE	HAMMER	CUSHION	PILE INFORMATION TYPE	TOTAL LENGTH (ft)	EMBEDDED LENGTH (ft)
DLTP-8	41.5 / 80.2	V-1	Oak	12H53	80.1 / 97.0	41.5 / 80.2
DTP-5	20.0 / 79.0	V-50C	Micarta	12 in. Pipe 0.179 in. wall	40.0 / 84.0	20.0 / 79.0
DRP-3	40.0 / 60.0	LB-312	Micarta	12H53	80.0 / 80.0	40.0 / 60.0
DTP-13	40.0 / 80.7	M-DE30	Oak	12 in. Pipe 0.179 in. wall	45.0 / 90.7	40.0 / 80.7
DTP-15	20.0 / 80.5	D-D12	German Oak	12H53	46.1 / 86.1	20.0 / 80.5

TABLE 3.5 SUMMARY OF MUSKEGON CASES SOLVED BY WAVE EQUATION

PILE I.D.	CASE	HAMMER	CUSHION	PILE INFORMATION: TYPE	TOTAL LENGTH (ft)	EMBEDDED LENGTH (ft)
MLTP-2	20.0 / 53.0	V-1	Oak	12 in. Pipe 0.250 in. wall	45.0 / 60.0	20.0 / 53.0
MLTP-9	72.0 / 127.0	V-80C	Micarta	12 in. Pipe 0.250 in. wall	80.0 / 134.0	72.0 / 127.0
MTP-12	30.5 / 70.8	LB-520	Micarta	12 in. Pipe 0.250 in. wall	40.0 / 80.0	30.5 / 70.8
MTP-11	69.5 / 150.0	M-DE40	Oak and Plywood	12 in. Pipe 0.250 in. wall	80.0 / 160.0	69.5 / 150.0
MLTP-8	31.0 / 178.0	D-D22	German Oak	12 in. Pipe 0.250 in. wall	40.0 / 185.0	31.0 / 178.0

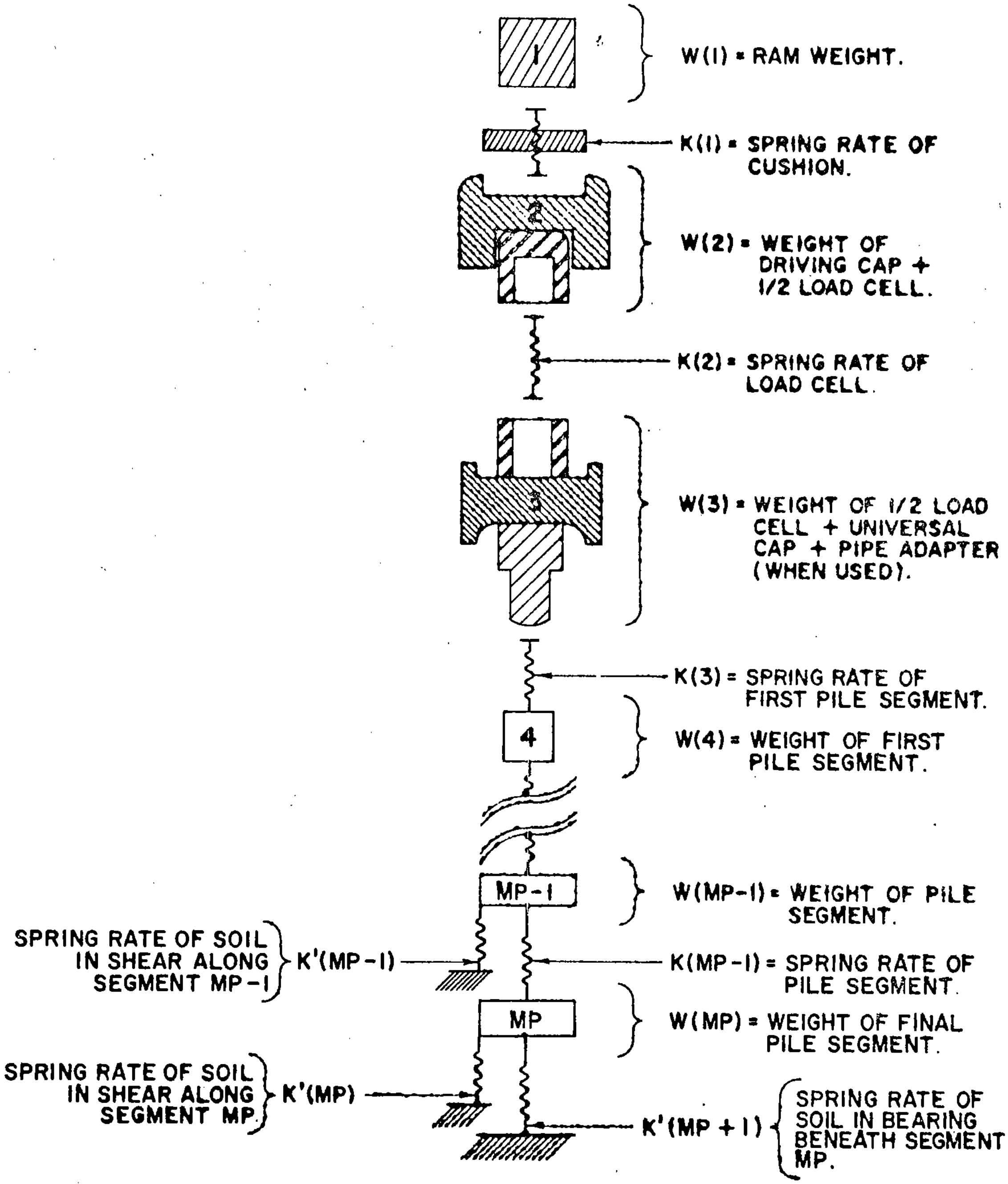


FIGURE 3.4 - IDEALIZATION OF A VULCAN HAMMER

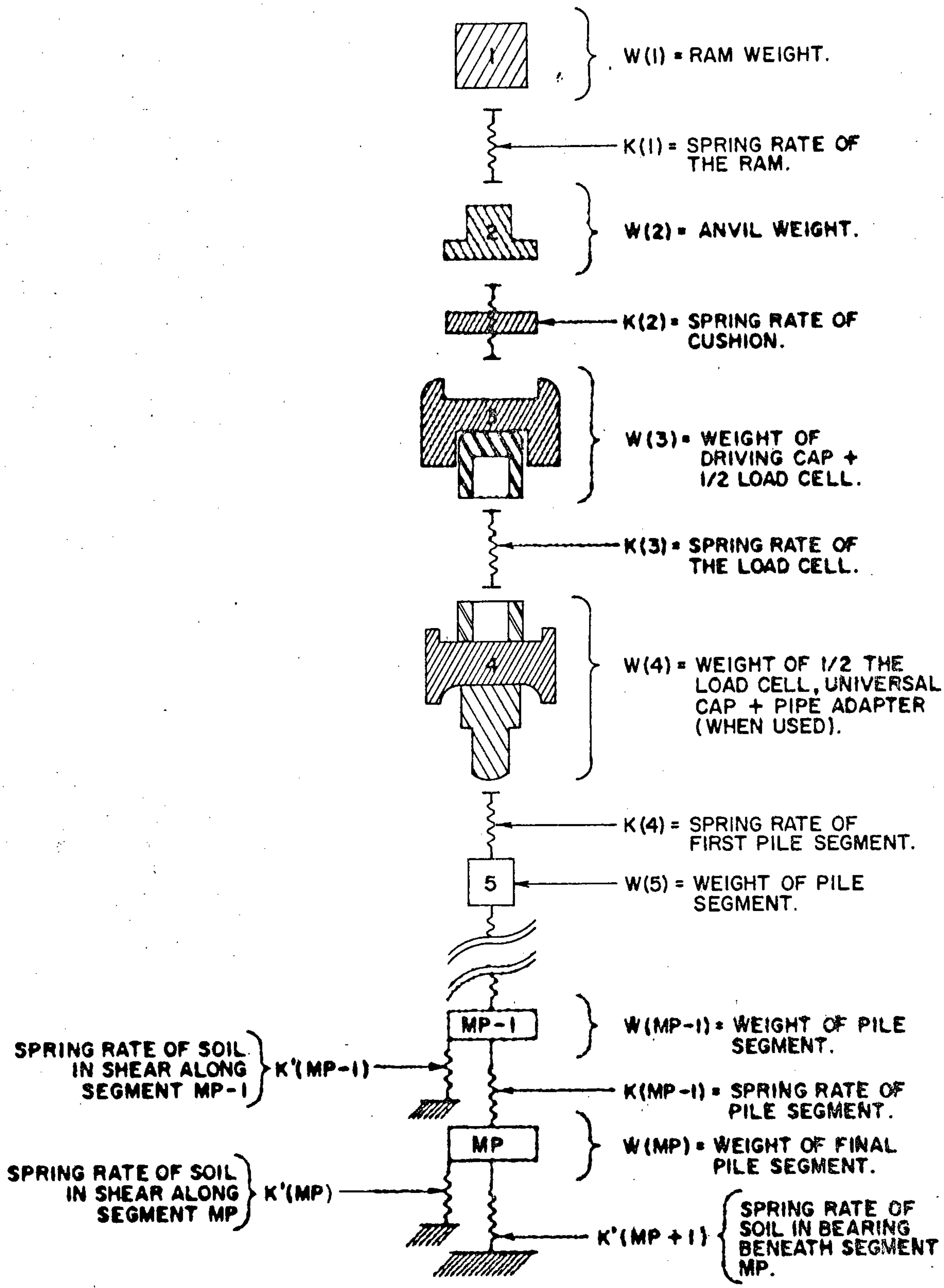


FIGURE 3.5 - IDEALIZATION OF A DIESEL HAMMER

completely documented and fully reported research projects published concerning pile driving, certain information was not reported which must be known in order to apply the wave equation. This omission was not the result of any failure in reporting the data, but rather stems from the fact that the methods of analysis used in the Michigan project could not have utilized the data, and they were therefore not obtained. Probably the best example is the lack of information concerning the stiffness of the cushion and how the stiffness varied during driving. Preferably, the spring rate of the cushion would have been given at each depth of penetration for which other data were reported.

Preliminary Studies and Conclusions

Since cushion-block information was not given, and because the cushion stiffness varies greatly during driving, a broad parameter study was made using the first case mentioned in Table 3.3. In this study the cushion stiffness was varied by a factor of 50, from 540 kip/in up to 27,000 kip/in. Also studies was the effect of varying the total soil resistance, RUT, using resistances of 30, 90, and 150 kip and ram velocities of 8, 12, and 16 ft/sec.

The results of this parameter study indicate what a powerful tool the wave equation is, and how helpful it can be in understanding how the multitude of parameters affect the pile-driving problem. The solutions from ENTHRU, FMAX, and LIMSET resulting from a change in the cushion stiffness, soil resistance, and ram velocities are given in Tables 3.6, 3.7, and 3.8 respectively. Whereas before it could not be determined "when, how, or how much," this study indicates that for this particular problem, 1) ENTHRU is nearly independent of the cushion block used, since the cushion stiffness was increased by a factor of 50 while influencing ENTHRU only slightly, 2) ENTHRU has a slight tendency to increase as the driving resistance increases, 3) FMAX is almost completely independent of the driving resistance, 4) FMAX is almost linearly related to the hammer velocity, 5) FMAX consistently increases as the cushion stiffness increases, and 6) LIMSET is only slightly changed regardless of the spring rate of the cushion block.

Thus for the first time, a number of trends may be established for various pile driving situations by using the wave equation.

Determination of Hammer Efficiency

In order to analyze the above problem, certain data

TABLE 3.6 EFFECT OF CUSHION STIFFNESS ON ENTHRU FOR BLTP-6; 10.0

ENTHRU (kip ft)

Ram Velocity (ft/sec)	RUT (kip)	Cushion Stiffness (kip/in)			
		540	1080	2700	27,000
8	30	3.0	3.0	3.0	2.9
	90	3.1	3.2	3.3	2.9
	150	3.0	3.2	3.3	3.0
12	30	6.6	6.4	7.1	6.4
	90	7.0	7.1	7.2	6.4
	150	6.9	7.2	7.4	6.7
16	30	11.8	11.9	12.2	11.3
	90	12.3	12.6	12.8	11.5
	150	12.4	12.9	13.2	11.4

TABLE 3.7 EFFECT OF CUSHION STIFFNESS ON FMAX FOR BLTP-6; 10.0

		FMAX (kip)			
Ram Velocity (ft/sec)	RUT (kip)	Cushion Stiffness (kip/in)			
		540	1080	2700	27,000
8	30	132	185	261	779
	90	137	185	261	779
	150	143	186	261	779
12	30	198	278	391	1,169
	90	205	278	391	1,169
	150	215	279	391	1,169
16	30	264	371	522	1,558
	90	275	371	522	1,558
	150	288	371	522	1,558

TABLE 3.8 EFFECT OF CUSHION STIFFNESS ON LIMSET FOR BLTP-6; 10.0

LIMSET (in)					
Ram Velocity (ft/sec)	RUT (kip)	Cushion Stiffness (kip/in)			
		540	1080	2700	27,000
8	30	1.09	1.08	1.08	1.13
	90	0.44	0.44	0.45	0.45
	150	0.32	0.33	0.33	0.33
12	30	2.21	2.14	2.19	2.25
	90	0.80	0.82	0.84	0.84
	150	0.55	0.57	0.58	0.58
16	30	3.62	3.59	3.63	3.68
	90	1.30	1.31	1.32	1.34
	150	0.85	0.87	0.88	0.90

given in the Michigan pile project will be used, for example the experimental values for ENTHRU, FMAX, and LIMSET. The information reported for each problem solved is listed in Table 3.9.

Figure 3.6 shows the relationship between the ram's kinetic energy at the instant of impact, termed EINPUT, and the energy measured at the load cell, ENTHRU, for case BLTP-6; 10.0. The first solution was based on a soil resistance of 30 kip and a cushion stiffness of 1080 kip/in. As shown earlier, ENTHRU is not particularly sensitive to either of these parameters, so the results will probably be sufficiently accurate.

As shown in Table 3.9, the Michigan study found ENTHRU for this case to be 6,380 ft/lb. Therefore, as illustrated in Figure 3.6, this known value of ENTHRU can be used to determine the actual energy output of the hammer. In this case, EINPUT is 11,000 ft lb, indicating that the hammer either lost 4,000 ft lb of its rated 15,000 ft lb because of friction and other causes or the hammer is over-rated. Also, another 4,620 ft lb (11,000 ft lb - 6,380 ft lb) was lost in the cushion and helmet assembly, since only 6,380 ft lb was transmitted through the load cell.

Thus, based on a rated energy output of 15,000 ft lb, the overall efficiency of the hammer during

TABLE 3.9 DATA REPORTED IN THE MICHIGAN STUDY[44]

Driving Location	Pile I.D.	Case	Rated Energy (ft lb)	ENTHRU (ft lb)	LIMSET (in)	Permanent Set (in)	Estimated Static Soil Resistance (kip)
Belleville	BLTP-6	10.0	15,000	6,380	0.76	0.48	48
		57.9	15,000	4,440	0.42	0.02	400
	BLTP-4	25.0	18,000	8,010	0.94	0.36	140
		66.4	18,000	11,200	0.92	0.02	690
	BRP-4	20.0	22,400	4,980	0.57	0.37	100
		50.0	22,400	4,470	0.41	0.12	320
	BLTP-5	15.0	22,500	9,040	1.86	1.43	80
		60.0	22,500	9,930	0.79	0.11	340
Detroit	DLTP-8	41.5	15,000	5,760	1.22	1.00	60
		80.2	15,000	4,540	0.54	0.50	360
	DTP-5	20.0	15,100	8,290	2.55	2.00	22
		79.0	15,100	11,420	0.82	0.09	235
	DRP-3	40.0	18,000	7,060	1.36	1.25	60
		60.0	18,000	6,620	1.41	0.77	76
	DTP-13	40.0	22,400	9,100	2.21	2.00	30
		80.7	22,400	9,480	1.12	0.07	265
	DTP-15	20.0	22,500	10,100	2.07	2.00	40
		80.5	22,500	5,480	0.58	0.25	120

TABLE 3.9 (Continued)

Driving Location	Pile I.D.	Case	Rated Energy (ft lb)	ENTHRU (ft lb)	LIMSET (in)	Permanent Set (in)	Estimated Static Soil Resistance (kip)
Muskegon	MLTP-2	20.0	15,000	7,210	1.42	1.00	80
		53.0	15,000	4,870	0.57	0.09	200
	MLTP-9	72.0	24,450	14,660	1.06	0.56	160
		127.0	24,450	13,110	1.03	0.23	470
	MTP-12	30.5	30,000	14,860	1.48	1.00	40
		70.8	30,000	13,140	1.02	0.77	156
	MTP-11	69.5	32,000	16,760	1.16	0.67	160
		150.0	32,000	17,900	1.41	0.05	500
	MLTP-8	31.0	39,700	25,500	2.35	1.25	40
		178.0	39,700	22,050	1.71	0.04	988

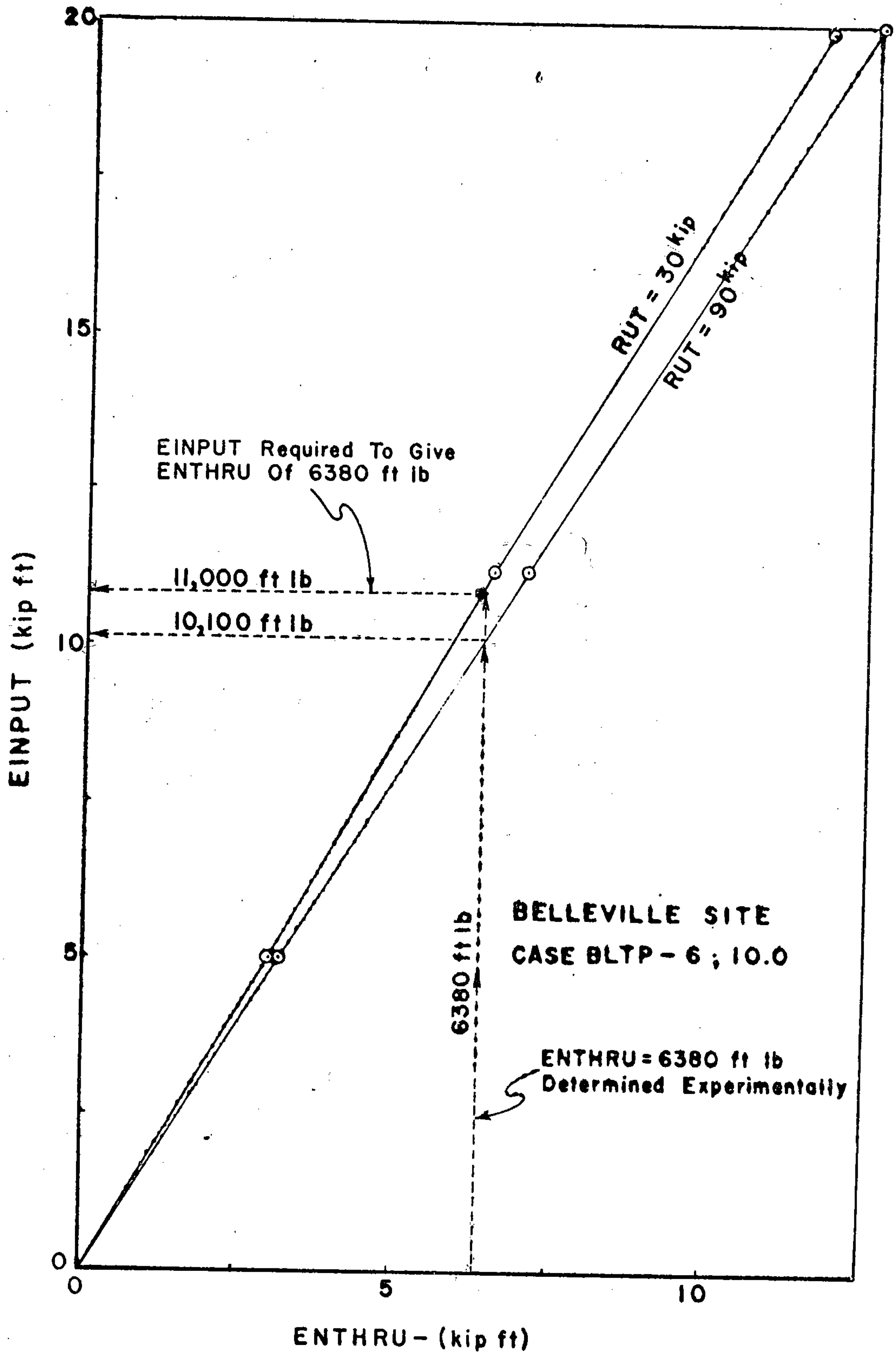


FIGURE 3.6 – EINPUT vs ENTHRU

this blow was $\frac{11,000 \times 100}{15,000}$ or about 73 percent while the efficiency of the cushion-helmet-load cell assembly, based on its ability to transmit the 11,000 ft lb delivered to it, was $\frac{6,380 \times 100}{11,000}$ or about 58 percent.

The ability to determine these efficiencies separately is important since it will indicate whether the pile driver or the cushion-helmet assembly should be improved to reduce energy losses during the pile-driving operation.

It is now a simple matter to determine the ram's velocity at impact by

$$V = \sqrt{\frac{(\text{EINPUT})(64.4)}{\text{Ram Weight}}} = \sqrt{\frac{(11,000)(64.4)}{5,000}} = 11.9 \text{ ft/sec}$$

Since Smith's numerical method gives the displacement curve for each segment, the maximum displacement of the bottom of the load cell (LIMSET) is known. Figure 3.7 shows how LIMSET is related to the ram velocity for various soil resistances. Table 3.9 gives an experimental value of LIMSET equal to 0.75 in. As shown in Figure 3.7, lines projecting from LIMSET = 0.75 in. and V = 11.9 ft/sec intersect at a point which indicates a driving resistance of about 90 kip. Since this is considerably greater than RUT = 30 kip, the problem was solved again using RUT = 90 kip. This re-

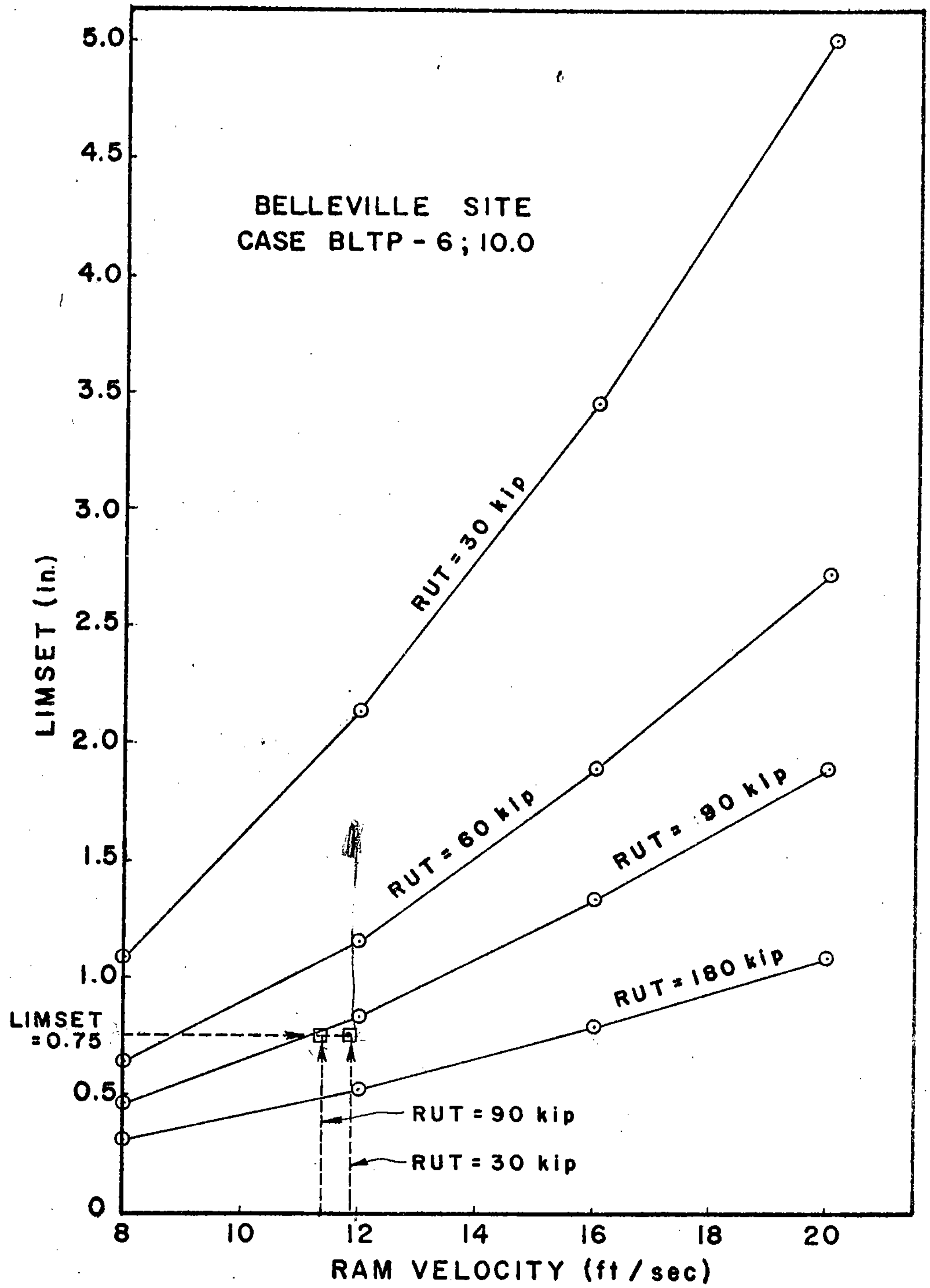


FIGURE 3.7 — RAM VELOCITY vs LIMSET

sults in the lower curve of Figure 3.6, and the new value of EINPUT is found to be 10,100 lt lb, which corresponds to an initial velocity of 11.4 ft/sec. When this velocity is substituted into Figure 3.7, the resulting value of RUT agrees closely with the assumed value of 90 kip. Thus, since the hammer output is 4,900 ft lb less than its rated 15,000 ft lb, and the cushion-helmet assembly lost another 3,720 ft lb, the hammer would be $\frac{10,100 \times 100}{15,000}$ = 67 percent efficient, while the cushion efficiency is $\frac{6,380 \times 100}{10,100}$ = 63 percent. It should be noted that even though RUT was off by a factor of 3, it made little difference in these efficiencies.

Correlation of Experimental and Theoretical Results

Comparisons between the experimental results and those by the wave equation for this case are shown in Figures 3.8 thru 3.11. These figures show the experimental and theoretical solutions for forces, accelerations, displacements, and work vs time, measured at the load cell. The correlations are reasonably accurate, expecially during the first 0.01 sec. Although the reflected compressive wave seems to be overestimated, as shown in Figure 3.8 at 0.014 sec. this did not greatly affect either the ENTHRU or displacement curves, although it may have caused the rather

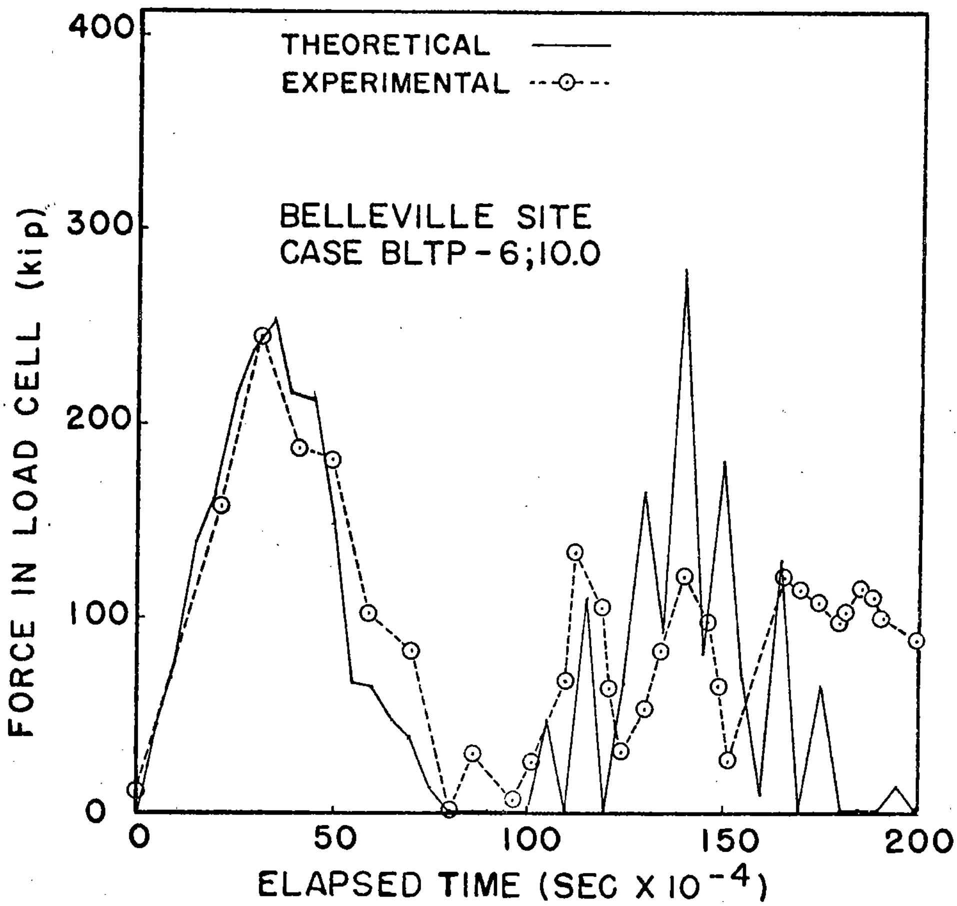


FIGURE 3.8 – COMPARISON OF THEORETICAL AND EXPERIMENTAL LOAD CELL FORCES

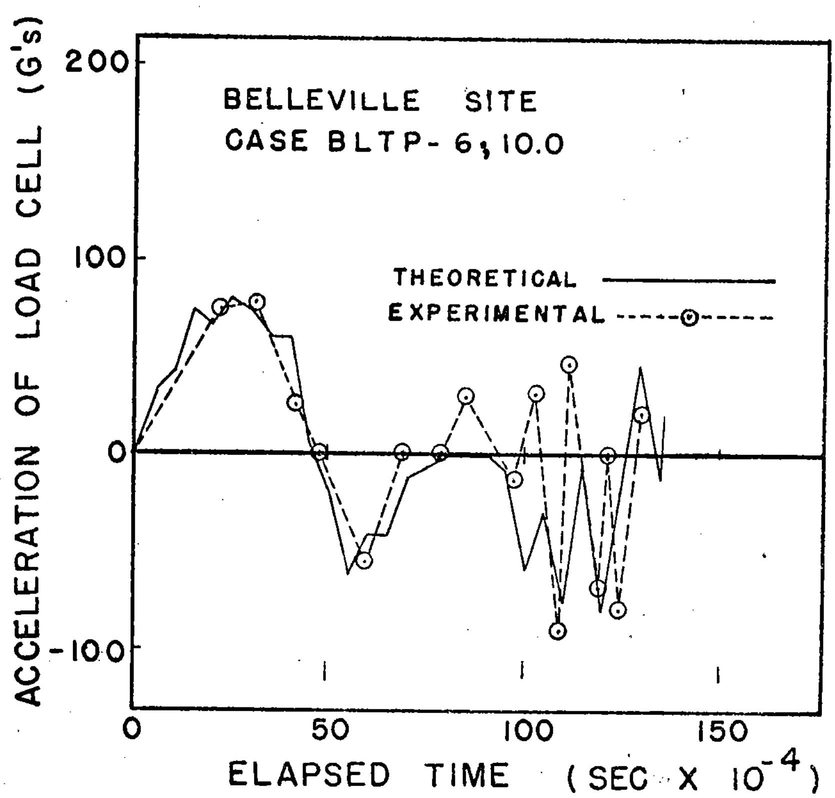


FIGURE 3.9 — COMPARISON OF THEORETICAL AND EXPERIMENTAL LOAD CELL ACCELERATIONS

TABLE 3.13 EFFECT ON ENTHRU RESULTING FROM REMOVING THE LOAD CELL ASSEMBLY

Driving Location	Pile I.D.	Case	ENTHRU (ft lb) With Load Cell	ENTHRU (ft lb) Without Load Cell	Increase in ENTHRU %
Belleville	BLTP-6	10.0	6380	7500	18
		57.9	4440	5300	19
	BLTP-4	25.0	8010	8800	10
		66.4	11200	12000	8
	BRP-4	20.0	4980	5750	15
		50.0	4470	6450	44
	BLTP-5	15.0	9040	10750	19
		60.0	9930	12300	24
Detroit	DLTP-8	41.5	5760	6900	21
		80.2	4540	5400	19
	DTP-5	20.0	8290	10000	23
		79.0	11420	12700	12
	DRP-3	40.0	7060	7600	13
		60.0	6620	7200	11
	DTP-13	40.0	9100	10850	13
		80.7	9480	11400	20
	DTP-15	20.0	10100	11500	14
		80.5	5480	6600	20
Muskegon	MLTP-2	20.0	7210	8800	23
		53.0	4870	5700	17
	MLTP-9	72.0	14660	17000	16
		127.0	13110	16000	22
	MTP-12	30.5	14860	17000	14
		70.8	13140	15000	14
	MTP-11	69.5	16760	22000	31
		150.0	17900	25300	41
	MLTP-8	31.0	25500	31000	22
		178.0	22050	26600	21

when the load cell assembly is removed for each of the problems solved.

As mentioned earlier, a complete parameter study would be greatly beneficial to the engineer, but only if the correct behavior of the numerous parameters were known. More accurate information concerning wave propagation must therefore be determined while looking for new tests and methods to determine the behavior of the many parameters which influence the problem.

Effects of Explosive Pressure

As noted earlier, the diesel explosive pressure was neglected in the previous solutions, since the explosive force is usually much smaller than the impact forces and have little effect on the driving stresses[41]. However, the ram velocity required to predict ENTHRU is often higher than that calculated from the free fall of the ram, even assuming 100 percent efficiency. As noted in Table 3.10, several ram velocities exceed 20 ft/sec. Also, the diesel hammer efficiencies found are higher than indicated by practical experience. Therefore, the diesel hammer cases were re-run to account for the explosive pressure in the hammer.

As recommended by Smith[23], the force on the anvil is assumed to reach some maximum due to the ram's impact, and then decrease. The diesel explosive pressure then maintains a given minimum force between the ram and anvil for 0.01 seconds, after which the force tapers to zero at 0.0125 seconds as shown in Figure 3.14. The explosive forces assumed to be acting within the diesel hammers are listed in Table 3.14.

In previous solutions, it was an easy matter to solve for the total energy of the ram at impact since only its kinetic energy (EINPUT) was involved. Since explosive pressure is included, the total energy output is changed.

This total energy output, ENTOTL, is used in two ways: to transmit energy to the anvil and pile, and to raise the ram for the next blow. The energy transmitted to the anvil (ENTHRU I) is calculated by the same method as was used for ENTHRU at the load cell, and the kinetic energy of the ram after impact is equal to $WV^2/64.4$, where W is the weight of the ram and V is the rebound velocity of the ram after impact.

A number of runs were first made to bracket the results given in the Michigan report, and also to determine the influence of certain variables such as ram velocity and ultimate soil resistance. The ef-

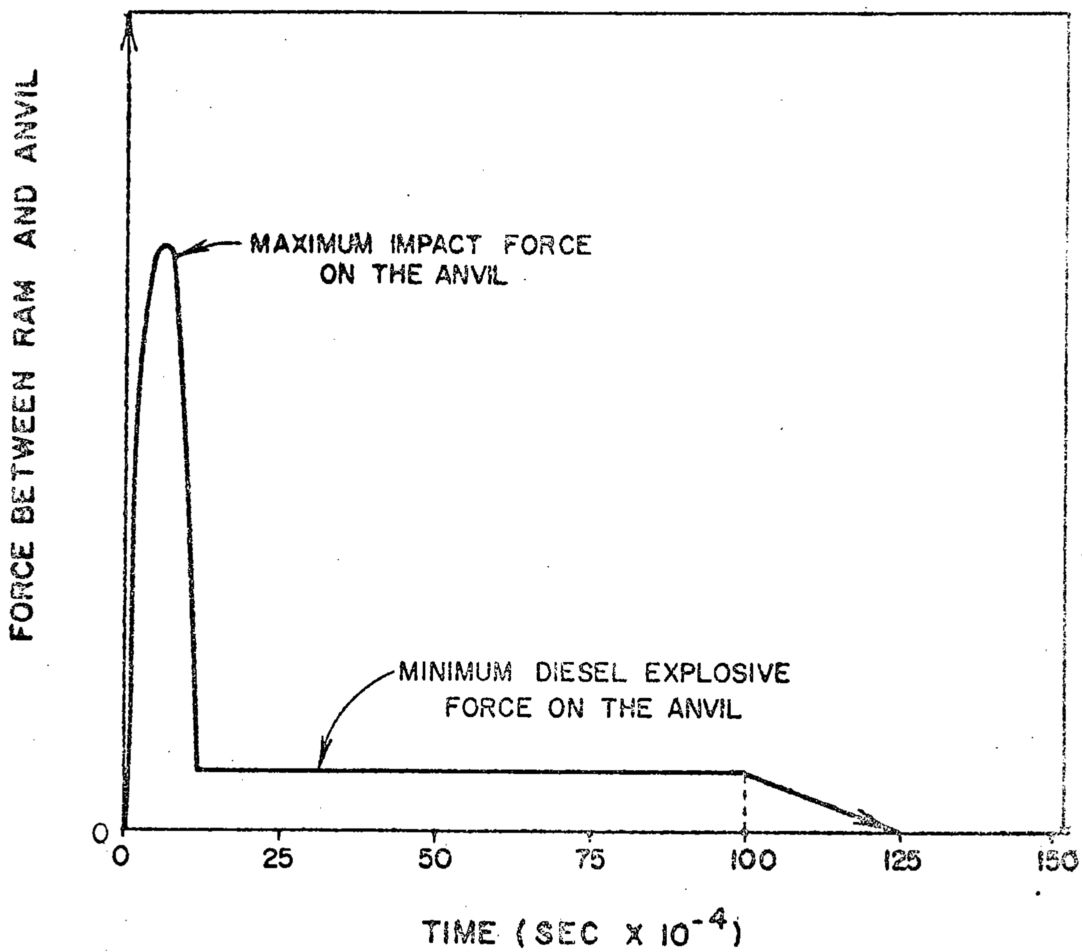


FIGURE 3.14 - TYPICAL FORCE VS TIME CURVE WHEN DIESEL EXPLOSIVE PRESSURE IS INCLUDED

TABLE 3.14 RESULTS INCLUDING DIESEL HAMMER EXPLOSIVE PRESSURE IN THE WAVE EQUATION ANALYSIS

Driving Location	Pile I.D.	Case	Hammer Type	Minimum Explosive Force on Anvil (kip)	Total Energy Output of Hammer (ft lb)	Hammer Efficiency %	Cushion Assembly Efficiency %	Ram Velocity at Impact (ft/sec)	RUT (kip)	Ratio of Reported RUT to Theoretical RUT %
Belle-Ville	BLTP-4	25.0	LB-312	98.0	10,630	59	75	8.2	70	50
		66.4			16,030	89	70	6.4	250	36
	BRP-4	20.0	M-DE30	98.0	9,450	42	53	9.8	100	100
		50.0			9,100	41	49	10.6	200	63
	BLTP-5	15.0	D-D12	93.7	13,000	58	69	12.8	40	50
		60.0			14,730	66	67	15.0	400	118
De-troit	DRP-3	40.0	LB-312	98.0	9,270	52	76	9.8	45	75
		60.0			13,900	77	48	5.2	60	79
	DTP-13	40.0	M-DE30	98.0	14,390	64	63	13.7	35	117
		80.7			15,280	68	62	15.1	120	45
	DTP-15	20.0	D-D12	93.7	15,270	68	66	15.2	45	112
		80.5			9,430	42	58	11.6	110	92
Muske-gon	MTP-12	30.5	LB-520	98.0	22,140	74	67	16.4	75	187
		70.8			21,260	71	62	14.4	70	45
	MTP-11	69.5	M-DE40	138.0	32,800	102	50	20.6	150	94
		150.0			36,850	115	49	21.5	250	50
	MLP-8	31.0	D-D22	158.7	31,600	80	81	17.8	70	175
		178.0			27,300	69	81	17.1	300	30

ficiencies and ram velocities noted in Table 3.14 were found by plotting ENTHRU and ENTOTL vs the initial ram velocity as shown in Figure 3.15. By plotting the values of LIMSET for varying soil resistance vs ram velocity as in Figure 3.16, the total soil resistance predicted by the wave equation was then determined. This revised procedure was followed on all the diesel hammer cases, and the results are summarized in Table 3.14.

Effects of Cushion Properties on Driving

The general effects of cushioning materials on pile driving is discussed in Chapter IV. The following discussion is given since it deals with the Michigan pile study.

As previously noted, the Michigan report states that the cushion properties influence the values of ENTHRU significantly, although "how, when, or how much" ENTHRU was affected could not be determined. It was thought that ENTHRU could be increased by using a more resistant cushion block, in the case of the Vulcan 1 and McKiernan-Terry DE-30 hammers. Although this conclusion seems reasonable, results given by the wave equation did not seem to agree. For example, as seen in Table 3.6, ENTHRU does not always increase with increasing cushion stiffness, and furthermore, the maximum increase in

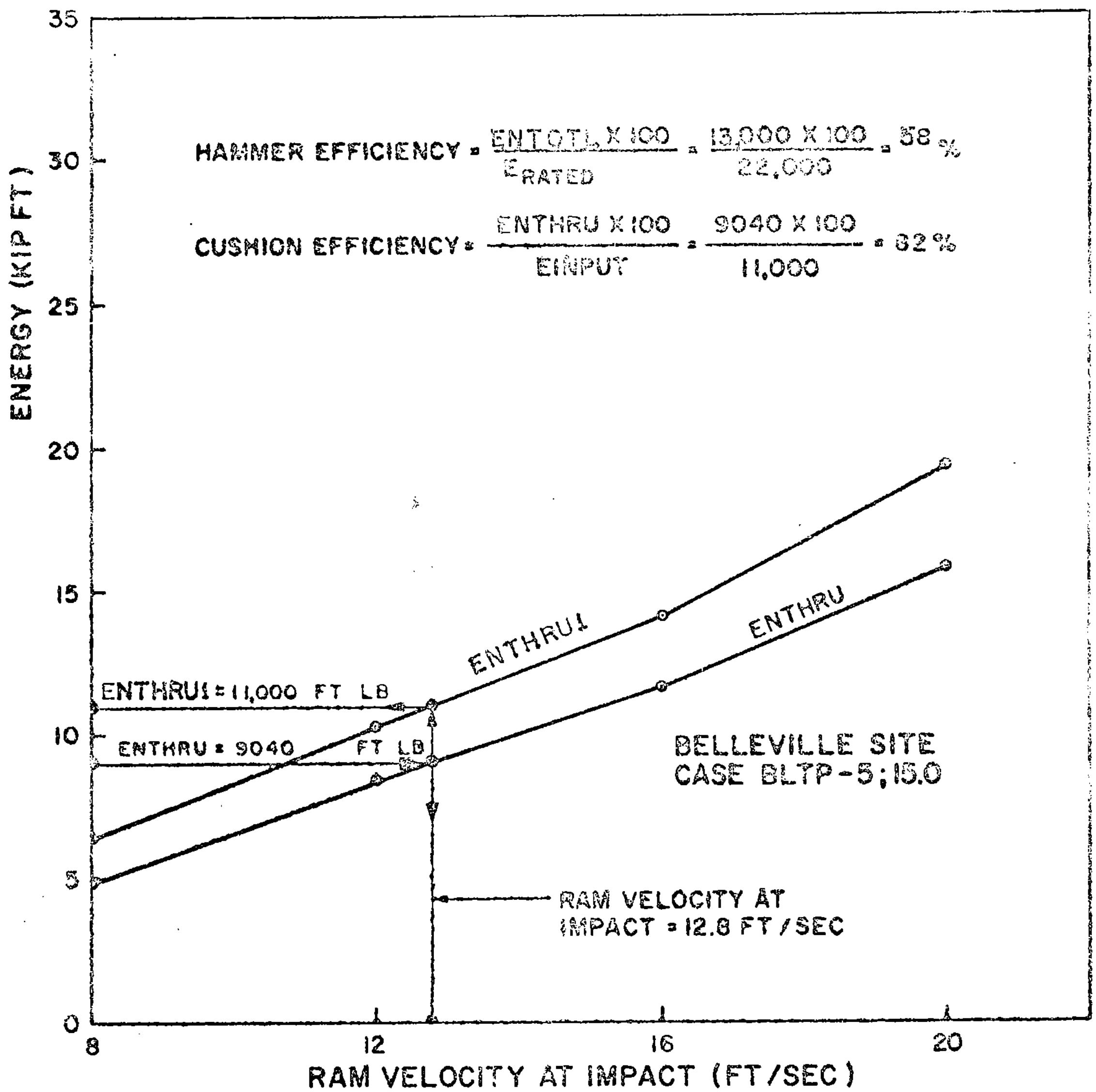


FIGURE 3.15 - ENTHRU1 & ENTHRU VS RAM VELOCITY INCLUDING EXPLOSIVE PRESSURE

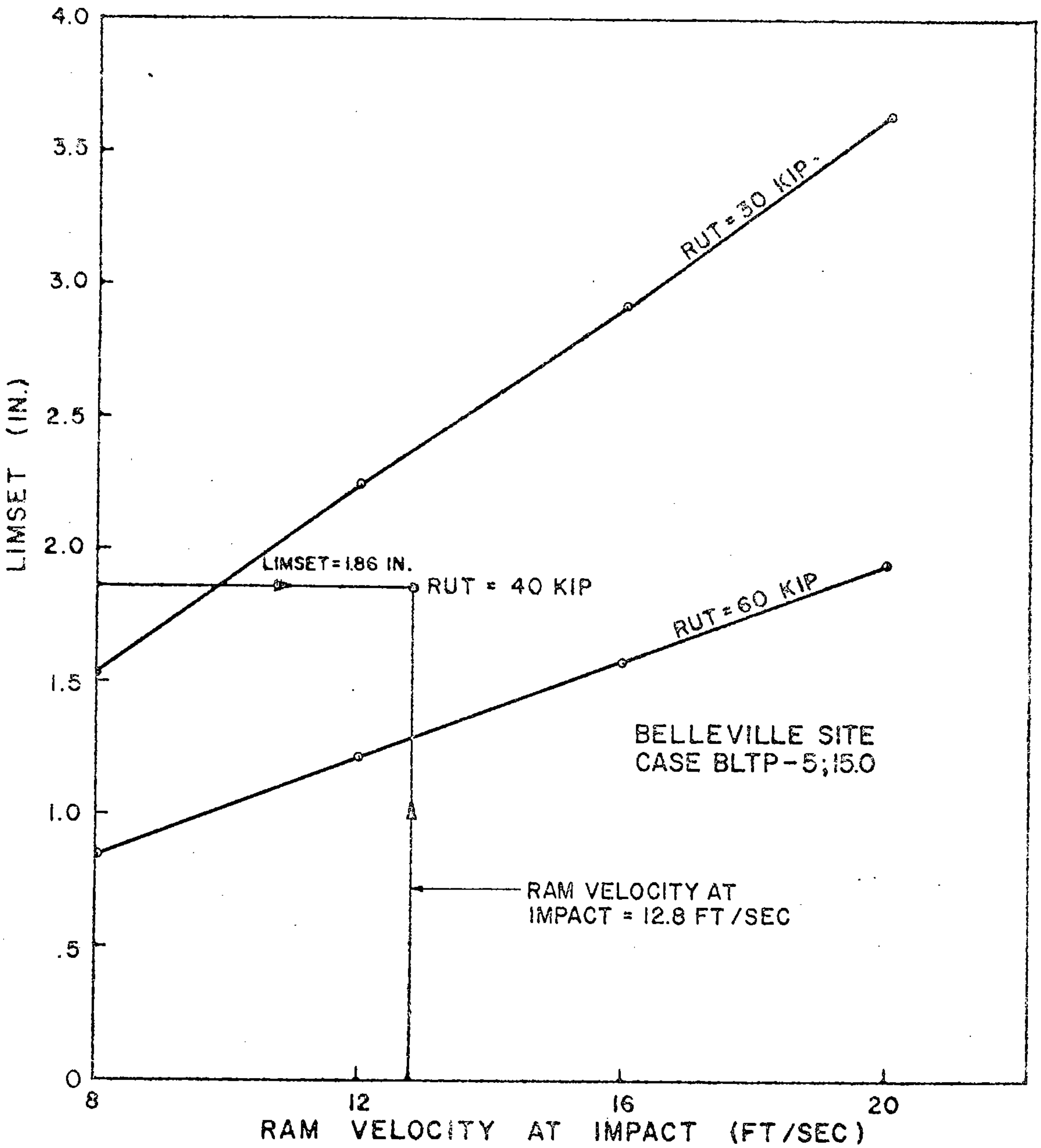


FIGURE 3.16 - LIMSET VS RAM VELOCITY
INCLUDING EXPLOSIVE PRESSURE

ENTHRU noted here is relatively small - only about 10 percent. This effect can also be seen in Table 3.15, in which the cushion stiffness varies greatly, while the displacement of the pile point changes less than 10 percent.

However, if a different cushion is used, the coefficient of restitution will probably change too. Since the coefficient of restitution of the cushion may affect ENTHRU, a number of cases were solved with "e" ranging from 0.2 to 0.6. As shown in Tables 3.16 and 3.17, an increase in "e" from 0.2 to 0.6 normally increases ENTHRU from 18 to 20 percent, while increasing the permanent set from 6 to 11 percent. Thus, for the case shown, the coefficient of restitution of the cushion has a greater influence on rate of penetration and ENTHRU than does its stiffness. This same effect was noted in the other solutions, and the cases shown in Tables 3.16 and 3.17 are typical of the results found in other cases.

As was noted in Table 3.7, any increase in cushion stiffness also increases the driving stresses. Thus, according to the wave equation, increasing the cushion stiffness to increase the rate of penetration (for example by not replacing the cushion until it has been beaten to a fraction of its original height or by

TABLE 3.15 EFFECT OF CUSHION STIFFNESS ON MAXIMUM POINT DISPLACEMENT FOR CASES BLTP-6; 10.0 and 57.9

Pile I.D.	RUT	Ram Velocity	Maximum Point Displacement (in) 540	1080	2700	27,000	Maximum Change
	(kip)	(ft/sec)	Cushion Stiffness (kip/in)				%
BLTP-6; 10.0	30	12	2.20	2.14	2.22	2.26	5
		16	3.54	3.47	3.52	3.70	6
		20	4.66	4.93	5.00	5.01	7
BLTP-6; 57.9	150	12	0.45	0.48	0.38	0.48	6
		16	0.72	0.76	0.76	0.79	9
		20	1.06	1.10	1.11	1.15	8

TABLE 3.16 EFFECT OF COEFFICIENT OF RESTITUTION ON ENTHRU FOR CASE BLTP-6; 10.0 and 57.9

Pile I.D.	RUT (kip)	Ram Velocity (ft/sec)	ENTHRU (kip ft) e = 0.2	ENTHRU (kip ft) e = 0.4	ENTHRU (kip ft) e = 0.6	Maximum Change (%)
BLTP-6; 10.0	30	12	6.0	6.5	7.3	18
		16	10.5	11.8	12.8	18
		20	16.5	17.4	20.0	17
BLTP-6; 57.9	150	12	6.7	7.2	8.2	18
		16	11.6	12.7	14.5	20
		20	18.2	19.7	22.4	19

TABLE 3.17 EFFECT OF COEFFICIENT OF RESTITUTION ON MAXIMUM POINT DISPLACEMENT FOR CASE BLTP-6; 10.0 and 57.9

Pile I.D.	RUT (kip)	Ram Velocity (ft/sec)	Maximum Point Displacement (in)			Maximum Change (%)
			e = 0.2	e = 0.4	e = 0.6	
BLTP-6; 10.0	30	12	2.13	2.14	2.36	10
		16	3.38	3.47	3.58	6
		20	4.73	4.93	5.17	8
BLTP-6; 57.9	150	12	0.46	0.48	0.50	8
		16	0.73	0.76	0.81	10
		20	1.05	1.10	1.18	11

omitting the cushion entirely) is both poor practice because of the high stresses induced in the pile, and inefficient. It would be better to use a cushion having a high coefficient of restitution and a low cushion stiffness in order to increase ENTHRU and to limit the driving stresses.

This suggests that a long micarta cushion having a relatively low spring rate, and a high coefficient of restitution may be very effective.

Comparison of Various Hammers Driving the Same Pile

One of the objectives of the Michigan pile study was to determine just how effective the various hammers actually were during driving. Therefore, every attempt was made to equalize any variables which would affect the results, such as choosing the driving location to give comparable driving conditions. However, it would be impossible to test several hammers without having some variations occur, perhaps in the soil resistance or hammer condition. Since the wave equation does not have this limitation, it could be used to advantage here.

As a basis for this comparison, Case BLTP-6; 57.9 was used, with the load cell and extra helmet omitted, and with a soil resistance of 300 kips. This pile was then studied to determine its penetration per blow when driven by each of the hammers listed in Table 3.10. In each case, the soil and pile parameters were held constant. Thus, for example, even though the values of the soil damping constant or quake may not be exact, they remain constant for each problem, while the experimental results will vary unless Q and J are constant at each new driving location.

Certain quantities must be known for each hammer before the wave equation can be applied. For example, the ram velocity at impact must be known, as well as the dynamic behavior of the cushion, the diesel explosive pressure in the hammer, and the length of time it exerts a force on the pile. Unfortunately, the above data were not directly measured for the Michigan research program, which means that they must be calculated from other data reported.

The ram velocities at impact and explosive forces on the pile for the diesel hammers were based on the results given in Table 3.14, assuming the explosive force to be acting as shown in Figure 3.12. The Vulcan hammer properties were based on Table 3.10. The results of driving this pile with the eight different hammers are listed in Table 3.18 in the form of permanent set of the pile per blow and blows per inch.

TABLE 3.18 STUDY OF VARIOUS HAMMERS DRIVING THE SAME PILE

Hammer	Ram Velocity (ft/sec)	Explosive Force (kip)	Maximum Point Displacement (in)	Permanent Set of Pile Per Blow (in)	Blows Per Inch
Vulcan-1	10.0	0	0.125	0.025	8
Vulcan-50C	14.5	0	0.284	0.184	3
Vulcan-80C	12.5	0	0.360	0.260	2
Link Belt 312	7.0	98.0	0.119	0.019	8
Link Belt 520	16.0	98.0	0.357	0.257	3
McKiernen-Terry DE-30	13.0	98.0	0.139	0.039	7
McKiernen-Terry DE-40	21.0	138.0	0.592	0.492	1
Delmag D-12	15.0	93.7	0.173	0.073	5
Delmag D-22	17.5	158.7	0.473	0.373	2

CHAPTER IV

CHARACTERISTIC CUSHION PROPERTIES

Introduction

Although a pile cushion serves several purposes, its primary function is to limit impact stresses in both the pile and hammer[50]. In general, it has been found that a wood or rope cushion is more effective in reducing the driving stresses than one of a relatively stiff material such as Micarta. However, a stiffer cushion is usually more durable and transmits a greater percentage of the hammer's energy to the pile.

For example, the results given in Table 3.11 show an overall average efficiency of 52 percent for cushion assemblies using wood, while the Micarta assemblies have an average efficiency of 66 percent. As shown in Table 3.7, an increase in cushion stiffness will also cause an increase in impact stresses which might damage the pile or hammer during driving. This increase in stress is particularly important when driving concrete or prestressed concrete piles.

Dynamic Stress-Strain Curves

In order to apply the wave equation to pile driving, Smith[51] assumes that the cushion's stress-strain curve is a series of straight lines as shown in Figure 4.1. Even though this curve might be sufficiently accurate to predict maximum compressive stresses in the pile, the shape of the stress wave often disagrees with that of the actual stress wave[52]. This discrepancy was at first though to be the result of inaccurate soil data, since very little was known concerning the soil behavior during driving. It was therefore decided to suspend several test piles horizontally above the ground[53] as shown in Figure 4.2 to eliminate the effects of soil resistance.

Table 4.1 lists the pertinent information concerning these piles. The cushion was then hit by the ram and the resulting strains were measured at six points along the pile. Displacements and accelerations of both the ram and the head of the pile were also measured. However, even though the soil resistance had now been excluded, the shape of the stress wave still did not agree with the theoretical shape, and so the device illustrated in Figure 4.3 was used to see if the cushion's stress-strain diagram was actually a straight line.

Using this method, the dynamic stresses and strains were measured for several cushion materials. It was later discovered that for a given material, the dynamic

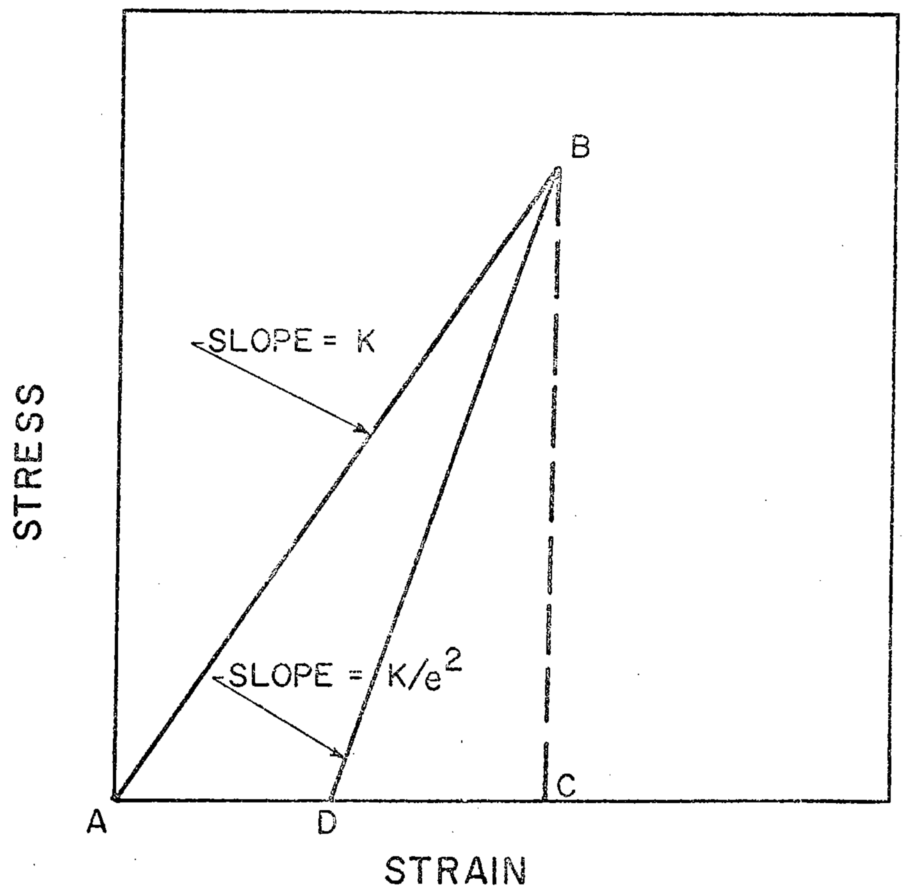


FIGURE 4.1 — STRESS – STRAIN CURVE FOR A CUSHION BLOCK. (AFTER REFERENCE 51)

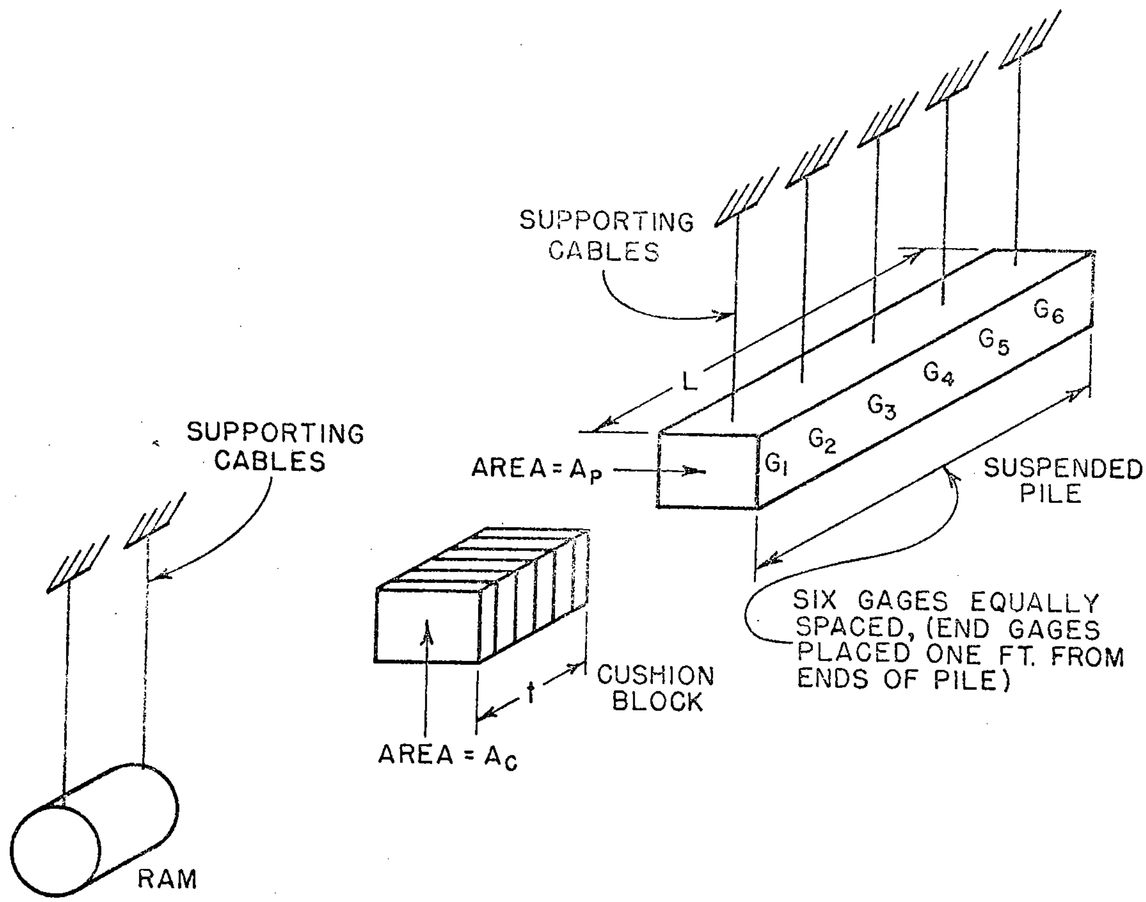


FIGURE 4.2 - TEST PILE SHOWING PLACEMENT OF STRAIN GAGES

TABLE 4.1 SUSPENDED PILE DATA

	Pile					Cushion			Ram	
Case	Material	E (psi)	Ap (in^2)	L (ft)	D (ft)	Material	Ac (in^2)	t (in)	Weight (lb)	Velocity (ft/sec)
LT-48	Class A Concrete	6.12×10^6	254	65	12.6	Fir	62.8	9.0	4160	13.91
LT-41	Class A Concrete	6.12×10^6	254	65	12.6	Micarta	89.1	9.0	4160	8.03
LT-39	Steel	30×10^6	21.46	85	16.6	Oak	225.0	7.5	2128	11.42
LT-15	Class Y Concrete	*3.96×10^6	225	65	12.6	Oak	225.0	9.5	2128	13.98

* $E_{sonic} = 4.64 \times 10^6$ psi

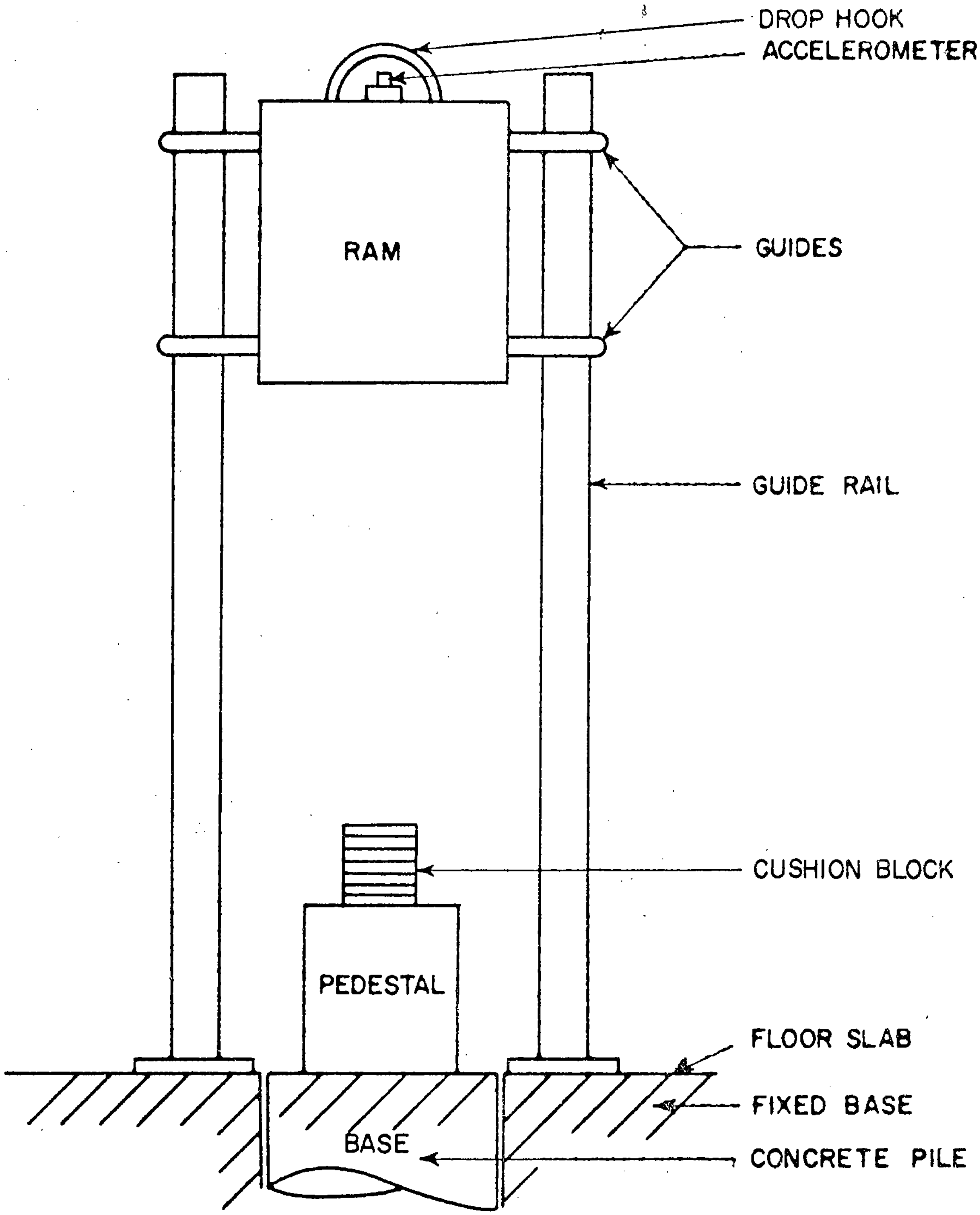


FIGURE 4.3 - CUSHION TEST STAND

stress-strain curves were almost identical to the corresponding static curves. This is demonstrated in Figure 4.4 in which the dynamic and static curves for a fir cushion are compared.

Since the stress-strain curves are not linear as assumed, the shape of the theoretical stress wave in the pile is not likely to agree with the experimental shape and so the "dynamic" curves were used.

Furthermore, it is not known how much the rigidity of the pedestal shown in Figure 4.3 affects the cushion's behavior. Therefore, the wave equation was used to check the results. The second method required the following information: 1) the stresses determined experimentally at the head of the pile vs time, 2) the velocity of the ram at impact, and 3) the physical properties of the pile system required for solution by the wave equation.

As shown in Figure 4.5, both the cushion and ram are omitted and the previously determined stresses measured experimentally at gage 1 (see Figure 4.2) are placed on the head of the pile. The wave equation is then used to determine the motion of the ram and the pile, from which the compression of the cushion at any instant of time is known. By plotting the measured cushion forces against the corresponding compressions of the cushion, the dynamic stress-strain curve may be

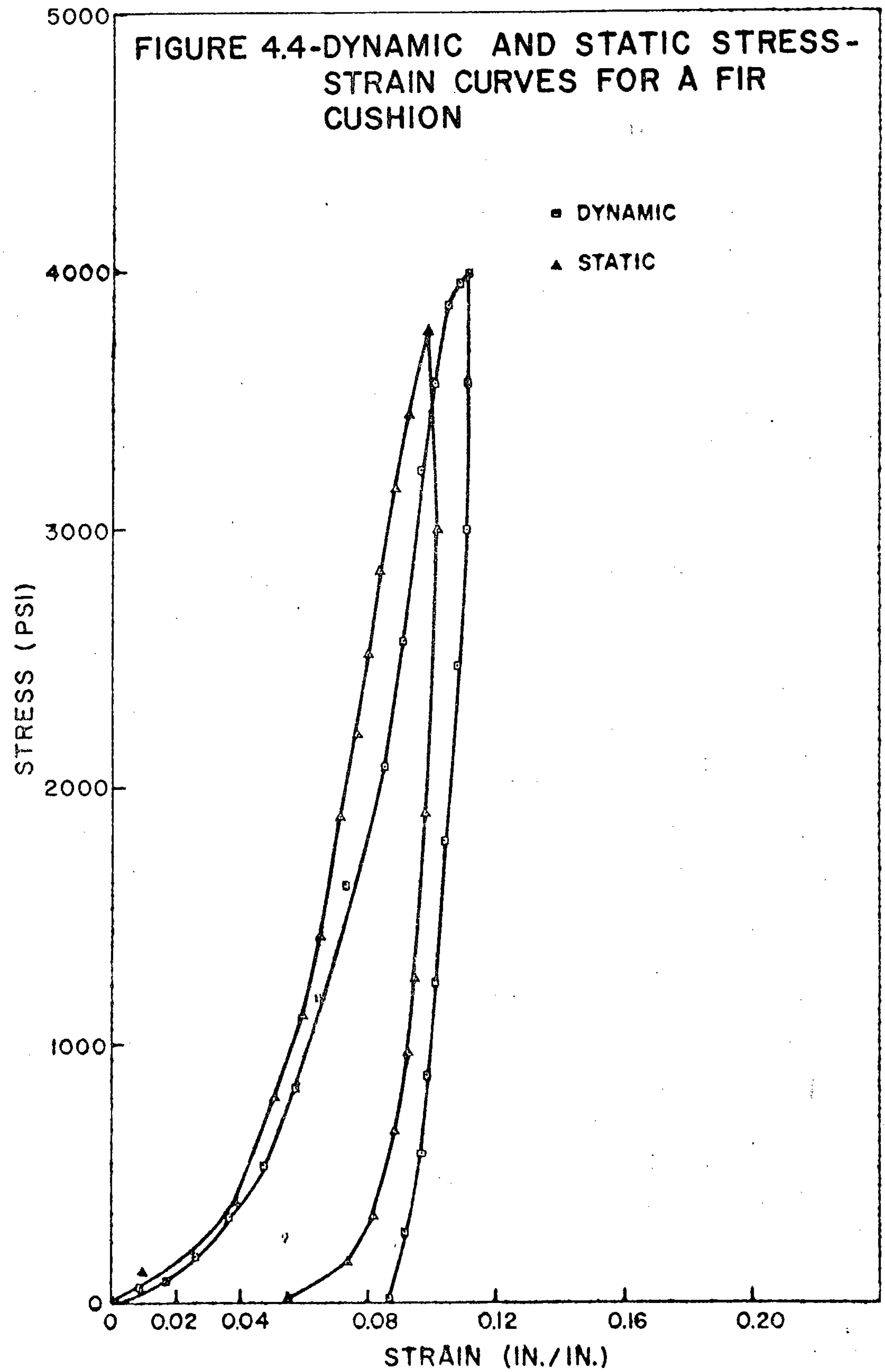


FIGURE 4.4-DYNAMIC AND STATIC STRESS-STRAIN CURVES FOR A FIR CUSHION
DYNAMIC
STATIC
5000
4000
3000
2000
1000
0
STRESS (PSI)
0
0.02
0.04
0.08
0.12
0.16
0.20
STRAIN (IN./IN.)

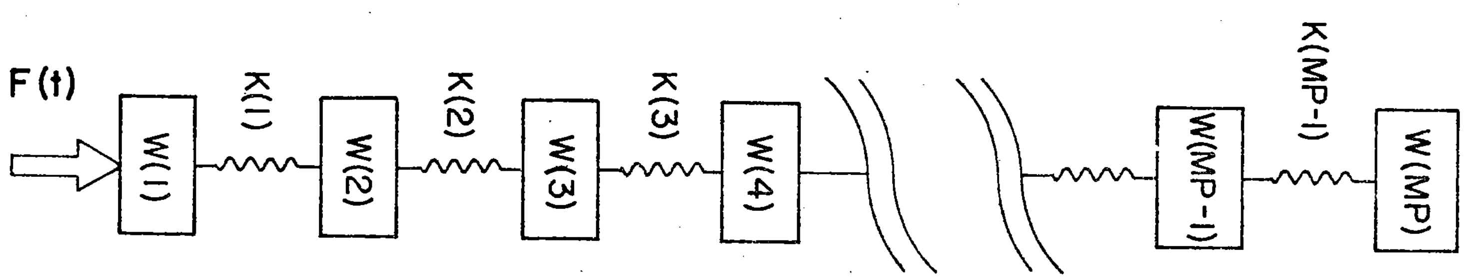


FIGURE 4.5 – IDEALIZED TEST PILE WITH KNOWN FORCES APPLIED AT HEAD OF PILE

determined. The curves obtained by this method are illustrated in Figures 4.6, 4.7, and 4.8. Comparing these with Figure 4.4, it is noted that the curves are generally similar in shape.

Dynamic Coefficient of Restitution

Although the cushion is needed to limit the driving stresses in both hammer and pile, it reduces the available hammer energy because of internal damping. The load diagram shown in Figure 4.1 illustrates this energy loss since the energy input is given by the area ABC while the energy output is given by area BCD. Usually this energy loss is accounted for by a coefficient of restitution of the cushion "e", in which

$$e = \sqrt{\frac{\text{Area under BCD}}{\text{Area under ABD}}}$$

When the dynamic stress-strain curve for the cushion is known, such as for the previous problem, the coefficient of restitution can be computed. As shown in Figure 4.6, the area under the dynamic curve ABC is computed by summing elemental areas ijkl until point B is reached (i.e., until the strain reaches a maximum), then the area under BCD is determined by summing elemental areas mnop until point D is reached.

Table 4.2 summarizes the results found for the

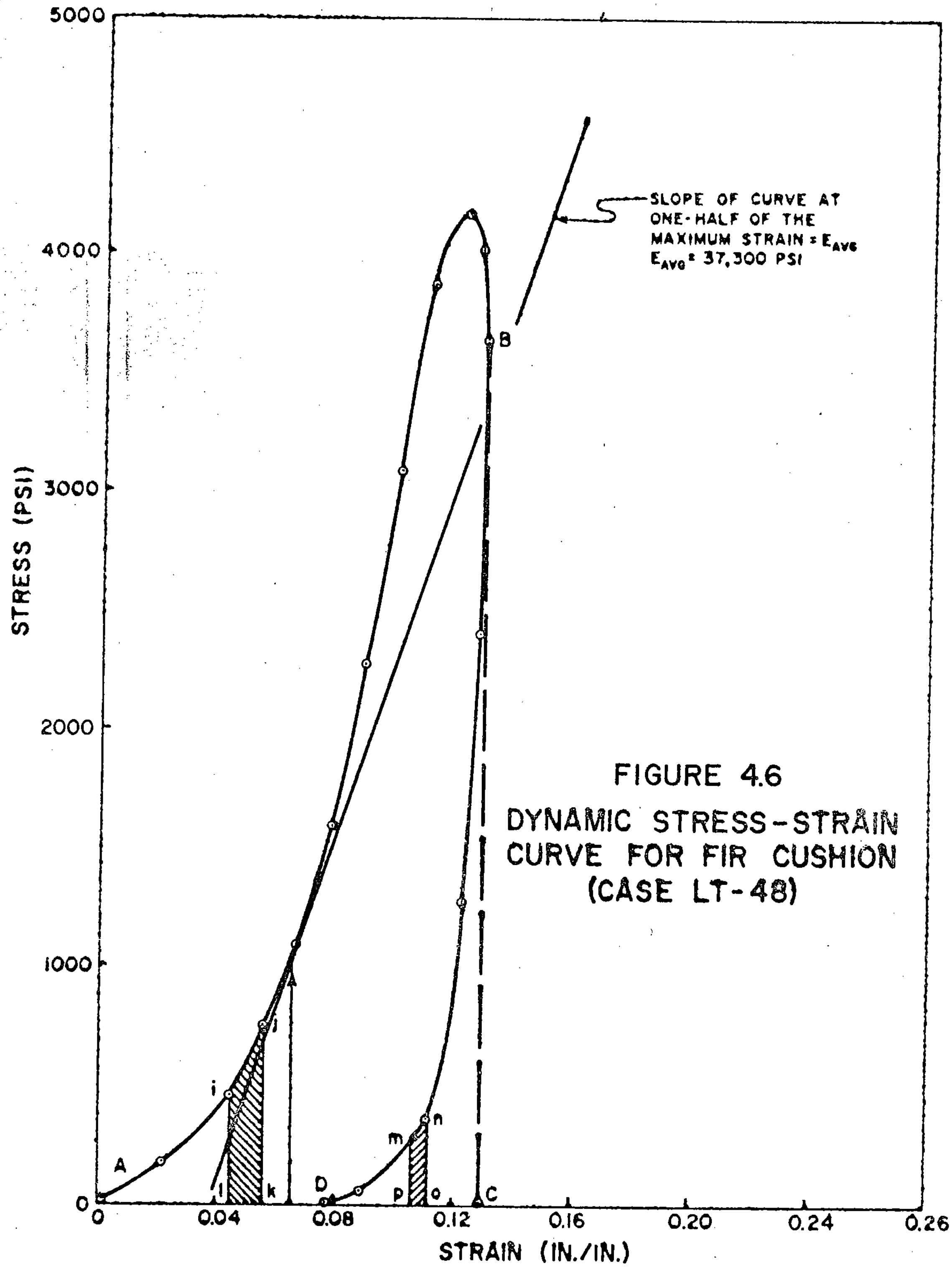


FIGURE 4.6
DYNAMIC STRESS-STRAIN CURVE FOR FIR CUSHION (CASE LT-48)

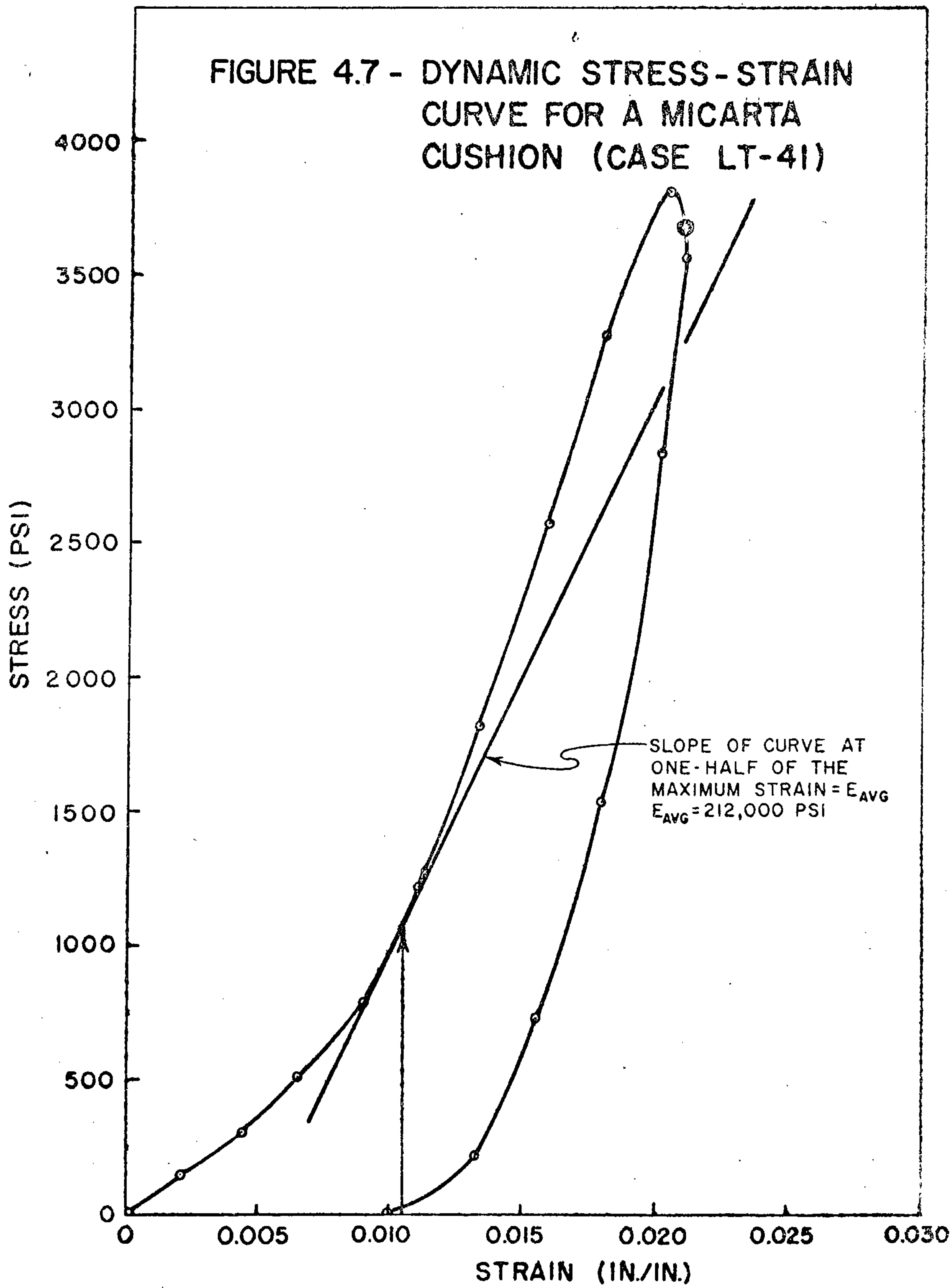


FIGURE 4.7 - DYNAMIC STRESS-STRAIN CURVE FOR A MICARTA CUSHION (CASE LT-41)
STRESS (PSI)
4000
3500
3000
2500
2000
1500
1000
500
0
0
0.005
0.010
0.015
0.020
0.025
0.030
STRAIN (IN./IN.)
SLOPE OF CURVE AT ONE-HALF OF THE MAXIMUM STRAIN = E_{AVG}
E_{AVG} = 212,000 PSI

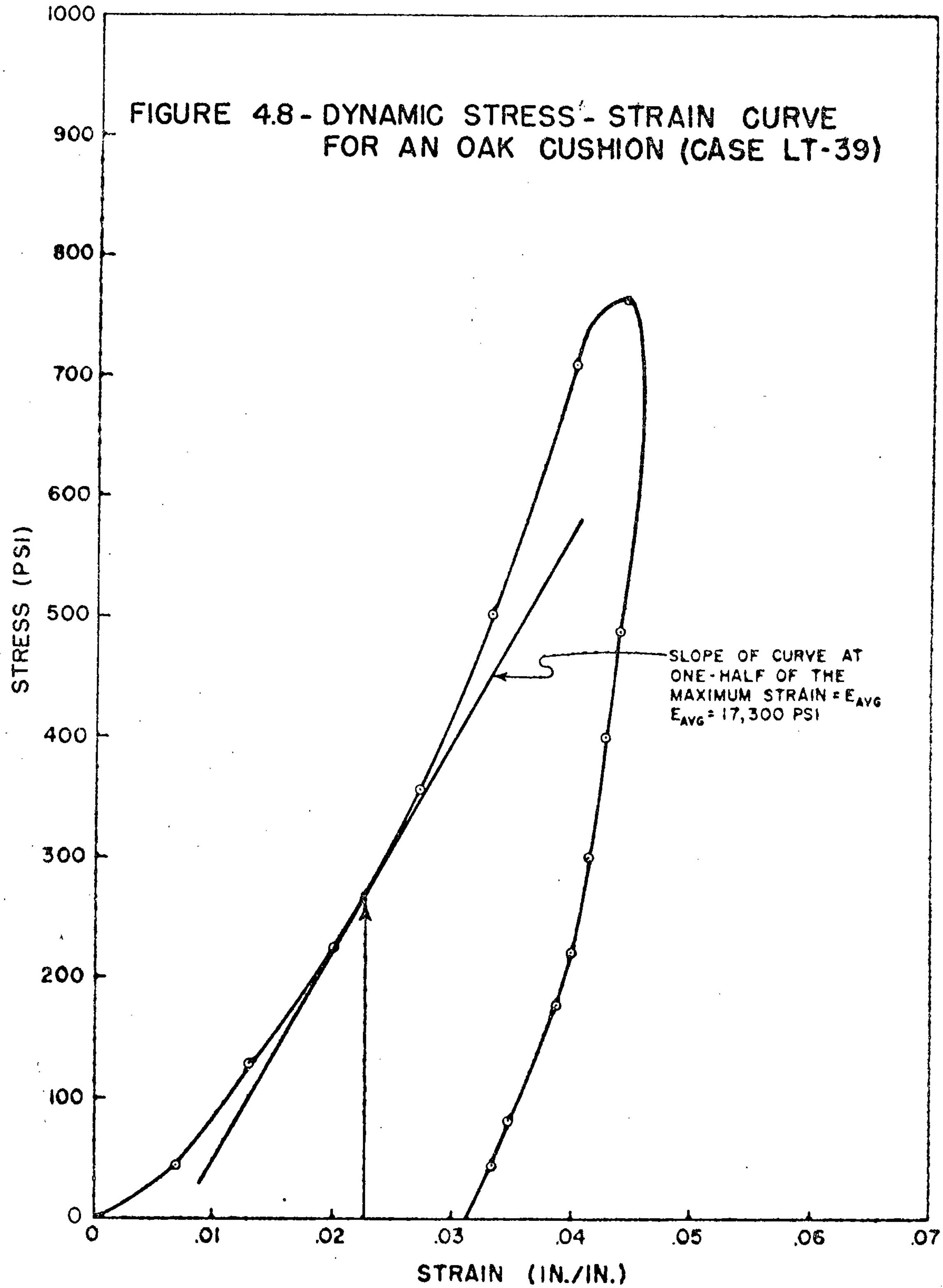

1000
900
800
700
600
500
400
300
200
100
0
STRESS (PSI)
FIGURE 4.8 - DYNAMIC STRESS - STRAIN CURVE
FOR AN OAK CUSHION (CASE LT-39)
SLOPE OF CURVE AT
ONE-HALF OF THE
MAXIMUM STRAIN = E_{AVG}
E_{AVG} = 17,300 PSI
0
.01
.02
.03
.04
.05
.06
.07
STRAIN (IN./IN.)

TABLE 4.2 DYNAMIC CUSHION PROPERTIES

Case	Cushion Material	Dynamic e	Commonly Recommended e
LT-48	Fir	0.35	0.40[55]
LT-41	Micarta	0.60	0.80
LT-39	Oak	0.47	0.48[55]

curves of Figures 4.6, 4.7, and 4.8. It may be noted that the coefficients of restitution agree closely with those recommended by Hirsch[55]. It is interesting that although e = 0.8 is commonly recommended for a micarta capblock, these experiments indicate that e is actually much lower, probably around 0.6.

Idealized Dynamic Stress-Strain Curves

The major difficulty in using the dynamic curves derived in the previous section is that numerous points on the curve must be specified in the input data, unless the curve can be input in equation form. Although the increasing load curve for each of the curves is nearly parabolic, the unloading segment is rather complex. Therefore, for convenience, the unloading segment will be approximated by a straight line having a slope such that the areas under the two curves result in the use of the correct coefficient of restitution for the cushion material being used.

Thus, the curve shown in Figure 4.9 can be defined by two different points on the loading curve (other than 0.0) and "e" of the material. The points on the curve are used to define the equation of the loading curve, and as long as the cushion strain increases, the increased input energy is computed as described earlier.

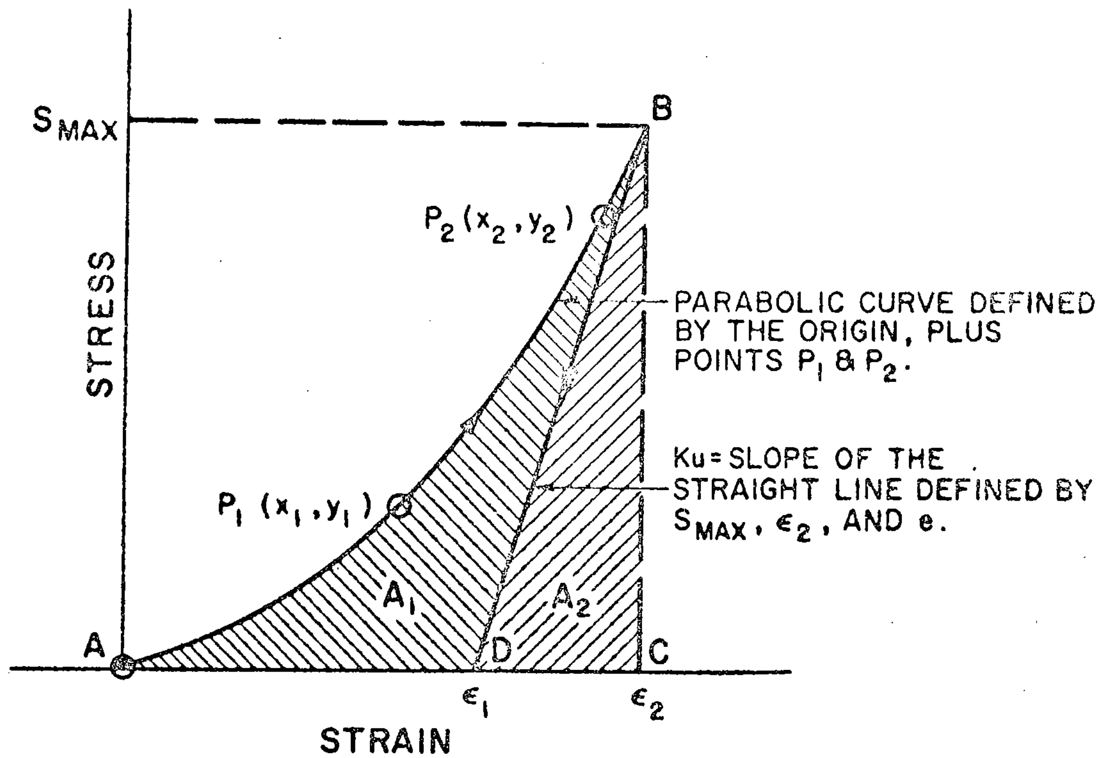


FIGURE 4.9 - IDEALIZED DYNAMIC STRESS-STRAIN CURVE FOR CUSHION (PARABOLIC)

When the strain in the cushion begins to decrease, the total input energy and the coefficient of restitution are used to determine the slope of the unloading curve in order to give the correct value of "e".

As shown in Figure 4.9, the total input energy is given by the area under the parabolic curve, $A_1 + A_2$, while the output energy is given by the area under the unloading curve, A_2. Since e is defined by

$$e^2 = A_2/(A_1+A_2),$$

then

$$A_2 = e^2(A_1+A_2).$$

But A_2 is also given by

$$A_2 = \left(\frac{Smax-0}{2}\right)(\varepsilon_2-\varepsilon_1)$$

$$e^2(A_1+A_2) = \left(\frac{Smax}{2}\right)(\varepsilon_2-\varepsilon_1)$$

$$(\varepsilon_2-\varepsilon_1) = \frac{2e^2(A_1+A_2)}{Smax}$$

Since the slope of the straight line BD is given by:

$$Ku = \frac{Smax}{(\varepsilon_2-\varepsilon_1)}$$

then

$$Ku = \frac{Smax^2}{2e^2(A_1+A_2)}$$

where Ku defines the slope of the unloading curve, e is the coefficient of restitution of the material, (A_1+A_2) is the total area under the curve ABD (calculated by the computer), and Smax is the maximum stress in the cushion determined by the wave equation.

Figures 4.10, 4.11, and 4.12 compare experimental force vs compression curves obtained for the first three cases listed in Table 4.1, with those resulting from the parabolic idealization of Figure 4.9, and the straight line shown in Figure 4.1. Note that the parabolic curves closely represent the actual force-displacement curves while the linear curves are not nearly so close. In each case the parabolic curves tend to "over-shoot" the true maximum force, while the linear curve does not. The effect this has on the stress wave in the pile will be discussed in Chapter V.

FIGURE 4.10 - DYNAMIC FORCE VS COMPRESSION CURVES FOR A FIR CUSHION (CASE LT-48)

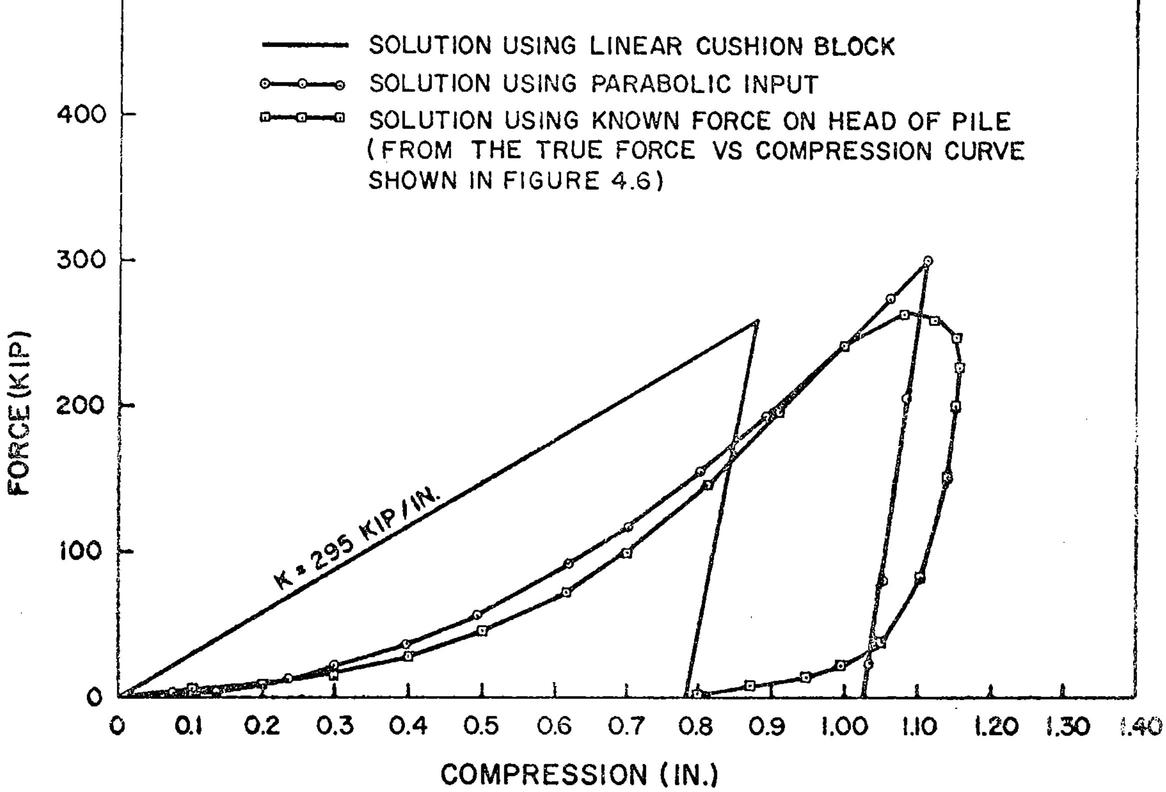
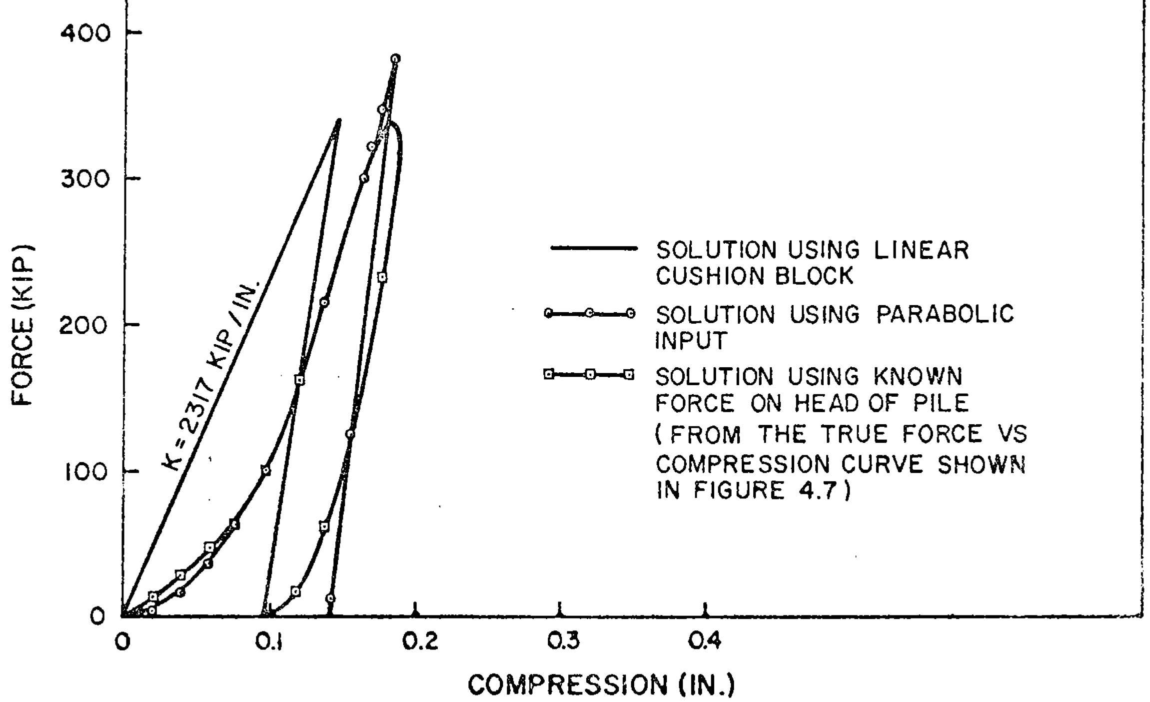


FIGURE 4.11 - DYNAMIC FORCE VS COMPRESSION CURVE FOR A MICARTA CUSHION (CASE LT-41)



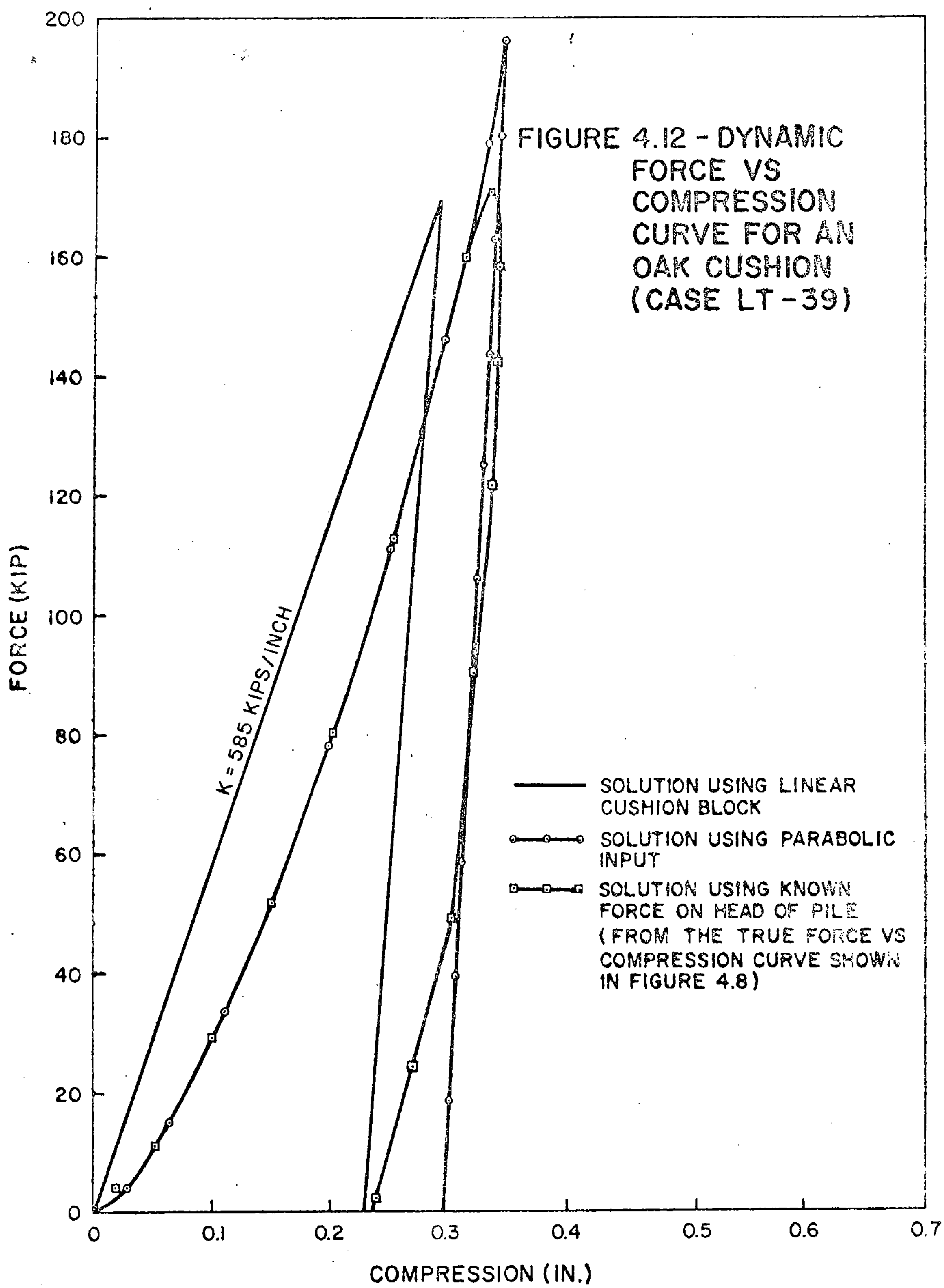


FIGURE 4.12 - DYNAMIC FORCE VS COMPRESSION CURVE FOR AN OAK CUSHION (CASE LT-39)
FORCE (KIP)
200
180
160
140
120
100
80
60
40
20
0
K = 585 KIPS/INCH
SOLUTION USING LINEAR CUSHION BLOCK
SOLUTION USING PARABOLIC INPUT
SOLUTION USING KNOWN FORCE ON HEAD OF PILE (FROM THE TRUE FORCE VS COMPRESSION CURVE SHOWN IN FIGURE 4.8)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
COMPRESSION (IN.)

CHAPTER V

STRESS WAVES IN PILING

Comparison of Actual and Experimental Stress Waves

As noted in Chapter IV, the shape and magnitude of the stress wave in a pile is greatly dependent upon the properties of the cushion used. This will become apparent by comparing the actual stress wave determined experimentally with results found by using the idealized cushion properties mentioned earlier.

The solution for stresses in the pile should be more accurate if the effects of the cushion and ram can be omitted. To accomplish this, the force measured at the head of the pile and the stresses at other gage points then determined by using the wave equation. The cases solved by this method are listed in Table 4.1. Comparisons between the experimental results and wave equation solutions at two points on the pile are shown in Figures 5.1 through 5.6.

One of the major factors which influenced these comparisons was the fact that the prestressed concrete test piles cracked while setting up the experiment. Therefore, any reflected tensile forces greater than the prestressing force opened a small gap at the crack such that the prestressing strands alone could transmit

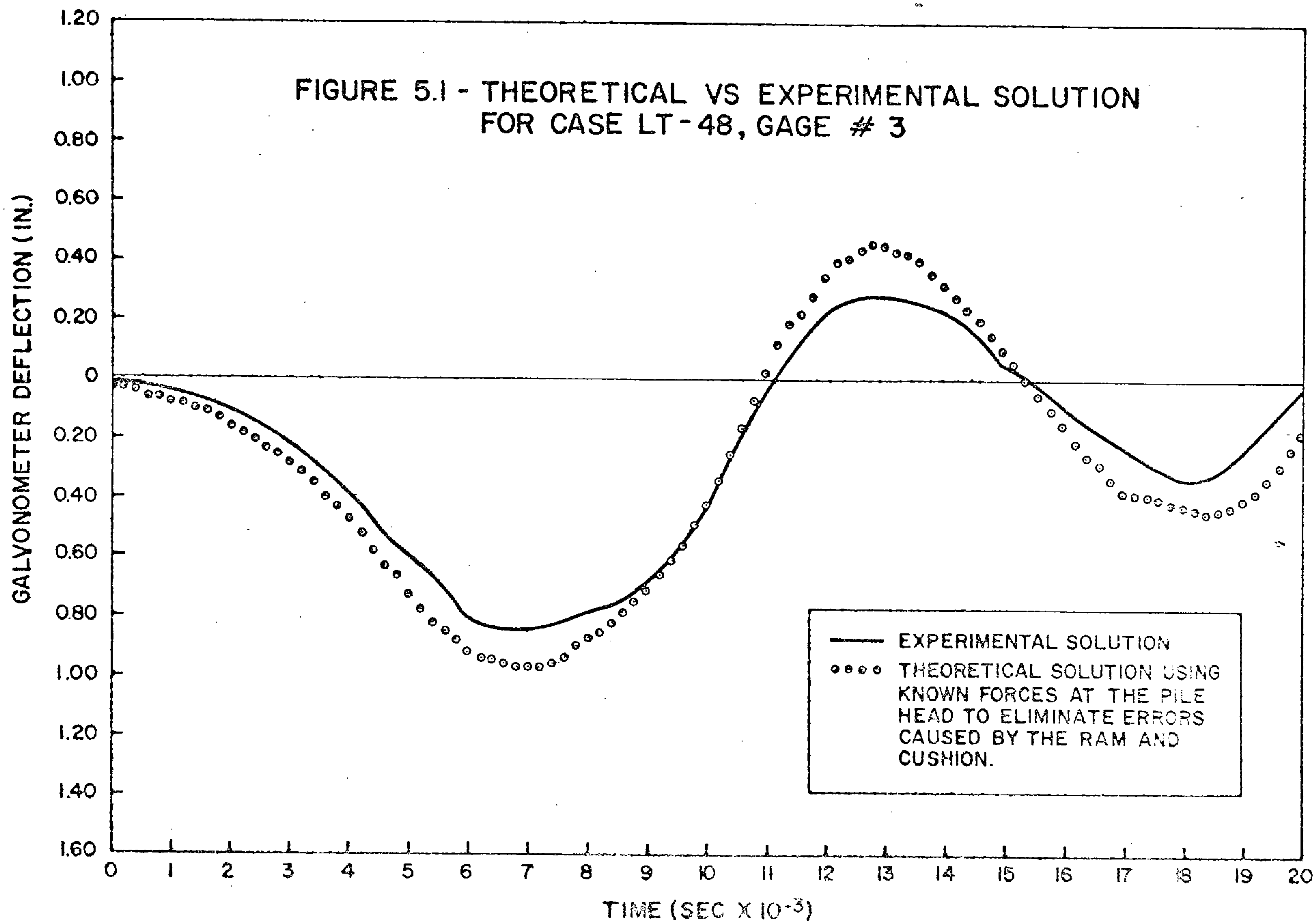


FIGURE 5.1 - THEORETICAL VS EXPERIMENTAL SOLUTION FOR CASE LT-48, GAGE # 3

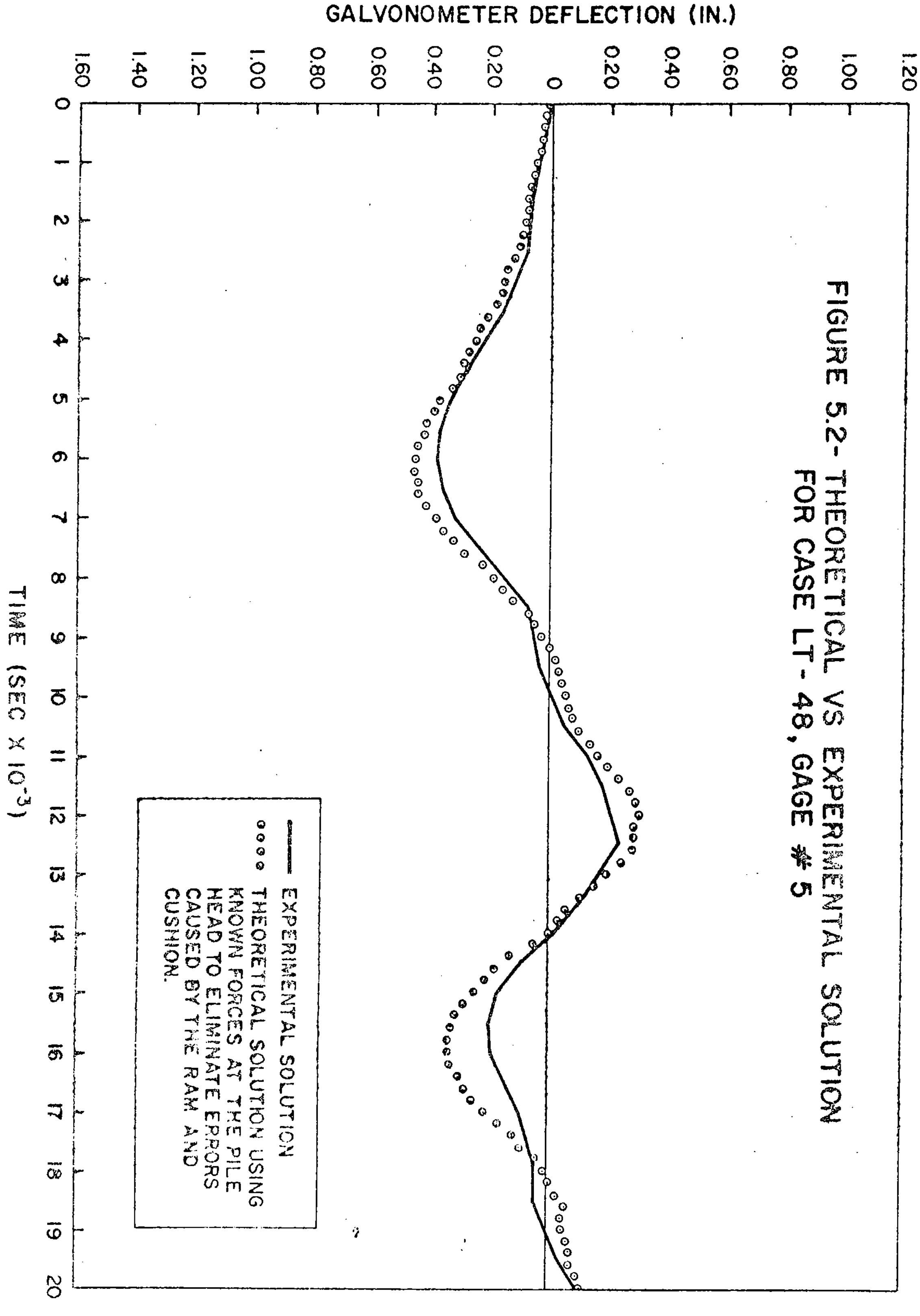


FIGURE 5.2- THEORETICAL VS EXPERIMENTAL SOLUTION FOR CASE LT - 48, GAGE # 5

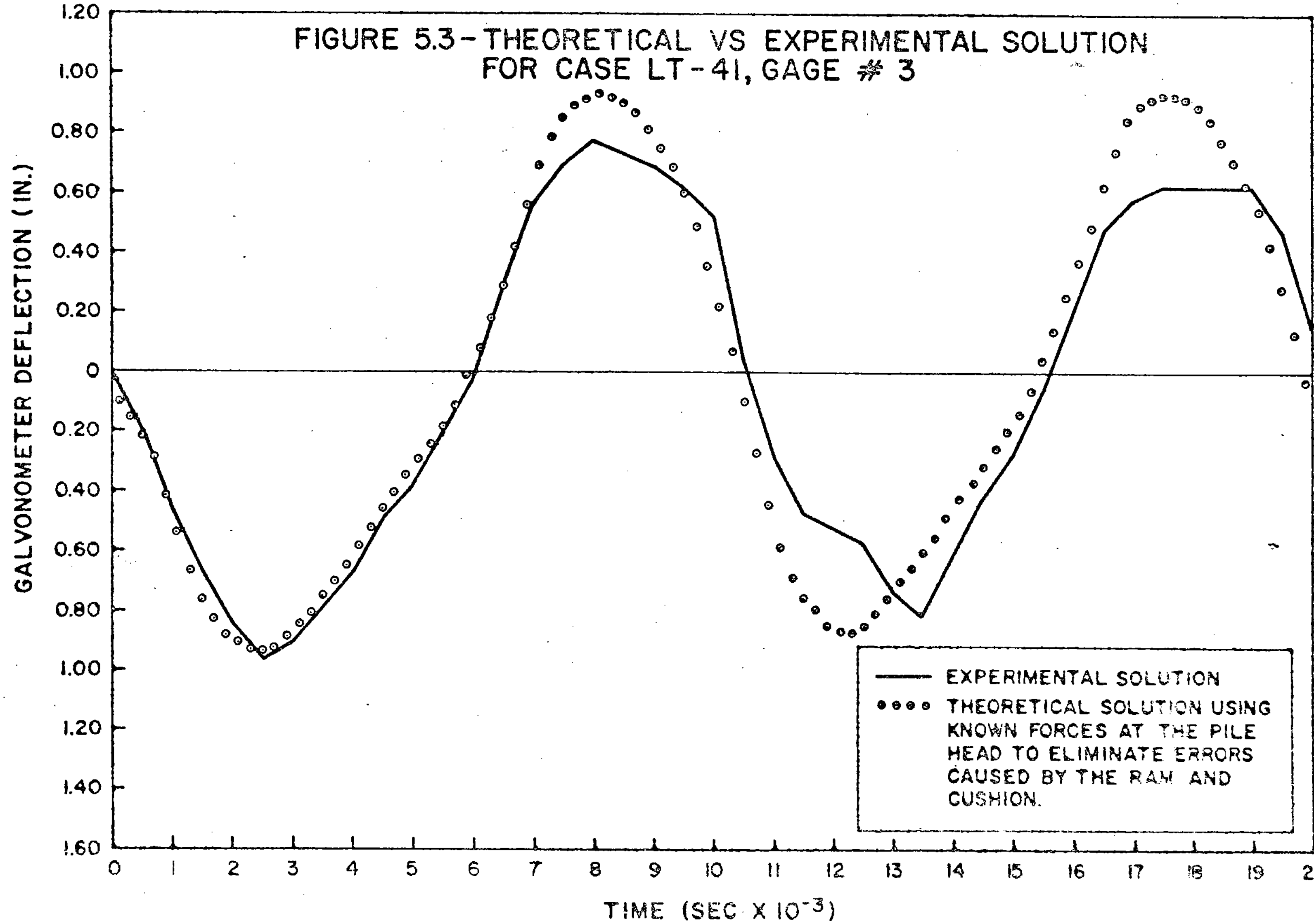


FIGURE 5.3 - THEORETICAL VS EXPERIMENTAL SOLUTION
FOR CASE LT-41, GAGE # 3
GALVONOMETER DEFLECTION (IN.)
1.20
1.00
0.80
0.60
0.40
0.20
0
0.20
0.40
0.60
0.80
1.00
1.20
1.40
1.60
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
TIME (SEC X 10-3)
EXPERIMENTAL SOLUTION
THEORETICAL SOLUTION USING KNOWN FORCES AT THE PILE HEAD TO ELIMINATE ERRORS CAUSED BY THE RAM AND CUSHION.

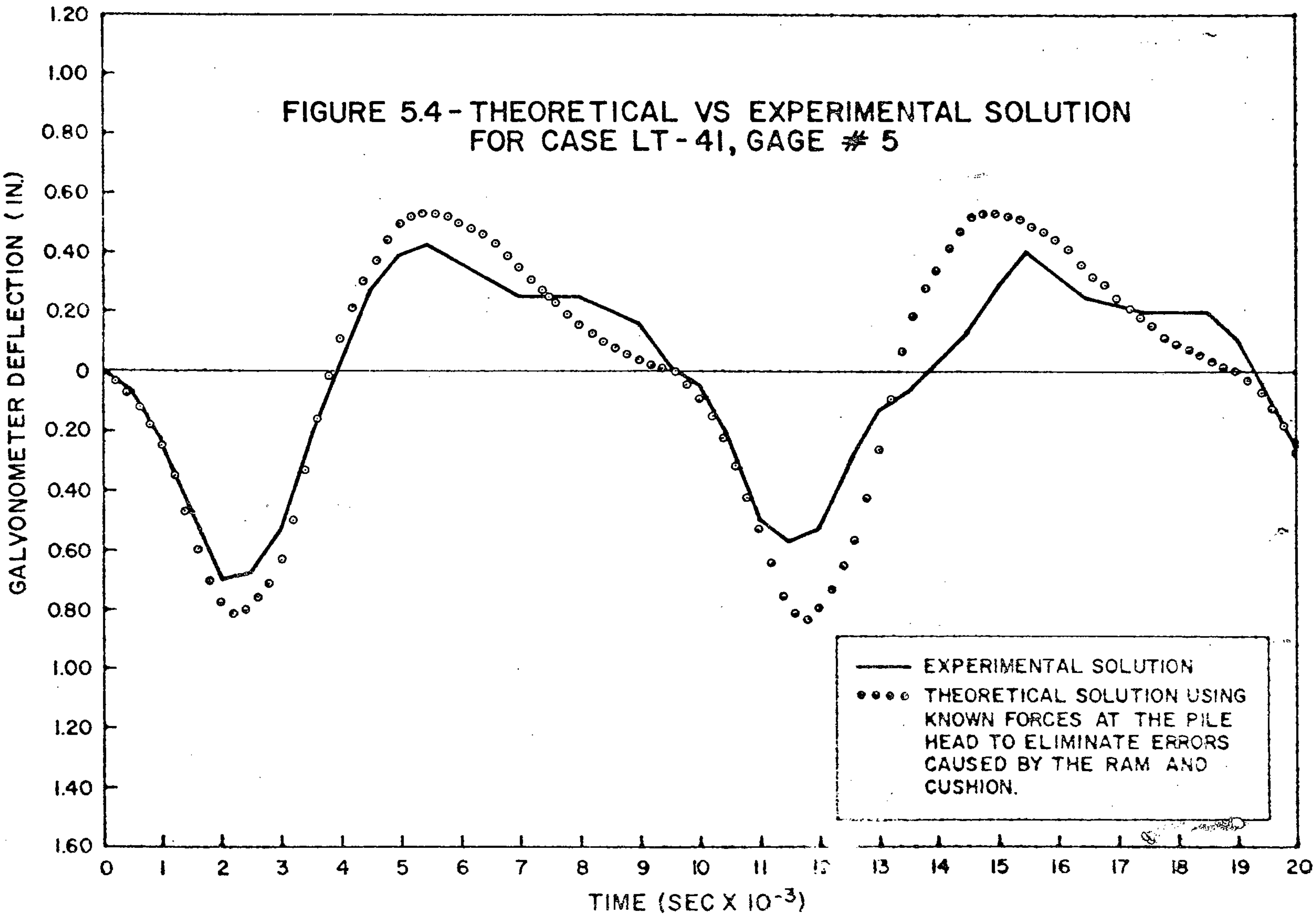


FIGURE 5.4 - THEORETICAL VS EXPERIMENTAL SOLUTION FOR CASE LT-41, GAGE # 5

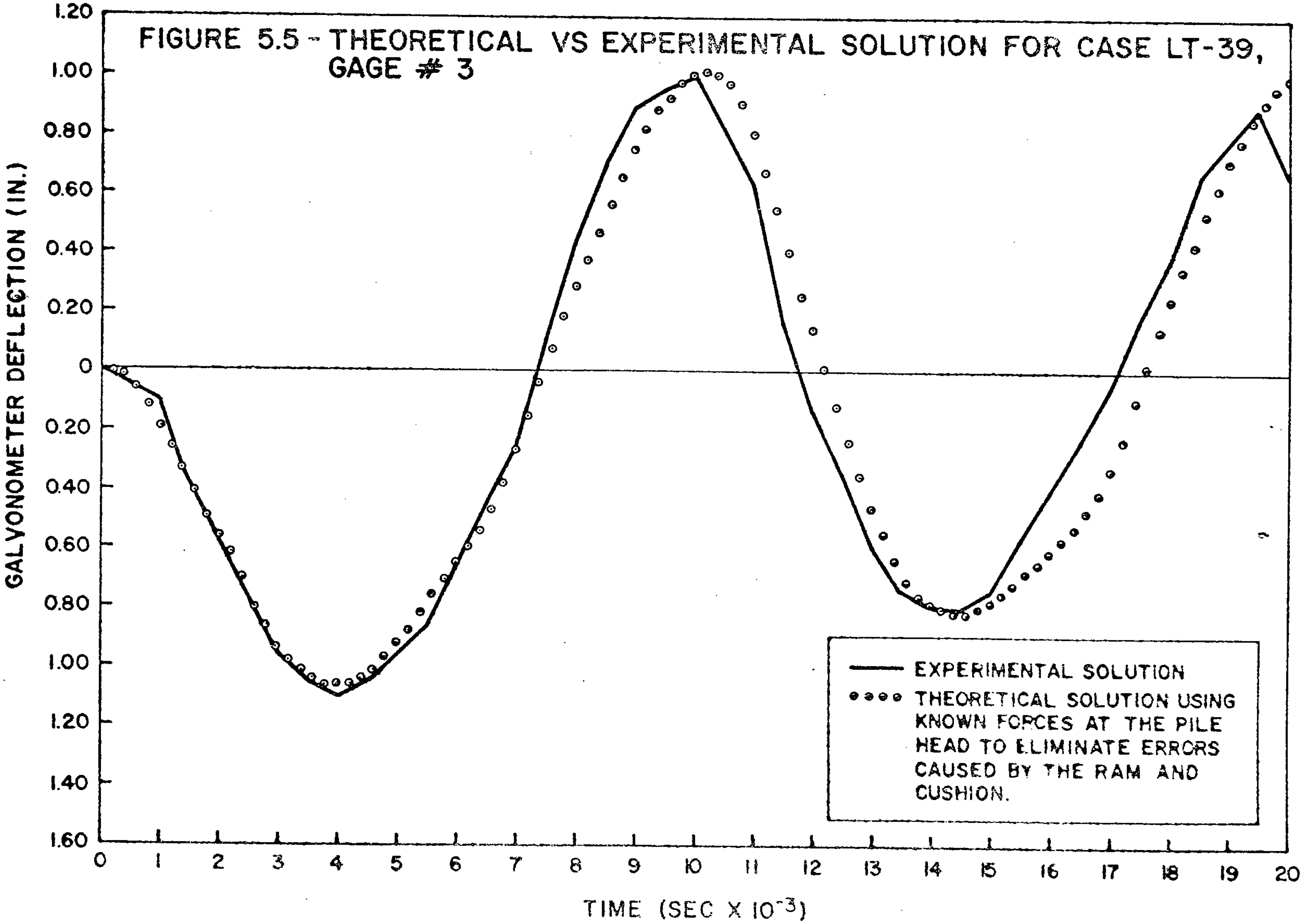


FIGURE 5.5 - THEORETICAL VS EXPERIMENTAL SOLUTION FOR CASE LT-39, GAGE # 3

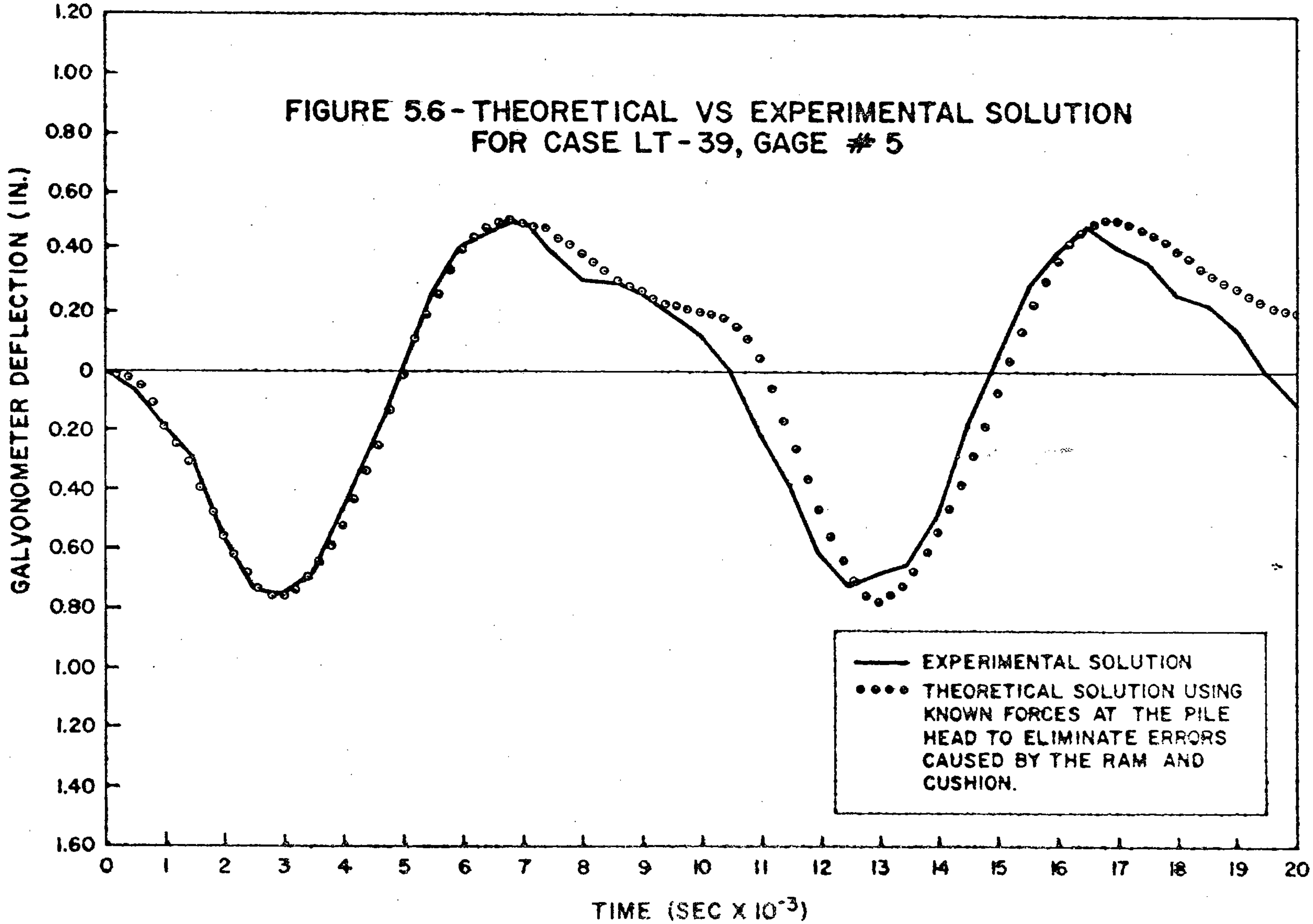


FIGURE 5.6 - THEORETICAL VS EXPERIMENTAL SOLUTION FOR CASE LT-39, GAGE # 5

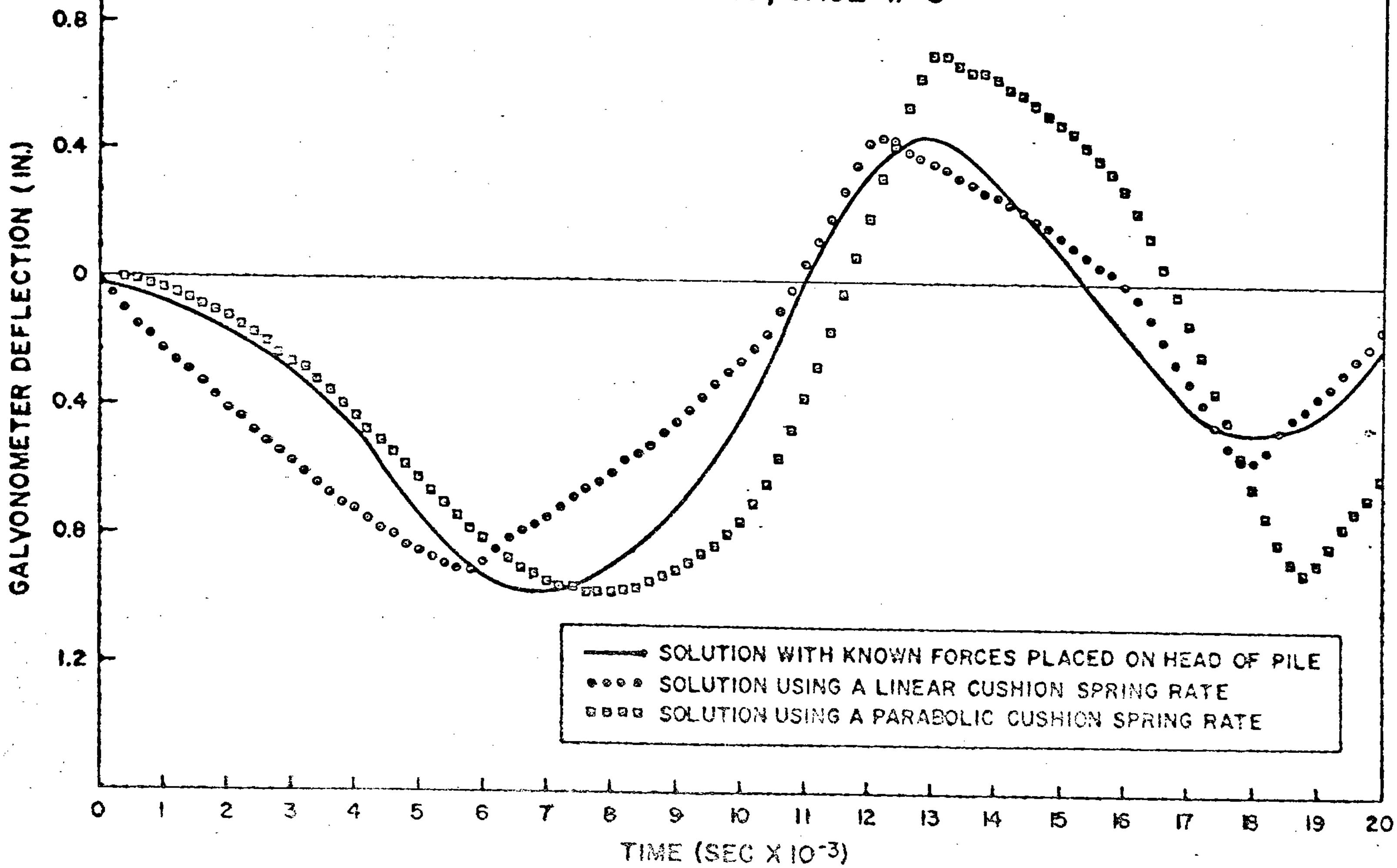
the tensile stress down the pile. This is seen by the relative agreement shown in Figures 5.1 through 5.6. Note that the stress-waves shown for the concrete piles (Figures 5.1 through 5.4) do not agree nearly so well as those for the steel pile (Figures 5.5 and 5.6).

Still, the results agree closely in each case, not only in magnitude, but also in the overall shape of the wave, thus indicating that the numerical solution to the wave equation is quite accurate. Further, any inaccuracies are likely due to faulty assumptions concerning the dynamic behavior of other variables such as the cushion, soil, etc.

As mentioned earlier, the stress-strain curve for the cushion is normally assumed to be linear as in Figure 4.1. The true stress-strain curves shown in Figures 4.6 through 4.8 indicate that the curves are not actually linear and this assumption might therefore cause inaccuracies.

To determine how the shape of the curve affects the solution, the previous three problems were run using the cushion stress-strain curves shown in Figures 4.1 (straight line), 4.6 through 4.8 (true stress-strain curves), and 4.9 (parabolic curve). These solutions are compared in Figures 5.7 through 5.12. In each case, it is noted that the straight line solution

FIGURE 5.7- THEORETICAL VS EXPERIMENTAL SOLUTION
FOR CASE LT-48, GAGE # 3



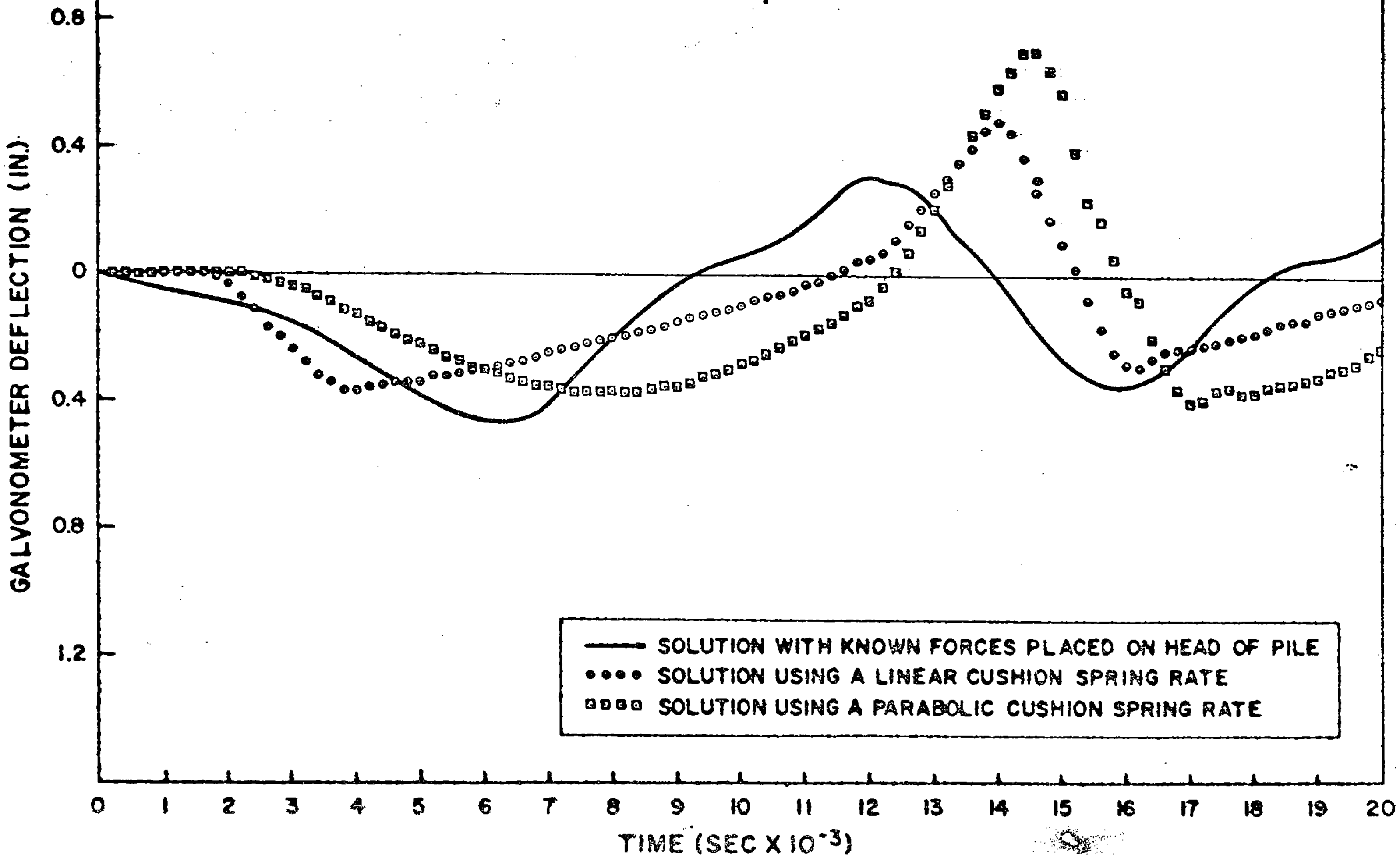


FIGURE 5.8 - THEORETICAL VS EXPERIMENTAL SOLUTION FOR CASE LT-48, GAGE # 5

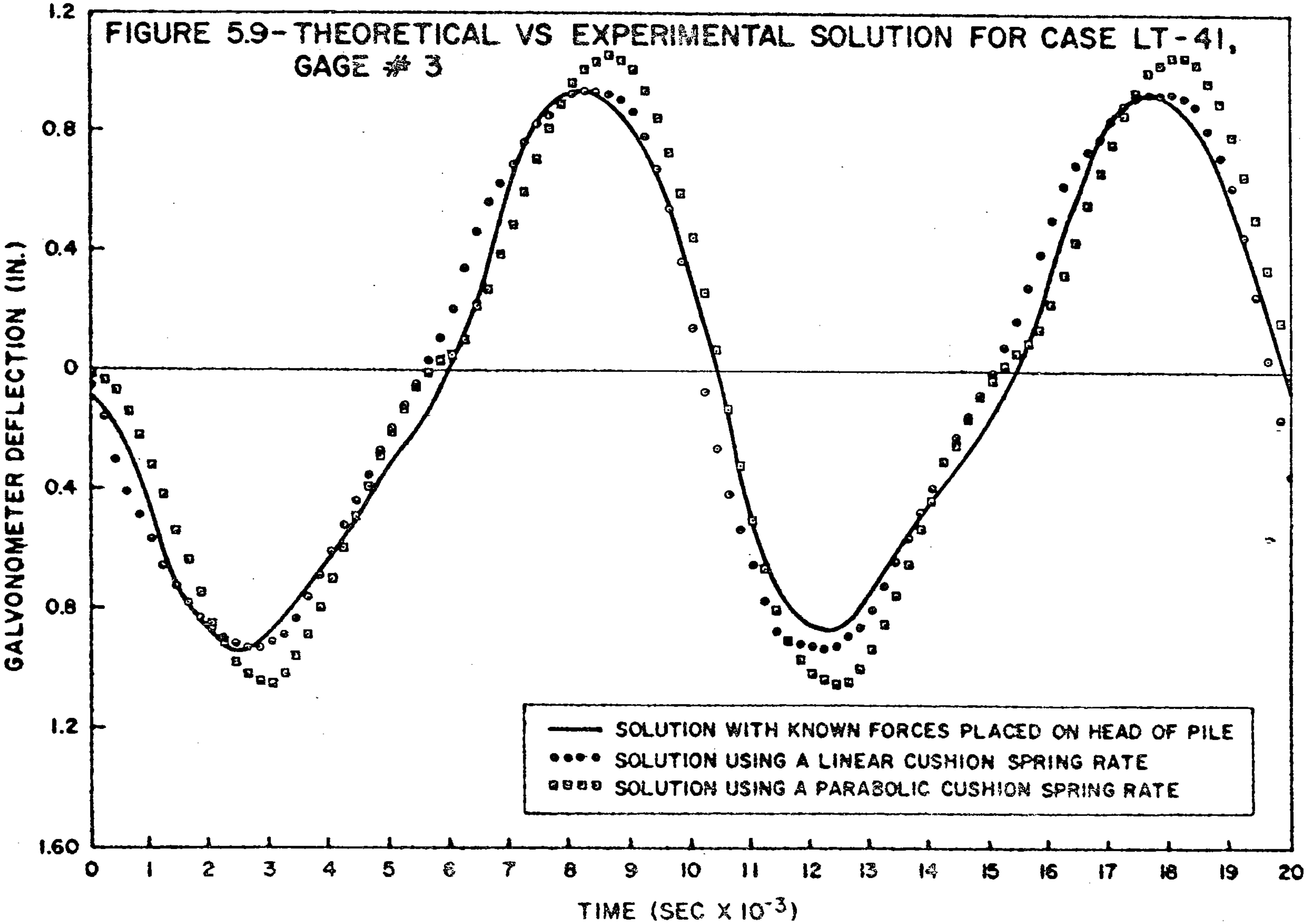


FIGURE 5.9 - THEORETICAL VS EXPERIMENTAL SOLUTION FOR CASE LT-41, GAGE # 3

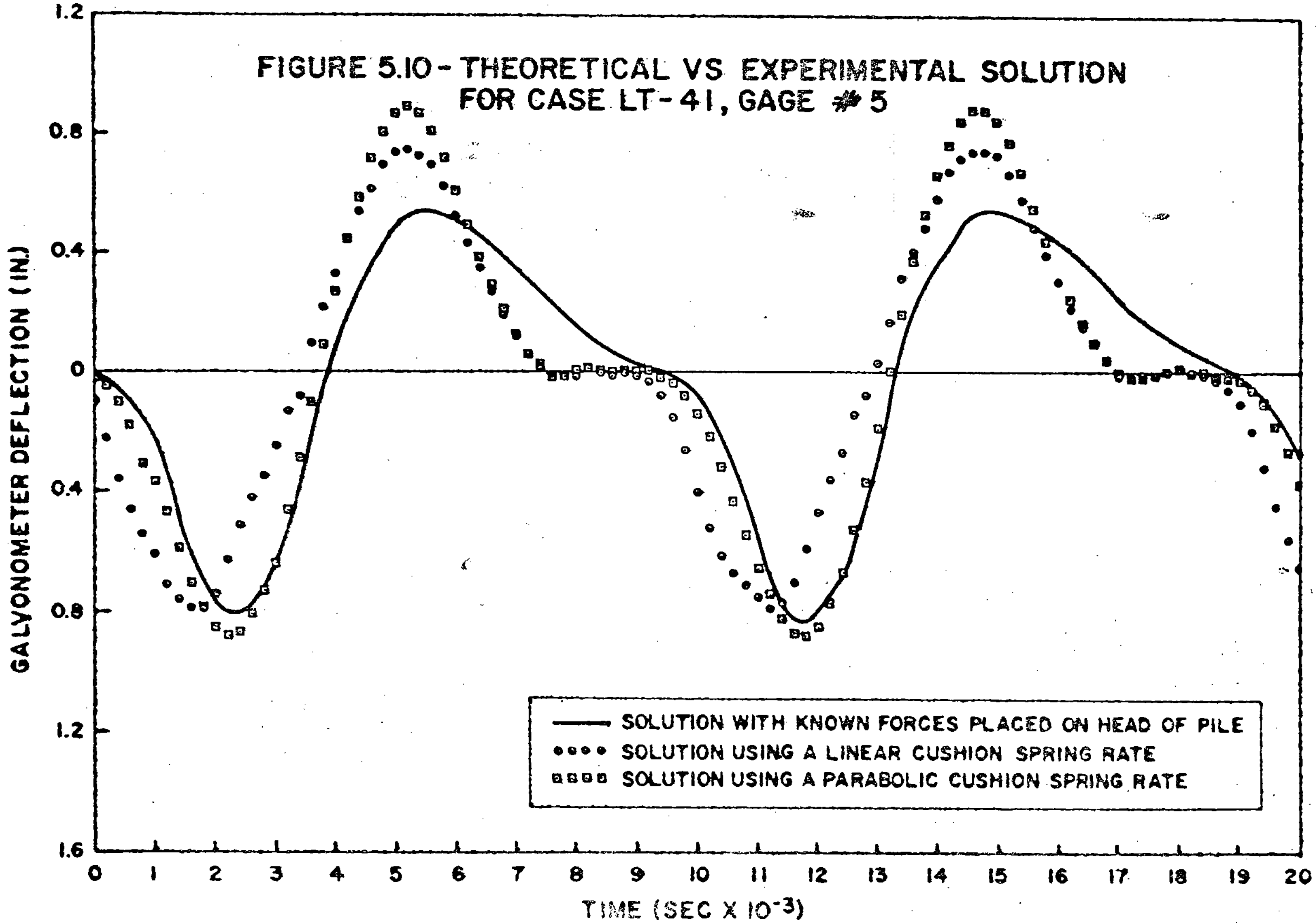


FIGURE 5.10 - THEORETICAL VS EXPERIMENTAL SOLUTION FOR CASE LT-41, GAGE #5

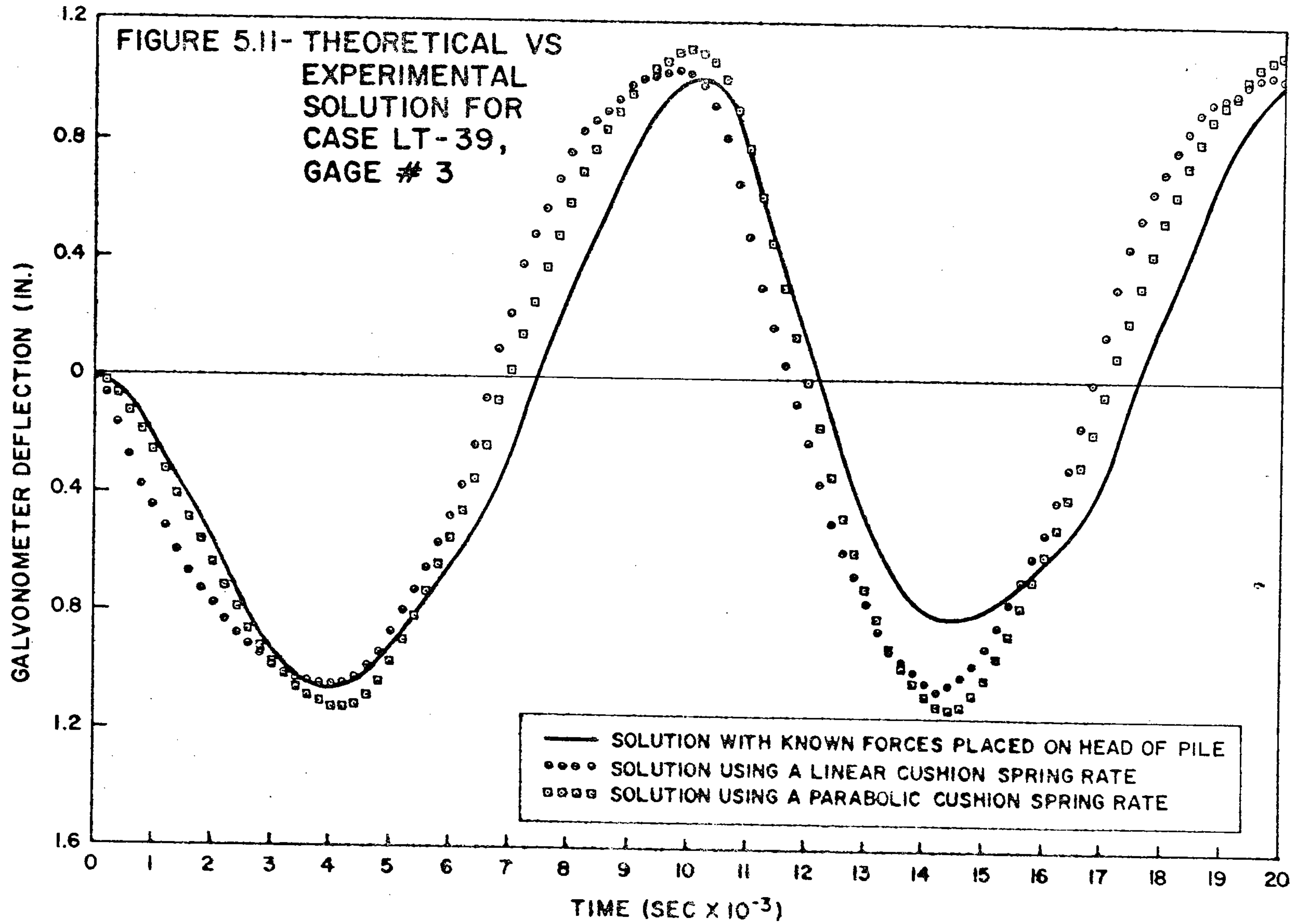


FIGURE 5.11- THEORETICAL VS EXPERIMENTAL SOLUTION FOR CASE LT-39, GAGE # 3

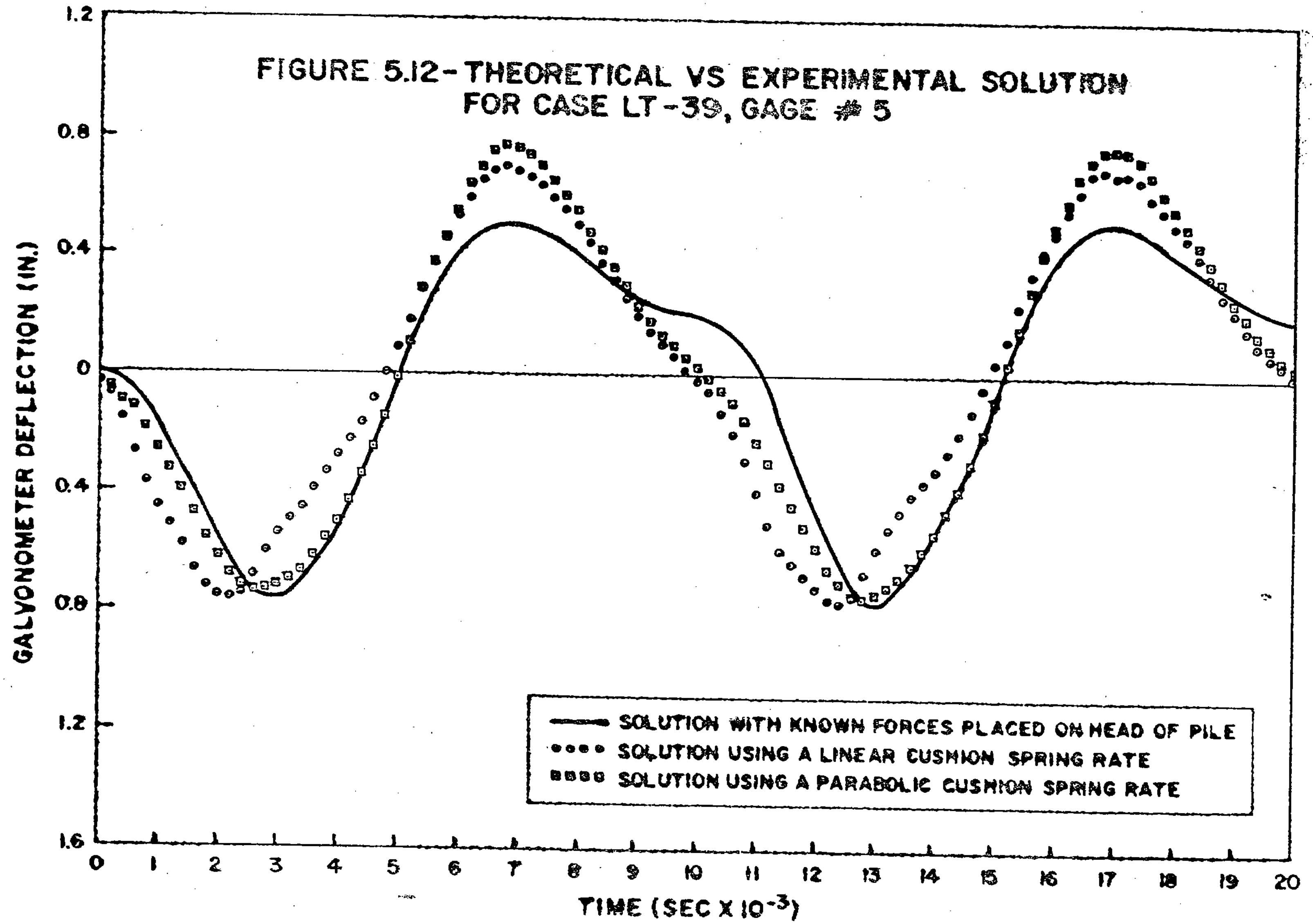


FIGURE 5.12 - THEORETICAL VS EXPERIMENTAL SOLUTION FOR CASE LT-39, GAGE # 5

is more accurate than the solution using the parabolic curve. This is because a single parabolic curve was used which, even though it agrees with the actual stress-strain curve most of the time, it cannot follow the reversed curvature at the peak of the actual curve and thus "over-shoots" the true peak force. Figures 4.10 through 4.12 show how closely the parabolic curves follow the true cushion forces, and also how far off the straight line assumption is. The parabolic curve always peaks above the true force vs compression curve, while the spring rate of the straight line can be raised or lowered so that the true maximum cushion force is not exceeded.

Thus the use of the straight-line assumption seems reasonable since it gives fairly accurate results. The linear spring constants used for the curves shown in Figures 5.7 through 5.12 were first varied between wide limits to obtain the most accurate maximum stresses. These spring rates were then used to determine what dynamic modulus of elasticity was required to give the desired spring rate, using the equation: Edynamic = (K cushion)(Length)/(Area of cushion). As shown in Table 5.1, these results give a lower value of E for oak than for fir, which in this case is correct since the fir capblock was highly stressed (4,170 psi) while

TABLE 5.1 DYNAMIC PROPERTIES OF NEW CUSHION BLOCKS OF VARIOUS MATERIALS

Case	Cushion Material	Linear Spring Rate - K (lb/in)	Depth of Cushion (in)	Area of Cushion (in^2)	$E_{dynamic}$ (psi)	Slope at Midpoint of Curve (psi)	SMAX in Cushion (psi)
LT-48	Fir	295,000	9.0	62.8	42,200	37,300	4170
LT-41	Micarta	2,320,000	9.0	89.1	234,000	212,000	3850
LT-39	Oak	585,000	7.5	225.0	19,500	17,300	765

the oak capblock was stressed only slightly (765 psi).

Further consideration of the dynamic stress-strain curves revealed that the dynamic modulus of elasticity of the capblock is almost exactly 10 percent greater than that given by the slope of the stress-strain curve (Figures 4.6 through 4.8) taken at a point halfway between zero and the maximum strain. As noted by Hirsch[62], the static and dynamic stress-strain curves are quite similar, so that curves like those shown in Figures 4.6 through 4.8 are easily determined for any other cushion material. It was also recommended that the dynamic modulus be increased as the cushion consolidated[63].

Internal Damping in Piling

As noted earlier, differences between experimental and theoretical results were assumed to be the result of inaccurate soil information. Other parameters were also varied in an attempt to obtain more accurate results[56], one of which was the material damping or internal damping capacity of the pile material.

Smith[57] first suggested that the internal damping in the pile might prove significant, and proposed the following equation by which hysterisis in the pile could be accounted for:

$$F(m,t) = C(m,t)K(m)+\frac{BK(m)}{12\Delta t}[C(m,t)-C(m,t-1)]$$

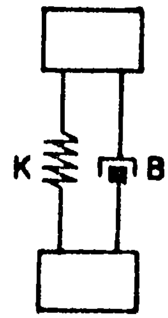
in which B is the internal damping constant. He also recommended that B be given a value of about 0.002 in order to produce a narrow hysteresis loop. This equation was derived from the model shown in Figure 5.13 (b) and if B is set equal to zero, no damping is present, as seen in Figure 5.13 (a).

The model shown in Figure 5.13 (c) has one major advantage over the previous model in that it is able to account for damping by considering the difference between the material's static modulus of elasticity E, and its sonic modulus of elasticity E_s. This is because a slowly applied load gives the dashpot time to relax without causing the spring K_s to exert a force, thereby resulting in a spring rate equal to K_0. However, when the loads are applied rapidly the dashpot has no chance to deform, resulting in a spring rate of K_0+K_s. Thus for the model of Figure 5.13 (c), K_0 is determined from the static modulus of elasticity E, while K_0+K_s would use the sonic value E_s.

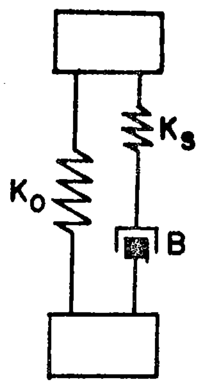
It is interesting to note that when K_s is infinitely large, model (c) becomes equivalent to model (b), and if $K_s = 0$, model (c) becomes equivalent to model (a).

In order to derive the equation, Figure 5.14 is provided. Figure 5.14 (a) illustrates the damping model

(a) NO DAMPING PRESENT



(b) INTERNAL DAMPING PROVIDED BY DASHPOT



(c) INTERNAL DAMPING PROVIDED BY AN ELASTIC SPRING AND DASHPOT CONNECTED IN SERIES

FIGURE 5.13 - VARIOUS IDEALIZATIONS FOR THE SPRING SEGMENT OF A PILE

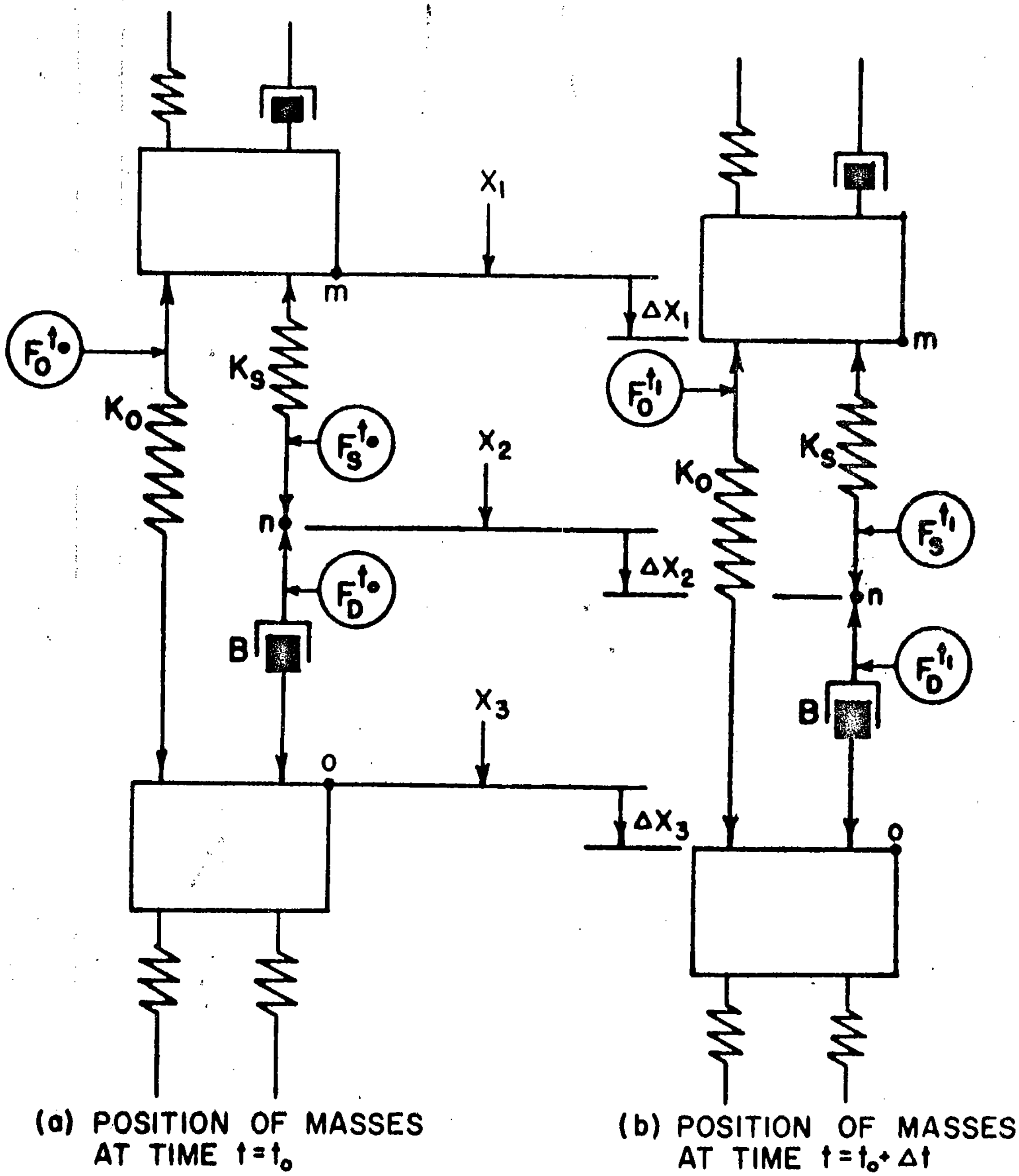


FIGURE 5.14. – IDEALIZED PILE SEGMENT WITH STANDARD LINEAR SOLID DAMPING

wherein point"m" (on the upper mass) has moved a distance x_1, point "n" (between the dashpot and spring) has moved a distance x_2, and point "o" (on the lower mass) has moved a distance of x_3. Assume that at time $t = t_o$ there exists a force $F_o{}^{t}o$ in the spring K_o. There is also a force in the spring K_s given by $F_s{}^{t}o$, and a force in the dashpot equal to $F_D{}^{t}o$.

As shown in Figure 5.14 (b), after a single time interval passes, point m moves an additional distance Δx_1, point na moves Δx_2, and point o moves Δx_3. At this time, $t = t_1 = t_o + \Delta t$, and the forces in K_o, K_s, and B are designated $F_o{}^{t}1$, $F_s{}^{t}1$, and $F_D{}^{t}1$ respectively.

At time t_o:

$$F_s{}^{t}o = K_s(x_1 - x_2). \qquad \text{Eq. 5.1}$$

At time $t_1 = t_o + \Delta t_1$:

$$F_s{}^{t}1 = K_s[(x_1 + \Delta x_1) - (x_2 + \Delta x_2)].$$

$$F_s{}^{t}1 = K_s[(x_1 - x_2) + (\Delta x_1 - \Delta x_2)]. \qquad \text{Eq. 5.2}$$

Substituting Equation 5.1 into 5.2:

$$F_s{}^{t}1 = F_s{}^{t}o + K_s(\Delta x_1 - \Delta x_2). \qquad \text{Eq. 5.3}$$

By definition, at all times:

$$F_D{}^{t}1 = B\frac{(\Delta x_2 - \Delta x_3)}{\Delta t}. \qquad \text{Eq. 5.4}$$

Because point n must be in equilibrium:

$$F_S t_1 = F_D t_1. \qquad \text{Eq. 5.5}$$

Substituting Equation 5.3 and 5.4 into 5.5:

$$F_D t_o + K_s(\Delta x_1 - \Delta x_2) = B\frac{(\Delta x_2 - \Delta x_3)}{\Delta t}$$

$$F_D t_o + K_s \Delta x_1 = K_s \Delta x_2 + \frac{B\Delta x_2}{\Delta t}\ \frac{B\Delta x_3}{\Delta t}$$

$$F_D t_o \Delta t + K_s \Delta x_1 \Delta t + B\Delta x_3 = \Delta x_2 (K_s \Delta t + B).$$

Solving for Δx_2:

$$\Delta x_2 = \frac{F_D t_o \Delta t + K_s \Delta x_1 \Delta t + B\Delta x_3}{K_s \Delta t + B}. \qquad \text{Eq. 5.6}$$

Substituting Equation 5.6 into 5.4 produces:

$$F_D t_1 = \frac{F_D t_o + K\ (\Delta x_1 - \Delta x_3)}{(K_s \Delta t / B) + 1} \qquad \text{Eq. 5.7}$$

The solution begins by setting $F_D t_o$ equal to zero, and calculating it for the next time interval from Equation 5.7. The quantity K_s is a constant and $(\Delta x_1 - \Delta x_3)$ is simply the change in compression during a single time interval. Therefore, returning to the earlier terminology, Equation 5.7 can be written:

$$DF(I,t+1) = \frac{DF(I,t) + DK(I)[C(I,t+1) - C(I,t)]}{[DK(I)\Delta t/B] + 1.0} \qquad \text{Eq. 5.8}$$

where DF(I,t) is the damping force in dashpot number "I" during time interval "t", DK(I) is the dynamic spring rate of damping spring "I", C(I,t) is the compression in spring I during time interval number t, Δt is the time increment, and B is a damping constant.

The static force in spring I will be computed as before, by

$$F(I,t+1) = K(I)[C(I,t+1)]. \qquad \text{Eq. 5.9}$$

Thus by adding the Equations 5.8 and 5.9, the total force acting on each mass can be determined for the next time interval.

Since as far as is known this derivation does not appear elsewhere, the boundary conditions for the damping force given by Equation 5.7 were checked. From Equation 5.7,

$$\text{(a) Letting } K_s = 0: F_D{}^{t_1} = \frac{F_D{}^{t_o}+0}{1+0} = F_D{}^{t_o}.$$

This is correct since F_D begins at zero and cannot increase in magnitude when $K_s = 0$.

$$\text{(b) Letting } K_s = \infty: \quad F_D{}^{t_1} = \frac{F_D{}^{t_o}+\infty}{\infty+1} = \infty/\infty.$$

Since this is indeterminate,

$$F_D{}^{t_1} = \lim_{K_s \to \infty} \frac{\frac{d}{dK_s}[F_s{}^{t_o}+K_s(\Delta x_1-\Delta x_3)]}{\frac{d}{dK_s}\left[\frac{K_s \Delta t}{B}+1\right]}$$

$$= \lim_{K_s \to \infty} \frac{0+(\Delta x_1-\Delta x_3)}{\Delta t/B+0} = \frac{B(\Delta x_1-\Delta x_3)}{\Delta t}.$$

This checks since it is the equation found when $K_s = \infty$ and only the dashpot remains. In this case the models of Figures 5.13 (b) and (c) would be identical

(c) Letting $B = 0: F_D{}^{t_1} = \dfrac{F_D{}^{t_o}+K_s(\Delta x_1-\Delta x_3)}{1+\frac{K_s \Delta t}{0}} = \dfrac{1}{\infty} = 0.$

This checks since if the dashpot has no damping ability, the damping force must be zero.

(d) Letting $B = \infty: F_D{}^{t_1} = \dfrac{F_D{}^{t_o}+K_s(\Delta x_1-\Delta x_3)}{\frac{K_s \Delta t}{\infty}+1}$

$$= F_D{}^{t_o}+K_s(\Delta x_1-\Delta x_3)$$

But $F_D{}^{t_o} = F_S{}^{t_o} = K_s(x_1-x_2)$.

Substituting this into the previous equation one finds

$$F_D{}^{t_1} = [K_s][(x_1-x_2)+(\Delta x_1-\Delta x_2)]$$

$$= [K_s][(x_1+\Delta x_1)-(x_2+\Delta x_2)]$$

$$= [K_s][\text{Total compression at time t}].$$

This is correct since it is the equation for the spring and when $B = \infty$, the dashpot is "locked" and no damping occurs.

(e) Letting $\Delta t = 0: F_D{}^{t_1} = \dfrac{F_D{}^{t_o}+K_s(\Delta x_1-\Delta x_3)}{0+1}$

This result agrees because it gives the same result as letting $B = \infty$. (See part (d) above.)

(f) Letting $\Delta t \to \infty : F_D^{t_1} = \frac{F_D^{t_o} + K_s(\Delta x_1 - \Delta x_3)}{\infty + B} = 0.$

This checks because the force stored in the damping spring would be released by relaxation of the dashpot if $\Delta t = \infty$.

(g) Let $\Delta x_1 = \Delta x_2$ and assume that the damping spring has an initial force stored at $t = t_o$. Although this force should diminish with time, it cannot go to zero during a single time interval, uless $\Delta t = \infty$.

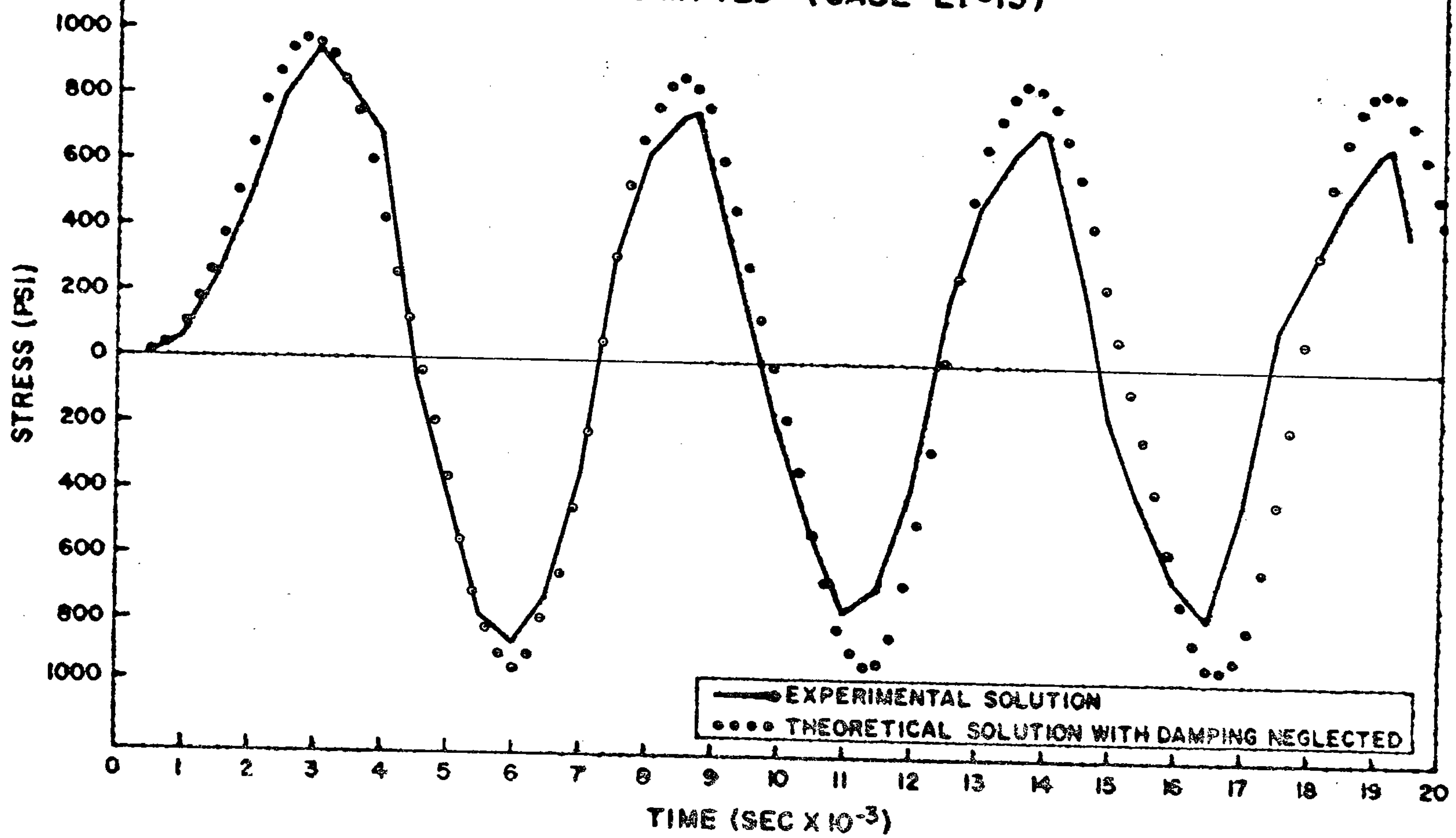
$$F_D^{t_1} = \frac{F_D^{t_o} + K_s(0)}{\frac{K_s \Delta t}{B} + 1.0} = \frac{F_D^{t_o}}{\frac{K_s \Delta t}{B} + 1.0} .$$

This is correct since the force in the spring is reduced, but will never actually reach zero unless $\Delta t = \infty$.

Figures 5.15 through 5.18 compare the effects of damping in a pile using the damping models shown in Figure 5.13. The results given are for test pile number LT-15 which is described in Table 4.1. This particular pile was of lightweight concrete with $E = 3.96 \times 10^6$ and $E_s = 4.63 \times 10^6$ psi. This problem was chosen since E_s was relatively larger than E, indicating the possibility of rather high damping.

However, one is often more interested in the maximum stresses found in the pile, which usually occurs during the first or second pass of the stress wave along the pile. During this time the effects of damping

FIGURE 5.15 - COMPARISON OF EXPERIMENTAL AND THEORETICAL SOLUTIONS FOR STRESSES AT GAGE # 3 WITH DAMPING OMITTED (CASE LT-15)



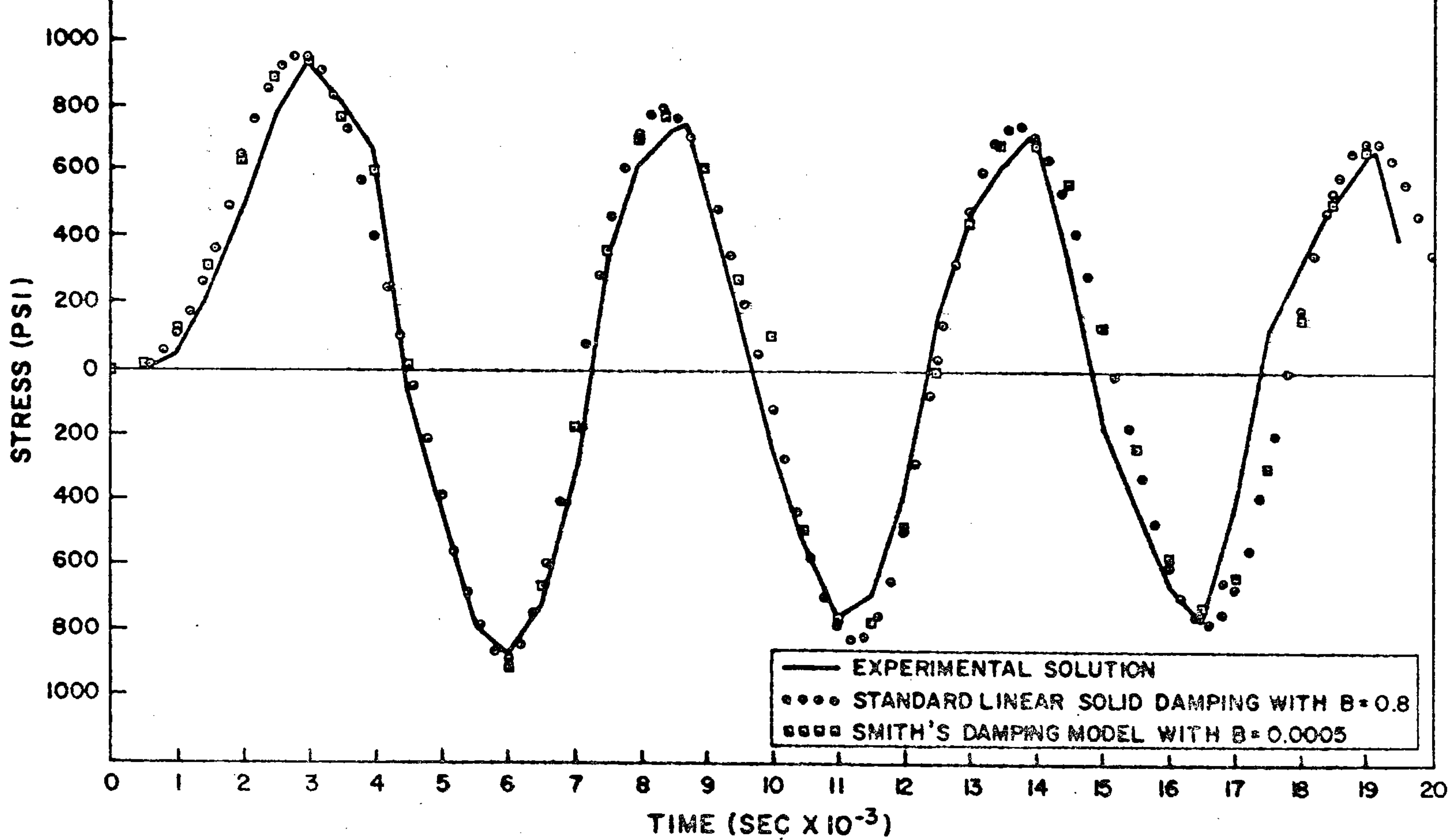


FIGURE 5.16 - COMPARISON OF EXPERIMENTAL AND THEORETICAL SOLUTIONS FOR STRESSES AT GAGE # 3 FOR DIFFERENT DAMPING MODELS (CASE LT-15)

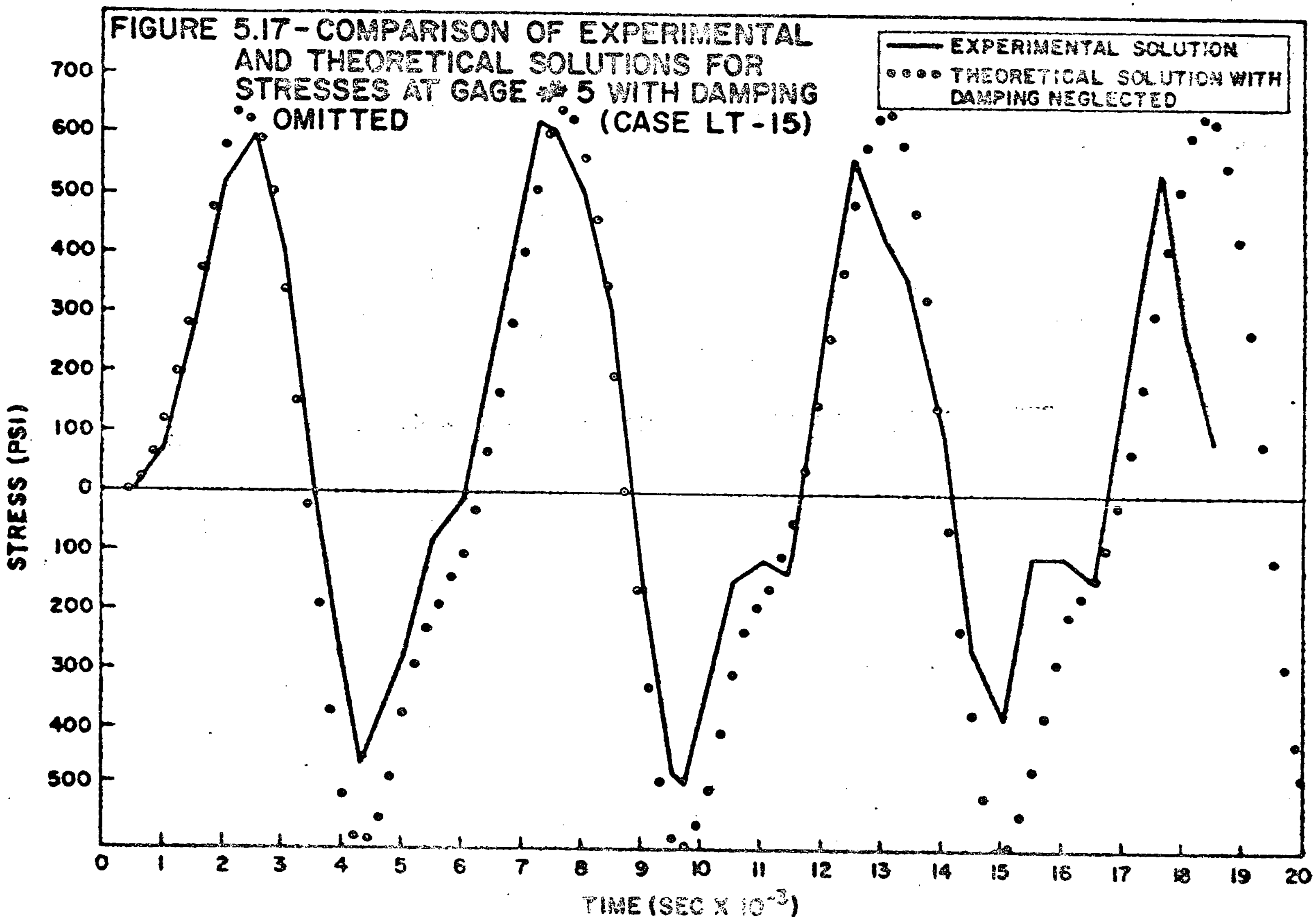


FIGURE 5.17 - COMPARISON OF EXPERIMENTAL AND THEORETICAL SOLUTIONS FOR STRESSES AT GAGE # 5 WITH DAMPING OMITTED (CASE LT-15)

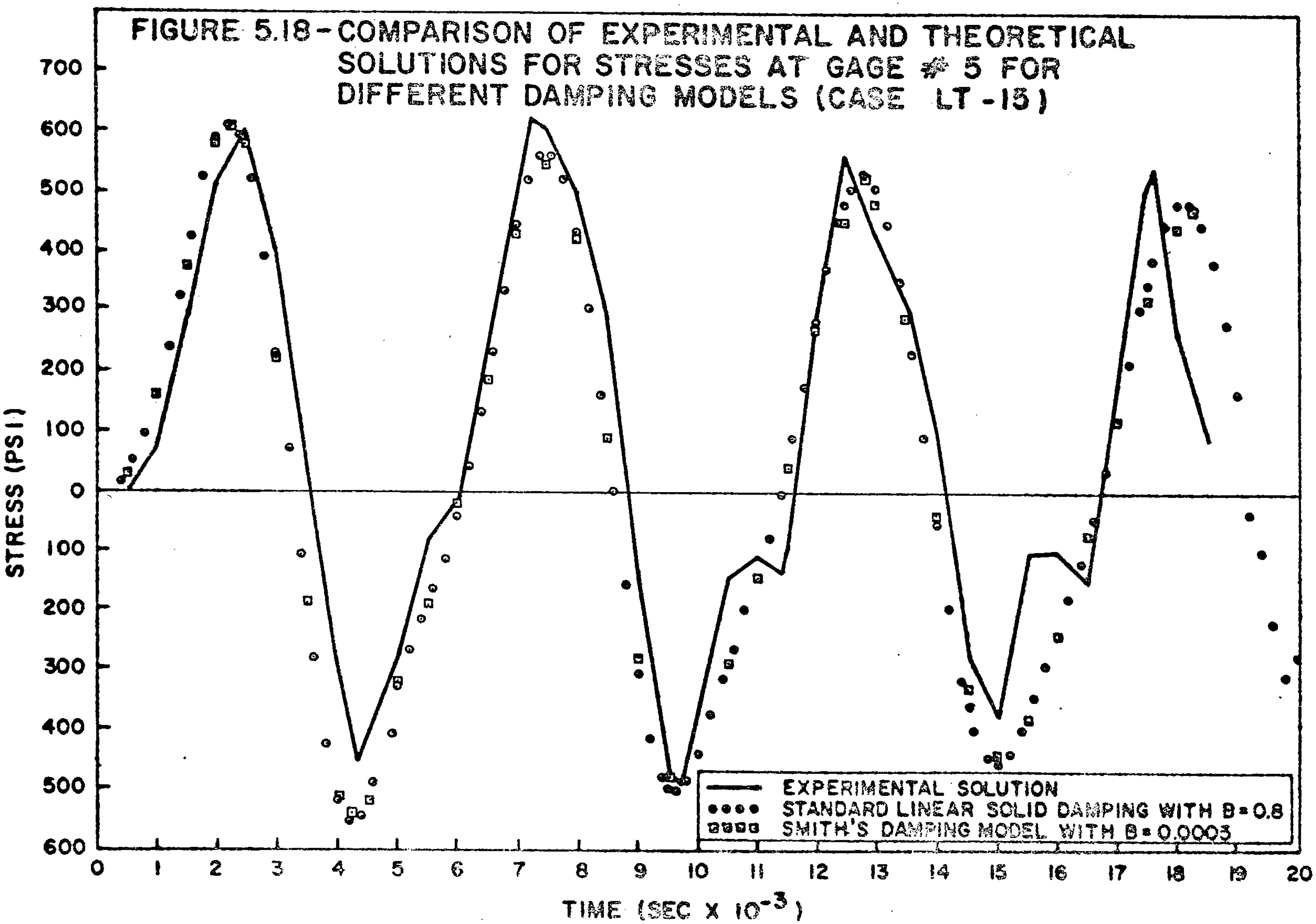


FIGURE 5.18 - COMPARISON OF EXPERIMENTAL AND THEORETICAL SOLUTIONS FOR STRESSES AT GAGE # 5 FOR DIFFERENT DAMPING MODELS (CASE LT-15)

are small and can usually be neglected.

This conclusion may not be accurate for timber piles since wood has a much higher damping capacity than either the steel or concrete piles for which experimental data was available. This higher damping capacity might affect the results earlier in the solution which might in turn lower the accuracy of the results. Nevertheless, if more testing should indicate that the damping models are accurate for timber piling too, then the problem, or rather the uncertainties of damping effects will no longer be a problem.

In any case, if the wave is to be studied for an extended period of time, damping in the pile cannot be neglected. This is illustrated in Figures 5.15 and 5.17 where fairly large errors resulted when damping was neglected. On the other hand, Figures 5.16 and 5.18 suggest that in certain cases damping should be accounted for using either of the damping models of Figure 5.13.

The most surprising result of this study is not the accuracy of the damping models, but rather that both models give nearly identical results even though Smith's model is extremely simple while the other is rather complex. Again, this may also prove incorrect for timber piling or other piling which has a large damping capacity. For example, one of the above methods might be more accurate than the other.

CHAPTER VI

SOIL PROPERTIES

Idealized Soil Resistance Curves

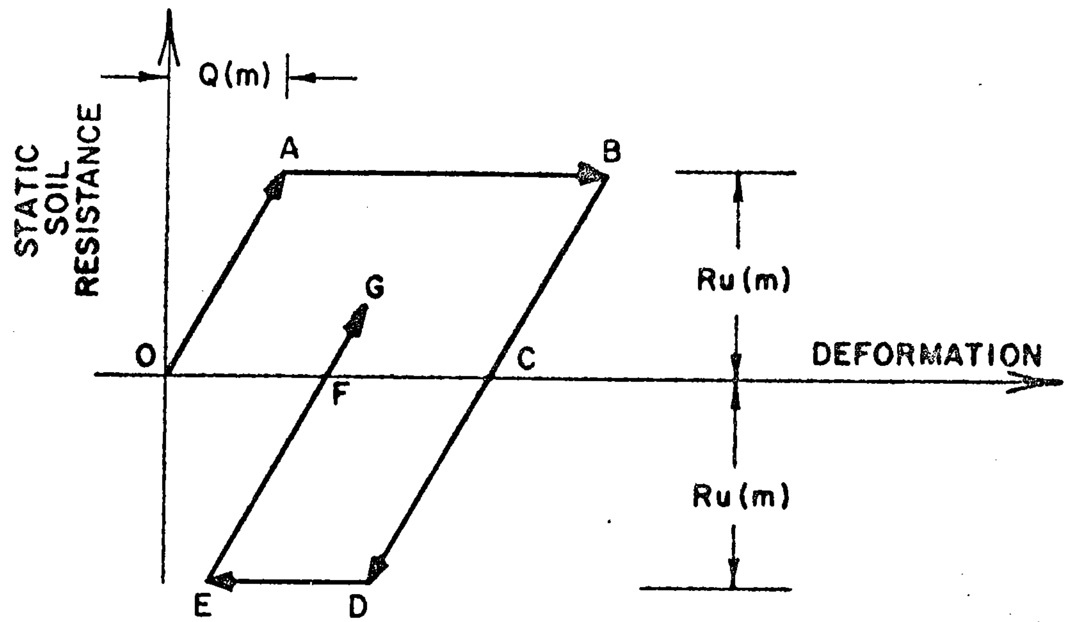
The load-deformation characteristics assumed for the soil in Smith's numerical solution are shown in Figure 6.1 (a). This curve excludes the damping effects of the soil caused by rapid loading, and illustrates only the soil resistance caused by static loading. As shown, the two parameters required to define the load-deformation curve are the ground quake "Q(m)" and the ultimate static soil resistance "Ru(m)".

When the soil is located along the side of the pile, it is assumed to resist any rebound of the pile as well as any downward motion. This is typified by the curve OABCDEFG. However, the soil located at the tip of the pile can only exert upward forces, as represented by the curve OABCFCB.

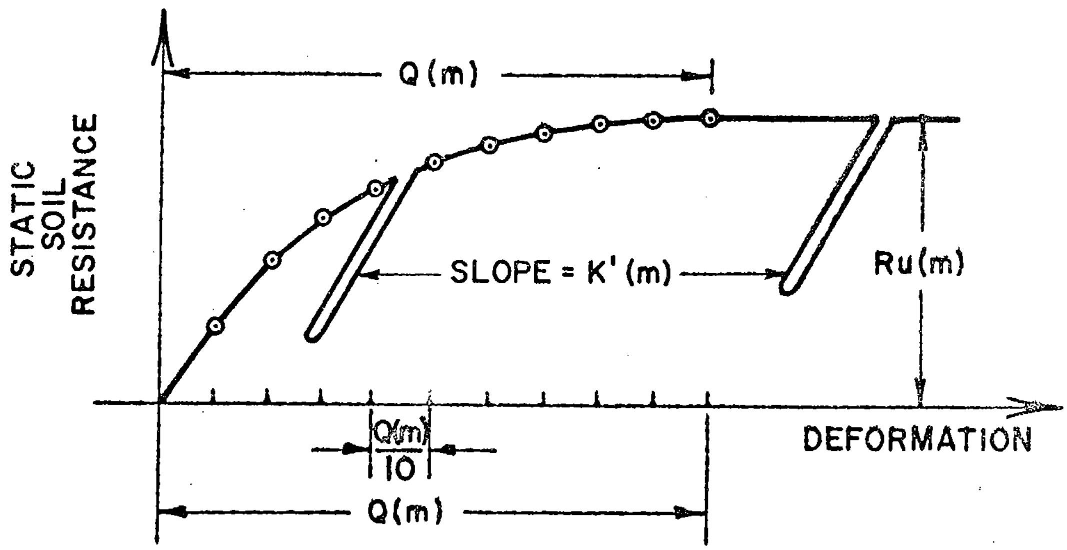
The spring rate for the curve between point O and A may now be determined from

$$K'(m) = \frac{Ru(m)}{Q(m)} .$$

In order to include the damping effects of the soil, a third variable J9m) is defined as the damping constant of soil spring "m". Thus the total resistance



(a) ELASTIC-PLASTIC OR "LINEAR" SOIL RESISTANCE CURVE



(b) GENERALIZED SOIL RESISTANCE CURVE

FIGURE 6.1 - LOAD-DEFORMATION CHARACTERISTICS ASSUMED FOR THE SOIL

of the soil, including the effect of loading rate, is given by

$$R(m,t) = [D(m,t) - D'(m,t)]\ K'(m)[1 + J(m)V(m,t-1)]$$

where m denotes the segment number of the pile, t is the time interval number, D(m,t) is the displacement of segment m at time interval number t, K'(m,t) is the plastic deformation of the soil, J(m) is the soil damping constant, K'(m) is the soil spring constant, V(m,t) is the velocity of mass number m at time interval number t, and R(m,t) is the soil resistance acting on that element at time t.

In cases in which more accurate soil data are available, the general soil resistance curve of Figure 6.1 (b) may be used to advantage. This curve also uses the variables Q(m) and Ru(m), but the curve no longer must be linear. In this case, the ground quake Q(m) is divided into ten equal segments, and the static soil resistances corresponding to these ten points comprise the input data required to establish the curve. Also, as shown in Figure 6.1 (b), the slope of the unloading curve is given by K'(m). A more complete discussion of the use of this method is given in the appendix.

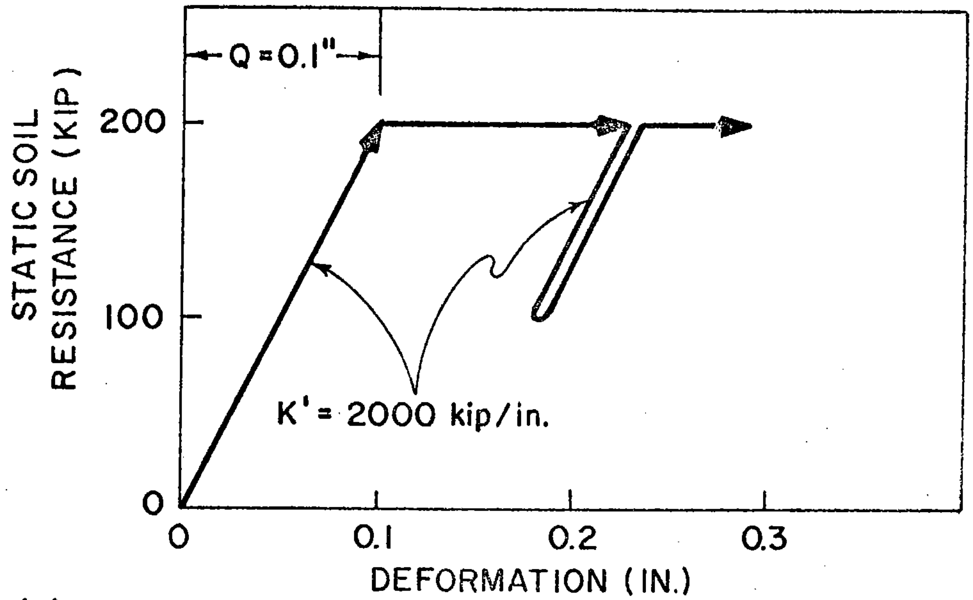
To check out the programming changes involved in this method, several problems were first solved using the regular elastic-plastic curve of Figure 6.1 (a).

These problems were then solved again using the generalized soil resistance method with soil resistance values lying on the same curve, the two solutions then being checked for identical results.

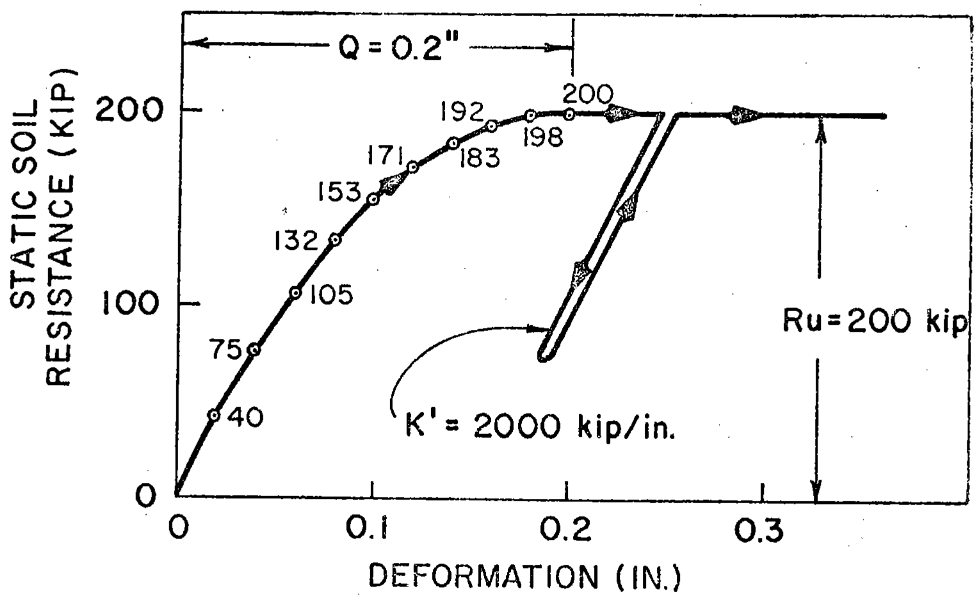
A number of other problems were also solved to see what changes might result when the shape of the soil resistance curve was altered. For example, the linear soil resistance curve used in a problem originally solved by Smith[58] is shown in Figure 6.2 (a). This problem was then solved using the nonlinear curve of Figure 6.2 (b).

The solutions for these two problems, shown in Table 6.1, are typical of the results found for the other cases studied, in that a rather large change in the soil curve changed the results only slightly, In this case, for example, although the soil quake was doubled and the curve made nonlinear, the maximum change in stress was less than 9 percent, and the permanent set increased less than 8 percent. Only a drastic change in the soil resistance curve was found to cause an appreciable difference in the solution.

Therefore, if the soil resistance curve for the problem even slightly resembles the curve of Figure 6.2 (a), the linear resistance equation will probably be satisfactory. Whenever it becomes necessary, the



(a) ELASTIC-PLASTIC SOIL RESISTANCE CURVE (AFTER REFERENCE 58)



(b) GENERALIZED SOIL RESISTANCE CURVE

FIGURE 6.2 - SOIL RESISTANCE VS DEFORMATION CURVES

TABLE 6.1 COMPARISON OF RESULTS FOUND BY USING ELASTIC-PLASTIC VS NON-LINEAR SOIL RESISTANCE CURVES

	Maximum Force			
Type of Soil Resistance	At Head of Pile (kip)	At Center of Pile (kip)	At Point of Pile (kip)	Maximum Point Displacement (in)
Elastic Plastic	290	300	405	0.203
Non-Linear	290	301	370	0.218
Percentage of Change	0.0	+0.3	-8.7	+7.4

nonlinear soil resistance can be used as explained in the appendix.

Significance of the Soil Quake "Q"

The properties of the soil under the action of dynamic loading are probably the least understood of the many variables affecting the problem[64]. Although a number of values for the soil quake may be used, the value Q = 0.1, recommended by Chellis[65] is probably the most widely accepted for general use, except when a more accurate value can be determined. As might be expected, the trouble stems mainly from the large number of variables influencing the value of Q at any given driving location, the most obvious of course being the type of soil encountered. Much work is presently being done to define these factors and to more accurately determine the actual values for both "Q" and "J" to increase the solution's accuracy[66,67].

While it is beyond the scope of the present research to attempt to determine values for Q, it is interesting to see how the value of Q affects the solution. After a number of the Michigan research problems with varying values of Q were studied, Case BLTP - 6; 57.9 was chosen as being fairly representative. The problems were solved with Q ranging from 0.1 to 0.5, as seen in Table

6.2. To determine whether Q would have similar effects at all magnitudes of soil resistance, Ru_{total} was also varied. The results of this parameter study are given in Table 6.2.

One of the trends noted in Table 6.2 is the small effect Q has on the maximum compressive force found in the pile. The effect on tensile force is more pronounced, although no conclusion could be reached as to whether the tensile stress will increase or decrease as Q changes since the results did not indicate an apparent trend. Maximum ENTHRU values are also relatively independent of the soil quake, with ENTHRU tending to decrease as the soil quake increases.

The most pronounced and consistant trend is the marked increase in maximum point displacement corresponding to increasing values of Q. It is also noted that the percent increase in maximum point displacement is relatively small for a small soil resistance, but greatly increases as the total soil resistance becomes large. This is also shown in Figure 6.3. Similar results were found for the other Michigan cases studied, except that the tensile force often varied substantially more than indicated for the case of Table 6.2.

TABLE 6.2 INFLUENCE OF SOIL QUAKE AT DIFFERENT SOIL RESISTANCES FOR CASE BLTP-6; 57.9 WITH NO SOIL DAMPING

Total Soil Resistance (kip)	Q (in)	Maximum Point Displacement (in)	Maximum ENTHRU (kip ft)	Maximum Compressive Force (kip)	Maximum Tensile Force (kip)
50	0.1	1.49	6.80	225	109
	0.2	1.51	6.80	222	109
	0.3	1.51	6.73	221	114
	0.4	1.54	6.71	221	119
	0.5	1.58	6.69	221	124
100	0.1	0.84	6.96	230	68
	0.2	0.88	6.88	224	85
	0.3	0.90	6.86	223	97
	0.4	0.93	6.84	222	98
	0.5	0.97	6.83	222	97
150	0.1	0.56	7.10	235	91
	0.2	0.57	7.05	227	90
	0.3	0.61	6.93	225	128
	0.4	0.64	6.88	223	163
	0.5	0.69	6.85	223	188
200	0.1	0.41	7.21	240	79
	0.2	0.44	7.13	230	67
	0.3	0.48	7.06	226	77
	0.4	0.52	6.99	224	107
	0.5	0.56	6.90	224	118

TABLE 6.2 (Continued)

Total Soil Resistance (kip)	Q (in)	Maximum Point Displacement (in)	Maximum ENTHRU (kip ft)	Maximum Compressive Force (kip)	Maximum Tensile Force (kip)
300	0.1	0.22	7.28	250	82
	0.2	0.30	7.24	234	108
	0.3	0.36	7.16	229	111
	0.4	0.42	7.10	225	59
	0.5	0.47	7.05	224	73
400	0.1	0.11	7.30	260	127
	0.2	0.21	7.28	239	114
	0.3	0.29	7.24	233	158
	0.4	0.36	7.18	228	158
	0.5	0.41	7.12	226	102

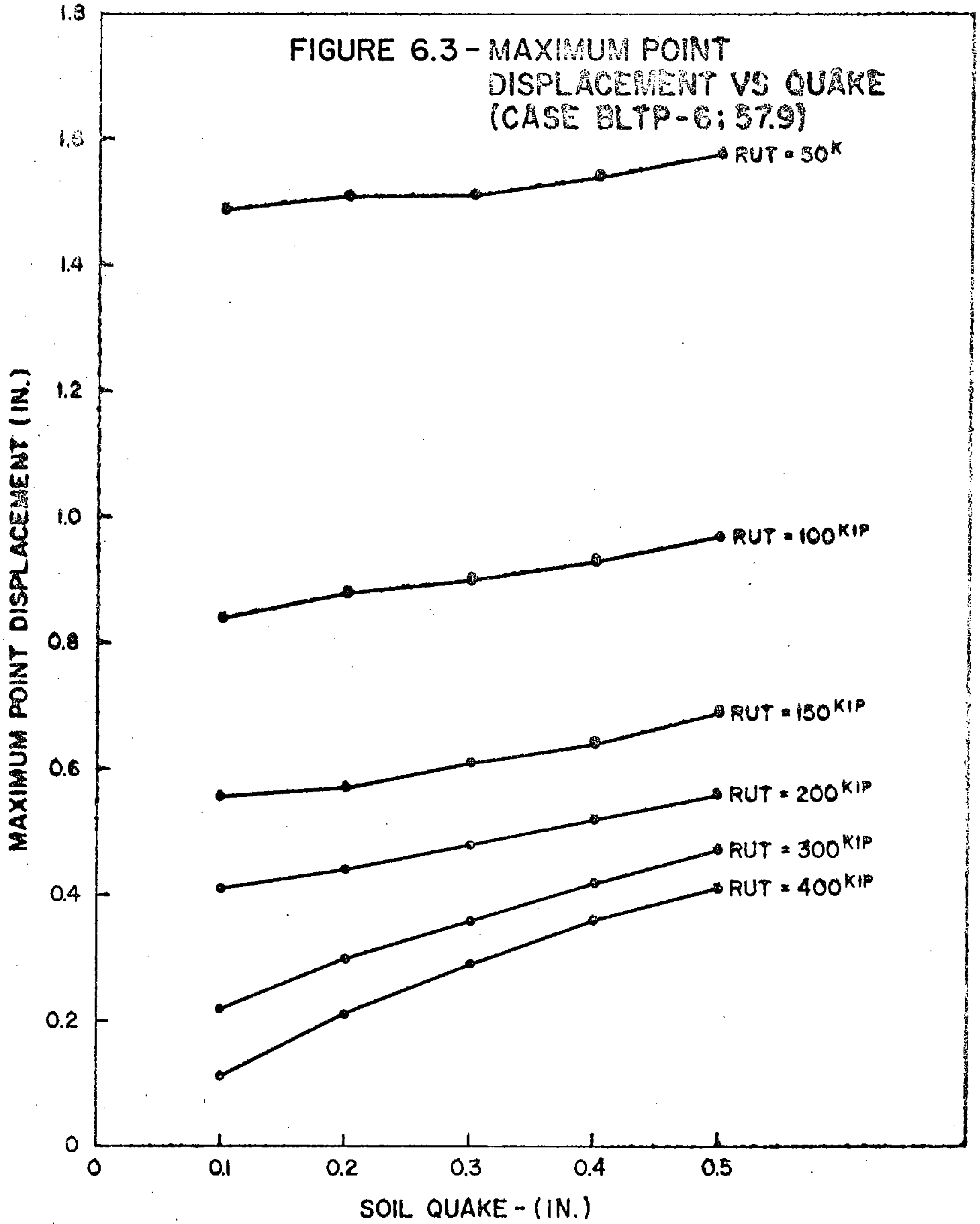


FIGURE 6.3 - MAXIMUM POINT DISPLACEMENT VS QUAKE (CASE BLTP-6; 57.9)
RUT = 50K
RUT = 100KIP
RUT = 150KIP
RUT = 200KIP
RUT = 300KIP
RUT = 400KIP
MAXIMUM POINT DISPLACEMENT (IN.)
SOIL QUAKE - (IN.)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
0.1
0.3
0.5

Significance of the Soil Damping Constant

Michigan Case BLTP - 6; 57.9 was also chosen to illustrate the damping effects of the soil. These damping constants were given values ranging from 0.0 to 0.05, and as was done in the previous section, the total soil resistance was varied from 50 to 400 kip to see if trends found at low resistances would also be noted when the soil resistance was large. Since the soil damping constants most commonly used are those recommended by Smith[68], i.e., a soil damping constant of 0.05 sec/ft along the side of the pile and 0.15 sec/ft at the point of the pile, the variation of J = 0.0 to 0.5 very likely covers the true values for most conditions and soils. These results are given in Table 6.3.

As was previously determined for Q, the soil damping constants also have little effect on the maximum ENTHRU values. The maximum compressive forces do have a tendency to increase as J increases, expecially when the soil resistance is large. While the tensile forces still do not follow any definite pattern, they are somewhat more regular than those determined by varying "Q".

The maximum point displacements again show the

TABLE 6.3 INFLUENCE OF SOIL DAMPING ON DIFFERENT SOIL RESISTANCES FOR CASE BLTP-6; 57.9 (Q = 0.1 FOR ALL CASES)

Total Soil Resistance (kip)	J (sec/ft)	Maximum Point Displacement (in)	Maximum ENTHRU (kip ft)	Maximum Compressive Force (kip)	Maximum Tensile Force (kip)
50	0.0	1.49	6.80	225	109
	0.1	1.11	6.89	221	68
	0.2	0.85	7.03	221	41
	0.3	0.72	7.21	221	18
	0.4	0.63	7.23	222	6
	0.5	0.56	7.25	222	5
100	0.0	0.84	6.96	230	68
	0.1	0.58	7.12	222	31
	0.2	0.49	7.20	223	11
	0.3	0.43	7.25	223	14
	0.4	0.38	7.27	224	12
	0.5	0.34	7.28	225	17
150	0.0	0.56	7.10	235	91
	0.1	0.42	7.23	223	23
	0.2	0.34	7.26	224	21
	0.3	0.28	7.28	225	26
	0.4	0.24	7.27	239	24
	0.5	0.21	7.26	251	22
200	0.0	0.41	7.21	223	79
	0.1	0.28	7.28	225	35
	0.2	0.22	7.28	239	37
	0.3	0.18	7.25	255	31
	0.4	0.15	7.22	267	27
	0.5	0.13	7.20	274	26
300	0.0	0.22	7.28	250	82
	0.1	0.12	7.23	272	53
	0.2	0.09	7.18	286	41
	0.3	0.08	7.14	293	33
	0.4	0.07	7.11	298	31
	0.5	0.07	7.07	302	30

TABLE 6.3 (Continued)

Total Soil Resistance (kip)	J (sec/ft)	Maximum Point Displacement (in)	Maximum ENTHRU (kip ft)	Maximum Compressive Force (kip)	Maximum Tensile Force (kip)
400	0.0	0.11	7.20	260	127
	0.1	0.07	7.13	308	61
	0.2	0.06	7.07	313	41
	0.3	0.05	7.02	314	35
	0.4	0.05	6.96	314	33
	0.5	0.05	6.90	314	33

most consistant trend as J is varied, as shown in Figure 6.4. The other cases studied showed this same trend, i.e., as J increases, the maximum displacement decreases rapidly.

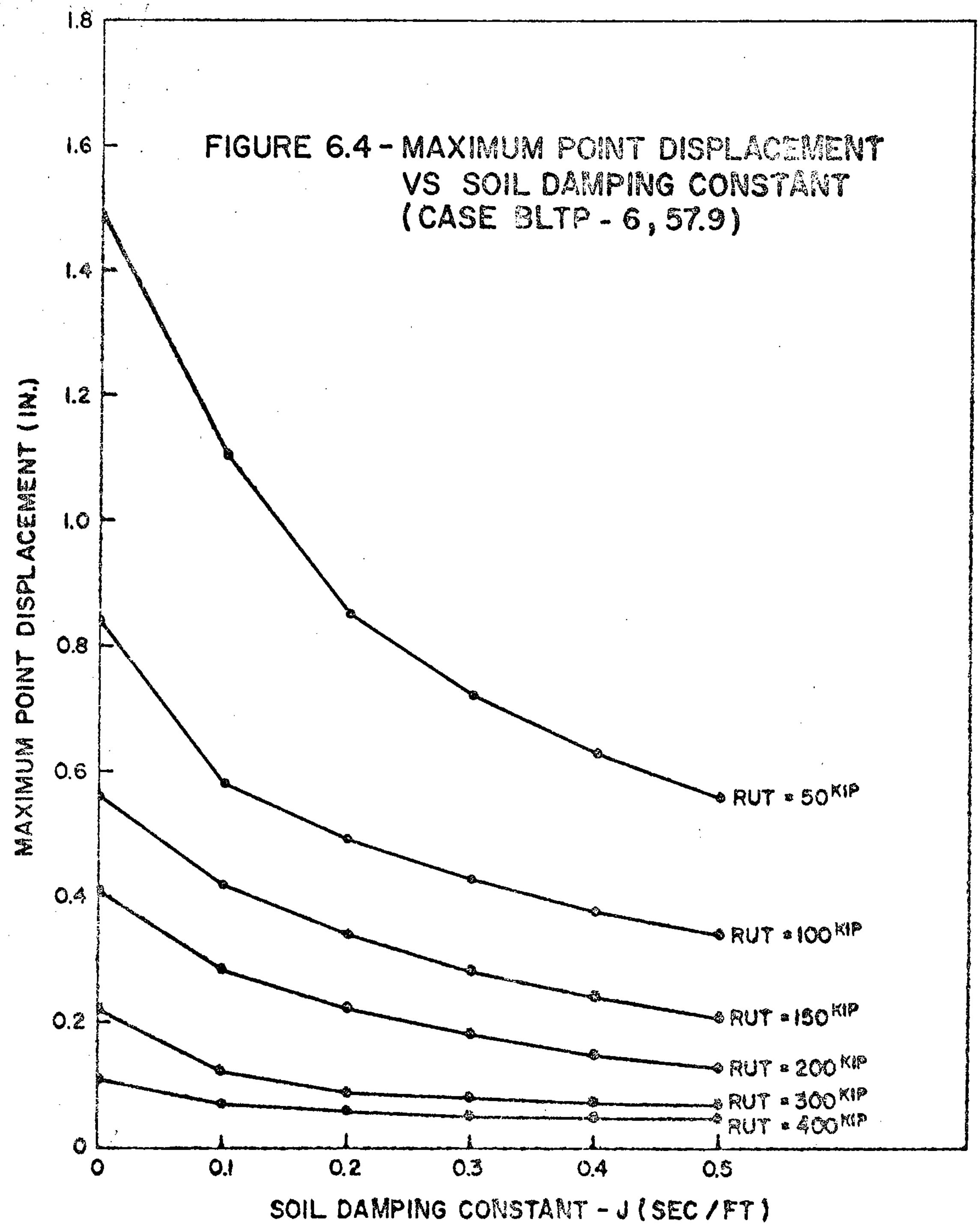


FIGURE 6.4 - MAXIMUM POINT DISPLACEMENT
VS SOIL DAMPING CONSTANT
(CASE BLTP - 6, 57.9)
MAXIMUM POINT DISPLACEMENT (IN.)
SOIL DAMPING CONSTANT - J (SEC / FT)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
0.1
0.3
0.5
RUT = 50 KIP
RUT = 100 KIP
RUT = 150 KIP
RUT = 200 KIP
RUT = 300 KIP
RUT = 400 KIP

CHAPTER VII

CONCLUSIONS

The correlation between the numerical solution and the experimental data presented in Chapter V indicates the potential accuracy of Smith's method, but the problem involves so many important parameters that it is extremely important to know as much as possible about their actual behavior.

As shown in Chapter III, it is possible to determine valuable information from the wave equation even though some of these parameters are unknown. For example, several problems can be solved in which the unknown parameter varies between some upper and lower values, as was done to determine the effect of the ram's elasticity in Chapter III. This study shows that only for steel on steel impact does the elasticity of the ram affect the solution.

In order to study the Michigan data over 5,000 problems had to be solved because certain key information such as the ram velocity was not reported. Still it was possible to study the behavior of the pile-driving hammers discussed. For example, the efficiency of the cushion assembly was remarkably consistent, in that they were nearly independent of the type of pile, pile length, and soil resistance. The correlation between the wave equation and the field data shown in Chapter III further illustrates that Smith's method is accurate, especially if all the required data is known and need not be assumed.

Much of the value of this method of analysis is its flexibility. As illustrated in Chapter III, the wave equation can be used for any number of studies which otherwise would not be possible.

It was shown that the stress-strain curve for a cushion is not straight line. Instead, it follows a curve which is closely parabolic. However, a straight line which has a slope equal to that of the true stress-strain curve taken at a point halfway between zero and the maximum strain gives accurate results. The cushion's dynamic coefficient of restitution was found to agree with commonly recommended values.

The effect of internal damping the the concrete and steel piles was shown to be negligible in these cases, although it can be accurately accounted for by the wave equation.

The parameter for which further research is most needed is probably the dynamic behavior of the soil.

CHAPTER VIII

RECOMMENDATIONS FOR FURTHER RESEARCH

The following areas are recommended for further research:

1. A complete evaluation of the data collected by the Michigan State Highway Commission, including correlation of hammer energy, permanent set of pile per blow, etc. This would require a major research effort because of the quantity of data reported. Also, because certain variables were not determined, several theoretical solutions must be solved for each attempt correlation until the unknown parameter can be "pinned down" with reasonable accuracy. For example, the solutions for over 5,000 problems were required to complete the 28 case study made in Chapter III.

2. A study to determine how to improve the efficiency of the pile-driving hammers presently in use. This type of research should be most interesting to the hammer manufacturers since present equipment could be optimized to drive piling faster and/or reduce the driving stresses during driving. The possibility that today's pile-driving hammers are as efficient as possible through trial and error is remote.

3. Further research is needed to insure that the

damping models proposed in Chapter IV are also accurate for timber piling, and to determine what damping constants should be used.

4. Major research efforts are needed to investigate every aspect of the soil resistance acting on the pile during driving.

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APPENDIX A

PROGRAM INPUT DATA

PROGRAM INPUT DATA

CARD 101 (Required)

ID1 and ID2 - All "ID" values are for identification only and can be either alphabetic or numeric.

$1/\Delta t$ - Time interval. If left blank, $\Delta tcr/2$ will be used. (1/sec)

MP - Total number of segments in the system to be analyzed.

VELMI - Initial velocity of the ram. (ft/sec)

MH - Element number of the first pile segment.

NR - Number of divisions of the ram.

EEM(NR) - Coefficient of Restitution of spring number NR, directly under ram.

EEM(NR+1) - Coefficient of Restitution of spring number NR+1.

GAMMA(NR) - The minimum force in the spring beneath the ram once that force has reached a maximum. (kip) For example, if the diesel hammer explosive pressure causes 158.7 kip minimum force in this spring, set GAMMA(NR) = 158.7 kip. If the minimum force the spring can transmit.

is zero (for example, when no tensile force can exist between the ram and anvil) set the corresponding GAMMA(I) = 0.0. If the spring represents a continuous body such as the spring between any two pile segments, it can transmit tensile forces between the elements. This is signified by setting GAMMA(I) equal to any negative value, usually -1.0 kip.

GAMMA(NR+1) - Same as above, but for spring number NR+1. 0

NSTOP - Total number of time intervals the program is to run. 1000

NOP(I)	VALUE	FUNCTION
NOP(1)		Used to read cards 103-106 and print out the data for problem identification.
	=1	No identification card is to be used. 1
	=2	Read and print a single ID card. (card 103)
	=3	Read and print two ID cards. (cards 103 and 104)
	=4	Read and print ID cards 103, 104, and 105.

NOP(I)	VALUE	FUNCTION
	=5	Read and print ID cards 103, 104, 105, and 106.
NOP(2)		Used to specify the input method for the segment weights WAM(I).
	=1	Read one weight for each segment (card series 200).
	=2	Read the segment weights for only the first five and last five segments of the pile system from a single card (card 200), and equate all remaining segment weights to the sixth weight in the system. (NOP(2) = 2 is used when a large number of equal weights are present except for the first or last few weights.)
NOP(3)		Used to specify the input method for the internal spring stiffness, (XKAM(I).
	=1	Read one stiffness for each internal spring from card series 300.

NOP(I)	VALUE	FUNCTION
	=2	Read the stiffness values for only the first five and last five internal springs on a single card 300, and assign the fifth value to all remaining internal springs.
		(NOP(3) = 2 is used under the same conditions as NOP(2) = 2.
NOP(4)		Used to specify what soil resistance distribution act along the pile.
	=1	Read RUM(I) for each element from card series 400, and set the point bearing soil resistance RUM(MP+1) equal to RUP.
	=2	Set all side resistances equal to zero, and set RUM(MP+1) = RUP.
	=3	Distribute RUT-RUP uniformly along the side of the pile from segment MO thru MP, and set RUM(MP+1) = RUP.
	=4	Distribute RUT-RUP triangularly along the pile between segments MO and MP and set RUM(MP+1) = RUP.

NOP(I)	VALUE	FUNCTION
	=5	Read one 450 series card for each mass upon which a nonlinear resistance vs displacement curve acts. If a linear curve also happens to be acting on an element, it must also be input on a 450 series card.
NOP(5)		Used to specify the input method for GAMMA(I). Note: The significance of GAMMA(I) is discussed in the "500 card series".
	=1,2	Read GAMMA1 and GAMMA2 from card 101 and assign GAMMA1 to internal spring number NR, and assign GAMMA2 to spring number NR+1. Then set GAMMA(I) of the remaining springs to -1.0.
	=3	Same as for NOP(5)=2, except that GAMMA(NR+2) is also set equal to 0.0
	=4	Same as for NOP(5)=2, except GAMMA(NR+2)=0.0 and GAMMA(NR+3)=0.0 This option is used when a large number of elements such as an anvil, follower,

NOP(I)	VALUE	FUNCTION
		load cell and pile cap are encountered, since these elements cannot transmit a tensile force to the next element. This option can be used to set up to eight consecutive values of GAMMA(I)=0.0 by setting NOP(5)=8.
	=9	Read GAMMA(I) for each spring from card series 500.
NOP(6)		Used to specify the input method for EEM(I).
	=1	Read EEM1 and EEM2 from card 101 set EEM(NR)=EEM1, and EEM(NR+1)=EEM(2). Then set EEM(I) for all other springs equal to 1.0 (perfectly elastic).
	=2	Read EEM(I) for each spring from card series 600.
NOP(7)		Used to specify the input method for BEEM(I).
	=1	Set all BEEM(I)=0.0.
	=2	Read BEEM(I) for each spring from

NOP(I)	VALUE	FUNCTION
		card series 700.
NOP(8)		Used to specify the input method for VEL(I).
	=1	Read VELMI from card 101 and set VEL(I,t=0) for all segments of the ram (usually one segment) equal to VELMI. Set all other VEL(I)=0.0.
	=2	Read VEL(I) for each segment from card series 800.
NOP(9)		Used to specify input method for Q(I).
	=1	Read QSIDE and QPOINT from card 102 and set all Q(I) along side of the pile equal to QSIDE. Set Q(MP+1) under pile tip equal to QPOINT.
	=2	Read Q(I) for each element including Q(MP+1) from card series 900.
NOP(10)		Used to specify input method for SJ(I).
	=1	Read SIDEJ and POINTJ from card 102. Set all SJ(I) along side of pile equal

NOP(I)	VALUE	FUNCTION
		to SIDEJ and SJ(MP+1) under pile tip equal to POINTJ.
	=2	Read SJ(I) for each element including SJ(MP+1) from card series 1000.
NOP(11)		Used to specify the input method for DYNAMK(I).
	=1	Set all DYNAMK(I)=0.0.
	=2	Read DYNAMK(I) for each spring from card series 1100.
NOP(12)		Used to specify input method for A(I).
	=1	Read AREA from card 102 and set all A(I) equal to AREA.
	=2	Read A(I) for each internal spring from card series 1200.
NOP(13)		Used to specify which method of internal damping is to be used in the pile.
	=1	Use Smith's method (refer to Figure 5.13b).

NOP(I)	VALUE	FUNCTION
	=2	Use standard linear solid method (refer to Figure 5.13c).
NOP(14)		Used to specify how the force in the cushion after impact is to be determined.
	=1	Calculate cushion forces from the wave equation applied to the moving ram after impact.
	=2	In this case, the force at the head of the pile at all times is known, probably by experimental methods, and this force curve is to be applied at the head of the pile. The force at each time interval FORCIN(t) is read from card series 1300 (kip).
	=3	Same as when NOP(14)=2, except that galvanometer readings rather than forces at each time interval are input and the cushion forces are determined by the computer. In this case, the information on the 1400 header card is

NOP(I)	VALUE	FUNCTION
		needed, followed by the galvanometer deflection at each time interval from card series 1400.
NOP(15)		Used to specify how gravity is to be accounted for in the solution
	=1	The effect of gravity is to be neglected.
	=2	Gravity is to be considered, with the initial displacement of each segment, D(I,0), and the initial soil resistances RAM(I,0) assumed to be zero.
	=3	Gravity is to be considered, and D(I,0) and RAM(I,0) are to be approximated by Smith's suggested method[63]
	=4	Gravity is to be considered, and the values for D(I,0) and RAM(I,0) are computed by Samson's suggested method[61].
NOP(16)		Used to specify the number of problems to be solved using the basic data given on cards 101 through the 1700 card series.

NOP(I)	VALUE	FUNCTION
	=1	Only one problem is to be solved using this set of data.
	=2	Run more than one problem with changes in this data as specified on card 1600.
NOP(17)		Used to specify whether the ultimate pile capacities predicted by various pile driving equations are desired.
	=1	No capacities are to be computed.
	=2	Using the information from card 1700 and the information provided by the wave equation solution, solve for the ultimate resistance to failure as predicted by several popular pile driving equations.

CARD 102 (Required)

ID3 - Identification.

ID4 - Identification.

RUT - The total static soil resistance acting on the pile. (kip)

RUP - The total static soil resistance acting beneath the point. (kip)

MO - Number of first element upon which soil resistance acts.

QSIDE - Soil quake along side of pile, if a single value exists. If not, set QSIDE=0.0. (in.)

QPOINT - Soil quake beneath pile point. (in.)

SIDEJ - Soil damping factor in shear along the side of the pile if a single value exists. If not, set SIDEJ=0.0. (sec/ft)

POINTJ - Soil damping factor in compression beneath the pile point. (sec/ft)

NUMR - Number of elements for which the soil spring does not have a linear stress-strain curve.

IPRINT - Print frequency. For example, if the solution at every 5th time interval is wanted, set IPRINT=5.

AREA - A constant used to convert the forces into stresses or other more convenient values (such as changing lb. to kip by setting AREA=1000.0).

NS1-NS6 - The element numbers for which solutions vs time interval will be printed. Maximum values and other information are always printed for each element after NSTOP time intervals have elapsed.

CARDS 103-106 (Required only if NOP(1)=2,3,4,5)

If NOP(1)=1, no identification card will be read. If NOP(1)=2, read card 103 containing 72 columns of alphabetic or numeric identification and print this information above the problem. If NOP(1)=3, read and print two identification cards, up to a maximum of four cards (NOP(1)=5).

200 CARD SERIES (Required)

IDW1, IDW2 - Throughout this Input, variables beginning with the letters "ID" are for identification, in this case to help identify what segment weights are being used.

WAM(I) - The weight of element number I (kip).
a) If NOP(2)=1, the computer will read MP segment weights, ten segment weights

to a card from cards 201-230, up to a maximum of 300 segments. For example, if the system is divided into 37 segments, four 200 series cards must be included in the data: 201 through 204.

b) If NOP(2)=2, in this case the pile must have a constant weight per foot along its length. Since the pile is usually divided into equal segment lengths, only a few of the element weights are different. Therefore, only the top five weights (the ram, anvil, ...) and the bottom five weights (..., pile segment, pile point) must be read from the card 200. The computer then sets all other element weights equal to the sixth value punched in the card.

300 CARD SERIES (Required)

IDK1, IDK2 - Identification.

XKAM(I) - The internal spring rate of spring I. (kip/in.)

a) If NOP(3)=1, the computer reads MP-1 spring rates from cards 301-330.

b) If NOP(3)=2, the first and last five

XKAM(I) are read from card 300, and the remaining XKAM(I) are set equal to the sixth XKAM(I) value, i.e., XKAM(MP-4).

400 CARD SERIES (Required if NOP(4)=1)

IDRL1, IDRL2 - Identification.

RUM(I) - The ultimate static resistance of the soil acting on pile segment I. (kip)

a) If NOP(4)=1, read MP ultimate soil resistances, from cards 401-430, and set RUM(MP+1) equal to RUP.

b) If NOP(4)=2, set all side friction =0.0 and set RUM(MP+1)=RUP.

c) If NOP(4)=3, distribute (RUT-RUP) uniformly along the pile starting from segment number MO to number MP, and set RUM(MP+1)=RUP.

d) If NOP(4)=4, distribute (RUT-RUP) triangularly between MO and MP and set RUM(MP+1)=RUP.

e) If NOP(4)=5, read NUMR cards, each of which can define a linear or non-linear force-displacement curve for the soil (see card series 450).

450 CARD SERIES (Required if NOP(4)=5)

When NOP(4)=5, the soil resistance vs displacement curve is nonlinear. This requires ten soil resistances to be read for each soil spring, one for each displacement corresponding to a multiple of Q/10. As shown on data card 451, I is the number of the element upon which the nonlinear resistance is acting, XKIM(I) is the unloading spring rate (kip/in.), and R(I,J) are the soil resistances (kip) at each of the displacements Q/10, 2Q/10, ..., 9Q/10, Q. Whenever NOP(4)=5, one 450 series card is required for each element upon which soil resistance acts.

500 CARD SERIES (Required when NOP(5)=2)

IDG1, IDG2 - Identification.

GAMMA(I) - The minimum force possible in spring I after a peak compressive force has passed, except that any negative GAMMA(I) is construed to mean that that spring can transmit a tensile force of any magnitude. (kip)

600 CARD SERIES (Required when NOP(6)=2)

IDE1, IDE2 - Identification.

EEM(I) - The coefficient of restitution for MP-1 internal springs. This determines the slope of the unloading curve. (dimensionless)

700 CARD SERIES (Required when NOP(7)=2)

IDB1, IDB2 - Identification.

BEEM(I) - The damping coefficient of the MP-1 internal springs. (in. sec/ft)

800 CARD SERIES (Required when NOP(8)=2)

IDV1, IDV2 - Identification.

VEL(I) - The initial velocities of each of the MP weights. (ft/sec)

900 CARD SERIES (Required when NOP(9)=2)

IDQ1, IDQ2 - Identification.

Q(I) - The soil "quake" for MP+1 soil springs. (in.)

1000 CARD SERIES (Required when NOP(10)=2)

IDJ1, IDJ2 - Identification.

SJ(I) - The soil damping factor for MP+1 soil spring. (sec/ft)

1100 CARD SERIES (Required when NOP(11)=2)

IDDK1, IDDK2- Identification.

DYNAMK(I) - The dynamic spring rate of MP-1 internal springs. (kip/in.)

1200 CARD SERIES (Required when NOP(12)=2)

IDA1, IDA2 - Identification.

A(I) - The cross-sectional area of each of the MP-1 internal springs. (in.^{2})

1300 CARD SERIES (Required when NOP(13)=2)

FORCIN(INTV) - The force acting on the head of the pile (kip) at time interval INTV, for NSTOP intervals with a maximum NSTOP equal to 100 time intervals.

1400 CARD SERIES (Required when NOP(14)=2)

CARD 1400 - Header Card

APILE - The area of the head of the pile. (in.^{2})

EMODUL - The modulus of elasticity of the pile. (kip/in.^{2})

RGAGE - The strain gage resistance. (ohm)

RCAL - Calibration resistance. (ohm)

ACTIVG - Number of active gages.

GFACTR - Gage factor for the gages used.

D1 - Displacement of the galvanometer trace when RCAL is thrown into the bridge at the head of the pile. (in.)

D2 Through

D5 - Galvo displacements corresponding to RCAL at any other four strain gage points. (in.)

CARDS 1401 UP TO 1410

DGALVI(INTV) - The galvanometer deflection for the gage at the head of the pile, at interval number INTV. (in.)

CARD 1500 (Required when NOP(15)=4)

F1 and F2 - Forces known to lie on the true dynamic force vs compression curve of the cushion. (kip)

C1 and C2 - The cushion compressions corresponding to F1 and F2, respectively. (in.)

CARD 1600 (Required when NOP(16)=2)

NOPP(I) - When a number of cases are to be solved for which only a few parameters will change, NOPP(I) designates which parameter to vary and how many different values it should be assigned. For example: NOPP(1)=5 indicates that five problems are to be solved, for which only the ram's initial velocity will vary. Each NOPP(I) controls a single variable as shown in Table A.1.

DV1 Through DK1 - These parameters control the percent change in the variables mentioned above. For example, assume that the

TABLE A.1 LIST OF PARAMETER VARIATIONS AND THEIR CONTROLLING OPTIONS

Controlling Option	Per Cent Increase in Original Value	Parameter Controlled
NOPP(1)	DV1	VELMI (Initial ram velocity)
NOPP(2)	DW1	W(1)
NOPP(3)	DW2	W(2)
NOPP(4)	DWI	W(3) through W(MP)
NOPP(5)	DK1	XKAM(1)
NOPP(6)	DK2	XKAM(2)
NOPP(7)	DKI	XKAM(3) through XKAM(MP-1)
NOPP(8)	DQI	QSIDE
NOPP(9)	DQP	QPOINT
NOPP(10)	DJI	SIDEJ
NOPP(11)	DJP	POINTJ
NOPP(12)	DRI	~~RUT~~
NOPP(13)	DRP	~~RUP~~
NOPP(14)	DRI	RUT & RUP
NOPP(15)	DE1	EEM(1)
NOPP(16)	DE2	EEM(2)

effects of ram velocities of 10, 12, 14, 16, 18, and 20 ft/sec are being studied. The value of DV1 would be

$$\frac{(12 \text{ ft/sec} - 10 \text{ ft/sec})}{10 \text{ ft/sec}}$$

or DV1=0.20. In this case, NOPP(1) would equal 6 since 6 separate problems are to be run.

The variables controlled by DV1 to DK1 are also listed in Table A.1.

CARD 1700 (Required when NOP(17)=2)

AREAP - Cross-sectional area of pile. (in.^{2})

XLONG - Length of pile. (ft)

ELAST - Modulus of elasticity of pile. (kip/in.^{2})

CENR - Value for use in ENR pile driving formula.

QAVG - Average ground "Quake". (in.)

WRAM - Ram weight. (kip)

WPILE - Pile weight. (kip)

ENERGY - Actual energy output of the ram. (ft lb)

APPENDIX B

EXAMPLE PROBLEM

Introduction

The following example problem is given to illustrate the steps necessary to arrive at a solution. In the previous chapters, the functional components involved were discussed separately; for example, the driving hammer, pile, soil properties, etc. However, the input data is more easily handled by grouping according to similar physical quantitites rather than functional quantities. For example, one series of cards is used to input all segment weights, another for the spring rates, and so onl The order in which the input data is set up for the example problems is by no means unique, but it probably should be followed until the programmer becomes familiar with the operations involved.

It should be noted that any variable without a decimal point (such as MP, MH, NR, NSTOP, and NOP(I) on card 101) is always an integer and must be entered as far to the right in its field as possible. Also, the decimal point does not have to be punched for any variable which has a decimal place already shown on the data sheet unless it is desired to change its position. For example, if the initial ram velocity (IVEL on card 101) is 13.48 ft/sec, the numbers 1, 3, 4, and 8 should be punched in columns 19 through 22, respectively. However, to enter a velocity of 127 ft/sec into IVEL, punch 1, 2, and 7 in columns 19, 20, and 21, and punch a decimal point in column 22.

Except for this last case, decimal points need never be punched.

Example Problem

Since case BLTP-6; 57.9 (from the Michigan Pile Study) was one of the problems most often used in this report, the input data required for its solution will be determined first. Figures 3.3 and 3.4 show the real system and the idealized system.

A. Given Information - Case BLTP-6; 57.9

1. Hammer Data-Vulcan #1

a. Manufacturer's Rated Energy = 15,000 ft lb, normal stroke = 3 ft.

b. Ram Weight = 5,000 lb, velocity at impact not measured.

c. Driving Cap Weight = 1,000 lb.

d. Cushion Data = Oak block, 6-1/4 in. deep by 11-1/4 in. in diameter, direction of grain unknown, condition of cushion unknown (somewhere between new and "crushed and badly burnt").

2. Pile Data-CBP 124 H-section

a. Area = 15.58 $in.^2$

b. Weight = 53 lb/ft

c. Total Length = 72.5 ft

d. Driven Length = 57.9 ft

e. Modulus of Elasticity = 30 x 10^6

3. Soil Data

a. Ultimate Soil Resistance = 400 kip (static value from load test after soil "set-up").

b. From driving log, 75 percent of the soil resistance is assumed point bearing and 25 percent side resistance.

c. Soil damping factor "J" and soil quake "Q" - not known.

4. Miscellaneous Data

a. Load Cell Weight = 580 lb.

b. Additional Helmet Weight = 1,080 lb.

B. Input Data Calculations

Card 101

1. ID1 - Identification Tag, use BLTP-6/

2. ID2 - Identification Tag, use 57.9.

3. Segment Lengths - Although segment lengths of 10 ft are usually satisfactory, a 5 ft length will be used to increase the accuracy of the solution.

4. Time Interval - The normal time interval

of 1/4000 to 1/5000 iterations/sec must be halved since the normal segment length of 10 ft was reduced by half. Therefore, use Δt = 1/10,000 sec or $1/\Delta t$ = 10,000.

5. MP - The total number of segments as shown in Figure 3.4 is 3 above the pile plus 14 pile segments. Thus, MP = 17.
6. Since the ram velocity at impact was not recorded, the following ram velocities will be studied: IVEL = 8, 12, 16, and 20 ft/sec.
7. MH - The first pile segment weight = 4.
8. NR - Number of divisions of the ram = 1.
9. EEM1 = Coefficient of restitution of cushion = 0.4, EEM2 = coefficient of restitution of load cell = 1.0.
10. Since springs 1, 2, and 3 cannot transmit tensile forces, GAMMA(1), (2), and (3) are 0.0. The remaining GAMMA (I) are set equal to -1.0. This is done by setting GAMMA1 = GAMMA2 = 0.0 and designating NOP(5) = 3 so that GAMMA(3) will also be set = 0.0.
11. To allow the wave time to make two com-

plete passes up and down the pile, NSTOP is set = 173 iterations. This is found from the velocity of travel of the stress wave and the value of Δt.

$$V_{wave} = E/\rho = \frac{30{,}000{,}000}{(0.283/386)} = 202{,}000 \text{ ips or}$$

$$V_{wave} = \frac{202{,}000}{12} = 16{,}800 \text{ ft/sec.}$$

Total distance wave must travel = 4(72.5) = 290 ft.

$$\text{Total time required} = \frac{290 \text{ ft}}{16{,}800 \text{ ft/sec}} = .0173 \text{ sec.}$$

$$\text{NSTOP} = \frac{\text{Total time}}{\Delta t} = \frac{.0173 \text{ sec}}{(1/10{,}000)\text{sec/iteration}}$$

= 173 iterations

Therefore, use NSTOP = 200 iterations.

12. Option Calculations - NOP(I)

a. NOP(1) - No header cards to be read in and printed out, so NOP(1) = 1.

b. NOP(2) - Read segment weights from card series 200 (long form), so NOP(2) = 1.

c. NOP(3) - Read spring constants from long form card series 300 (long form), so NOP(3) = 1.

d. NOP(4) - Assume triangular soil distribution along the side of the pile, so NOP(4) = 4.

e. NOP(5) - Since GAMMA(3) is to be set equal to 0.0, NOP(5) = 3.

f. NOP(6) - Since all the internal springs are considered perfectly elastic, except for the first one or two for which values of "e" are given by EEM1 and EEM2, set NOT(6) = 1 (short form, no series 600 cards).

g. NOP(7) - Assume zero internal damping in the steel pile, thus set NOP(7) = 1 and do not include the 700 card series.

h. NOP(8) - Only the ram has an initial velocity, so NOP(8) = 1, no 800 card series.

i. NOP(9) and NOP(10) - Since more exact soils information is not available, Smith's recommended values for Q and J will be used and input on card 102 (short form). ~~Tush~~ Thus, NOP(9) = NOP(10) = 1.

j. NOP(11) - No damping, set NOP(11) = 1.

k. NOP(12) - Use a single factor to change force to stress for all springs - NOP(12) = 1.

l. NOP(13) - Use the damping procedure illustrated in Figure 5.13(a), so NOP(13) = 1.

m. NOP(14) - Calculate the force at the pile head from the action of the ram so NOP(14) = 1.

n. NOP(15) - Neglect gravity effects - NOP(15) = 1.

o. NOP(16) - Since several parameters are to be varied, set NOP(16) = 2, and thus card 1600 must be included in the data.

p. NOP(17) - Do not calculate driving resistance predicted by pile driving equations. NOP(17) = 1.

Card 102

1. ID3 - Identification Tag, use 12H53.
2. ID4 - Identification Tag, use L = 72.
3. RUT - Since the Michigan Report noted a soil "set-up" of about 2.0, the static resistance actually encountered during driving was probably around half of the measured 400 kip, so RUT = 200 kip.
4. RUP - Assuming 75 percent of the total

soil resistance at the point, RUP = 150 kip.

5. MO - Since the length of pile in the ground was 57.9 ft, the first segment upon which soil resistance acts is given by:

$$MO = MP+1 - \frac{\text{Depth Driven}}{\text{Segment Length}}$$

$$= 17+1-\frac{57.9}{5.0}$$

$$= 18 - 11.6$$

$$= 18 - 12$$

so MO = 6

6. QSIDE and QPOINT - Smith's recommended value of 0.1 in. will be used due to lack of better soils data.

7. SIDEJ and POINTJ - For the same reasons above for values of Q, use SIDEJ = 0.05 sec/ft and POINTJ = 0.15 sec/ft.

8. NUMR - Since the soil springs all act as shown in Figure 6.1(a), NUMR = 0.

9. Set IPRINT = 5 to print out the solution at every 5th iteration.

10. AREA - A single factor will be used to change all forces from lb to kip, thus AREA - 1000.0.

11. NS1 through NS6 - In this case, the solutions for segments 1, 2, 3, 4, 11, and 17 are desired and, therefore, NS1 through NS6 are given these values.

Cards 201-202

Segment Weights - As shown in Figure 3.4, several weights normally present during driving have been added between the pile and the driving cap to obtain experimental data.

a. W(1) = Ram Weight = 5.0 kip.

b. W(2) = Driving Cap Weight + 1/2 of the load cell weight = 1.29 kip.

c. W(3) = 1/2 load cell weight + helmet = 1.37 kip.

d. W(4) through W(17) = pile segment weights = (53 lb/ft)(5 ft) = 0.265 kip.

Cards 301-302

Segment Stiffness

a. Because of the lack of data concerning cushion stiffness, several values of K(1) will be run: K(1) - 500, 1,000, and 1,500 kip. in.

b. The helmet was found to be extremely stiff compared to the load cell, so

K(2) was taken as the stiffness of the load cell alone. From dimensions of the load cell given in the Michigan Report and using K = AE/L, the spring rate of the load cell was found to be 86,500 kip/in.

c. The spring rate of each 5 ft pile segment is found by:

$$K = \frac{AE}{L} = \frac{(15.58)(30x10^3)}{5x12} = 7,790 \text{ kip/in.}$$

So K(3) through K(16) = 7,790 kip/in.

Card 1600

1. Parameter Options - NOPP(I) - Note that all values of NOPP(I) are set = 1 except when an option is used to vary its assigned parameter, in which case NOPP(I) can equal 2 through 9.

   a. Since IVEL is to be given the four values of 8, 12, 16, and 20 ft/sec, NOPP(1) = 4.

   b. NOPP(2) through NOPP(4) = 1 since no segment weights are to be varied.

   c. NOPP(5) = 3 since three different cushion stiffnesses are to be used (K(1) = 500,1,000, and 1,500 kip/in.)

d. NOPP(6) through NOPP(17) - 1 since no other parameter changes are required.

2. Parameter Change Constants - DV1, DE1, DE2, etc. These values specify the desired increase in a given parameter based on the parameter's original value. They may be calculated from the equation:

$$\text{Constant} = \frac{\text{Second Value-Initial Value}}{\text{Initial Value}}$$

Thus, since the initial value of IVEL is 8 ft/sec and the second value is 12 ft/sec, so

$$DV1 = \frac{12-8}{8} = \frac{4}{8} = 1.0$$

The value for DK1 is therefore given by

$$DK1 = \frac{1000-500}{500} = \frac{500}{500} = 1.0$$

All other values such as DW1, DW2, etc., May be left blank or given any value for later use since they are not used as long as the corresponding NOPP(I) = 1.

APPENDIX C

PROGRAM LISTING

```
$EXECUTE       IBJOB
$IBJOB
$IBFTC MAIN
C - PROGRAM CONSISTS OF APPROXIMATELY 1200 LINES OUTPUT
C - LINES/PROBLEM = 50 +2*MP +NSTOP/IPRINT (UNLESS J5 CHANGES)
C - RUN TIME FOR PROGRAM IS ABOUT 1 MINUTE
C - RUM TIME FOR ONE PROBLEM IS ABOUT = (MP*NSTOP)/60,000  (MINUTES)
C  NOP(1) = 0,1,NO IDENTIFICATION CARDS (SERIES 103)
C         = 2, READ IDENTIFICATION CARD 103 (72 COLS OF ALPHAMERIC POOP)
C         = 3, READ 2 IDENTIFICATION CARDS
C         = 4, ETC. UP TO 4 CARDS
C  NOP(2) = 0
C         = 1,READ NEW WAM(I),I=1,MP
C         = 2, READ CARD 200 MAXIMUM DIFFERENT WAM(I) = TEN
C  NOP(3) = 0
C         = 1,READ NEW XKAM(I),I=1,N
C         = 2, READ CARD 300 MAXIMUM DIFFERENT XKAM(I) = TEN
C  NOP(4) = 0,USE OLD SOIL RESISTANCE VALUES,STANDARD OR GENERAL METHOD
C         = 1,READ NEW STANDARD RUM(I),I=1,MPP
C         = 2,ZERO SIDE RESISTANCE, SET RUM(MPP) = RUT
C         = 2,ZERO SIDE RESISTANCE, SET RUM(MPP) = RUP
C         = 3,UNIFORM SIDE RESISTANCE(RUT-RUP) WITH RUM(MPP) = RUP
C         = 4,TRIANGULAR SIDE RESISTANCE(RUT-RUP) WITH RUM(MPP) = RUP
C         = 5,READ NUMR CARDS AND USE GENERAL SOIL BEHAVIOR ROUTINE
C  NOP(5) = 0,USE OLD GAMMA(I)
C         = 1,2 SET GAMMA(NR)=GAMMA1 AND GAMMA(NR+1)=GAMMA2 (SOP)
C         = 3, USE SOP ABOVE AND SET GAMMA(NR+2) = 0.0
C         = 4, USE SOP ABOVE AND SET GAMMAS(NR+2) AND (NR+3) = 0.0
C         = 4, ETC.
C         = 9, USE LONG FORM INPUT
C           NOTE THAT NOP(5) IS USED TO SET ADDITIONAL GAMMA(I)S = 0.0
C  NOP(6) = 0, USE OLD EEM(I),I=1,N
C         = 1,USE SHORT FORM INPUT
C         = 2, USE LONG FORM INPUT
C  NOP(7) = 0, USE OLD BEEM(I), I=1,N
```

```
C             = 1,USE SHORT FORM INPUT
C             = 2, USE LONG FORM INPUT
C   NOP(8) = 0,USE OLD VEL(I), I=1,MP
C             = 1,USE SHORT FORM INPUT
C             = 2, USE LONG FORM INPUT
C   NOP(9) = 0,USE OLD Q(I), I=1,MPP
C             = 1,USE SHORT FORM INPUT
C             = 2, USE LONG FORM INPUT
C  NOP(10) = 0,USE OLD SJ(I), I=1,MPP
C             = 1,USE SHORT FORM INPUT
C             = 2, USE LONG FORM INPUT
C  NOP(11) = 0,USE OLD DYNAMK(I), I=1,N
C             = 1,DYNAMK=0.0
C             = 2, USE LONG FORM INPUT
C  NOP(12) = 0,USE OLD A(I), I=1,N
C             = 1,USE SHORT FORM INPUT
C             = 2, USE LONG FORM INPUT
C  NOP(13) = 0,1,USE SMITHS EEM ROUTINE
C             = 2,  USE LINEAR SOLID DAMPING
C  NOP(14) = 0,1,USE FOM(MI) COMPUTED FROM RAMS BEHAVIOR
C             = 2,  READ NSTOP VALUES OF FORCIN(INTV) (CARD SERIES 1300)
C             = 3,READ HEADER CARD + NSTOP GALVO DEFLECTIONS(IN.) CARDS 1400
C             = 4,READ CARD 1500 AND USE PARABOLIC FOM(1) VS. CEEM(1)
C  NOP(15)    = 1,NO GRAVITY
C               = 2,GRAVITY WITH DEM(I,0) = 0.0
C               = 3,GRAVITY WITH DEM(I,0) BY SMITH
C               = 4,GRAVITY WITH DEM(I,0) BY EXACT
C               = 5,GRAVITY WITH DEM(I,0) AS USED FOR PREVIOUS PROBLEM
C  NOP(16) = 0,1,NO PARAMETER CHANGES
C             = 2,  READ CARD 1600 WITH PARAMETER CHANGES
C   NOP(17) =0,1,NO PILE DRIVING FORMULA OUTPUT
C             = 2, READ CARD 1700 WITH PILE DRIVING CONSTANTS
C
C
C  NUMBER OF CASES = NOPP(1)*NOPP(2)* ... * NOPP(14)
```

```
C
C NOPP(1) = 1,RAM VELOCITY = VELMI
C         = 2,RAM VELOCITY=VELMI,(1.0+DV1)*VELMI
C         = 3,RAM VELOCITY=VELMI,(1.0+DV1)*VELMI,(1.0+2.*DV1)*VELMI
C         = 4,ETC.
C NOPP(2) = WAM(1) CHANGE
C NOPP(3) = WAM(2) CHANGES
C NOPP(4) = WAM(3,MP) CHANGES
C NOPP(5) = XKAM(1) CHANGES
C NOPP(6) = XKAM(2) CHANGES
C NOPP(7) = XKAM(3,N) CHANGES
C NOPP 8) = QSIDE CHANGES
C NOPP(9) = QPOINT CHANGES
C NOPP(10) = SIDEJ CHANGES
C NOPP(11) = POINTJ CHANGES
C NOPP(12) = RUM(1,MP) CHANGES
C NOPP(13) = RUM(MPP) CHANGES
C NOPP(14) = BOTH RUM(1,MP) AND RUM(MPP) CHANGE
C NOPP(15) = EEM(1) CHANGES
C NOPP(16) = EEM(2) CHANGES
C
C
C
      COMMON    WAM(100),  XKAM(100),   RUM(100),  BEEM(100),   EEM(100)      1
      COMMON  GAMMA(100),  XKIM(100),CEEMAS(100),  NFOM(100),  XDEM(100)      2
      COMMON    DEM(100), XCEEM(100),  CEEM(100),   FOM(100),  XFOM(100)      3
      COMMON    VEL(100),   DIM(100),   RAM(100),  RMAX(100), RSTAT(100)      4
      COMMON R(100,10)  , ITRIG(100),     Q(100),FORCIN(100),  DFOM(100)      5
      COMMON  FOMAX(100),IFOMAX(100), FOMIN(100),IFOMIN(100),     A(100)      6
      COMMON  DEMAX(100),IDEMAX(100),    SJ(100),   NOP( 22),DYNAMK(100)      7
      COMMON CEEMIN(100),HOLDEM(100),ANSVEC( 50),SE(50,51)  ,  IROW( 51)      8
      COMMON   RUMA(100),  WAMC(100), XKAMC(100),    QA(100),   SJA(100)      9
      COMMON   ICOL( 51),  NOPP( 20),ENTHRU(100),ENTMAX(100),   IDS( 50)     10
      COMMON QSIDE , QPOINT, SIDEJ , POINTJ, NQDIV , NORAMS, NSTOP           50
      COMMON INTV   , ISECTN, NUMR  , F1    , F2    , C1    , C2             51
```

```
      COMMON IPRINT, DELTEE, EEM1   , EEM2   , GAMMA1, GAMMA2, INT
      COMMON INIT  , I     , ITST   , IX     , NR    , MO    , MP
      COMMON NPAGE , N     , QUAKE  , RUP    , RUT   , VELMI , ID1
      COMMON ID2   , ID3   , ID4    , IDW1   , IDW2  , IDK1  , IDK2
      COMMON IDRL1 , IDRL2 , IDG1   , IDG2   , IDE1  , IDE2  , IDB1
      COMMON IDB2  , IDV1  , IDV2   , IDQ1   , IDQ2  , IDJ1  , IDJ2
      COMMON IDDK1 , IDDK2 , IDA1   , IDA2   , KGRADD, J5    , TMIN
      COMMON TMAX  , SMIN  , SMAX   , NOPNTS, AREA  , NS1   , NS2,NS6
      COMMON NS3   , NS4   , NS5    , IDEEM  , MH    , VEL1  , ACCEL
      COMMON B     , C     , AREAP  , XLONG  , ELAST , ACELMX
      COMMON DV1,DE1,DE2,DRI,DRP,DQI,DQP,DJI,DJP,DW1,DW2,DWI,DK1,DK2,DKI
C
C
C
C
      NPAGE =0
    9 CONTINUE
      NS1 = 0
      CALL INPUT
      MP = MP
      MO = MO
      NR = NR
      MH=MH
      N = MP-1
      MPP = MP+1
C                                        INITIALIZE PARAMETER CONSTANTS
      DELTAA = DELTEE
      WAMA = WAM(1)
      WAMB = WAM(2)
      XKAMA = XKAM(1)
      XKAMB = XKAM(2)
      DO 1 I=1,MP
      RUMA(I) = RUM(I)
      WAMC(I) = WAM(I)
      XKAMC(I) = XKAM(I)
```

```
      QA(I) = Q(I)
      SJA(I) = SJ(I)
    1 CONTINUE
      NOPA = NOPP( 1)
      NOPB = NOPP( 2)
      NOPC = NOPP(3 )
      NOPD = NOPP( 4)
      NOPE = NOPP(5 )
      NOPF = NOPP( 6)
      NOPG = NOPP( 7)
      NOPH = NOPP( 8)
      NOPI = NOPP( 9)
      NOPJ = NOPP(10)
      NOPK = NOPP(11)
      NOPL = NOPP(12)
      NOPM = NOPP(13)
      NOPN = NOPP(14)
      NOPO = NOPP(15)
      NOPQ = NOPP(16)
C                                       BEGIN PARAMETER VARIATIONS
      DO 98 IQ = 1,NOPQ
      DO 98 IO = 1,NOPO
   11 DO 98 IN = 1,NOPN
      IM = IN
      IL = IN
      DO 98 IK = 1,NOPK
      DO 98 IJ = 1,NOPJ
      DO 98 II = 1,NOPI
      DO 98 IH = 1,NOPH
      DO 98 IG = 1,NOPG
      DO 98 IF = 1,NOPF
      DO 98 IE = 1,NOPE
      DO 98 ID = 1,NOPD
      DO 98 IC = 1,NOPC
      DO 98 IB = 1,NOPB
```

```
      DO 98 IA = 1,NOPA
      DELTEE = DELTAA
      DO  4 I=1,MP
      VEL(I) = 0.0
      WAM(I) = WAMC(I)      *(1.0 + FLOAT(ID-1) * DWI)
      XKAM(I) = XKAMC(I)    *(1.0 + FLOAT(IG-1) * DKI)
      Q(I) = QA(I)          *(1.0 + FLOAT(IH-1) * DQI)
      SJ(I) = SJA(I)        *(1.0 + FLOAT(IJ-1) * DJI)
      RUM(I) = RUMA(I)      *(1.0 + FLOAT(IL-1) * DRI)
    4 CONTINUE
      DO 3 I=1,NR
      VEL(I) = VELMI        *(1.0 + FLOAT(IA-1) * DV1)
    3 CONTINUE
      VEL1 = VEL(1)
      WAM(1) = WAMA         *(1.0 + FLOAT(IB-1) * DW1)
      WAM(2) = WAMB         *(1.0 + FLOAT(IC-1) * DW2)
      XKAM(1) = XKAMA       *(1.0 + FLOAT(IE-1) * DK1)
      XKAM(2) = XKAMB       *(1.0 + FLOAT(IF-1) * DK2)
      Q(MPP) = QPOINT       *(1.0 + FLOAT(II-1) * DQP)
      SJ(MPP) = POINTJ      *(1.0 + FLOAT(IK-1) * DJP)
      RUM(MPP) = RUP        *(1.0 + FLOAT(IM-1) * DRP)
      EEM(NR)= EEM1         *(1.0 + FLOAT(IO-1) * DE1)
      EEM(NR+1)= EEM2       *(1.0 + FLOAT(IQ-1) * DE2)
      IF(NOP(4)-5)13,16,13
   13 DO 15 I=1,MPP
   15 XKIM(I) = RUM(I)/Q(I)
   16 CONTINUE
C  IF DELTEE IS LEFT BLANK, 1/2 THE CRITICAL TIME INTERVAL WILL BE USED
      IF(DELTEE)32,32,31
   32 DO 33 I=1,N
   33 DELTEE = AMAX1(DELTEE,39.296*SQRT(XKAM(I)/WAM(I)),
     1                      39.296*SQRT(XKAM(I)/WAM(I+1)))
   31 CONTINUE
C                                     END PARAMETER VARIATIONS
      C1PC2 = 0.0
```

```
      ACELMX = 0.0
      CALL PRINT 1
      CALL REP 1
      J5 = IPRINT
      KXT=1
      INTV = 0
      INTT =1
      MP = MP
      N = MP-1
      MPP = MP+1
      NOP15P = NOP(15)+1
      GO TO(50,50,49,48,47,43,50,50,50),NOP15P
   43 DO 42 I = 1,MP
   42 DEM(I) = HOLDEM(I)
   44 RAM(MP) = DEM(MP)*XKIM(MP)
      RAM(MP+1) = DEM(MP)*XKIM(MP+1)
      HOLDEM(MP) = DEM(MP)
      HOLDEM(1) = DEM(1)
      CEEM(1) = DEM(1) - DEM(2)
      FOM(1) = CEEM(1)*XKAM(1)
      DO 45 I = 2,N
      HOLDEM(I) = DEM(I)
      CEEM(I) = DEM(I)-DEM(I+1)
      FOM(I) = CEEM(I)*XKAM(I)
   45 RAM(I) = FOM(I-1)-FOM(I)+WAM(I)
      GO TO 49
   47 CALL EXACTG
      GO TO 49
   48 CALL SMITH
   49 CONTINUE
      WRITE(6,8002)(DEM(I),I=1,MP)
      WRITE(6,8001)(DIM(I),I=1,MP)
      WRITE(6,8003)(FOM(I),I=1,MP)
      WRITE(6,8004)(CEEM(I),I=1,N)
      WRITE(6,8005)(RAM(I),I=1,MPP)
```

```
      WRITE(6,8006)(XKIM(I),I=1,MPP)
   50 CONTINUE
      NSM = MP-1
      NSM=MINO(NS6,NSM)
      WRITE(6,1104)NS1,NS2,NS3,NS4,NS5,NSM,NS1,NS2,NS3,NS4,NS5,NS6,MPP
C                                                          BEGIN ITERATION LOOP
   12 CALL REP N
      INTT=INTT
      GO TO(22,9 ),INTT
   22 CONTINUE
      CMAX = 0.0
      DO 24 I=NR,N
   24 CMAX = CMAX+CEEM(I)
      C1PC2 = AMAX1(C1PC2,CMAX)
      IF(INTV-999)25,23,25
   23 J5 = 25
   25 CONTINUE
      IF(((INTV/J5)*J5)-INTV)94,26,94
   26 CONTINUE
   27 FOMA = FOM(NS1)/A(NS1)
      FOMB = FOM(NS2)/A(NS2)
      FOMC = FOM(NS3)/A(NS3)
      FOMD = FOM(NS4)/A(NS4)
      FOME = FOM(NS5)/A(NS5)
      FOMF = FOM(NSM)/A(NSM)
      RAMP = RAM(MP)/1000.0
C     WRITE(6,99)INTV ,FOMA,FOMB,FOMC,FOMD,FOME, CEEM(1),DEM(NS3),
C    1  DEM(NS4),DEM(NS5),DEM(NS5P),(ENTHRU(I),I=2,4),ENTHRU(N),ACCELR
      WRITE(6,99)INTV,FOMA,FOMB,FOMC,FOMD,FOME,FOMF,DEM(NS1),DEM(NS2),
     1DEM(NS3),DEM(NS4),DEM(NS5),DEM(NS6),RAMP
   94 CONTINUE
      IF(INTV-NSTOP )12,14,14
   14 WRITE(6,105)
      MP = MP
      N = MP-1
```

```
      MH = MH
      DO20I=1,N
      FOMAX(I) = FOMAX(I)/A(I)
      FOMIN(I) = FOMIN(I)/A(I)
      WRITE(6,106)I,IFOMAX(I),FOMAX(I),IFOMIN(I),FOMIN(I),
     1            ENTHRU(I),ENTMAX(I)
   20 CONTINUE
C     BLOWS = 1.0/DIM(MP)                    OLD STATEMENT
C     WRITE(6,2107)DIM(MP),BLOWS             OLD STATEMENT
      WRITE(6,2108)DEMAX(MH-1),DEMAX(MP)
      SMIN = SMIN/12.0
      SMAX = SMAX/12.0
      ERES1 = SQRT(SMIN/SMAX)
      WRITE(6,109)SMIN,SMAX,ERES1
      EINPUT = (WAM(1)*VEL1**2)/64.4
      WRITE(6,110)EINPUT
      WRITE(6,111)ACELMX
C                                           BEGIN ULTIMATE LOAD FORMULAS
      IF(NOP(17)-1)98,98,5
    5 CONTINUE
      C4 = 0.1
      AEL = AREAP*ELAST/XLONG
      NRP = NR+1
      C3 = QAVG
      S = DIM(MPP)
      W =WRAM
      U = ENERGY
      P = WPILE
      RWAVE = 0.0
      DO 6 I=1,MPP
      RWAVE = RWAVE+RUM(I)/1000.0
    6 CONTINUE
      SEGL = XLONG/(FLOAT(MP-MH+1))
      SUMR = 0.0
      DO 10 I=MH,MP
```

```
   10 SUMR = SUMR+RUM(I )*SEGL*(FLOAT(I -MH)+0.5)
      SUMR = SUMR+RUM(MPP)*XLONG
      HILEYL = SUMR/RWAVE
      RENEWS = U/(S+CENR)
      REYTEL = U/(S+(C4*P/W))
      RTERZG = AEL*(-S+SQRT(S**2+(2.0*U*(W+P*EEM1**2)/(AEL*(W+P)))))
      REDTEN = AEL*(-S+SQRT(S**2+(2.0*U*W/(AEL*(W+P)))))
      RHILYD =AEL*(-(S+C3)+SQRT((S+C3)**2+(2.0*U*(W+P*EEM1**2)/
     1        (AEL*(W+P)))))
      RHILYC=U*(W+P*EEM1**2)/((S+0.5*(C1PC2+C3))*(W+P))
      RCOAST =(AEL/2.)*(-S+SQRT(S**2+(4.*U*(W+P*EEM1**2)/(AEL*(W+P)))))
      WRITE(6,107)
      WRITE(6,108)RENEWS,REYTEL,RTERZG,REDTEN,RHILYD,RHILYC,RCOAST,RWAVE
C                                                 END ULTIMATE LOAD FORMULAS
   98 CONTINUE
      GO TO 9
   99 FORMAT(1X,I3,6F9.2,6F9.3,F9.1)
C  99 FORMAT(1X,  I3,  5F10.2,  5F11.7,F9.1)
  105 FORMAT(1H0,//, 18X,  63HMAXIMUM COMPRESSIVE AND TENSILE STRESSES (
     1PSI) IN THE SEGMENTS    ,//,19X,  7HSEGMENT  ,  1X, 5H TIME  ,
     2 3X,  6HSTRESS  ,  5X,  4HTIME,3X,6HSTRESS,7X,6HENTHRU,7X,
     3 10HMAX ENTHRU    ,    //)
  106 FORMAT(20X,I4,I8,F9.1,I9,F9.1,2F13.1)
  107 FORMAT(    16X,30H ULTIMATE PILE LOADS (KIPS)          )
  108 FORMAT(    21X,25H BY ENG NEWS FORMULA  =  ,   F15.3,/ ,
     1    22X,25H BY EYTELWEIN          =  ,   F15.3,/ ,
     2    22X,25H BY TERZAGHI           =  ,   F15.3,/ ,
     3    22X,25H BY REDTENBACHER       =  ,   F15.3,/ ,
     4    22X,25H BY HILEY (DUNHAM)     =  ,   F15.3,/ ,
     5    22X,25H BY HILEY (CHELLIS)    =  ,   F15.3,/ ,
     6    22X,25H BY PACIFIC COAST      =  ,   F15.3,/ ,
     7    22X,25H BY THE WAVE EQUATION =  ,   F15.3)
  109 FORMAT(17X,7HSMIN = F10.1, 7HSMAX = F10.1, 10HERES(1) = F10.7)
  110 FORMAT(16X,18H EINPUT (FT LBS) = F9.1)
  111 FORMAT(16X,24H MAX ACCELERATION (GS) = F9.1)
```

```
 1104 FORMAT(3H  T,6(6X,1HF,I2),1X, 6(6X,1HD,I2)  ,6X,1HR,I2,//)
C1104 FORMAT(115H TIME F(1)   F(2)   F(3)   F(4)   F(5)   D(2)  D(3)  D(
C     14)  D(5)  D(P)  ENT(2)  ENT(3)  ENT(4)  ENT(N) ACC(MH-1)        )
C1104 FORMAT(5H TIME,5(2X,4HFOM( I3, 1H) ) ,5(3X,4HDEM( I3, 1H) )  ,
C     1 3X, 12HENTHRU (1)      //)
 2107 FORMAT(1H /    ,17X,24HPERMANENT SET OF PILE = F13.8,8H  INCHES/
     1    ,17X,27HNUMBER OF BLOWS PER INCH = F13.8)
 2108 FORMAT(1H /    ,17X,24HLIMSET FOR (MH-1)      = F13.8,8H  INCHES/
     1    ,17X,27HMAX DISPLACEMENT OF POINT= F13.8)
 8001 FORMAT(33HOINITIAL VALUES FOR DIM(I),I=1,MP /(6E19.8))
 8002 FORMAT(33HOINITIAL VALUES FOR DEM(I),I=1,MP /(6E19.8))
 8003 FORMAT(33HOINITIAL VALUES FOR FOM(I),I=1,MP /(6E19.8))
 8004 FORMAT(33HOINITIAL VALUES FOR CEEM(I),I=1,N /(6E19.8))
 8005 FORMAT(35HOINITIAL VALUES FOR RAM(I),I=1,MP+1 /(6E19.8))
 8006 FORMAT(38HOCONSTANT VALUES FOR XKIM(I),I=1,MP+1 /(6E19.8))
      END
$IBFTC INPUTT
      SUBROUTINE INPUT
      COMMON    WAM(100),  XKAM(100),   RUM(100),  BEEM(100),   EEM(100)
      COMMON  GAMMA(100),  XKIM(100),CEEMAS(100),  NFOM(100),  XDEM(100)
      COMMON    DEM(100), XCEEM(100),  CEEM(100),   FOM(100),  XFOM(100)
      COMMON    VEL(100),   DIM(100),   RAM(100),  RMAX(100), RSTAT(100)
      COMMON R(100,10)  , ITRIG(100),      Q(100),FORCIN(100),  DFOM(100)
      COMMON  FOMAX(100),IFOMAX(100), FOMIN(100),IFOMIN(100),     A(100)
      COMMON  DEMAX(100),IDEMAX(100),     SJ(100),   NOP( 22),DYNAMK(100)
      COMMON CEEMIN(100),HOLDEM(100),ANSVEC( 50),SE(50,51)  ,  IROW( 51)
      COMMON   RUMA(100),  WAMC(100), XKAMC(100),    QA(100),   SJA(100)
      COMMON   ICOL( 51),  NOPP( 20),ENTHRU(100),ENTMAX(100),   IDS( 50)
      COMMON QSIDE , QPOINT, SIDEJ , POINTJ, NQDIV , NORAMS, NSTOP
      COMMON INTV  , ISECTN, NUMR  , F1    , F2    , C1    , C2
      COMMON IPRINT, DELTEE, EEM1  , EEM2  , GAMMA1, GAMMA2, INT
      COMMON INTT  , I     , ITST  , IX    , NR    , MO    , MP
      COMMON NPAGE , N     , QUAKE , RUP   , RUT   , VELMI , ID1
      COMMON ID2   , ID3   , ID4   , IDW1  , IDW2  , IDK1  , IDK2
      COMMON IDRL1 , IDRL2 , IDG1  , IDG2  , IDE1  , IDE2  , IDB1
```

```
      COMMON ID82  , IDV1  , IDV2  , IDQ1  , IDQ2  , IDJ1  , IDJ2
      COMMON IDDK1 , IDDK2 , IDA1  , IDA2  , KGRADD, J5    , TMIN
      COMMON TMAX  , SMIN  , SMAX  , NOPNTS, AREA  , NS1   , NS2,NS6
      COMMON NS3   , NS4   , NS5   , IDEEM , MH    , VEL1  , ACCELR
      COMMON B     , C     , AREAP , XLONG , ELAST , ACELMX
      COMMON DV1,DE1,DE2,DRI,DRP,DQI,DQP,DJI,DJP,DW1,DW2,DWI,DK1,DK2,DKI
C
      READ(5,100)ID1,ID2,DELTEE,MP,VELMI,MH,NR,EEM1,EEM2,GAMMA1,
     1 GAMMA2,NSTOP,(NOP(I),I=1,20)
      READ(5,101)ID3,ID4,RUT,RUP,MO,QSIDE,OPOINT,SIDEJ,POINTJ,NUMR,
     1 IPRINT,AREA,NS1,NS2,NS3,NS4,NS5,NS6
      RUT = RUT*1000.0
      RUP = RUP*1000.0
      NR = MAX0(NR,1)
      N = MP-1
      MPP = MP+1
      WAM(MPP) = -0.0
      XKAM(MP) = -0.0
      XKAM(MPP) = -0.0
      IF(NOP(1)-2)9,7,7
    7 NOIDS = 12*(NOP(1)-1)
      READ(5,103)(IDS(I),I=1,NOIDS)
    9 CONTINUE
      IF(NOP(2)-1) 1,1,14
    1 READ(5,102)IDW1,IDW2,(WAM(I),I=1,MP)
      GO TO 2
   14 NRP1 = NR+1
      NRP5 = NR+5
      NRP6 = NR+6
      MPM3 = MP-3
      READ(5,111)IDW1,IDW2,WAM(1),(WAM(I),I=NRP1,NRP5),
     1           (WAM(I),I=MPM3,MP)
  111 FORMAT(A5,A4,-3P10F6.4)
      DO 76 I=1,NR
   76 WAM(I) = WAM(1)
```

```
      DO 77 I=NRP6,MPM3
   77 WAM(I) = WAM(NRP5)
    2 CONTINUE
      IF(NOP(3)-1) 3,3,15
    3 READ(5,104)IDK1,IDK2,(XKAM(I),I=1,N)
      GO TO 4
   15 NRM1 = NR-1
      NRP5 = NR+5
      NRP6 = NR+6
      MPM3 = MP-3
      READ(5,112)IDK1,IDK2,XKAM(1),(XKAM(I),I=NR,NRP5),
     1           (XKAM(I),I=MPM3,N)
  112 FORMAT(A5,A4,-3P10F6.0)
      DO 78 I=1,NRM1
   78 XKAM(I) = XKAM(1)
      DO 79 I=NRP6,MPM3
   79 XKAM(I) = XKAM(MPM3)
    4 CONTINUE
      IF(NOP(4)-1)22,5,5
    5 NOP4 = NOP(4)
      DO 6 I=1,MP
    6 RUM(I) = 0.0
      RUM(MPP) = RUP
      GO TO(10,22,11,13,17,22,22,22,22),NOP4
   10 READ(5,106)IDRL1,IDRL2,(RUM(I),I=1,MPP)
C INPUT RUM(I) IN UNITS OF KIPS - THE COMPUTER WILL CONVERT TO LBS.
      GO TO 22
   11 RCONST = (RUT-RUP)/FLOAT(MPP-MO)
      DO 12 I=MO,MP
   12 RUM(I) = RCONST
      GO TO 22
   13 DO 16 I=MO,MP
   16 RUM(I) = (2.0*(RUT-RUP)*(FLOAT(I-MO)+0.5))/(FLOAT(MPP-MO))**2
      GO TO 22
C GENERAL R(I,J) INPUT
```

```
   17 DO 20 I=1,MPP
   20 XKIM(I) = 0.0
      DO 21 K=1,NUMR
   21 READ(5,115)I, XKIM(I),(R(I,J),J=1,10)
   22 CONTINUE
C THE R(I,J) INPUT CARDS CAN BE IN RANDOM ORDER
C THE R(I,J) ARRAY NEED NOT BE ZEROED SINCE IF XKIM(I)=0 THE GENERAL
C SOIL RESISTANCE ROUTINE FOR SEGMENT(I) IS NOT CONSIDERED
C NUMR = TOTAL NUMBER OF SEGMENTS W/GEN. R  (DONT FORGET TO ADD MPP)
CC I = THE SEGMENT NUMBER FOR WHICH R(I,J) VALUES ARE BEING INPUT
C R(I,J) = STATIC RESISTANCE ON SEGMENT I AT EACH OF TEN POINTS J
      IF(NOP(5)-1)29,27,26
   26 IF(NOP(5)-9)24,25,24
   25 READ(5,106)IDG1,IDG2,(GAMMA(I),I=1,N)
      GO TO 29
   24 IGAMMA = NOP(5)+NR-1
      DO 23 I=1,N
   23 GAMMA(I) = -1000.0
      DO 19 I=NR,IGAMMA
   19 GAMMA(I) = 0.0
      GAMMA(NR) = GAMMA1
      GAMMA(NR+1) = GAMMA2
      GO TO 29
   27 DO 28 I=1,N
   28 GAMMA(I) = -1000.0
      GAMMA(NR) = GAMMA1
      GAMMA(NR+1) = GAMMA2
   29 GAMMA(MP) = -0.0
      GAMMA(MPP) = -0.0
      IF(NOP(6)-1)33,31,30
   30 READ(5,107)IDE1,IDE2,(EEM(I),I=1,N)
      GO TO 33
   31 DO 32 I=1,N
   32 EEM(I) = 1.0
      EEM(NR) = EEM1
```

```
      EEM(NR+1) = EEM2
   33 EEM(MP) = -0.0
      EEM(MPP) = -0.0
      IF(NOP(7)-1)37,35,34
   34 READ(5,107)IDB1,IDB2,(BEEM(I),I=1,N)
      GO TO 37
   35 DO 36 I=1,N
   36 BEEM(I) = 0.0
   37 BEEM(MP) = -0.0
      BEEM(MPP) = -0.0
C DO NOT TRY TO USE LAST PROBLEMS VALUES OF VEL(I)
      IF(NOP(8)-1)39,39,38
   38 READ(5,108)IDV1,IDV2,( VEL(I),I=1,MP)
      GO TO 71
   39 DO 40 I=NR,MPP
   40 VEL(I) = 0.0
      DO 41 I=1,NR
   41 VEL(I) = VELMI
   71 VEL(MPP) = -0.0
      IF(NOP(9)-1)45,43,42
   42 READ(5,107)IDQ1,IDQ2,(Q(I),I=1,MPP)
      GO TO 45
   43 DO 44 I=1,MPP
      Q(I) = QSIDE
   44 CONTINUE
      Q(MPP) = QPOINT
   45 IF(NOP(10)-1)49,47,46
   46 READ(5,107)IDJ1,IDJ2,(SJ(I),I=1,MPP)
      GO TO 49
   47 DO 48 I=1,MP
   48 SJ(I) = SIDEJ
      SJ(MPP) = POINTJ
   49 IF(NOP(11)-1)53,51,50
   50 READ(5,104)IDDK1,IDDK2,(DYNAMK(I),I=1,N)
      DO 72 I=1,N
```

```
   72 DYNAMK(I) = DYNAMK(I)-XKAM(I)
      GO TO 53
   51 DO 52 I=1,N
   52 DYNAMK(I) = 0.0
C STATEMENT 52 SETS DYNAMK(I) = 0.0 SO SMITHS ROUTINE WILL BE USED
   53 DYNAMK(MP) = -0.0
      DYNAMK(MPP) = -0.0
      IF(NOP(12)-1)57,55,54
   54 READ(5,109)IDA1,IDA2,(A(I),I=1,N)
      GO TO 57
   55 DO 56 I=1,N
   56 A(I) = AREA
   57 A(MP) = -0.0
      A(MPP) = -0.0
      IF(NOP(4)-1)61,58,59
   58 IF(NOP(4)-5)59,61,61
   59 DO 60 I=1,MPP
   60 XK(M(I) = R[illegible](I)/Q(I)
   61 CONTINUE
      NOP14 = NOP(14)+1
      GO TO(65,65,62,63,65),NOP14
C READ NSTOP VALUES OF FOM(1,T)  -  MAXIMUM NSTOP = 300
   62 READ(5,120)(FORCIN(I),I=1,NSTOP)
      GO TO 65
   63 READ(5,122)AREAP,EMODUL,RGAGE,RCAL,ACTIVG,GFACTR,D1,D2,D3,D4,D5
      READ(5,121)(FORCIN(I),I=1,NSTOP)
      CE = (AREAP*EMODUL*RGAGE*1000.0)/(ACTIVG*GFACTR*RCAL)
      A(NS1) = CE/D1
      A(NS2) = CE/D2
      A(NS3) = CE/D3
      A(NS4) = CE/D4
      A(NS5) = CE/D5
      DO 64 I=1,NSTOP
   64 F[illegible] = FO[illegible]CIN(I)*A(1)
   65 [illegible]
```

```
      IF(NOP(14)-4)67,66,67
   66 READ(5,123)F1,F2,C1,C2
   67 CONTINUE
      DO 90 I=1,20
   90 NOPP(I) = 1
      IF(NOP(16)-2)69,68,69
   68 READ(5,124)(NOPP(I),I=1,20),DV1,DW1,DW2,DWI,DK1,DK2,DKI,DOI,
     1   DQP,DJI,DJP,DRI,DRP,DE1,DE2
   69 CONTINUE
      DO 8 I=1,20
      NOPP(I) = MAX0(NOPP(I),1)
    8 CONTINUE
      IF(NOP(17)-1)74,74,73
   73 READ(5,125)AREAP,XLONG,ELAST,CENR,QAVG,WRAM,WPILE,ENERGY
      XLONG = XLONG*12.0
   74 CONTINUE
  100 FORMAT(A5,A4,F6.0,I3,F4.2,2I3,2F4.3,2F6.0,I4,20I1)
  101 FORMAT(A5,A4,2F7.2,I3,4F4.3,2I3,F6.2,6I3)
  102 FORMAT(A5,A4,-3P10F6.4,/(9X,-3P10F6.4))
  103 FORMAT(12A6)
  104 FORMAT(A5,A4,-3P10F6.0,/(9X,-3P10F6.0))
  106 FORMAT(A5,A4,-3P10F6.1,/(9X,-3P10F6.1))
  107 FORMAT(A5,A4,    10F6.5,/(9X,    10F6.5))
  108 FORMAT(A5,A4,    10F6.3,/(9X,    10F6.3))
  109 FORMAT(A5,A4,    10F6.2,/(9X,    10F6.2))
  115 FORMAT(I3,-3P11F6.1)
  120 FORMAT(-3P10F6.1)
  121 FORMAT(    10F6.4)
  122 FORMAT(F7.2,3F7.0,7F4.2)
  123 FORMAT(-3P2F6.1,0P2F6.5)
  124 FORMAT(20I1,17F3.2)
  125 FORMAT(F6.2,F5.2,F7.2)
      RETURN
      END
$IBFTC PRINT
```

```
C     PRINT 1 IS A SUBROUTINE TO PRINT INPUT DATA.
C
      SUBROUTINE PRINT 1
      COMMON    WAM(100),  XKAM(100),   RUM(100),  BEEM(100),    EEM(100)     1
      COMMON  GAMMA(100),  XKIM(100),CEEMAS(100),  NFOM(100),   XDEM(100)     2
      COMMON    DEM(100), XCEEM(100),  CEEM(100),   FOM(100),   XFOM(100)     3
      COMMON    VEL(100),   DIM(100),   RAM(100),  RMAX(100),  RSTAT(100)     4
      COMMON R(100,10)   , ITRIG(100),     Q(100),FORCIN(100),  DFOM(100)     5
      COMMON  FOMAX(100),IFOMAX(100), FOMIN(100),IFOMIN(100),      A(100)     6
      COMMON  DEMAX(100),IDEMAX(100),     SJ(100),   NOP( 22),DYNAMK(100)     7
      COMMON CEEMIN(100),HOLDEM(100),ANSVEC( 50),SE(50,51)   ,   IROW( 51)     8
      COMMON   RUMA(100),  WAMC(100), XKAMC(100),     QA(100),    SJA(100)     9
      COMMON   ICOL( 51),  NOPP( 20),ENTHRU(100),ENTMAX(100),    IDS( 50)    10
      COMMON QSIDE , QPOINT, SIDEJ , POINTJ, NODIV , NORAMS, NSTOP           50
      COMMON INTV  , ISECTN, NUMR  , F1    , F2    , C1    , C2              51
      COMMON IPRINT, DELTEE, EEM1  , EEM2  , GAMMA1, GAMMA2, INT             52
      COMMON INTT  , I     , ITST  , IX    , NR    , MO    , MP              53
      COMMON NPAGE , N     , QUAKE , RUP   , RUT   , VELMI , ID1             54
      COMMON ID2   , ID3   , ID4   , IDW1  , IDW2  , IDK1  , IDK2            55
      COMMON IDRL1 , IDRL2 , IDG1  , IDG2  , IDE1  , IDE2  , IDB1            56
      COMMON IDB2  , IDV1  , IDV2  , IDQ1  , IDQ2  , IDJ1  , IDJ2            57
      COMMON IDDK1 , IDDK2 , IDA1  , IDA2  , KGRADD, J5    , TMIN            58
      COMMON TMAX  , SMIN  , SMAX  , NOPNTS, AREA  , NS1   , NS2,NS6         59
      COMMON NS3   , NS4   , NS5   , IDEEM , MH    , VEL1  , ACCELR          60
      COMMON B     , C     , AREAP , XLONG , ELAST , ACELMX                  61
      COMMON DV1,DE1,DE2,DRI,DRP,DQI,DCP,DJI,DJP,DW1,DW2,DWI,DK1,DK2,DKI
C
      NPAGE = NPAGE+1
      WRITE(6,102)NPAGE
      IF(NOP(1)-2)3,2,2
    2 NOIDS = 12*(NOP(1)-1)
      WRITE(6,101)
      WRITE(6,103 )(IDS(I),I=1,NOIDS)
      WRITE(6,101)
    3 CONTINUE
```

```
      MPP=MP+1
      RCT = 0.0
      DO 6 I= 1,MPP
      RCT = RCT+RUM(I)/1000.0
    6 CONTINUE
      RCP = RUM(MPP)/1000.0
      WRITE(6,105)DELTEE,NOP(1),NOP(16)
      DELTEE = 1.0/DELTEE
      WRITE(6,106)MP,NOP(2),NOP(17)
      WRITE(6,107)ID1,ID2,VELMI,NOP(3),NOP(18)
      WRITE(6,108)ID3,ID4,NSTOP,NOP(4),NOP(19)
      WRITE(6,110)IDW1,IDW2,RCT,NOP(5),NOP(20)
      WRITE(6,111)IDK1,IDK2,RCP,NOP(6)
      WRITE(6,112)IDRL1,IDRL2,MO,NOP(7)
      WRITE(6,113)IDG1,IDG2,QSIDE,NOP(8)
      WRITE(6,114)IDE1,IDE2,QPOINT,NOP(9)
      WRITE(6,115)IDB1,IDB2,SIDEJ,NOP(10)
      WRITE(6,116)IDV1,IDV2,POINTJ,NOP(11)
      WRITE(6,117)IDQ1,IDQ2,NUMR,NOP(12)
      WRITE(6,118)IDJ1,IDJ2,IPRINT,NOP(13)
      WRITE(6,119)IDDK1,IDDK2,AREA,NOP(14)
      WRITE(6,120)IDA1,IDA2,NR,NOP(15)
      WRITE(6,101)
      WRITE(6,121)
      MPP = MP+1
      LINES = 19
      DO 5 I=1,MPP
      WRITE(6,122)I,WAM(I),XKAM(I),RUM(I),GAMMA(I),EEM(I),BEEM(I),
     1 VEL(I),Q(I),    SJ(I),DYNAMK(I),A(I)
      LINES = LINES+1
      IF(LINES-58)5,4,4
    4 NPAGE = NPAGE
      LINES = 5
      WRITE(6,102)NPAGE
      WRITE(6,101)
```

```
      WRITE(6,121)
    5 CONTINUE
      IF(NOP(4)-5)30,7,30
    7 IF(LINES-50)9,9,8
    8 NPAGE = NPAGE
      LINES = -1
      WRITE(6,102)NPAGE
      GO TO 10
    9 WRITE(6,101)
   10 WRITE(6,123)(J,J=1,10)
      LINES = LINES+6
      LINADD = NQDIV/10
      IF(NQDIV-LINADD*10)13,14,13
   13 LINADD = LINADD+1
   14 LINADD = LINADD+1
      DO 29 I=1,MPP
      IF(XKIM(I)-0.0)29,29,20
   20 LINES = LINES+LINADD
      IF(LINES-59)24,24,23
   23 NPAGE = NPAGE
      WRITE(6,102)NPAGE
      WRITE(6,123)(J,J=1,10)
      LINES = 6
   24 WRITE(6,124)I,(R(I,J),J=1,10)
   29 CONTINUE
      WRITE(6,101)
      LINES = LINES+2
   30 WRITE(6,101)
      LINES = LINES+2
      LINADD = MP/8
      IF(MP-LINADD*8)40,41,40
   40 LINADD = LINADD+1
   41 LINADD = LINADD+2
  101 FORMAT(1H0)
```

```
102 FORMAT(1H1, 20H                                  66X,7HPROBLEM I4)
103 FORMAT(1X,12A6)
123 FORMAT(85H    R(M,N) = STATIC SOIL RESISTANCE FOR GIVEN SEGMENTS -
   1 OTHERS HAVE R(I,J) = 0.0       // 5X,10(9X,I2)  )
105 FORMAT(4X,29H CARD    ID1    ID2  1/DELTEE =  F8.0,12H    NOP(1) =
   1 I2, 12H   NOP(16) =    I2)
106 FORMAT(28X, 5H MP = I8,12H    NOP(2) =I2,12H   NOP(17) =I2)
107 FORMAT(11H       101     A6,A4,12H      VELMI =F8.2,12H    NOP(3) =
   1 I2, 12H   NOP(18) =    I2)
108 FORMAT(11H       102     A6,A4,12H      NSTOP = I8 ,12H    NOP(4) =
   1 I2, 12H   NOP(19) =    I2)
110 FORMAT(11H       WAM  A6,A4,12H        RUT =F8.1,12H    NOP(5) =I2,
   1 12H   NOP(20) =    I2)
111 FORMAT(11H      XKAM  A6,A4,12H        RUP =F8.1,12H    NOP(6) =I2)
112 FORMAT(11H       RUM  A6,A4,12H         MO =I8  ,12H    NOP(7) =I2)
113 FORMAT(11H     GAMMA  A6,A4,12H      QSIDE =F8.4,12H    NOP(8) =I2)
114 FORMAT(11H       EEM  A6,A4,12H     QPOINT =F8.4,12H    NOP(9) =I2)
115 FORMAT(11H      BEEM  A6,A4,12H      SIDEJ =F8.4,12H   NOP(10) =I2)
116 FORMAT(11H       VEL  A6,A4,12H     POINTJ =F8.4,12H   NOP(11) =I2)
117 FORMAT(11H         Q  A6,A4,12H       NUMR =I8  ,12H   NOP(12) =I2)
118 FORMAT(11H     SOILJ  A6,A4,12H     IPRINT =I8  ,12H   NOP(13) =I2)
119 FORMAT(11H    DYNAMK  A6,A4,12H       AREA =F8.2,12H   NOP(14) =I2)
120 FORMAT(11H         A  A6,A4,12H         NR =I8  ,12H   NOP(15) =I2)
121 FORMAT(116H   M    WAM(M)    XKAM(M)    RUM(M)  GAMMA(M)   EEM(M)
   1  BEEM(M)     VEL(M)      Q(M)  SOILJ(M) DYNAMK(M)      A(M)     /,
   2 116H         (KIPS)  (KIPS/IN)    (KIPS)    (KIPS)     (NONE) (SECIN/
   3FT)   (FT/SEC)       (IN)  (SEC/FT) (KIPS/IN)    (SQ IN)      )
122 FORMAT(I4,-3PF10.4,3F10.1,0P2F10.6,F10.3,2F10.6,-3PF10.3,0PF12.3)
124 FORMAT(/4H 7 = I3,2X,10F10.1,(/9X,10F10.1))
    RETURN
    END
$IBFTC REPONE
    SUBROUTINE REP1
    COMMON   WAM(100),  XKAM(100),   RUM(100),  BEEM(100),   EEM(100)        1
    COMMON  GAMMA(100),  XKIM(100),CEEMAS(100),  NFOM(100),  XDEM(100)       2
```

```
      COMMON    DEM(100), XCEEM(100),  CEEM(100),   FOM(100),  XFOM(100)      3
      COMMON    VEL(100),    DIM(100),   RAM(100),  RMAX(100), RSTAT(100)     4
      COMMON R(100,10)   , ITRIG(100),       Q(100),FORCIN(100),  DFOM(100)   5
      COMMON  FOMAX(100),IFOMAX(100), FOMIN(100),IFOMIN(100),      A(100)     6
      COMMON  DEMAX(100),IDEMAX(100),     SJ(100),    NOP( 22),DYNAMK(100)    7
      COMMON CEEMIN(100),HOLDEM(100),ANSVEC( 50),SE(50,51)   ,  IROW( 51)    8
      COMMON   RUMA(100),   WAMC(100), XKAMC(100),     QA(100),   SJA(100)    9
      COMMON   ICOL( 51),   NOPP( 20),ENTHRU(100),ENTMAX(100),   IDS( 50)   10
      COMMON QSIDE , QPOINT, SIDEJ , POINTJ, NODIV , NORAMS, NSTOP           50
      COMMON INTV  , ISECTN, NUMR  , F1    , F2    , C1    , C2              51
      COMMON IPRINT, DELTEE, EEM1  , EEM2  , GAMMA1, GAMMA2, INT             52
      COMMON INTT  , I     , ITST  , IX    , NR    , MO    , MP              53
      COMMON NPAGE , N     , QUAKE , RUP   , RUT   , VELMI , ID1             54
      COMMON ID2   , ID3   , ID4   , IDW1  , IDW2  , IDK1  , IDK2            55
      COMMON IDRL1 , IDRL2 , IDG1  , IDG2  , IDE1  , IDE2  , IDB1            56
      COMMON IDB2  , IDV1  , IDV2  , IDQ1  , IDQ2  , IDJ1  , IDJ2            57
      COMMON IDDK1 , IDDK2 , IDA1  , IDA2  , KGRADD, J5    , TMIN            58
      COMMON TMAX  , SMIN  , SMAX  , NOPNTS, AREA  , NS1   , NS2,NS6         59
      COMMON NS3   , NS4   , NS5   , IDEEM , MH    , VEL1  , ACCFLR          60
      COMMON B     , C     , AREAP , XLONG , ELAST , ACELMX                  61
      COMMON DV1,DE1,DE2,DRI,DRP,DQI,DQP,DJI,DJP,DW1,DW2,DWI,DK1,DK2,DKI
C
      MP = MP
      MPP = MP+1
      SMAX = 0.0
      SMIN = 0.0
      DO 64 I = 1,MPP
      ITRIG(I) = 1
      DEM(I) = 0.0
      XDEM(I) = 0.0
      DEMAX(I) = 0.0
      IDEMAX(I) = 0
      CEEM(I) = 0.0
      XCEEM(I) = 0.0
      CEEMAS(1) = 0.0
```

```
      FOM(I) = 0.0
      XFOM(I) = 0.0
      FOMAX(I) = 0.0
      FOMIN(I) = 0.0
      IFOMAX(I) = 0
      IFOMIN(I) = 0
      NFOM(I) = 1
      RAM(I) = 0.0
      RMAX(I) = 0.0
      RSTAT(I) = 0.0
      DIM(I) = 0.0
      ENTHRU(I) = 0.0
      ENTMAX(I) = 0.0
   64 CONTINUE
      IF(NOP(14)-4)18,65,18
   65 CONTINUE
      C = (F1*C2 - F2*C1)/(C1*C2*(C1-C2))
      B = (F2*C1**2 - F1*C2**2)/(C1*C2*(C1-C2))
      IF(B)22,22,18
   22 IF(F1-F2)24,23,23
   23 C = F1/C1**2
      GO TO 25
   24 C = F2/C2**2
   25 B = 0.0
      WRITE(6,104)
  104 FORMAT(47HOPARABOLA BASED ON F2 AND C2 ONLY MUST BE USED          )
   18 CONTINUE
      RETURN
      END
$IBFTC REPREP
      SUBROUTINE REP N
      COMMON    WAM(100),  XKAM(100),   RUM(100),  BEEM(100),   EEM(100)     1
      COMMON  GAMMA(100),  XKIM(100),CEEMAS(100),  NFOM(100),  XDEN(100)     2
      COMMON    DEM(100), XCEEM(100),  CEEM(100),   FOM(100),  XFOM(100)     3
      COMMON    VEL(100),   DIM(100),   RAM(100),  RMAX(100), RSTAT(100)     4
```

```
      COMMON R(100,10)   , ITRIG(100),      Q(100),FORCIN(100),   DFOM(100)
      COMMON  FOMAX(100),IFOMAX(100), FOMIN(100),IFOMIN(100),      A(100)
      COMMON  DEMAX(100),IDEMAX(100);     SJ(100),    NOP( 22),DYNAMK(100)
      COMMON CEEMIN(100),HOLDEM(100),ANSVEC( 50),SE(50,51)   ,   IROW( 51)
      COMMON   RUMA(100),   WAMC(100), XKAMC(100),     QA(100),   SJA(100)
      COMMON   ICOL( 51),   NOPP( 20),ENTHRU(100),ENTMAX(100),    IDS( 50)
      COMMON QSIDE , QPOINT, SIDEJ , POINTJ, NQDIV , NORAMS, NSTOP
      COMMON INTV  , ISECTN, NUMR  , F1    , F2    , C1    , C2
      COMMON IPRINT, DELTEE, EEM1  , EEM2  , GAMMA1, GAMMA2, INT
      COMMON INTT  , I     , ITST  , IX    , NR    , MO    , MP
      COMMON NPAGE , N     , QUAKE , RUP   , RUT   , VELMI , ID1
      COMMON ID2   , ID3   , ID4   , IDW1  , IDW2  , IDK1  , IDK2
      COMMON IDRL1 , IDRL2 , IDG1  , IDG2  , IDE1  , IDE2  , IDB1
      COMMON IDB2  , IDV1  , IDV2  , IDQ1  , IDQ2  , IDJ1  , IDJ2
      COMMON IDDK1 , IDDK2 , IDA1  , IDA2  , KGRADD, J5    , TMIN
      COMMON TMAX  , SMIN  , SMAX  , NOPNTS, AREA  , NS1   , NS2,NS6
      COMMON NS3   , NS4   , NS5   , IDEEM , MH    , VEL1  , ACCELR
      COMMON B     , C     , AREAP , XLONG , ELAST , ACELMX
      COMMON DV1,DE1,DE2,DRI,DRP,DQI,DQP,DJI,DJP,DW1,DW2,DWI,DK1,DK2,DKI
C
      INTV = INTV+1
      MP=MP
      MPP = MP+1
      NOP(4) = NOP(4)
      NOP(13) = NOP(13)
      NOP(14) = NOP(14)
      NOP(15) = NOP(15)
      ITEST1 = 1
      ITESTP = 1
      DO 68 I = 1, MP
      I=I
      IF(I-MP)18,17,18
   17 ITESTP = 2
   18 CONTINUE
      XDEM(I) = DEM(I)
```

```
      DEM(I) = XDEM(I) +VEL(I)*12.0*DELTEE
      IF(DEMAX(I)-DEM(I))20,21,21
   20 DEMAX(I)= DEM(I)
      IDEMAX(I) = INTV
   21 GO TO(34,19),ITESTP
   34 XCEEM(I) = CEEM(I)
C     STATEMENT 34 MUST USE A COMPUTED VALUE FOR THE ACTUAL DEM(I+1)
      CEEM(I) = DEM(I) -DEM(I+1) -VEL(I+1)*12.0*DELTEE
      XFOM(I) = FOM(I)
      IF(BEEM(I)-0.000001)36,36,30
   30 IF(DYNAMK(I))31,31,32
C                                                SMITHS DAMPING METHOD
   31 DFOM(I) = BEEM(I)*XKAM(I)*(CEEM(I)-XCEEM(I))/(DELTEE*12.0)
      GO TO 33
C                                          STANDARD LINEAR SOLID DAMPING
   32 DFOM(I) = (DFOM(I)+DYNAMK(I)*(CEEM(I)-XCEEM(I)))/
     1           (1.0+DYNAMK(I)*DELTEE/(1000.0*BEEM(I)))
   33 FOM(I) = CEEM(I)*XKAM(I) + DFOM(I)
      GO TO 43
   36 IF(0.99999-EEM(I))38,38,39
   38 FOM(I) = CEEM(I)*XKAM(I)
      CEEMAS(I) = AMAX1(CEEMAS(I),XCEEM(I))
      GO TO 43
   39 CEEMAS(I) = AMAX1(CEEMAS(I),XCEEM(I))
      CEEMIN(I) = AMIN1(CEEMIN(I),XCEEM(I))
      IF(CEEM(I))13,43,5
    5 IF(CEEM(I)-CEEMAS(I))11,11,38
   11 FOM(I)=AMAX1(XKAM(I)*(CEEMAS(I)-(CEEMAS(I)-CEEM(I))/EEM(I)**2),0.)
      GO TO 43
   13 IF (CEEM(I)-CEEMIN(I))38,14,14
   14 FOM(I)=AMIN1(XKAM(I)*(CEEMIN(I)-(CEEMIN(I)-CEEM(I))/EEM(I)**2),0.)
   43 CONTINUE
C                                 IF NOP(14)=2, SET FOM(1) = FORCIN(INTV)
      GO TO(1,16),ITEST1
    1 NOP14 = NOP(14)+1
```

```
      GO TO(6,6,2,2,6),NOP14
    2 FOM(1) = FORCIN(INTV)
      IF(FOM(1)-1.0)3,3,4
    3 DEM(1) = XDEM(1)
      CEEM(1) = XCEEM(1)
      GO TO 16
C IF NOP(14) = 4, USE PARABOLIC FOM(1) VS. CEEM(1) CURVE
C THE RAM MUST BE A SINGLE MASS IF FOM VS. DEM IS PARABOLIC
    6 IF(NOP(14)-4)4,7,4
    7 IF(CEEM(1) - CEEMAS(1))9,8,8
    8 FOM(1) = C*CEEM(1)**2 + B*CEEM(1)
      GO TO 12
    4 IF(CEEM(1)-CEEMAS(1))16,12,12
    9 FOMAX(1) = AMAX1(XFOM(1),FOMAX(1))
      FOM(1) = FOMAX(1)-((CEEMAS(1)-CEEM(1))*FOMAX(1)**2)/(2.0*SMAX*
     1     EEM(1)**2)
      GO TO 16
   12 SMAX = SMAX+((FOM(1)+XFOM(1))/2.0)*(CEEM(1)-XCEEM(1))
   16 CONTINUE
      IF(GAMMA(I))46,44,45
   44 FOM(I) =AMAX1 (.0, FOM(I))
      GO TO 46
   45 IF(FOM(I) - XFOM(I))48,47,47
   48 NFOM(I) = 2
   47 IX = NFOM(I)
      GO TO (46,49),IX
   49 HOLDF = FOM(I)
      FOM(I) = AMAX1(FOM(I),GAMMA(I))
COMMENT THE .01 HOLDS MIN.PRESSURE AT GAMMA(I) FOR .01 SECONDS WHILE THE
COMMENT .0025 REDUCES THE PRESSURE TO ZERO IN .0025 ADDITIONAL SECONDS.
      TINT = INTV
      IF(TINT - .01/DELTEE)46,46,90
   90 FOM(I) = AMAX1(0.0, GAMMA(I)*(1.0-(DELTEE*TINT-.01)/.0025),HOLDF)
   46 CONTINUE
      ENTHRU(I) = ENTHRU(I)+(FOM(I)+XFOM(I))*(DEM(I+1)-XDEM(I+1))/24.0
```

```
      ENTMAX(I) = AMAX1(ENTMAX(I),ENTHRU(I))
      GO TO(22,19),ITEST1
   22 IF(CEEM(1) - CEEMAS(1))15,19,19
   15 SMIN = SMIN-((FOM(1)+XFOM(1))/2.0)*(CEEM(1)-XCEEM(1))
   19 CONTINUE
      IF(NOP(4)-5)29,28,29
C                                               GENERALIZED SOIL RESISTANCE
   28 CALL GENRAM
      GO TO 55
   29  CONTINUE
C                                                    SMITHS SOIL RESISTANCE
      IF(XKIM(I))50,155,50
  155 GO TO(55,156),ITESTP
  156 IF(XKIM(MPP ))50,55,50
   50 IF(DIM(I) -DEM(I) +Q(I) )51,52,52
   51 DIM(I) = DEM(I) -Q(I)
   52 CONTINUE
   70 IF(DIM(I) -DEM(I) -Q(I) )53,53,54
   54 DIM(I) = DEM(I) +Q(I)
   53 CONTINUE
      DIM(MPP ) =AMAX1 (DIM(MP),DIM(MPP ))
      ITST = ITRIG(I)
      GO TO(10,57),ITST
   10 IF(DEM(I) -DIM(I) -Q(I) )56,57,57
   56 RAM(I) = (DEM(I)-DIM(I))*XKIM(I)*(1.0+(SJ(I) *VEL(I)))
      GO TO(55,171),ITESTP
  171 RAM(MP) = RAM(MP)+(DEM(MP)-DIM(MPP ))*XKIM(MPP )*
     1 (1.0+(SJ(MPP)*VEL(MP)))
C                                  SEGMENT MP HAS RAM(MP) + RAM(MP+1) APPLIED
C                                               RAM(MP+1) MAY BE TENSILE
      GO TO 55
   57 RAM(I) = (DEM(I)-DIM(I)+ SJ(I) *Q(I) *VEL(I))*XKIM(I)
      ITRIG(I) = 2
      GO TO(55,172),ITESTP
  172 RAM(MP)=RAM(MP)+(DEM(I)-DIM(MPP)+SJ(MPP)*Q(MPP)*VEL(MP))*XKIM(MPP)
```

```
 55 CONTINUE
    GO TO(58,72),ITEST1
 58 VEL(1) = VEL(1)-(FOM(1) +RAM(1))*32.17*DELTEE/WAM(1)
    ITEST1 = 2
    GO TO 59
 72 VEL(I) = VEL(I)+(FOM(I-1) -FOM(I) -RAM(I))*32.17*DELTEE/WAM(I)
 59 CONTINUE
    IF(NOP(15)-1)85,85,83
 83 VEL(I) = VEL(I) + 32.17*DELTEE
 85 CONTINUE
 65 IF(FOMAX(I)-FOM(I))67,67,66
 67 FOMAX(I) = FOM(I)
    IFOMAX(I) = INTV
 66 IF(FOMIN(I)-FOM(I))68,69,69
 69 FOMIN(I) = FOM(I)
    IFOMIN(I) = INTV
 68 CONTINUE
    IF(VEL(2)/VEL1  -2.1)61,60, 60
 60 WRITE(6,105)
    INTT = 2
    RETURN
105 FORMAT(76HO THE RATIO OF THE VELOCITY OF W(2)  TO THE VELOCITY OF
   1THE RAM EXCEEDS 2.1. )
 61 IF(VEL(MP)/VEL1  -2.1)63,62,62
 62 WRITE(6,106)
106 FORMAT(76HO THE RATIO OF THE VELOCITY OF W(P)  TO THE VELOCITY OF
   1THE RAM EXCEEDS 2.1. )
    INTT = 2
    RETURN
 63 CONTINUE
    LDCELL = MH-1
    ACCELR = (FOM(LDCELL-1)-FOM(LDCELL))/WAM(LDCELL)
 71 ACELMX=AMAX1(ACELMX,ACCELR)
 73 CONTINUE
    RETURN
```

```
      END
$IBFTC RAMGEN
      SUBROUTINE GENRAM
C
C  NQDIV = NO. OF EQUAL SEGMENTS INTO WHICH Q(I) IS DIVIDED = 10
C  RSTAT(I) = STATIC SOIL RESISTANCE NEGLECTING THE SOIL DAMPING EFFECTS
C  RMAX(I) = A TEMPORARY MAXIMUM STATIC SOIL RESISTANCE
C  PERCQ = DISTANCE FROM ZERO DISPLACEMENT TO DEM(I) IN UNITS (1.732,..)
C
      COMMON    WAM(100),  XKAM(100),   RUM(100),  BEEM(100),   EEM(100)     1
      COMMON  GAMMA(100),  XKIM(100),CEEMAS(100),  NFOM(100),  XDEM(100)     2
      COMMON    DEM(100), XCEEM(100),  CEEM(100),   FOM(100),  XFOM(100)     3
      COMMON    VEL(100),   DIM(100),   RAM(100),  RMAX(100), RSTAT(100)     4
      COMMON R(100,10)  , ITRIG(100),    Q(100),FORCIN(100),  DFOM(100)      5
      COMMON  FOMAX(100),IFOMAX(100), FOMIN(100),IFOMIN(100),     A(100)     6
      COMMON  DEMAX(100),IDEMAX(100),    SJ(100),   NOP( 22),DYNAMK(100)     7
      COMMON CEEMIN(100),HOLDEM(100),ANSVEC( 50),SE(50,51)  ,  IROW( 51)     8
      COMMON   RUMA(100),  WAMC(100), XKAMC(100),    QA(100),    SJA(100)    9
      COMMON   ICOL( 51),  NOPP( 20),ENTHRU(100),ENTMAX(100),   ID5( 50)     10
      COMMON OSIDE , QPOINT, SIDEJ , POINTJ, NODIV , NORAMS, NSTOP          50
      COMMON INTV  , ISECTN, NUMR  , F1    , F2    , C1    , C2             51
      COMMON IPRINT, DELTEE, EEM1  , EEM2  , GAMMA1, GAMMA2, INT            52
      COMMON INTT  , I     , ITST  , IX    , NR    , MO    , MP             53
      COMMON NPAGE , N     , QUAKE , RUP   , RUT   , VELFI , ID1            54
      COMMON ID2   , ID3   , ID4   , IDW1  , IDW2  , IDK1  , IDK2           55
      COMMON IDRL1 , IDRL2 , IDG1  , IDG2  , IDE1  , IDE2  , IDB1           56
      COMMON IDB2  , IDV1  , IDV2  , IDQ1  , IDQ2  , IDJ1  , IDJ2           57
      COMMON IDDK1 , IDDK2 , IDA1  , IDA2  , KGRADD, J5    , TMIN           58
      COMMON TMAX  , SMIN  , SMAX  , NOPNTS, AREA  , NS1   , NS2,NS6        59
      COMMON NS3   , NS4   , NS5   , IDEEM , MH    , VEL1  , ACCELR         60
      COMMON B     , C     , AREAP , XLONG , ELAST , ACELMX                 61
      COMMON DVJ,DE1,DE2,DRI,DRP,DQI,DQP,DJI,DJP,DW1,DW2,DWI,DK1,DK2,DKI
C
      [illegible] = MP
      K = 1
```

```
      QDIV = 10.0
      NQDIV = 10
      IF(XKIM(K)-0.1) 1,2,2
    1 RAM(K) = 0.0
      GO TO 70
    2 IF(DEM(K)-DIM(K))32,3,3
    3 DIM(K) = DEM(K)
      IF(DEM(K)-Q(K))7,6,6
    6 RSTAT(K) = R(K,NQDIV)
      GO TO 50
    7 PERCQ = DEM(K)/(Q(K)/QDIV)
      IPERCQ = PERCQ
      XPERCQ = IPERCQ
      IF(IPERCQ)8,8,9
    8 RSTAT(K) = PERCQ*R(K,1)
      GO TO 50
    9 RSTAT(K) = R(K,IPERCQ)+(PERCQ-XPERCQ)*(R(K,IPERCQ+1)-R(K,IPERCQ))
      GO TO 50
   32 RMAX(K) =AMAX1(RMAX(K),RSTAT(K))
      RSTAT(K) = RMAX(K)-(DIM(K)-DEM(K))*XKIM(K)
C THE STATIC FORCE SHOULD REALLY LEAVE THE XKIM(I) SLOPE AND REMAIN
C CONSTANT IF RMAX(I)+RSTAT(I) EVER EXCEEDS 0.0
      IF(RMAX(K)+RSTAT(K))39,50,50
   39 WRITE(6,200)RMAX(K),RSTAT(K),K
  200 FORMAT(11HORMAX(I) = F10.2,  6X, 12H RSTAT(I) = F10.2,6X,4H I =I6)
C  STATEMENTS 50 THRU 70 INCLUDE THE SOIL DAMPING EFFECT
   50 ITST = ITRIG(K)
      GO TO(51,57),ITST
   51 IF(DEM(K)-Q(K))56,57,57
   56 RAM(K) = RSTAT(K)+RSTAT(K)*SJ(K)*VEL(K)
      GO TO 70
   57 RAM(K) = RSTAT(K)+R(K,NQDIV)*SJ(K)*VEL(K)
      ITRIG(K) = 2
      GO TO 70
   70 IF(K-[illegible])80,71,73
```

```
 71 CONTINUE
    K = MP+1
    IF(XKIM(K)-0.01)80,80,72
 72 DEM(K) = DEM(MP)
    VEL(K) = VEL(MP)
    GO TO 2
 73 CONTINUE
C 73 IF(RAM(K))74,75,75      (OLD STATEMENT)
C 74 RAM(K) = 0.0 (OLD STATEMENT)      1
 74 CONTINUE
 75 RAM(MP) = RAM(MP)+RAM(MP+1)
C                                     RAM(MP+1) CAN    GO INTO TENSION
 80 RETURN
    END
$IBFTC EXCTG
    SUBROUTINE EXACTG
    COMMON    WAM(100),  XKAM(100),   RUM(100),  BEEM(100),   CEM(100)
    COMMON  GAMMA(100),  XKIM(100),CEEMAS(100),  NFOM(100),  XDEM(100)
    COMMON    DEM(100), XCEEM(100),  CEEM(100),   FOM(100),  XFOM(100)
    COMMON    VEL(100),   DIM(100),   RAM(100),  RMAX(100), RSTAT(100)
    COMMON R(100,10)   , ITRIG(100),     Q(100),FORCIN(100),  DFOM(100)
    COMMON  FOMAX(100),IFOMAX(100), FOMIN(100),IFOMIN(100),     A(100)
    COMMON  DEMAX(100),IDEMAX(100),    SJ(100),   NOP( 22),DYNAMK(100)
    COMMON CEEMIN(100),HOLDEM(100),ANSVEC( 50),SE(50,51)  ,  IROW( 51)
    COMMON   RUMA(100),  WAMC(100), XKAMC(100),    QA(100),   SJA(100)
    COMMON   ICOL( 51),  NOPP( 20),ENTHRU(100),ENTMAX(100),   IDS( 50)
    COMMON QSIDE , QPOINT, SIDEJ , POINTJ, NCDIV , NORANS, NSTOP
    COMMON INTV  , ISECTN, NUMR  , F1    , F2    , C1    , C2
    COMMON IPRINT, DELTEE, EEM1  , EEM2  , GAMMA1, GAMMA2, INT
    COMMON INTT  , I     , ITST  , IX    , NR    , MO    , MP
    COMMON NPAGE , N     , QUAKE , RUP   , RUT   , VELMI , ID1
    COMMON ID2   , ID3   , ID4   , IDW1  , IDW2  , IDK1  , IDK2
    COMMON IDRL1 , IDRL2 , IDG1  , IDG2  , IDE1  , IDE2  , IDR1
    COMMON IDR2  , IDV1  , IDV2  , IDQ1  , IDQ2  , IDJ1  , IDJ2
    COMMON IDDK1 , IDDK2 , IDA1  , IDA2  , KGRADU, J5    , TMIN
```

```
      COMMON TMAX   , SMIN   , SMAX   , NOPNTS, AREA   , NS1    , NS2;NS6
      COMMON NS3    , NS4    , NS5    , I[illegible] , MH     , VEL1   , ACCELP
      COMMON B      , C      , AREAP  , XLONG  , ELAST  , ACELMX
      COMMON DV1,DE1,DE2,DRI,DRP,DQI,DQP,DJI,DJP,DW1,DW2,DWI,DK1,DK2,DKI
C
      MP = MP
      MO = MO
      MMO = MO-1
      MMOO = MO - 2
      MAO = MP - MO
      NSDD = MP-MO+1
      DO 6 NSEW = 1,NSDD
      DO 6 NSE = 1,NSDD
    6 SE(NSEW,NSE) = 0.0
      SE(1,1) = XKAM(MO) + XKIM(MO)
      SE(2,1) = -XKAM(MO)
      DO 13 K = 2,MAO
      NN = K + MMOO
      NNN = K + MMO
      SE(K-1,K) = SE(K,K-1)
      SE(K,K) = XKAM(NN) + XKAM(NNN) + XKIM(NNN)
      SE(K+1,K) = -XKAM(NNN)
   13 CONTINUE
      SE(MAO,NSDD) = SE(NSDD,MAO)
      SE(NSDD,NSDD) = XKAM(MP-1)+XKIM(MP) + XKIM(MP+1)
      DET = TAMINV(SE,ICOL,NSDD,50,0.00001)
      IF(0.00001 - ABS(DET))14,12,12
   12 WRITE(6,100)DET
  100 FORMAT(33H0THE VALUE OF THE DETERMINANT =  F10.7)
      INTT = 2
      RETURN
   14 CONTINUF
      WARTI = 0.0
      DO 15 NSEW = 2,MO
   15 WARTL = WARTL + [illegible]K(NSEW)
```

```
      SE(1,NSDD+1) = WAMTL
      DO 16 NSEW = 2,NSDD
      NUTZ = MMO+NSEW
   16 SE(NSEW,NSDD+1) = WAM(NUTZ)
      DO 17 IANS = 1,NSDD
   17 ANSVEC(IANS) = 0.0
      DO 23 IM1 = 1,NSDD
      DO 23 IM2 = 1,NSDD
   23 ANSVEC(IM1) = ANSVEC(IM1)+SE(IM1,IM2)*SE(IM2,NSDD+1)
      NAT = 0
      DO 26 NST = MO,MP
      NAT = NAT+1
      DEM(NST) = ANSVEC(NAT)
      HOLDEM(NST) = DEM(NST)
   26 CONTINUE
      WOS = 0.0
      DO 27 NST = 2,MMO
      WOS = WOS+WAM(NST)
      CEEM (NST) = WOS/XKAM(NST)
      FOM(NST) = WOS
   27 CONTINUE
      DO 28 NST = 1,MMO
      NEL = MO-NST
      DEM(NEL) = DEM(NEL+1) + CEEM(NEL)
      HOLDEM(NEL) = DEM(NEL)
   28 CONTINUE
      MAM = MP-1
      DO 29 NST = MO,MAM
      CEEM(NST) = DEM(NST) - DEM(NST+1)
      FOM(NST) = CEEM(NST)*XKAM(NST)
      RAM(NST) = DEM(NST)*XKIM(NST)
   29 CONTINUE
      RAM(MP) = DEM(MP)*XKIM(MP)
      RAM(MP+1) = DEM(MP)*XKIM(MP+1)
      RETURN
```

```
      END
$IBFTC SMTH
      SUBROUTINE SMITH
      COMMON    WAM(100),  XKAM(100),   RUM(100),  BEEM(100),   EEM(100)
      COMMON  GAMMA(100),  XKIM(100),CEEMAS(100),  NFOM(100),  XDEM(100)
      COMMON    DEM(100), XCEEM(100),  CEEM(100),   FOM(100),  XFOM(100)
      COMMON    VEL(100),   DIM(100),   RAM(100),  RMAX(100), RSTAT(100)
      COMMON R(100,10)   , ITRIG(100),     Q(100),FORCIN(100),  DFOM(100)
      COMMON  FOMAX(100),IFOMAX(100), FOMIN(100),IFOMIN(100),     A(100)
      COMMON  DEMAX(100),IDEMAX(100),    SJ(100),   NOP( 22),DYNAMK(100)
      COMMON CEEMIN(100),HOLDEM(100),ANSVEC( 50),SE(50,51)   ,  IROW( 51)
      COMMON   RUMA(100),  WAMC(100), XKAMC(100),    QA(100),   SJA(100)
      COMMON   ICOL( 51),  NOPP( 20),ENTHRU(100),ENTMAX(100),   IDS( 50)
      COMMON QSIDE , QPOINT, SIDEJ , POINTJ, NODIV , NORAMS, NSTOP
      COMMON INTV  , ISECTN, NUMR  , F1    , F2    , C1    , C2
      COMMON IPRINT, DELTEE, EEM1  , EEM2  , GAMMA1, GAMMA2, INT
      COMMON INIT  , I     , ITST  , IX    , NR    , NO    , MP
      COMMON NPAGE , N     , QUAKE , RUP   , RUT   , VELMI , ID1
      COMMON ID2   , ID3   , ID4   , IDW1  , IDW2  , IDK1  , IDK2
      COMMON IDRL1 , IDRL2 , IDG1  , IDG2  , IDF1  , IDE2  , IDD1
      COMMON IDB2  , IDV1  , IDV2  , IDQ1  , IDQ2  , IDJ1  , IDJ2
      COMMON IDDK1 , IDDK2 , IDA1  , IDA2  , KGRADD, J5    , TMIN
      COMMON TMAX  , SMIN  , SMAX  , NOPNTS, AREA  , NS1   , NS2,NS6
      COMMON NS3   , NS4   , NS5   , IDEEM , MH    , VEL1  , ACCELR
      COMMON B     , C     , AREAP , XLONG , ELAST , ACELMX
      COMMON DV1,DE1,DE2,DRI,DRP,DQI,DQP,DJI,DJP,DW1,DW2,DWI,DK1,DK2,DKI
C
      MP = MP
      N = MP-1
      WAMTL = 0.0
      RAMTL = 0.0
      DO 5 JT = 2,MP
      WAMTL = WAMTL + WAM(JT)
    5 RAMTL = RAMTL + RUM(JT)
      RAMTL = RAMTL + RUM(MP+1)
```

```
      DO 8 JT = 2,N
      RAM(JT) = (RUM(JT)*WAMTL)/RAMTL
    8 FOM(JT) = FOM(JT-1)+WAM(JT)-RAM(JT)
      RAM(1) = RUM(1)*WAMTL/RAMTL
      RAM(MP) = RUM(MP)*WAMTL/RAMTL
      RAM(MP+1) = RUM(MP+1)*WAMTL/RAMTL
      DEM(MP) = (RAM(MP)+RAM(MP+1))/(XKIM(MP)+XKIM(MP+1))
      HOLDEM(MP) = DEM(MP)
      DO 11 JT = 1,N
      JTM = MP-JT
      CEEM(JTM) = FOM(JTM)/XKAM(JTM)
      DEM(JTM) = DEM(JTM+1) + CEEM(JTM)
      HOLDEM(JTM) = DEM(JTM)
      DIM(JTM)=DEM(JTM)-WAMTL*Q(JTM)/RAMTL
   11 CONTINUE
      RETURN
      END
$DATA
```