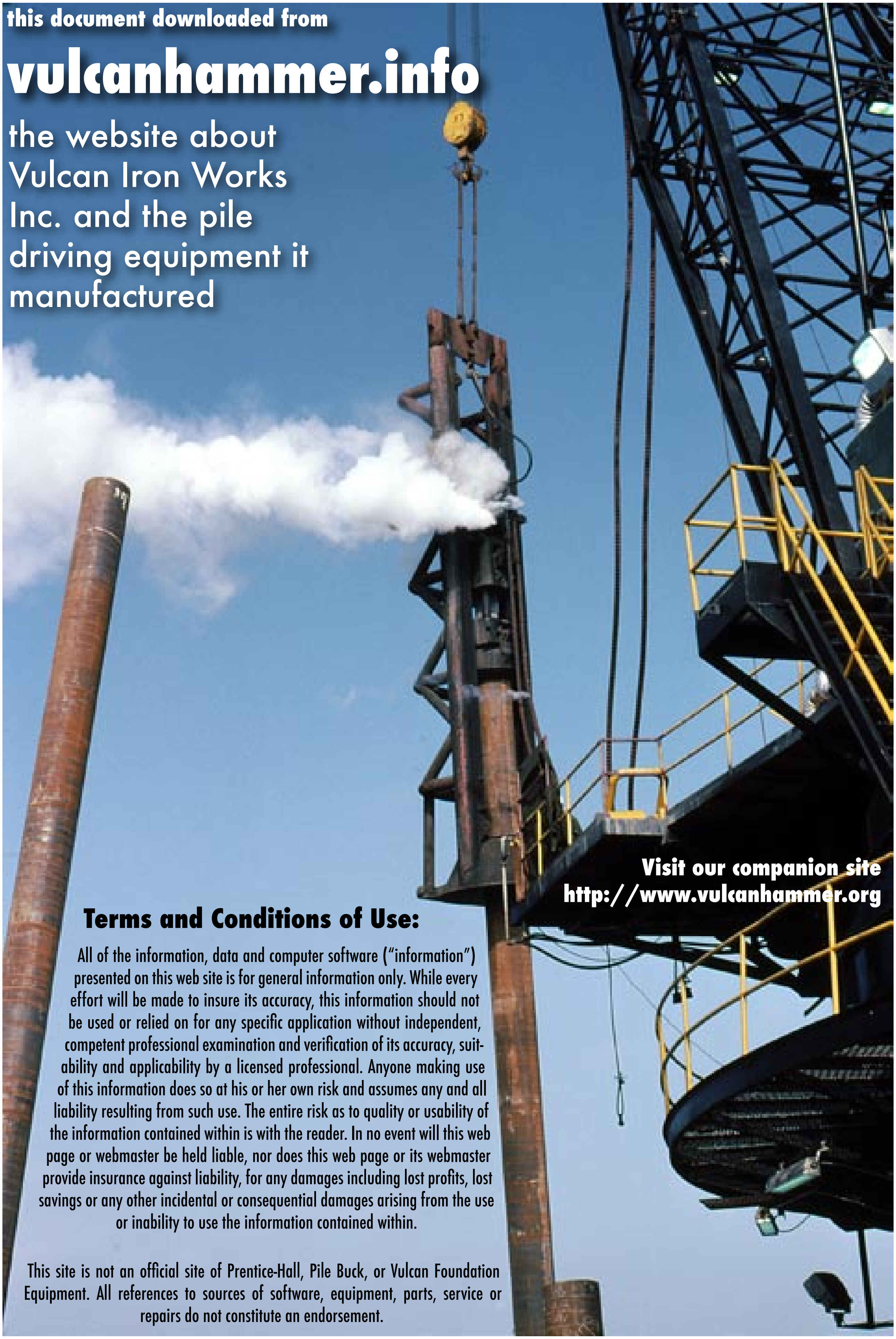

VOL. 3. No. 9. SEPTEMBER 1931

THE JOURNAL OF THE INSTITUTION OF ENGINEERS AUSTRALIA

INCORPORATING
THE TRANSACTIONS OF THE INSTITUTION

SCIENCE HOUSE,
GLOUCESTER & ESSEX STREETS,
SYDNEY.—N.S.W.

Registered at G.P.O., Sydney, for Transmission by Post as a Periodical.

TRANSACTIONS OF THE INSTITUTION

Reinforced Concrete Pile Formulae.

BY

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Summary.—Existing formulae for the calculation of safe loads for reinforced concrete piles are critically examined, and errors in theory are shown. A new theory of the phenomena of driving is advanced, and a set of graphs based on the theory, is prepared for application to reinforced concrete pile driving. Allowances are made for weight and drop of hammer, size and length of pile, size and weight of dolly or helmet, degree of cushioning in the helmet (or strength of concrete in pile), and type of supporting strata.

I.—INTRODUCTION.

(*a*) *The purpose of the paper.*—The purpose of this paper is to find a method by which safe loads for reinforced concrete piles can be calculated from the observed conditions of driving. Many formulae at present available attempt to achieve this, but there are large differences in the results they give, and there is little to show which, if any, is to be trusted.

(*b*) *Outstanding differences between concrete piles and timber piles.*—So much of the knowledge of pile driving has been based on the behaviour of timber piles, that it is important to note that: (1) The structural and elastic characteristics of timber piles and concrete piles are very dissimilar. (2) In drop hammer driving, timber piles are usually driven direct, with a hammer of weight nearly as great as, or greater than that of the pile; concrete piles are usually driven using a cushion and helmet, and a hammer only a fraction of the weight of the pile. (3) Concrete piles are relatively more expensive, and a good gauge of their capacity is therefore relatively more valuable.

(*c*) *Driving resistance, and bearing resistance.*—A pile partly driven, and left overnight, is often much harder to move when first struck next morning. In some cases the reverse is true. It is thus important to distinguish between the *driving resistance* (R)—the actual ground resistance to driving—and the *bearing resistance* (U)—the ultimate resistance to a static load applied after the ground has become quiescent subsequent to driving. An attempt to relate the two will be made in the paper.

(*d*) *Factor of safety.*—The factor of safety to be applied to a determined driving resistance or ultimate bearing resistance must of necessity depend on the degree of accuracy likely in such determined resistance, and on the type of loading to be borne. It is thus controlled by the quality of the supporting strata, and may usually be made less where subsurface conditions have been well determined; and while vibrating loads require larger factors of safety than static loads, these factors also may be reduced where the total penetration is such that the supporting strata are below the region of lateral vibration in the pile, and where the previously explored supporting strata are such as not to be greatly affected by vertical vibrations.† Whereas impact is usually omitted from foundation calculations, it should be included in the load calculated for any pile. Impactive or live loading of long duration requires more allowance than such loading of short duration.

In some classes of ground, a single pile may support much more than the average load supported by a number of piles driven fairly close together. In ground where the sets in driving different piles are very variable, an increase should be made in the factor of safety.

An often-quoted table of factors of safety for bearing resistances is the following:—

	Soil	to	Silt
Steady loads	2	to	6
Vibrating loads, machinery, etc.	4	to	12

It should be remembered, however, that these are not true factors of safety, but include a "factor of ignorance." The author suggests that when the ultimate resistance of any pile has been determined, in fixing the factor of safety (as given in the above table) the most unfavourable conditions possible in the supporting strata should be judged (the range of conditions possible being narrowed with better knowledge of the subsurface conditions and of the possibility of disturbance from extraneous sources) and a proportion of the factor of safety—a "factor of ignorance"—then allowed in respect of these possible conditions, the manner of determining the ultimate load, and the type of loading to be borne. The remaining proportion of the factor of safety—or *true* margin of safety—should be approximately constant for all classes of loadings and foundation conditions involving the same value of loss in case of failure; and the overall factor of safety (S) will then be equal to the product of the true factor of safety with the "factor of ignorance." For permanent work the true factor of safety should not be less than $1\frac{1}{2}$.

The quotient obtained by dividing the ultimate resistance (R or U) by an appropriate factor of safety (S) is the bearing value (B) to be assigned to any pile; and the use of any formula which always includes the same arbitrary factor of safety is therefore to a certain extent to be deprecated. The ultimate bearing resistance found by a static load test is considered to be the maximum for which no progressive settlement is noted. For some classes of foundation the time element is large, and it may be necessary to allow the test load to remain on for several days, even though no settlement appears at first.

(*e*) *Height of fall of hammer.*—In driving with a drop hammer it is usually convenient to work direct on to the

*This paper, No. 370, originated in the Melbourne Division of the Institution.

†See *Engineering News Record*, Vol. 104, No. 22, May 29th, 1930, p. 900. It is there concluded that, due to very small vibrations caused by street or railway traffic, nearby foundations may settle, settlements being largest in plastic soils, and almost negligible in sand.

winch, instead of using a tripper. Friction and inertia of the winch parts then seriously reduce the velocity which the hammer acquires in its fall, and observations by different observers indicate consistently that the velocity of the hammer as it hits the pile is only approximately $\sqrt{1.15\,gH}$, i.e., the equivalent free fall is

$$h = \tfrac{3}{4}\mathrm{H}$$

where H feet is the vertical fall.

With new equipment h may be less than $\frac{3}{4}$**H**. The equivalent free fall (or an actual value of the free fall) should always be used in formulae.

If the height of fall measured obliquely on raking guides inclined θ to the vertical (in driving batter piles) be H feet,

$$\mathrm{H} = H\,(\cos\,\theta - \mathbf{0.1}\,\sin\,\theta)$$
$$\text{and}\quad h = \tfrac{3}{4}H\,(\cos\,\theta - \mathbf{0.1}\,\sin\,\theta)$$

(*f*) *Mathematical treatment.*—Where mathematical treatment is included, the leading results are summarized in *italics* for facility in following the paper, and the essentially mathematical text is printed in smaller type.

(*g*) *Notation.*—The notation given below includes every symbol used in the paper. The symbols are mainly phonetic or standard. Further explanations are given in the text as required.

(*i*) *Physical Quantities.*

$\mathbf{A}$ =	Area of cross section of parallel pile;	(square inches)
A =	Equivalent area of cross section of tapered pile;	(square inches)
$\mathbf{B}$ =	Working bearing load of pile;	(lb.)
$\mathbf{C}$ =	Equivalent compression of pile plus cushion in the Hiley formula;	(inches)
$\mathbf{c}$ =	Compression of cushion and dolly alone;	(inches)
$\mathbf{D}$ =	Total weight of helmet or cap, and dolly;	(lb.)
$\mathbf{d}$ =	Total depth of penetration of pile;	(feet)
$\mathbf{E}$ =	Young's modulus for concrete;	(lb. per sq. in.)
ε =	Coefficient of restitution;	
$\mathbf{F}$ =	Maximum compressive stress on pile head cross section, during driving;	(lb. per sq. in.)
$\mathbf{f}$ =	Equivalent driving resistance compressive stress on pile cross section;	(lb. per sq. in.)
$\mathbf{G}$ =	Stiffness of cushion in helmet;	(lb. per in.)
g =	Acceleration due to gravity;	(ft. per sec. per sec.)
$\mathbf{H}$ =	Height of fall of monkey;	(feet)
H =	Height of fall of monkey measured along raking guides;	(feet)
$\mathbf{h}$ =	Equivalent height of free fall of hammer;	(feet)
h =	Equivalent height of free fall of monkey;	(feet)
i =	Magnitude of an impulse at any instant;	
$\mathbf{L}$ =	Equivalent length of pile;	(feet)
L =	Actual length of pile;	(feet)
$\mathbf{l}$ =	Length of hammer;	(feet)
l =	Length of monkey;	(feet)
λ =	Length of timber dolly or striking piece;	(feet)
$\mathbf{M}$ =	Weight of monkey;	(lb.)
$\mathbf{m}$ =	Weight of monkey per foot of length;	(lb. per ft.)
μ =	A quantity proportional to the logarithmic decrement of a longitudinal wave;	
$\mathbf{N}$ =	A ratio of driving resistances under different conditions;	
$\mathbf{P}$ =	Weight of pile;	(lb.)
$\mathbf{p}$ =	Weight of pile per foot of length;	(lb. per ft.)
p =	Percentage of steel reinforcement;	
$\mathbf{Q}$ =	Ratio of cushioned to ideally impactive compressive stress;	
q =	Ratio of driving resistance compressive stress to ideally impactive compressive stress;	
$\mathbf{R}$ =	Driving resistance of pile;	(lb.)
ϱ =	Ratio of velocity of sound in the hammer to the velocity in the pile;	
$\mathbf{S}$ =	Factor of safety;	
$\mathbf{s}$ =	Set per blow in driving;	(inches)
σ =	Perimeter of pile;	(inches)
$\mathbf{t}$ =	Time required for sound to travel the length of the hammer;	(seconds)
t =	Time generally;	
θ =	Angle of inclination of pile machine guides to the vertical;	
$\mathbf{U}$ =	Ultimate bearing capacity of pile after a period of rest;	(lb.)
u =	Velocity of pile after impact;	(ft. per sec.)
$\mathbf{V}$ =	Velocity of sound in pile;	(ft. per sec.)
$\mathbf{v}$ =	Velocity of hammer as impact occurs with helmet or dolly;	(ft. per sec.)
v =	Velocity of hammer after impact;	(ft. per sec.)
$\mathbf{W}$ =	Weight of hammer;	(lb.)
$\mathbf{w}$ =	Weight of hammer per ft. length;	(lb. per ft.)

(*ii*) *Mathematical Quantities.*

$a = \dfrac{1 - \mathbf{r}}{k}$

$b = \dfrac{\mathbf{r}}{Xk}$

$\beta = \sqrt{b - \frac{1}{4}a^2}$

D = The operator $\dfrac{d}{dx}$

e = Base of the Naperian logarithms;

$\gamma = \dfrac{\beta}{\sin\,\eta} = \sqrt{b}$

η = A phase constant equal to $\tan^{-1}\dfrac{2\beta}{a}$

$K = \dfrac{1}{\beta k}$

$k = \dfrac{\mathbf{c}}{12\,\mathbf{v}\mathbf{Q}}$ and a constant having the dimension of time;

$\varkappa$ = A constant;

$\mathcal{J}$ = A constant;

$\mathbf{r} = \dfrac{\mathbf{p}}{\varrho\mathbf{w} + \mathbf{p}}$

r = Modified value of $\mathbf{r}$ such that $r = \frac{1}{2}\log_e\{1 - 2\mathbf{r}/(1 - \mathbf{r})\}$

X = Unit value of x

x = Abscissa in wave diagram;

Y = Unit value of $\mathbf{y}$ or y

$\mathbf{y}$ = Ordinate in wave diagram measured +ve downwards;

y = Ordinate in wave diagram measured +ve upwards;

z = Ordinate in wave diagram corresponding with a reflected wave;

φ = A phase constant;

ξ = A phase constant;

ζ = A phase constant.

II. Formulae in Use.

(*a*) *Formulae dealt with.*—Numerous formulae are in use in America and Europe, for concrete piles. A few of the most widely used will be examined. They are:—

- (i) Wellington (American)
- (ii) Gow (American)
- (iii) Brix (Continental)
- (iv) Eytelwein (Continental)
- (v) Hiley (British)
- (vi) Formulae of the Rankine type which depend on a knowledge of the supporting strata only.

(*b*) *Wellington Formulae.*—The Wellington formula for timber piles and a drop hammer of weight M lb. is usually quoted as

$$\text{Safe Load (lb.)} = \frac{2\mathrm{M}h}{\mathrm{s}+1}$$

with a factor of safety nominally equal to 6, a free fall (h feet) assumed for the monkey, and s the average set in inches for the last 5 blows at least.*

Wellington's explanation of the formula is that, assuming the resistance to penetration during each blow of the monkey to start at a certain value and rapidly drop to a much lower value, which remains constant for the remainder of the penetration, the equivalent set (for the lower resistance) should be increased; and "based on extensive observations of the behaviour of piles in driving, and on many years' experiment and study as to the general laws of friction," the required amount of increase in set is given as 1 in. Equating (in in.-lb.) the work stored in the monkey, to the work done in producing an equivalent set

$$12Mh = (s + 1) \times \text{resistance overcome.}$$

Since the value 1 in. is based on actual bearing values (U) of piles after a period of rest subsequent to driving, the resistance overcome is really not a driving, but a bearing resistance, so that, using a factor of safety of 6

$$\text{Bearing Capacity, } B = \frac{U}{6} = \frac{2Mh}{s+1} \qquad \ldots\ldots (1)$$

The factor of safety 6, is nominally supposed to include the effects of brooming of the head of the pile, and other similar sources of loss, and it is intended to be applied only where the subsurface conditions are unknown. Under known subsurface conditions a factor of 4 or a little less is recommended. *The results of Goodrich's investigations on timber piles show that the resistance of the ground to penetration is, in the average case, almost constant from start to finish of any movement under one blow of the hammer. The Wellington formula should therefore be taken merely as a good empirical one, based on the behaviour of timber piles, and applicable to them provided (as Wellington asserts) the set per blow is not less than ½ in. or ¼ in. as an extreme limit. Remembering the differences in driving phenomena noted in section I (b), one should, therefore, on existing knowledge, be chary of using the formula for concrete pile work, although it is often recommended as being quite satisfactory for such use.* Wellington's modification of the above formula, for use with steam hammers is:

$$\text{Safe load} = \frac{2Mh}{s + 0.1} \qquad \ldots\ldots (2)$$

but tests have indicated that the figure 0.3 instead of 0.1 would be better for single acting hammers, and approximately 0.2 has been suggested for double acting hammers. In double acting hammers the effect of the steam pressure on the piston would have to be included. Comparing accelerations during fall of the hammer with and without steam

$$h = \text{Actual fall} \times \frac{1}{M}\{M + \text{Mean net total steam pressure on piston} - \text{Total friction}\}$$

for double acting hammers

and h = approx. {Actual fall — 2 in.}
for single acting hammers, when allowance is made for friction.

(*c*) *Gow Formula.*—Wellington's formulae have been modified by Gow, to take account of the heavy weights (P lb.) of concrete piles.

For a drop hammer the Gow formula is

$$\text{Safe load} = \frac{2Mh}{s + \frac{P}{M}} \qquad \ldots\ldots (3)$$

and corresponding to Wellington's steam hammer formula

$$\text{Safe load} = \frac{2Mh}{s + 0.1\frac{P}{M}} \qquad \ldots\ldots (4)$$

The United States Navy Department uses

$$\text{Safe load} = \frac{2Mh}{s + 0.3\frac{P}{M}} \qquad \ldots\ldots (5)$$

It appears as though the alterations to Wellington's formulae have been made rather as a guess, but the guess would appear reasonable. Safe results appear to have been given in America, but this is inconclusive as to the correctness of the formulae, because they may merely be conservative. The remarks concerning factor of safety and minimum set for the Wellington formulae apply to the Gow formulae also.

(*d*) *Brix Formula.*—Equating the momentum of a drop hammer which strikes a pile with velocity V ft. per second to the combined momenta of pile and hammer after impact

$$MV = Pv + Mv$$

$$\text{or} \quad v = \frac{M}{M+P} \cdot V \qquad \ldots\ldots (6)$$

The coefficient of restitution for the impact is here assumed zero, and the velocities of both pile and hammer after impact are therefore taken as v ft. per sec. The hammer is now assumed to play no further part in the action, and the energy available for penetration, due to the motion of the pile, is written as

$$\frac{1}{2g} \times P \times \left\{\left(\frac{M}{M+P}\right)V\right\}^2 \times 12 \text{ in. lb.}$$

which equals Rs in. lb., the work done in penetration against a driving resistance R lb., i.e.,

$$R = \frac{12}{2g} \cdot \frac{M^2P}{s(M+P)^2} \cdot V^2$$

and since $V^2 = 2gh$

$$R = \frac{12M^2Ph}{s(M+P)^2}$$

or the safe bearing load in lb. with factor of safety S is

$$B = \frac{12M^2Ph}{Ss(M+P)^2} \qquad \ldots\ldots (7)$$

The formula is thus a driving resistance formula which attempts to include the effects due to pile weight and lack of restitution at the head of the pile. No allowance is made for the following obvious sources of loss

(i) work done in compressing the pile cap;

(ii) work done in compressing the pile;

but to offset these

(iii) the available energy of the drop hammer after impact is neglected.

*See *Foundations of Bridges and Buildings*, 2nd Edition, by Jacoby and Davis

One would expect the gains and losses to balance for sets somewhere between $\frac{1}{4}$ in. and 1 in.

Put $\mathbf{P} = \mathbf{M}$, and the formula reduces to

$$R = 3.00\frac{Mh}{s} \quad \ldots\ldots(8)$$

Goodrich's very thorough theoretical and practical investigations into the behaviour of timber piles* led him to adopt

$$R = \frac{10}{3} \cdot \frac{MH}{s} \quad \ldots\ldots(9)$$

for the driving resistance provided the sets are not too small, and the weight of the hammer is at least as great as that of the pile. This formula has proved safe and reliable for timber piles and should therefore serve as a check on formula (8).

When $\frac{4}{3}h$ is substituted for H, Goodrich's formula takes the form

$$R = 4.44\frac{Mh}{s} \quad \ldots\ldots(10)$$

Thus the Brix formula, judged by the Goodrich standard is conservative for timber piles, giving values only $\frac{2}{3}$ as great.

On the above information only, the Brix formula might be trusted as a driving resistance formula for sets between $\frac{1}{4}$ in. and 1 in.

(*e*) *Eytelwein Formula.*—If the same conditions are postulated as were assumed in deriving the Brix formula, but advantage is taken of the energy left in the hammer after impact, following on equation (6) the energy available for penetration is found to be

$$\frac{1}{2g} \times (M + P)\left\{\frac{M}{M + P} \cdot v\right\}^2 \times 12 \text{ in. lb.}$$

whence the Eytelwein formula is obtained

$$B = \frac{12Mh}{Ss\left(1 + \frac{P}{M}\right)} \quad \ldots\ldots(11)$$

Losses i and ii of the Brix formula still remain and there is no gain to offset them.

Putting $\mathbf{P} = \mathbf{M}$, and thus making the formula fit timber piles

$$R = 6.00\frac{Mh}{s} \quad \ldots\ldots(12)$$

which gives results for timber piles about 35% higher than those given by the Goodrich formula (10). *Arguments so far advanced would thus make the Eytelwein formula unsafe as a driving resistance formula except for sets above about $1\frac{1}{2}$ in.*

(*f*) *Hiley Formula.*†—The following merely outlines the theory:

Let ε = coefficient of restitution of pile head or cap;
V = velocity of hammer before impact;
v = velocity of hammer after impact;
u = velocity of pile after impact.

Equating the momenta before and after impact

$$MV = Mv + Pu \quad \ldots\ldots(13)$$

and relating the velocities before and after impact

$$-\varepsilon V = v - u \quad \ldots\ldots(14)$$

Multiplying (14) by M and subtracting from (13)

$$MV(1 + \varepsilon) = u(M + P)$$

or

$$u = \frac{M}{M + P}(1 + \varepsilon)V \quad \ldots\ldots(15)$$

and multiplying (14) by P and adding to (13)

$$v = \frac{M - \varepsilon P}{M + P} \cdot V \quad \ldots\ldots(16)$$

Hiley assumes that if the hammer does not rise after the impact, the whole energy of the hammer as well as that of the pile can be utilized, and the condition for this from equation (16) is

$$M \not< \varepsilon P$$

The energy of the hammer after the impact is

$$\frac{1}{2g} \times M \times \left\{\frac{M - \varepsilon P}{M + P} \cdot V\right\}^2 \times 12 \text{ in. lb.}$$

and of the pile is

$$\frac{1}{2g} \times P \times \left\{\frac{M + \varepsilon M}{M + P} \cdot V\right\}^2 \times 12 \text{ in. lb.}$$

and hence the total energy is

$$\frac{12MV^2}{2g} \times \frac{1}{(M + P)^2}\{M^2 + MP + P\varepsilon^2(M + P)\} \text{ in. lb.}$$

$$= \frac{12MV^2}{2g}\left\{\frac{M + P\varepsilon^2}{M + P}\right\} \text{ in. lb.} \quad \ldots\ldots(17)$$

$\left\{\frac{M + P\varepsilon^2}{M + P}\right\}$ is termed the efficiency of the blow.

Putting $\varepsilon = 1$, and subtracting (17) the loss of energy due to impact

$$= \frac{12MV^2}{2g}\left(\frac{1 - \varepsilon^2}{1 + \frac{M}{P}}\right) \text{ in. lb.} \quad \ldots\ldots(18)$$

Hiley then assumes, as a result of his experience in pile driving, that

(i) Errors in centring the pile, and longitudinal vibrations result in a loss of energy equal to $\frac{1}{4}$ the energy required to compress the whole pile up to a stress of f lb. per sq. in. corresponding to the driving resistance.

(ii) A further like amount is absorbed if a dolly is used.
The energy remaining is then absorbed in compressing the padding on top of the pile, compressing the pile, and causing penetration.

(iii) The compression of the pile calculated by taking $E = 2{,}500{,}000$ lb. per sq. in. is

$$\frac{12fL}{2{,}500{,}000} \text{ inches}$$

(iv) The minimum values for the compression of the pile capping are taken as 0.10 in. per 1000 lb. per sq. in. of driving resistance (i.e., are equal to 0.0001 f inches) but the value is to be increased 50% if the dolly is long and by 100% if the padding is not very stiff.

Combining i, ii, iii and iv under the one heading as a loss due to an equivalent compression of C inches, against a driving resistance varying uniformly from 0 to R lb.

$$\text{Loss} = \frac{RC}{2} \text{ in. lb.}$$

$$\text{where } C = 1\tfrac{1}{2} \times \frac{12fL}{2{,}500{,}000} + \frac{f}{10{,}000} \quad \ldots\ldots(19)$$

is the minimum likely compression using a helmet and dolly.

Hiley gives tables for the quantity C for various conditions and values of f. Equating the value of energy available, in equation (17), to the work done in penetration, and losses in general,

$$\frac{12MV^2}{2g}\left\{\frac{M + P\varepsilon^2}{M + P}\right\} = Rs + R\frac{C}{2}$$

---

*Trans. Am. Soc. C.E., Vol. 48, page 180, August, 1902.

†Treatment of this formula will be found in:—
Engineering, June, 1922—"The Effect of the Hammer Blow";
May and June, 1925—"A Rational Pile Driving Formula."
Transactions of the Society of Engineers, 1923—"The Impact of Imperfectly Elastic Bodies in Pile Driving."

Whence, remembering that $v^2 = 2gh$

$$R = \frac{12Mh}{s + \frac{C}{2}} \times \frac{M + P\varepsilon^2}{M + P} \quad \text{..........................(20)}$$

The value of R cannot be found without first finding C; hence in practice, a value is assumed for C, R is then found, and since $f = \frac{R}{A}$ (A being the area of cross section of the pile in square inches) a fair value of C can be found, from (19), and R recalculated. The process is repeated until the results are consistent.

If C is put $= 0$, and $\varepsilon = 0$

$$R = \frac{12M^2h}{s(M + P)}$$

which is Eytelwein's formula.

Hiley gives a number of values of ε for varying conditions. Those applicable for concrete piles are:

- Steel D.A. steam hammer direct on concrete piles ... ... $\varepsilon = 0.5$
- Cast iron S.A. steam hammer direct on concrete piles ... ... $\varepsilon = 0.4$
- Cast iron S.A. steam hammer, or drop hammer on wood cap of helmet ... $\varepsilon = 0.25$
- Deteriorated conditions of wood cap of helmet or of packing in helmet ... $\varepsilon = 0.0$

The author cannot see why Hiley should take the whole energy of the hammer after impact as available if $M \nless P\varepsilon$, but concedes that the results would not be far out where ε is small, if the rest of the mathematics were in agreement with physical facts. For most cases of driving ε is small, and hence the efficiency of the blow may be taken as being correct.

Hiley assumes his formula to apply to values of $s = 0$, and on theoretical grounds this can be accepted as a limiting case for u approaching zero.

In the absence of further information, the Hiley formula may therefore be assumed, for the present, to give a reliable estimate of the driving resistance for all values of the set. The next main section of this paper is directed to showing that the phenomena of driving are very different from the comparatively simple ones assumed by Hiley and others, but no account has been taken of this in forming the above judgment.

(*g*) *Formulae depending on a knowledge of supporting strata.*—Formulae which depend on a knowledge of supporting strata allow a certain frictional resistance per sq. ft. on the sides of the pile, and a certain pressure under the toe, and can only be approximate.

Side friction often varies in amount at different depths in the same material, and is a very variable quantity in any case, as Table I shows.

TABLE I.

Material	Ultimate friction in lb. per sq. ft.
Sand	400 to 800
Sand and clay ...	300 to 1,000
Clay	250 to 1,500, or more
Silt and mud ...	200 to 500
Gravel	300 to 1,000

A fair estimate, however, should be possible if samples of the strata are available for examination. For soft materials, the allowable foundation pressure could be calculated in certain cases by Rankine's earth pressure theory, but it would be preferable to allow for cohesion in an amount consistent with the side friction allowed.*

The author considers it would be wise to check the load found for any pile by a static load test, or drop hammer test, by this method, if the underground conditions are known, as unexpected local variations in the foundations might thus be detected. Such a check is particularly necessary where sand is the supporting stratum.

(*h*) *Comparison of results given by the various formulae.*—About 1,500 comparative results have been worked out, checked, and plotted in Figs. 1, 2, 3, and 4. Curves of

*See *A Summary of Soil Mechanics*, by T. A. Farrent, THE JOURNAL, Vol. 3, No. 3, p. 107.

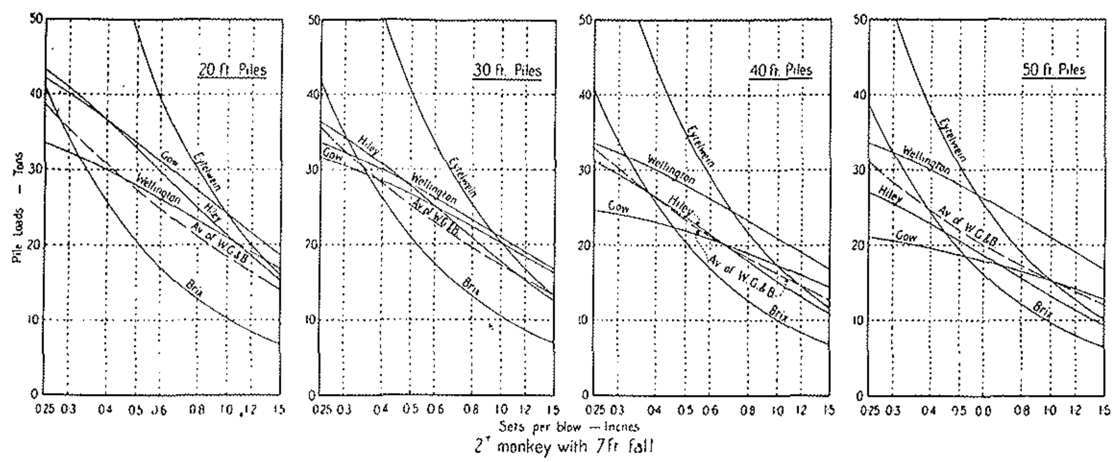


Fig. 1.—*Comparative Graphs for* 12 *in.* x 12 *in. Piles.*

load supportable, against set, have been plotted for a factor of safety of 4, in the cases of the following formulae:

(1) Wellington
(2) Gow
(3) Brix
(4) Eytelwein
(5) Hiley, for $\varepsilon = 0$
(6) $\frac{1}{2}$ of the Hiley load for double the drop of monkey for those cases where f $\not>$ 3,000 lb. per sq. in.
(7) Average of Wellington, Gow and Brix.

Eytelwein has no correction C;

Hiley has correction C;

"Hiley ($\frac{1}{2}$)" effectively has a correction C allowed twice over, i.e., shows the trend of results for a more easily compressed packing on top of the pile.

The effect of packing on top of the pile may thus be gauged on the basis of the mathematics used in deriving the Hiley formula.

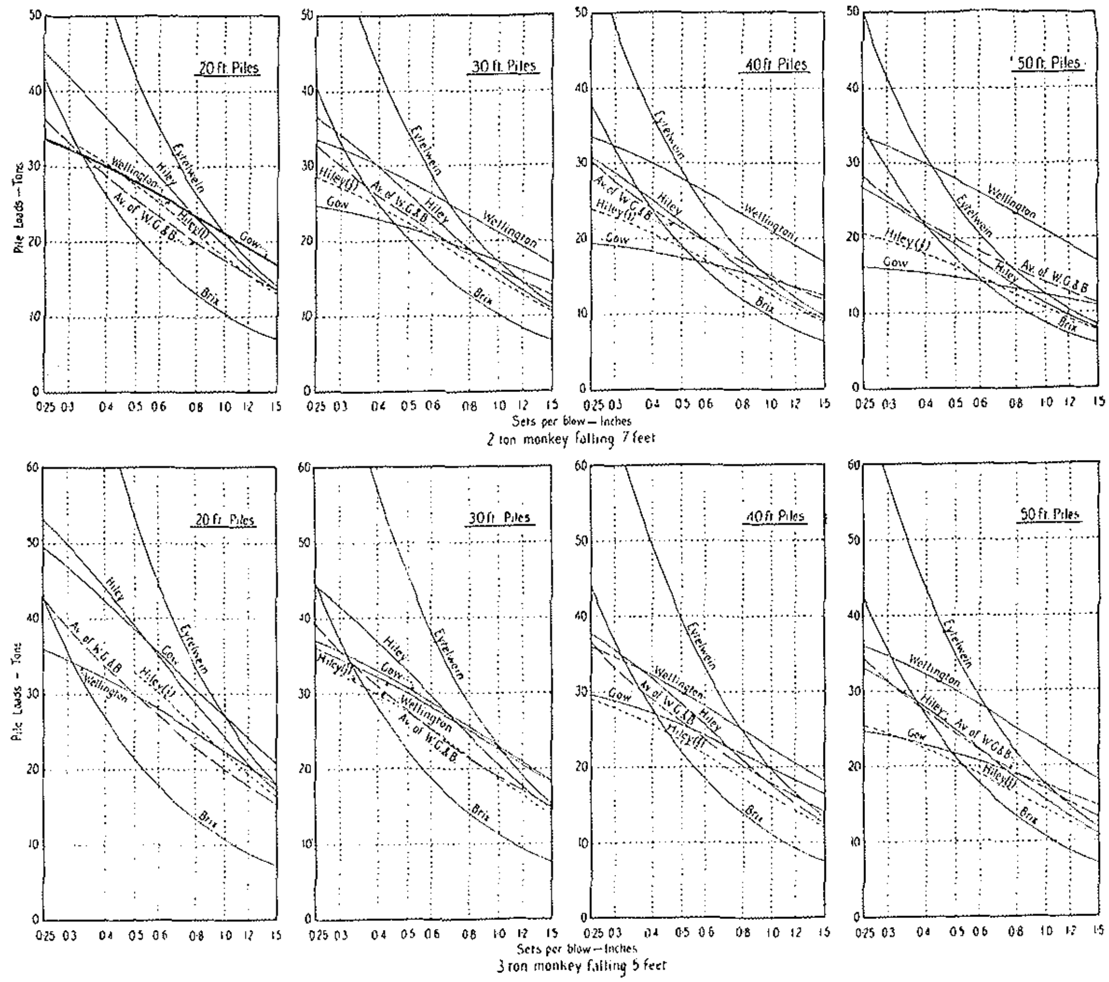


Fig. 2.—Comparative Graphs for 14 *in. x* 14 *in. Piles.*

Curves for (6) are noted in the figures as "Hiley ($\frac{1}{2}$)." The values of C for the Hiley formula were taken from Hiley's tables.

The Hiley values apply only for the exact conditions stated, but altering the drop of the monkey merely alters the loads for the other formulae, in proportion. All drops are taken as free drops.

Curves for the Eytelwein, Hiley, and "Hiley ($\frac{1}{2}$)" are related as follows:

The great divergence of results shown on the graphs shows how necessary it is that there should be some investigation of the subject.

It will be seen that there is in general a rough agreement between the Hiley values and the averages of the Wellington Gow, and Brix values, and considering previous opinions passed on these formulae, it would appear that the Hiley formula is the best so far developed. It should be remembered that the Hiley formula gives the driving resistance.

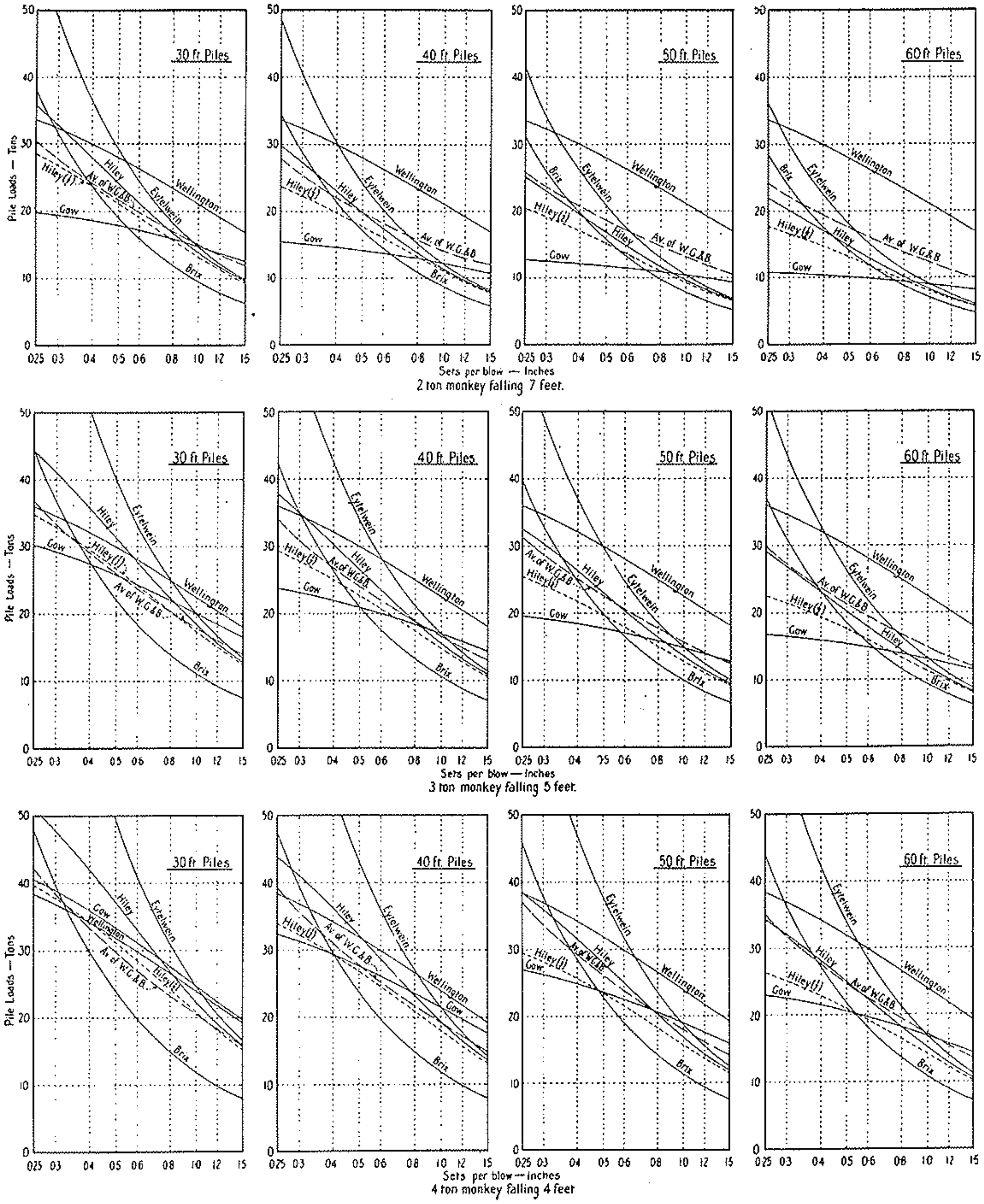


Fig. 3.—Comparative Graphs for 16 in. x 16 in. Piles.

III. Longitudinal Wave Movements in Piles.

(*a*) *Introductory.*

All the driving resistance formulae so far considered have assumed the pile to act as a unit, when struck by the hammer. The author carried out a simple experiment to test a theory which disagrees with this assumption.

Several straight pieces of ¼ in. diameter bright mild steel were prepared, with ends finished very slightly convex. Each could be suspended at each end in special light clips, to act as a pendulum swinging in a fixed plane, with the axis of the piece always perfectly horizontal. Impact experiments were carried out with these rods striking endwise in pairs, with corrections for damping. The results shown in Table II were obtained :—

TABLE II.

Conditions of experiment	Velocity of striking piece immediately before impact, ft. per sec.	Velocity of struck piece immediately after impact, ft. per sec.	Velocity of struck piece by calculation by ordinary theory, ft. per sec.
3 in. length striking 3 in. length	1.700	1.613	1.613
3 in. length striking 4½ in. length	1.700	1.256	1.290
3 in. length striking 6 in. length	1.700	0.988	1.075
3 in. length striking 7½ in. length	1.700	0.798	0.922
3 in. length striking 9 in. length	1.700	0.681	0.807

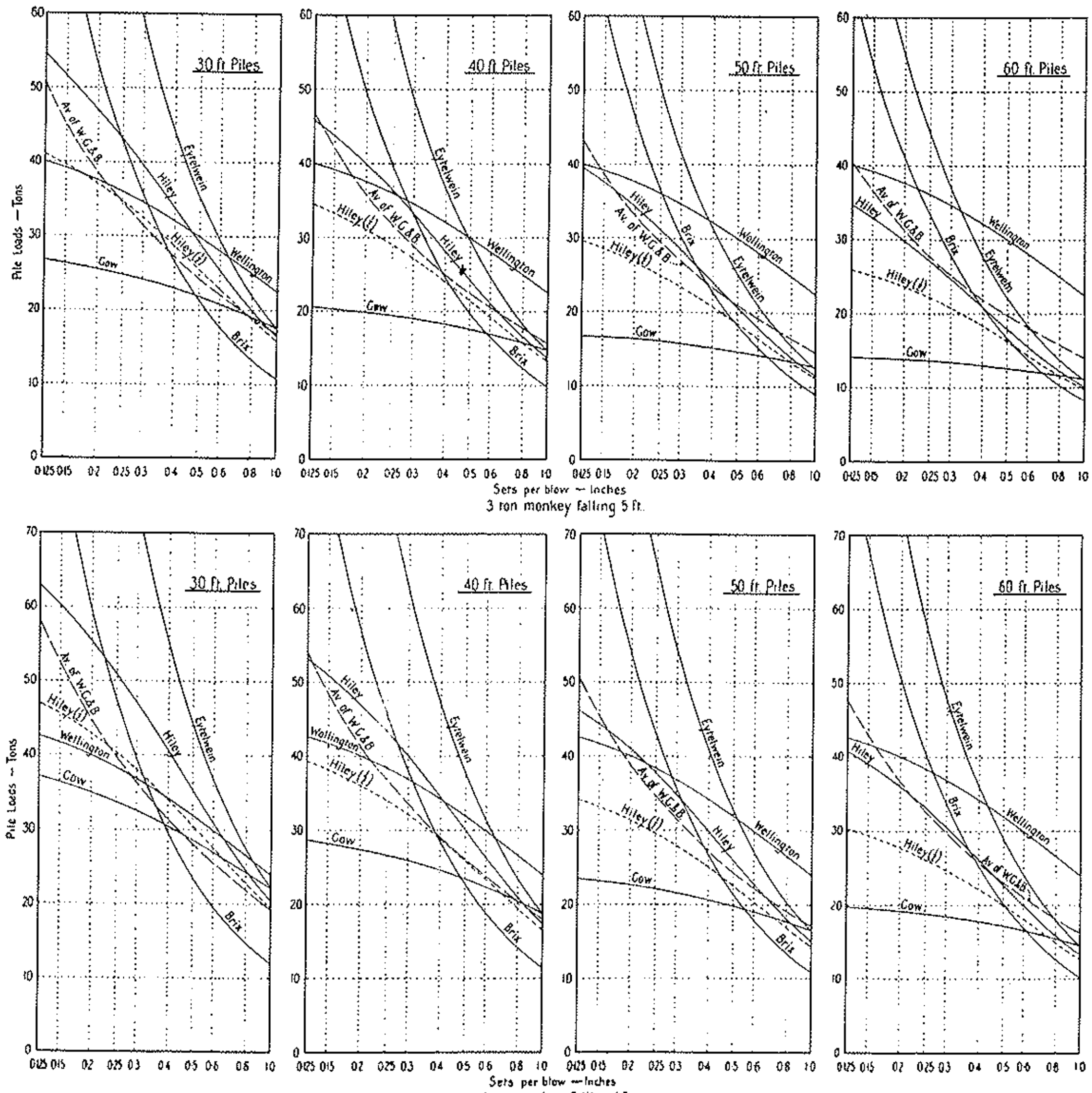


Fig. 4.—Comparative Graphs for 18 in. x 18 in. Piles.

From the first row of results the coefficient of restitution is found (from formula 15) to be $\left(\frac{2 \times 1.613}{1.700} - 1\right) = 0.897$ which is approximately the accepted result for steel on steel. The last column gives the velocities of the struck pieces by formula (15), with ε assumed 0.897 in all cases. The differences between theory and experiment are far too marked to be attributable to experimental errors, and the writer therefore submits his theory of impactive action by which results such as the above may be explained. *The theory results in rather complex calculations, but sufficient of these calculations have been made to obviate any more being required in practice.*

Reference to a good text book on physics is advised for facts concerning wave motion in elastic media.

(*b*) *Application of wave theory to pile driving.*

(i) *Ideal uncushioned impact of a long hammer on a long pile.*

Consider a long elastic hammer of mass **w** lb. per ft. length, approaching a long elastic pile of mass **p** lb. per ft. length, with velocity **v** ft. per sec.; and assume the hammer to hit the pile head in perfectly even contact over the whole area of the head. Assume the velocity of sound (or any longitudinal impulse) in the hammer to be ϱ times the corresponding velocity **V**, in the pile.

If, during any very short period of time δt, during impact of the hammer on the pile, the pressure exerted between the two causes impulses of a given magnitude i per second to be sent along the pile, the total change of momentum induced in a length of pile $V\delta t$, is $i\delta t$, producing a velocity

$$\frac{i\delta t}{pV\delta t} = \frac{i}{pV} \text{ ft. per sec.}$$

in that part of the pile in contact with the hammer. The corresponding velocity in the hammer must obviously be $\frac{i}{w(\varrho V)}$ ft. per sec., or the total velocity induced is

$$\frac{i}{V}\left\{\frac{1}{p} + \frac{1}{\varrho w}\right\}$$

$$= \frac{i}{pV}\left\{\frac{\varrho w + p}{\varrho w}\right\} \text{ ft. per sec.}$$

Consider the first instant δt after impact begins, and consider pile and hammer in perfect contact during this instant,

$$v = \frac{i}{pV}\left(\frac{\varrho w + p}{\varrho w}\right)$$

$$\text{i.e.,}\quad i = vpV\left(\frac{\varrho w}{\varrho w + p}\right)$$

giving the particles, both in the head of the pile, and in the base of the hammer a velocity

$$\frac{i}{pV} = v\left(\frac{\varrho w}{\varrho w + p}\right) \text{ ft. per sec.}$$

This is the velocity of the head of the pile relatively to the rest of the pile; and the velocity of the base of the hammer relatively to the rest of the hammer is

$$\frac{i}{w\varrho V} = v\left(\frac{p}{\varrho w + p}\right) \text{ ft. per sec.}$$

These two relative velocities allow the impact to continue for a time δt, during which time short sections only of the hammer and pile have become affected. It should be obvious, that a succession of equal valued compression impulses must occur, keeping the velocity of the pile head and hammer base $v\left(\frac{\varrho w}{\varrho w + p}\right)$ ft. per second, and that as time proceeds portions of the pile more and more remote from the head pick up this velocity, so that after a time t from the beginning of the impact, a length Vt of the pile, from the head downwards, is all moving with a velocity $v\left(\frac{\varrho w}{\varrho w + p}\right)$ ft. per sec.; but no other part of the pile is affected.

The quantity $\frac{p}{\varrho w + p}$ will be given the symbol **r**.

Then $\frac{\varrho w}{\varrho w + p} = (1 - r)$

(ii) *Ideal uncushioned impact of a short hammer on a long pile.*

When the impulsive relative velocity **vr** in the hammer reaches the top end, it is reflected as a tension impulse **vr**, so that, for portions of the hammer near to the top where both impulses now overlap, the relative velocity is **2vr**, and the stress is zero. By the time the reflected wave has reached the bottom of the hammer the whole hammer has been given a relative velocity of **2vr** and all stress in the hammer has been eliminated, so that the hammer now has the same relationship to the pile as a hammer possessed of velocity (**v — 2vr**), *and just beginning an impact.* A new cycle thus starts, and the process is repeated. If **l** be the length of the hammer, the time required for a complete cycle will be

$$2t = \frac{2l}{\varrho V} \text{ seconds.}$$

The velocity of the base of the hammer and the head of the pile at the beginning of the impact is thus $v(1 - r)$ feet per second; **2t** seconds later it is $v(1 - r)(1 - 2r)$; **4t** seconds later it is $v(1 - r)(1 - 2r)^2$; **6t** seconds later it is $v(1 - r)(1 - 2r)^3$, and so on. The compression wave propagated along the pile is thus of stepped form as shown in Fig. 5, where the ordinates represent velocities or intensities of compression, and abscissae, measured from the line OO to any portions of the wave, represent the time during which the wave has been travelling, or the distance travelled along the pile.

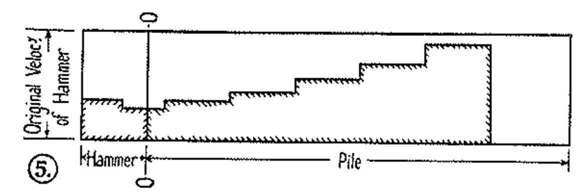
(iii) *Cushioned impact of a long hammer on a long column.*

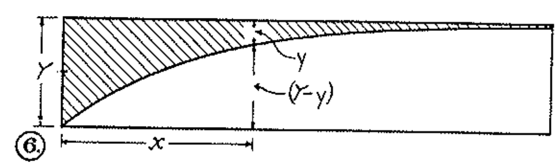
Duplicating the conditions assumed in section (i) (except that the pile is now assumed capped with an elastic cushion, through which any impulse travels so quickly that the time taken may be ignored) it is noted:

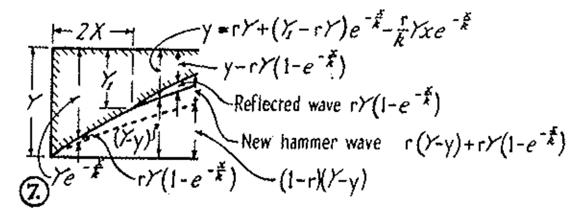
(α) The rate of compression of the padding equals the relative velocity of the base of the hammer with respect to the head of the pile.

(β) The magnitude of the impulses sent into both hammer and pile varies as the compression of the packing.

Plot the value of the velocity **v** as an ordinate Y in Fig. 6, and the relative velocity of the base of the hammer to the head of the pile as **y** measured downwards from the upper horizontal line. Ab-







Figs. 5, 6, and 7.

scissae x represent time from the beginning of impact. Then $\int_0^x ydx$ is the compression of the padding at x, and since the *total* relative velocity being developed by the compression in the padding is $(Y - y)$

$$\int ydx = k(Y - y) \quad \ldots\ldots(22)$$

k being a constant depending on the elasticity of the padding,

i.e., $$y = -k\frac{dy}{dx}$$

giving $$y = Ye^{-\frac{x}{k}} \quad \ldots\ldots(23)$$

i.e., the velocity of the head of the pile at the time represented by x is

$$Y(1 - r)\left(1 - e^{-\frac{x}{k}}\right)$$

and the velocity by which the base of the hammer has been reduced is

$$Yr\left(1 - e^{-\frac{x}{k}}\right)$$

(iv) *Cushioned impact of a short hammer on a long pile.*

Let X be the value of x for which $x = t$. Then, for the case of a short hammer, the action at the beginning of impact is precisely as in section (iii) above, till time t has elapsed. Then, from X to $2X$, the wave $Yr\left(1 - e^{-\frac{x}{k}}\right)$ is being reflected back as a tension wave from the top of the hammer.

From $2X$ to $4X$ the reflected wave is again reflected—this time from the bottom upwards as a compression wave—and therefore the action of the impact is altered. See Fig. 7.

From now onwards, y *will be considered as that relative velocity* between hammer and pile, *which results from compression in the padding alone.*

Thus there is obtained, in lieu of equation (22)

$$\int_0^{2X} Ye^{-\frac{x}{k}}dx + \int \left\{y - rY\left(1 - e^{-\frac{x}{k}}\right)\right\}dx = k(Y - y) \quad \ldots\ldots(24)$$

as the equation for the period $2X$ to $4X$,

$$\left\{y - rY\left(1 - e^{-\frac{x}{k}}\right)\right\}$$

being the relative velocity of the hammer and pile at any instant (of which velocity $rY\left(1 - e^{-\frac{x}{k}}\right)$ is the part twice reflected), and $\int_0^{2X} Ye^{-\frac{x}{k}}dx$ *being the compression of the padding* already brought about during the first cycle.

The solution of this equation is

$$y = rY + (Y_1 - rY)e^{-\frac{x}{k}} - \frac{r}{k}Yxe^{-\frac{x}{k}} \quad \ldots\ldots(25)$$

with origin at $2X$, *where* Y_1 = the value of y at $2X$

i.e., $Y_1 = e^{-\frac{2X}{k}}$

The corresponding hammer wave is (from Fig. 7)

$$r(Y - y) + rY\left(1 - e^{-\frac{x}{k}}\right)$$

$$= r\left(2Y(1 - 2r) - (Y + Y_1 - rY)e^{-\frac{x}{k}} + \frac{rY}{k}xe^{-\frac{x}{k}}\right)$$

Reflection of this hammer wave can similarly be shown to cause a wave during the period $4X$ to $6X$ such that

$$y = rY(2 - r) + \{Y_2 - rY(2 - r)\}e^{-\frac{x}{k}} -$$
$$-\frac{r}{k}\{Y_1 + Y(1 - r)\}xe^{-\frac{x}{k}} \quad \ldots\ldots(26)$$

together with a term

$\frac{r^2}{2k^2}Yx^2e^{-\frac{x}{k}}$ which can be shown to be negligible in all practical cases.

Y_2 = the value of y at $4X$ and the origin is at $4X$.

Neglecting terms involving $\frac{r^2x^2}{k^2}$ and higher powers, it is thus found

From $6X$ to $8X$

$$y = rY(3 - 3r + r^2) + \{Y_3 - rY(3 - 3r + r^2)\}e^{-\frac{x}{k}} -$$
$$-\frac{r}{k}\{Y_2 + Y_1 + Y(1 - 3r + r^2)\}xe^{-\frac{x}{k}} \quad \ldots\ldots(27)$$

From $8X$ to $10X$

$$y = rY(4 - 6r + 4r^2 - r^3) +$$
$$+\{Y_4 - rY(4 - 6r + 4r^2 - r^3)\}e^{-\frac{x}{k}} -$$
$$-\frac{r}{k}\{Y_3 + Y_2 + Y_1 + Y(1 - 6r + 4r^2 - r^3)\}xe^{-\frac{x}{k}}$$
$$\ldots\ldots(28)$$

From $10X$ to $12X$

$$y = rY(5 - 10r + 10r^2 - 5r^3 + r^4) +$$
$$+\{Y_5 - rY(5 - 10r + 10r^2 - 5r^3 + r^4)\}e^{-\frac{x}{k}} -$$
$$-\frac{r}{k}\{Y_4 + Y_3 + Y_2 + Y_1 + Y(1 - 10r + 10r^2 - 5r^3 + r^4)\}xe^{-\frac{x}{k}}$$
$$\ldots\ldots(29)$$

From $12X$ to $14X$

$$y = rY(6 - 15r + 20r^2 - 15r^3 + 6r^4 - r^5) +$$
$$+ \text{etc.} \quad \ldots\ldots(30)$$

and so on in an obvious sequence.

From the previous reasoning, the shaded areas (in Figs. 6 and 7) which represent the integrals of equations (22) and (24), represent the compression of the padding, and hence at any point x, the ordinate $(Y - y)$ is proportional to the shaded area up to point x.

In the case of a very short hammer, from Fig. 8, the compression of the padding (represented by the shaded area) is

$$Yx - \int(1 - r)ydx - \iint\frac{ry}{X}dx\,dx$$

$\iint\frac{ry}{X}dx\,dx$ representing the area between the curve $(1 - r)y$, and the lower side of the shaded area. X represents a unit value of x, and is inserted to preserve the correct dimensions for the integral. It should be noted that y *is now measured upwards from the bottom* of the diagram.

The relationship

$$Yx - \int(1 - r)ydx - \iint\frac{ry}{X}dx\,dx = ky$$

now holds, giving

$$\frac{d^2y}{dx^2} + \left(\frac{1 - r}{k}\right)\frac{dy}{dx} + \frac{r}{Xk}y = 0$$

Putting $\frac{1 - r}{k} = a$, and $\frac{r}{Xk} = b$

$$(D^2 + aD + b)y = 0 \quad \ldots\ldots(31)$$

The values of a and b are so related in the cases relevant to pile driving, that $b > \frac{1}{4}a^2$, and under these conditions the solution of equation (31) is

$$y = Ke^{-\frac{1}{2}ax}\sin\beta x \quad \ldots\ldots(32)$$

where $\beta = \sqrt{b - \frac{1}{4}a^2}$

and $K = \frac{Y}{\beta k}$

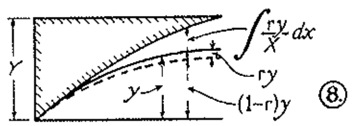
The values given by equations (25), (26), (27), (28), etc., can be shown to give a smooth curve, whose form is almost indistinguishable from that of one long, sweeping curve for all simultaneous values of r, k and x, which are relevant to the paper; and a comparison of the results given by these equations (25), (26), (27), etc., shows a close correspondence with the values given by equation (32).

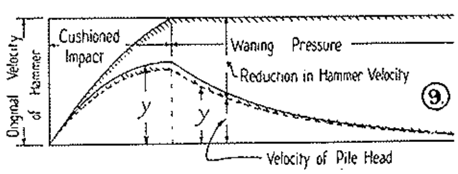
For the remainder of the work, then, equation (32) will be taken as giving the initial part of the wave form, for a short hammer striking a long cushioned pile. This wave form is seen in Fig. 8.

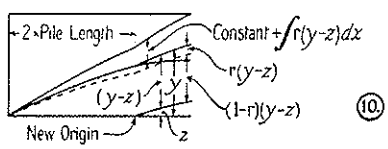
(v) *Waning pressure of contact.*

At the value if x for which equation (32) gives a maximum value to y, the compression of the padding (which, from the derivation of equation (32) is proportional to y) becomes a maximum also. For rope, sawdust, shavings, etc., which are commonly used as "padding," it might be expected that the padding would recover relatively so slowly from each impactive compression, that ε could be taken as 0. Equation (32) therefore does not apply beyond the point of maximum compression; but this equation may be used to derive the correct one. It is more instructive, however, to proceed as follows.

Consider a hammer and cushioned long pile at any stage beyond the stage of maximum compression in the packing (beyond which stage, the packing is assumed to remain of invariable thickness, since $\varepsilon = 0$). At first, consider the hammer to act as a unit (so that all parts of it move with the same velocity).







Figs. 8, 9, *and* 10.

Then, measuring y as for equation (31), if y equals the velocity of both hammer and head of pile, $-\frac{dy}{dx}$ represents the rate of change of velocity of the hammer, and $\frac{y}{X}$ represents the rate of change of velocity of the pile head.

Therefore, $-\frac{dy}{dx} = \frac{r}{1-r}\left(\frac{y}{X}\right)$

whence, $$y = J e^{-\frac{rx}{(1-r)X}} \quad \text{.........(33)}$$
where J is a constant.

Thus at time $2X$ after y has the value J, y has the value $J\left(e^{-\frac{2r}{1-r}}\right)$

and at time $4X$ after, $J\left(e^{-\frac{2r}{1-r}}\right)^2$

and at time $6X$ after, $J\left(e^{-\frac{2r}{1-r}}\right)^3$

Comparing with the results in section III (*b*) ii., it is seen that the value $e^{-\frac{2r}{1-r}}$ corresponds with $(1 - 2r)$ for the stepped curve of that section. The difference between these values is slight for the practical values of r (which are not greater than about 0.13). Actually no steps occur, as might be expected from section III (*b*) ii, as initial pulsations existing in the hammer at the time the action begins just cancel them out. This could be shown by proceeding via equation (32), instead of as above. Taking $x = 0$ where the curve begins and writing r as the equivalent value of $\frac{r}{1-r}$ to give a curve passing down the centres of the hypothetical steps, the equation

$$y = J e^{-\frac{rx}{X}} \quad \text{.........(34)}$$

thus represents the "waning curve" form, where

$$r = \tfrac{1}{2} \log_e \{1 - 2r/(1 - r)\} \quad \text{.........(35)}$$

and J is a constant equal to the maximum value of y in equation 32. The "waning curve" gives the form of wave generally applicable after the padding has reached its maximum compression, in the case of a short hammer striking a long pile. The form of wave so far deduced is thus represented in Fig. 9, which also shows between the hatched lines, the progressive curve of reduction in velocity of the hammer.

(vi) *Loss of amplitude during progress of wave.*

So far, consideration has centred round the wave form being propagated along the pile *from the head of the pile*. As the propagated wave passes along the pile it progressively loses amplitude. In what follows, the value of a unit impulse which has travelled for a time corresponding to x on the wave diagram will be considered to have a value $e^{-\mu\frac{x}{X}}$ where e is the base of the Naperian logarithms and μ is a constant. It should be noted that the value of the impulse to be taken is its *stress* value; and therefore, for example, if in its passage along the pile a compression wave encounters a tension wave passing along the pile in the opposite direction, the loss in propagation at any point will be proportional to the residual stress. The actual calculation of propagation losses is thus complicated, and an approximate solution only will be adopted, which assumes the loss to be always proportional to the amplitude of the wave under consideration. The final results which concern this paper will then be nearly consistent, provided the value of μ is small, or passing waves of opposite stress are considerably different in absolute value

(vii) *Action at the toe of a parallel pile.*

The ordinate Y of the wave diagram corresponds with a definite stress existing in the pile head, and this stress would be reduced in value by propagation to the toe of the pile. At the toe of the pile a stress due to resistance of the ground to driving is introduced, of value, say, q times the toe stress corresponding with Y. It should be obvious that the ground resistance is thus defined as a magnitude purely relative as far as it concerns the wave diagram.

A compression impulse of instantaneous value i, on reaching the toe of the pile, and being reflected as a compression impulse of value i and opposite sense of initial movement, entails a stress impulse at the toe of the pile, corresponding with (*a*) destroying the original impulse and (*b*) creating an opposite one, i.e., corresponding with an impulse of $2i$ from the toe of the pile. At the same time, the toe of the pile suffers no movement.

If the original impulse i meets with no resistance from the toe of the pile and thus becomes reflected as a tension impulse i having initial movements in the same sense as the original impulse had, then during the process of reflection the toe of the pile is moving forward with a velocity corresponding with an impulse of $2i$.

Thus, writing y as an instantaneous compression value of the wave diagram, and qY as corresponding with the ground resistance (α) where $2y \not> qY$ no penetration of the pile toe occurs, and the reflected wave is a compression wave of value

y. (β) Where $2y > qY > y$, penetration proceeds at a velocity corresponding with $(2y - qY)$, and the reflected wave is a compression one of value $(qY - y)$. (γ) Where $y > qY$, penetration proceeds at a velocity corresponding with $(2y - qY)$, and the reflected wave is a tension one of value $(y - qY)$.

(viii) *Reflected waves during cushioning.*

Where the packing on top of the pile is liberal in amount, waves reflected from the toe of the pile may reach the head before cushioning (as given by equation 32) is complete. Calling the instantaneous value of the reflected wave z (positive values applying to a tension wave, which impels the head of the pile to greater forward movement) the compression of the padding becomes (see Fig. 10)

$$\text{Constant} + Yx - \int (1-\mathbf{r})(y-z)dx - \iint \frac{\mathbf{r}(y-z)}{X}\,dxdx - \int z\ dx$$

which equals $k\,(y - z)$

$(Y - y)$ being the ordinate representing the relative velocity of the pile head and hammer.

Hence in lieu of equation (31), the following is obtained:

$$(D^2 + aD + b)\,(y - z) = -\frac{1}{k}Dz \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(36)$$

Allowing for losses in propagation, z will be of the form

$$z = K_1 e^{-\frac{1}{2}ax} \sin \beta x + \text{Constant}$$

whence the solution of equation (36) can be shown to be of the form

$$y = K_2\, e^{-\frac{1}{2}ax} \sin(\beta\, x + \xi) - K_3\, e^{-\frac{1}{2}ax}\, x \cos(\beta x + \zeta) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(37)$$

where K_2, K_3, ξ and ζ are constants.

The solution in this form is tedious in application, but the general form of equation is readily solved for practical purposes by a closely approximate arithmetical method.

(Put $y = y_1 - y_2 + z$ where $y_1 = Ke^{-\frac{1}{2}ax} \sin \beta x$ as in section III (*b*) iv, and then

$$z - (1 - \mathbf{r})y_2 - \mathbf{r}\int \frac{y_2}{X}\,dx = k\ D\ y_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(38)$$

which allows arithmetic calculation in steps).

(x) *Reflected waves during waning contact.*

Reflected waves always affect the curve of waning contact to some degree. Using the same ideas of y as in Section III (*b*) v and of z as in the last section, the rate of change of velocity of the pile head, due to compression in the padding is $\left(\dfrac{y - z}{X}\right)$

whence as in section III (*b*) v

$$-\frac{dy}{dx} = r\left(\frac{y - z}{X}\right)$$

$$\text{or } Dy = -\frac{r}{X}(y - z) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(39)$$

a differential equation readily solved arithmetically or mechanically. If at any point Dy becomes so great that the compression in the padding exceeds its former maximum value, equation (36) then rules.

(*xi*) *Multiple impact.*

The values of z may be such that the value of Dy in equation (39) reaches zero (just as z reaches y). If z still increases, the pile head travels with velocity z, and the hammer separates from the padding, travelling onwards with a velocity z_1 (constant for all practical purposes) which equals that attained just as separation occurred. When a point is reached where $\int zdx$ equals z_1x (x being zero where separation begins) a new contact begins, with the initial value of y equal to z_1, and from then onwards

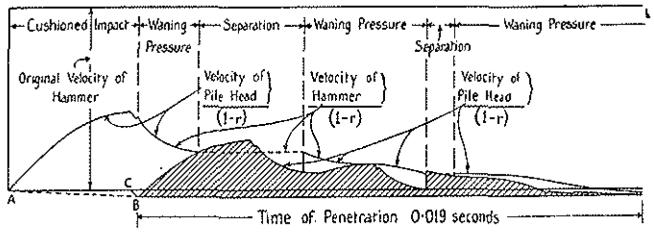


Fig. 11.—*Wave Diagram for $L = 20$, $r = 0.13$, $k = 20$, $l = 3\frac{3}{4}$, $q = 0.1$; i.e., a 20 ft. Pile, well cushioned, a light hammer, and medium Driving.*

$$Dy = -\frac{r}{X}(y - z) \text{ as before.}$$

Such a case occurs twice in Fig. 11.

(*xii*) *The general wave form.*

Plotting the appropriate values of y, $y/(1 - \mathbf{r})$, or $\{y + \mathbf{r}z/(1 - \mathbf{r})\}$ for the various cases so far considered, the general wave form is obtained in Fig. 11. *This figure represents a particular case only, and other cases may be quite different in detail; but the two main features exist*

(α) *the cushioned blow.*

(β) *the waning contact.*

(xiii) *Cushioning expressed mathematically.*

The velocity produced in the pile equals

$$\frac{\mathbf{g} \times \text{Force acting in lb.}}{\mathbf{pV}} \text{ ft. per sec.}$$

$\mathbf{pV}$ being the mass of the pile affected per second.

Hence the total relative velocity produced is

$$\frac{\mathbf{g} \times \text{Force acting in lb.}}{(1 - \mathbf{r})\mathbf{pV}} \text{ ft. per sec.}$$

If $\mathbf{G}$ is the stiffness of the padding in lb. per in. compression, the force acting is $\mathbf{12G} \times$ (compression in feet).

Now, the relation expressed in equation (22), and in many of the following equations is

Compression $= k$ times velocity being produced

and the reasoning immediately above gives

$$\text{Compression} = \frac{(1 - \mathbf{r})\mathbf{pV}}{\mathbf{12gG}} \times \text{velocity being produced,}$$

$$\text{hence } k = \frac{(1 - \mathbf{r})\mathbf{pV}}{\mathbf{12gG}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(40)$$

which has the dimensions of *time*.

Consider a hammer of infinite length striking a padded pile of infinite length, with a velocity $\mathbf{v}$ ft. per sec. In infinite time the hatched part of Fig. 6 has an area

$$\int_0^\infty Ye^{-\frac{x}{k}}\,dx = Yk$$

corresponding to an actual compression of $\mathbf{v}k$ feet, when the total velocity being produced is $\mathbf{v}$ ft. per sec.

Therefore, suppose the maximum value of y from equation (32), or the maximum value of $(y - z)$ from equation (36) equals $\mathbf{Q}Y$; then at the times corresponding, the compression of the padding is $\mathbf{Q}Y\mathbf{v}$ feet.

Hence, if $\mathbf{c}$ inches is the actual maximum compression of the padding

$$\mathbf{c} = \mathbf{12Q}k\mathbf{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(41)$$

$$\text{or} \quad \mathbf{c} = 96.2\ \mathbf{Q}k\sqrt{\mathbf{h}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(42)$$

and $\mathbf{Q}k$ thus represents the time taken to reach a compression $\mathbf{c}$ inches if compression proceeds at a uniform velocity of $\mathbf{v}$ ft. per sec.

The physical significance of k is thus the time required, if compression were to occur at a velocity of $\mathbf{v}$ ft. per second, to compress the padding to an ideally impactive value (which value is the value of the compressive stress reached in a perfect uncushioned impact); and k is thus proportional to a certain extent to the compressibility of the packing.

(xiv) *Compressive stress during driving.*

The following will be assumed:

Weight of concrete pile = 150 lb. per c. ft.
Weight of hammer = 460 lb. per c. ft.
Young's modulus—concrete = 2,500,000 lb. per sq. in.
Young's modulus—hammer = 17,000,000 lb. per sq. in.

Whence velocity of sound in concrete = 8,800 ft. per sec.
velocity of sound in hammer = 13,200 ft. per sec.
giving ϱ = 1.50

If the area of the concrete pile be A sq. in. the weight per ft. run in lb., $p = 1.04A$
hence from equation (40)

$$k = \frac{(1-r) \times 1.04A \times 8{,}800}{12 \times 32.2G}$$

$$= 23.7\,(1-r)\,\frac{A}{G} \quad \text{.......................................(43)}$$

and from equation (41)

$$k = \frac{c}{12Qv}$$

But $\frac{cG}{A}$ equals the maximum compressive stress F attained in the head of the pile during impact and $v = 8.02\sqrt{h}$

hence $F = 284\,(1-r)Qv$ (lb. per sq. in.) ....(44)

i.e., $F = 2{,}280\,(1-r)Q\sqrt{h}$ (lb. per sq. in.) ....(45)

More generally, and from first principles

$$\frac{\text{Velocity being produced in pile head}}{V} = \frac{F}{E}$$

whence, $F = 284 \times$ Velocity being produced in pile head
$= 2{,}280\,(1-r)\,Q\sqrt{h}$, as before.

The intensity of stress set up in the head of the pile, due to impact of the hammer thus varies as the square root of the fall of the hammer, and is readily calculated, when Q is known; and Q can be shown to vary slowly with variations of c, the compression of the padding.

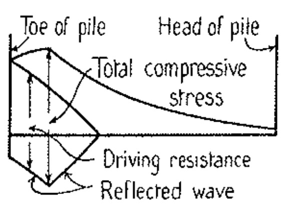


Fig. 12.

The initial impulse reflected from the toe of the pile in driving is always compressive, and in Fig. 12, such a compressive impulse is shown with its maximum value coincident with the maximum value of the outgoing wave from the pile head. Due account should be taken of the losses in propagation, in calculating this absolute maximum value of the compressive stress. The compressive stress at the head, however, will usually be more severe on the pile, as it may not be so evenly distributed over the pile cross section, and the longitudinal reinforcement is there ineffective.

(xv) *Tensile stress during driving.*

In many cases the combined actions of an outgoing compressive wave and a returning tensile wave produce a tension in the pile. The net tension values may be high, and will be discussed in section III (*c*) iv.

(xvi) *Driving resistance.*

Substituting the value q [of section III (*b*) vii] instead of Q in equation (45), the equivalent value at the head of the pile of the driving resistance, is

$$2{,}280\,(1-r)\,q\sqrt{h} \text{ lb.}$$

With a pile of length L feet, the factor $e^{-\mu\left(X\frac{L}{Vt}\right)}$ allows for the loss in propagation along the pile whence, in lb.

$$R = e^{-\mu\left(X\frac{L}{Vt}\right)} \times 2{,}280\,(1-r)\,q\sqrt{h} \quad \text{.........(46)}$$

a formula giving the driving resistance, for a given value of q.

(xvii) *Set per blow.*

For a given value of q, and known values of r, k, L, and l, the exact wave form may be plotted as in Fig. 11. The shaded area in that figure represents the product of time with half the velocity of the toe—or more precisely $\left(\frac{1}{1-r}\right)$ times the product of x and the value $\frac{1}{2}(2y - qY)$ of section III (*b*) vii.

When due allowance has been made for propagation losses, the penetration of the toe per blow is thus represented by $2 \times (1-r) \times$ *Shaded area of Fig.* 11 $\times\ e^{+\mu\left(X\frac{L}{Vt}\right)}$

$$\text{..............................(47)}$$

(the positive correction for propagation losses being made since Fig. 11 already gives a double negative correction, in order to make the values of the reflected waves applicable to the head of the pile).

After the end of the penetration due to one hammer blow, the toe of the pile exerts an upward force on the foundations. This force can never be greater than $\frac{1}{2}$ the driving resistance and may be practically ignored in the case of the set because if the toe does rise owing to it (thereby reducing the set for that blow) the first part of the penetration under the next blow is much easier and an approximate compensation is introduced.

When piles are driven to rock, through soft strata, the last blow of the hammer may thus cause the toe of the pile to separate a little from its foundation, and the author therefore considers that two or three light taps with the hammer should be used to complete driving under such conditions.

(xviii) *The effectiveness of the hammer*

It is noted (see section III (*b*) iv) that

$$a = \frac{1-r}{k}\ , \quad b = \frac{r}{Xk}$$

and $\beta = \sqrt{b - \frac{1}{4}a^2}$

and for constant values of the maximum value of y from equation (32)

$$\frac{a^2}{b} = \text{constant} = \frac{(1-r)^2X}{kr}$$

Also, from equations (33), (34) and (35), if $\frac{r}{(1-r)X}$ = constant, the equation for the waning curve is practically unvaried.

Thus $k \propto (1-r)$

in order to give an unvarying waning curve starting from the same value; and for this condition, the value of x for which cushioning reaches its maximum point varies as $\frac{1}{\beta}$, i.e., varies as $\frac{1}{a}$ and hence is constant, while k is also adjusted to suit the above equations by being simultaneously varied in the ratio $(1-r)$. Now, for the same stiffness of the padding, and the same pile, equation (40) shows that k varies as $(1-r)$. Therefore, for the same pile, with padding stiffness unaltered, and different hammers of such size that $\frac{r}{l(1-r)}$ and hence $\frac{r}{X(1-r)}$ is the same for all, the wave forms are practically similar, and the sets for the same driving resistances are equal for all the hammers.

$$\text{Now}\quad \frac{\mathbf{r}}{\mathbf{l}(1-\mathbf{r})} = \frac{\mathbf{p}}{\mathbf{l}\varrho\mathbf{w}}$$

$$= \frac{\mathbf{p}}{\varrho\mathbf{W}}$$

$$= \frac{1.04\mathbf{A}}{1.5\mathbf{W}}$$

$$\mathbf{r} \text{ being equal to } \frac{\mathbf{p}}{\varrho\mathbf{w}+\mathbf{p}},$$

and hence, for all practical purposes, where paddings have the same Young's modulus, the ratio $\left(\frac{\mathbf{W}}{\mathbf{A}}\right)$ *i.e. the weight of hammer per square inch of pile cross section, determines the wave form, and hence the set, and the maximum stresses set up in the pile during driving.*

(xix) *Corrections for dolly.*

Adequate consideration of the phenomena connected with a dolly, helmet or cap, would require far more space than can be devoted here.

Figure 13 shows diagrammatically a driving cap of form between that of two types commonly employed. The timber striking piece acts approximately as an equivalent part of the pile, and the metal section as part of the monkey. If ε for the striking piece be zero, the average velocity of the monkey plus helmet, as a unit, after impact from the monkey, is $\frac{\mathbf{M}}{\mathbf{M}+\mathbf{D}}$ times the velocity of the monkey before impact (**D** being the weight of the helmet and dolly and **M** being the weight of the monkey alone). Actually, oscillations occur in the helmet and dolly, which are superimposed on the wave forms already considered, but the following approximate rules may be applied. (See Fig. 13).

(α) *The equivalent height of free fall of the hammer (which is considered as being the helmet, dolly and monkey acting as a unit) is*

$$\mathbf{h} = \left(\frac{\mathbf{M}}{\mathbf{M}+\mathbf{D}}\right)^2 h \quad \text{.......................(48)}$$

$$\text{i.e.,}\quad \mathbf{h} = 0.75\left(\frac{\mathbf{M}}{\mathbf{M}+\mathbf{D}}\right)^2 \mathbf{H} \quad \text{................(49)}$$

where **H** *ft. is the actual drop of the monkey worked direct from a friction winch.*

(β) *The weight per ft. run of the hammer* (**w** *lb. per ft.*), *equals the weight per foot run of the monkey* (**m** *lb. per foot*), *and the length of the hammer* **l** *ft. should be taken as* $l\left(\frac{\mathbf{M}+\mathbf{D}}{\mathbf{M}}\right)$ *feet, the monkey alone being l ft. long.*

The equivalent length of a pile of actual length L feet is

$$\mathbf{L} = L + \lambda \quad \text{..................................(50)}$$

λ feet being the length of the striking piece (Fig. 13).

(xx) *Driving resistance supplied by side friction.*

When an attempt is made to modify the previous theory to allow for side friction, many difficulties are encountered, and an approximate solution only is practically possible.

It can be shown that the line *CB* changes approximately to the dotted line *AB* of Fig. 11, when side friction is encountered for the full length of the pile and this friction increases slightly towards the bottom of the pile. Equation (36) thus becomes

$$(D^2 + aD + b)(y - z) = -\frac{1}{k}\,D\varkappa z \quad \text{...................(51)}$$

where $\varkappa$ is a constant

and the solution of this is

$$(y-z) = Ke^{-\frac{1}{2}ax}\sin(\beta x - \varphi) + \frac{\varkappa}{kb} \quad \text{..............(52)}$$

$$\text{where } \sin\varphi = \frac{\varkappa\beta}{b}$$

The maximum value of $(y - z)$—which practically gives the maximum value of the compression on the pile head—is rarely more than 5% in excess of the corresponding value of $(y - z)$ in equation (36).

It is then noted

(α) Of the total energy reaching the pile head, part is expended in driving the pile forward, part in propagation losses, part in compressing the pile, and part in uselessly setting clinging earth in motion.

(β) Comparing propagation losses when the driving resistance is encountered at the point of the pile, and when encountered at the sides, a small decrease in the losses might be expected for the latter case; but actually, owing to the upper sections of the pile tending to move faster than the toe for the corresponding portions of most waves, temporary locked up stresses occur. An increase of the propagation losses might therefore be expected.

(γ) The compression losses in the pile are not large, and would remain practically unaltered, any change being a reduction.

(δ) Goodrich's investigations into timber piles, when applied to concrete ones, indicate that clinging earth is unlikely to increase the weight of any concrete pile cross section by more than 15%. Assuming added weight on any cross section to absorb velocity, but not to return it, by analogy with the impact equation (18) for $\varepsilon = 0$, a loss of not more than 15% of the energy reaching the pile could be expected.

By taking account of the modified cushion losses (by means of equation 52) and of the other factors enumerated above, an approximate solution is thus obtained. *The driving resistance calculated as applying to the toe of the pile is multiplied by a factor* N, *to convert to the driving resistance where side friction is encountered. Based on the work described in section* III (*c*) ii, *the factor may be given as :*

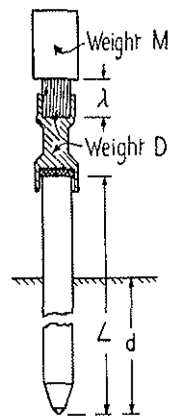


Fig. 13.

Where the supporting strata are fairly uniform for the full depth of penetration (d feet, of Fig. 13),

$$\mathbf{N} = \left(1 - 0.3\,\frac{\mathbf{d}}{L}\right) \quad \text{.......................(53)}$$

Where the main supporting strata are in the lower half of the length d feet,

$$\mathbf{N} = \left(1 - 0.2\,\frac{\mathbf{d}}{L}\right) \quad \text{.......................(54)}$$

(xxi) *Tapered piles.*

The action in tapered piles is involved, and would require a special investigation ; but general considerations would indicate that if the area of cross section be assumed A square inches so that a parallel pile of A square inches cross sectional *area would be of the same total weight as the tapered pile, the final relation between set and driving resistance will not be far out.*

(*c*) *Application of Theory.*

(i) *Calculation of curves.* The mathematical calculation of the curves so far described was found tedious, and arithmetical methods prove faster; but by far the fastest method of attack was a mechanical method.

The author designed and constructed a curve drawing machine, which considering all factors involved, saved some weeks of work, and allowed results to be obtained with a minimum possibility of practical error.

A diagram of the machine is given in Fig. 14.

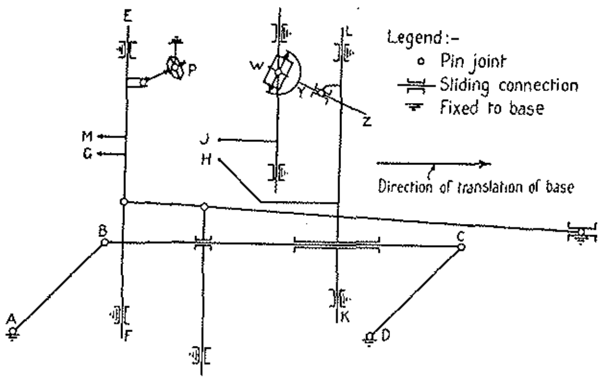


Fig. 14.—*Diagram illustrating Principle of Curve Drawing Machine*

ABCD is a pin jointed parallelogram attached to a base which causes the motions of pointers *G* and *M* to be drawn by a pen at *H*, to a smaller vertical scale corresponding to propagation losses. The base moves sideways against a straight edge, and *M* is moved to follow a curve already delineated.

W is a sharp edged heavy wheel, which by virtue of its weight and setting, compels the arm *YZ* to slide always in the direction *YZ*, as the base is translated. When *G* and *H* become level *YZ* is moved by the slide *KL*, into a position parallel to the direction of translation of the base, and a pen at *J* touches the pen at *H*. Thus the inclination of *YZ* to the direction of translation is proportional to the difference in position of *G* and *H*. The distance *GM* is proportional to the driving resistance being considered, and diagrams are drawn with base lines under the zero position of *G*. Thus, if *M* follows a curve x, y corresponding to the wave form leaving the pile head, the actual movement of *M* corresponds with that of a reflected wave uncorrected for propagation losses, and the movement of *H* is similar, but is corrected for these losses, and the pen at *H* draws the curve x, z. The pen at *J* obviously draws a curve such that its slope is proportional to the difference in level of *G* and *H* and thus equation (39) is plotted by this pen.

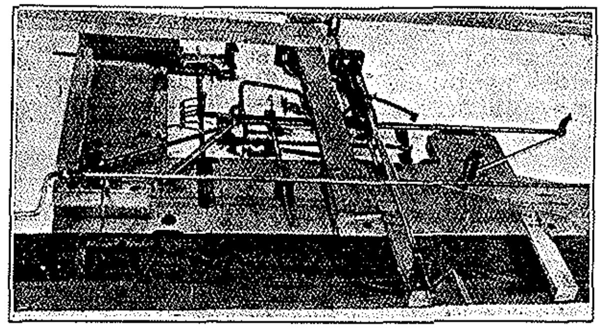
Thus, provided the curve for cushioned impact is first drawn for *M* to follow, the machine draws the remainder, with *M* following a machine-drawn curve in all parts except those corresponding to cushioned impact.

P is a specially constructed small planimeter which can be set to operate so that, if set at zero where the hammer leaves the pile, it returns to zero where contact begins again; and this latter point is thus readily noted during operation of the machine.

The average time taken to draw a curve completely, by means of the machine described above, was less than 10 minutes. Two views of the machine are shown. Fig. 15 shows it completely set up, and Fig. 16 shows it in a different position, with the wheel *W* and its attachments lifted out and placed to the right. Several pointers are shown corresponding with different driving resistances. A standard planimeter is also shown in these views, set so as to measure automatically the area giving the set per blow, but, as these areas were all checked, it was found quicker to do all the planimeter work separately.

All adjustments to allow for different length piles, different values of r, etc., were made into carefully set centres, which allowed accurate repeated settings.

In constructing the machine, care was taken in all details requiring accuracy, and the writer believes that errors due to the machine alone are too small to be of any account.



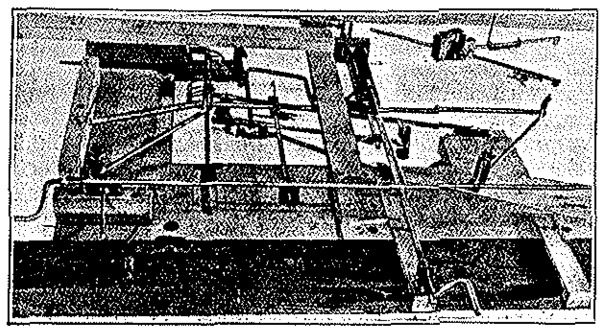


Fig. 15.—*Curve Drawing Machine Completely Assembled.*
Fig. 16.—*Showing Curve Drawing Unit Separate from Machine.*

(ii) *Graphs for practical application.*

The 192 combinations possible with the following values of the variables were investigated with the curve drawing machine

Values of r : 0.07, 0.10 and 0.13
Values of k : 4.00, 6.67, 11.11, and 20.00
Values of L : 20, 30, 45 and 60
Values of q : 0.45, 0.30, 0.18 and 0.10

The value of l was taken as $3\frac{3}{4}$, and propagation losses were assumed such that an impulse travelling 60 ft. through a pile would be reduced to 75% of its original value. This assumption is discussed in section III (*c*) v.

From the results the graphs of Figs. 17, 18, and 19 were drawn.

Fig. 17 *relates the set per blow to the compressive stress corresponding with the pile point driving resistance, for various values of the compression of the cushion.*

Fig. 18 *relates the weight of hammer per square inch of pile cross section and compressive stress set up in the pile head (for sets above the limit mentioned in the next sub-section)*

Fig. 19 *gives the percentage* (p) *of steel required in the upper two-thirds of the pile length, for various values of the driving resistance, length of pile, and weight of the hammer per square inch of pile cross section.* The steel is required to prevent opening of cracks in the pile due to tension as mentioned in section III (*b*) xv.

(iii) *Maximum compressive stress in the pile.*

Provided the driving resistance is not more than $\frac{3}{4}$ of the compressive value for the head, given in Fig. 17, this value is the criterion for strength of the pile in driving.

For very hard driving, with the shorter piles, the region near the toe of the pile becomes stressed more than the head of the pile. (See section III (*b*) xiv).

An indefinitely repeated stress of a little more than 75% of the ultimate strength of concrete will cause failure. In practice there will be a certain amount of uneven distribution of stress over the pile head, but the number of repetitions of stress will not be great, and except for hard driving the stress will only become serious for the last few blows. *The effective strength of the concrete in the head might thus be assumed about* 80% *of the ultimate strength for ordinary driving, and possibly down to about* 65% *for very hard driving. As the maximum safe drop of the hammer increases as the square of the strength of the concrete in the pile, high strength concrete shows a marked advantage.*

(iv) *Stress in tensile reinforcement.*

The maximum tension exists for a few thousandths of a second only, during which period a stress even somewhat beyond the yield point of the steel would be expected to cause very little actual yielding. Small scale experiments by the author indicate that very little yielding does take place under such conditions. A rapidly applied and rapidly removed stress 25% in excess of the yield point stress is therefore assumed no more severe than a slowly applied yield point stress. A small amount of resilience in the helmet cushion will reduce the tensile stress, as will also slight yielding under stress, and the presence of surrounding concrete. Large tensile stresses are always followed by large compressive stresses which would nullify any practical stretch in the steel. The steel itself, in propagating compression waves, tends to propagate these more rapidly than the concrete and hence blurs the waves and reduces the maximum values. In view of all these factors, the author would set the danger point of *theoretical* stress for mild steel at 56,000 lb. per sq. in.; and since the stress in the steel is dependent on that in the concrete, and the concrete is assumed never to reach an average of more than 80% of its ultimate strength, a factor of safety of $1\frac{1}{4}$ for the steel should suffice. *A* theoretical *working stress of* 45,000 *lb. per sq. in. was therefore adopted for calculating the steel percentages. The values apply for* 3,000 *lb. per sq. in. concrete, and may be adjusted proportionally for other strengths (higher strengths requiring higher percentages). The lower limit for this percentage of reinforcement should be about* $1\frac{1}{2}$%.

(v) *Losses in propagation.*

There is no definite information regarding propagation losses. On the one hand, phenomena such as the transmission of vibrations through buildings and even through fairly soft earth show that the damping cannot be excessive; and on the other hand the relative freedom from impact in the main members of concrete bridges, and the absence of "ring" in concrete when struck show that the damping is of practical magnitude. In piles, the propagation losses correspond with a rapid shortening of the pile each blow, and a slow recovery. The author's test for propagation losses, is that the results for long piles and large sets by the new theory should approximate to the results given by existing formulae. If such is the case, the results for the shorter piles and heavier loads should be little affected by an error in the assumed propagation losses.

The comparisons in Table III are made for 2,400 lb. per sq. in. developed in the pile head during driving, and N taken as $(\mathbf{1} - \mathbf{0.2} \times \frac{3}{4}) = \mathbf{0.85}$. They show that the assumptions made regarding propagation losses cannot be far out.

TABLE III.

18 *in.* × 18 *in. Concrete Piles, Factor of Safety* 4. *Helmet Weight* 600 *lb.*

Method or Formula	3 ton monkey, 5 ft. free fall; 0.625 in. set per blow		4 ton monkey, 4 ft. free fall; 0.75 in. set per blow	
	45 ft. pile	60 ft. pile	45 ft. pile	60 ft. pile
Author's	$19\frac{1}{2}$ tons	$14\frac{1}{2}$ tons	$19\frac{1}{2}$ tons	14 tons
Wellington ...	$27\frac{1}{2}$ „	$27\frac{1}{2}$ „	27 „	27 „
Gow	15 „	12 „	$19\frac{1}{2}$ „	$15\frac{1}{2}$ „
Brix	15 „	$12\frac{1}{2}$ „	15 „	$13\frac{1}{2}$ „
Eytelwein	$21\frac{1}{2}$ „	$17\frac{1}{2}$ „	$23\frac{1}{2}$ „	19 „
Hiley	$18\frac{1}{2}$ „	$15\frac{1}{2}$ „	$20\frac{1}{2}$ „	17 „
"Hiley $\frac{1}{2}$" ...	17 „	14 „	19 „	16 „

The values given above were obtained from Figures 3, 4, 17 and 18.

(vi) *A Relation between Driving Resistance and Bearing Resistance.*

That it is almost impossible to develop a relationship between driving and bearing resistances, unless the soil conditions are well known, may be judged from the information given in "Jacoby and Davis," articles 34 and 35, but a rough relationship may be struck as follows.

Wellington's timber pile formula is reckoned reliable for bearing resistances. Goodrich's timber pile formula was closely investigated by its author* for driving resistances. A comparison of the two formulae for a factor of safety of 4 gives coefficients of M*h* for safe loads, as in Table IV.

TABLE IV.

	Values of Set per blow						
	$\frac{1}{4}$ in.	$\frac{1}{2}$ in.	$\frac{3}{4}$ in.	1 in.	$1\frac{1}{2}$ in.	2 in.	3 in.
Wellington	2.40	2.00	1.71	1.50	1.20	1.00	0.75
Goodrich	3.33	1.67	1.11	0.83	0.56	0.42	0.28
Ratio $\frac{\text{Wellington}}{\text{Goodrich}} = \frac{\mathbf{3.6s}}{\mathbf{s+1}}$	0.72	1.20	1.54	1.80	2.16	2.40	2.70

Neither Wellington's nor Goodrich's formula is considered reliable below about $\frac{1}{2}$ in. set. Just below $\frac{1}{2}$ in. set the values given by them become coincident, and this is confirmatory. Goodrich's formula being designed for a restrained fall of 15 ft. for the monkey, the values are assumed to apply for that drop, and for a monkey of 2,000 lb. weight (corresponding, say, with a softwood pile 50 ft. long and weighing 40 lb. per ft.). Under such conditions Goodrich's formula for $\frac{1}{2}$ in. set gives $\mathbf{R} = 200{,}000$ lb.

Assuming somewhat less increase in friction per square foot for concrete piles and remembering the Wellington formulae for drop hammers and steam hammers, the author proposes the following formulae (which include their own factors of ignorance).

For drop hammers:

$$\frac{\mathbf{U}}{\mathbf{R}} = \frac{\mathbf{3}}{\mathbf{1} + \dfrac{\mathbf{R}}{\mathbf{50\sigma d}}} \quad \text{............(55)}$$

* *Transactions of the American Society of Civil Engineers*, Vol. 48, p. 180, August, 1902.

For S.A. steam hammers : $$\frac{U}{R} = \frac{3}{1 + \dfrac{R}{150\sigma d}} \quad \text{.........(56)}$$

For D.A. steam hammers : $$\frac{U}{R} = \frac{3}{1 + \dfrac{R}{250\sigma d}} \quad \text{.........(57)}$$

where σ inches is the perimeter of the pile and no value of $\frac{U}{R}$ is taken less than 1. The formulae should not be used unless it is known that the earth will cohere moderately well round the piles. Pending further investigation they could be used for (1) damp, but not saturated, fine and very fine sands, (2) soft clay, (3) soft clay and fine or medium sand mixtures, including stiff muds, and damp, but not saturated, silts ; but they would be inapplicable for (1) dry, and saturated sands of all sizes, (2) coarse sand, (3) gravel, (4) stiff and medium clays, (5) stiff and medium clays combined with sand or gravel. If the driving resistance is found for the first few blows after a period of rest, this resistance will be close to the bearing resistance, and the formulae are inapplicable. In stiff and medium clays, bearing resistance may be reduced by water percolating down the sides of piles.

Assuming a softwood pile as above, with $\mathbf{d} = 35$ ft. and $\sigma = 40$ in., formula (55) becomes

$$\frac{U}{R} = \frac{3s}{s + 1.43}$$

which is safer than the ratios $\frac{3.6s}{s + 1}$ given in the above table.

Where possible, tests should be made after a period of rest as well as at the end of driving, as the tests after rest more nearly give the bearing resistance, and a true relationship is then established between driving resistance and bearing resistance, and this may be applied to ordinary driving results.

(vii) *Measurement and effect of variables.*

A little care in measurement will have to be taken in finding the exact drop of the monkey for the test blows. The set per blow should be averaged for at least 5 blows ; and when testing after a period of rest two or three sets of 3 to 5 blows each should be taken to obtain an estimate of the set per blow for the *first blow* after rest. The impactive compression of dolly and cushion may require special means of measurement, and (as uneven impact over a struck surface corresponds with a non-restitutive compression) *the value of* c *should be taken not less than*

$$c = \text{measured compression} + 0.1 \text{ in.} \quad \text{.........(58)}$$

A heavy monkey is advantageous because :

(1) An increase in the weight of the monkey causes a greater proportional increase in the set, for the same maximum compression developed in the pile head and the larger sets can be more accurately measured.

(2) A heavy monkey involves less tensile stress in the pile during driving.

Where possible the weight of the monkey should be not less than 25 lb. per sq. in. of the pile cross section, irrespectively of the pile length. Very little is gained by increasing the weight over 30 lb. per sq. in.

Cushioning should not be much more than that necessary to keep the pile head from breaking, as a large proportion of lost energy for all sets consists of temporary compression losses. For heavy hammers and very hard driving, losses due to excessive cushioning are such that short piles suffer *less* set per blow than piles of somewhat greater length.

IV. Conclusion.

(*a*) *Likely accuracy of results.*—Errors in measurement of the conditions of driving may result in errors of about $\pm$ 15%. The author considers in view of the comparisons of formulae given in sections III (*c*) v and IV (*c*), and the assumptions made in the various methods, that his own method, including errors of measurement, should give driving resistances with errors not greater than about $\pm$ 30%, and would allow a factor of ignorance of $1\frac{1}{4}$ in any application of the method.

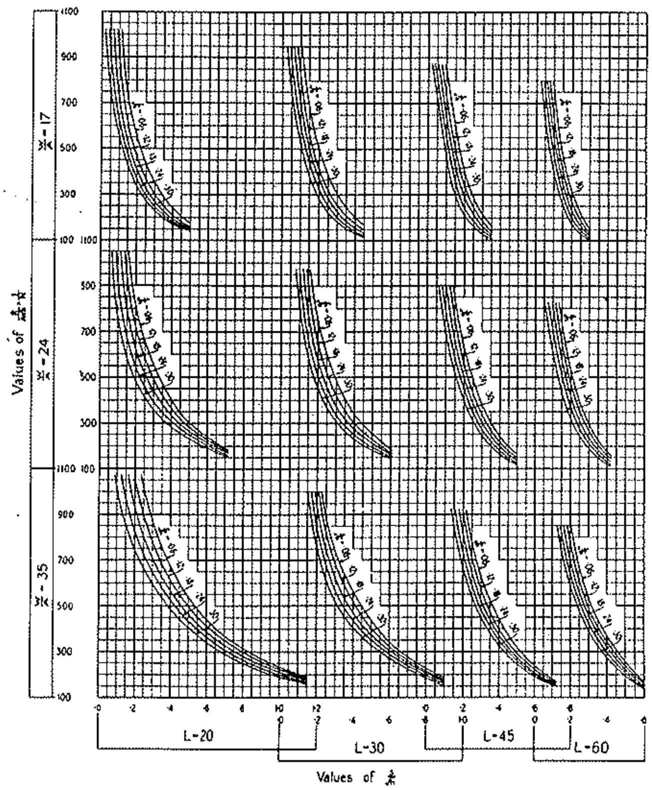


Fig. 17.—*Graphs giving Driving Resistances for given Sets per Blow.*

(*b*) *Summary of formulae, and illustrative example :*

The following formulae will enable all problems to be attacked. (See Figs. 13, 17, 18, and 19.)

(Section III (*b*) xix) $\mathbf{W} = \mathbf{M} + \mathbf{D}$ ....(59)

(Sections III (*b*) xix and I (*e*))

$$\mathbf{h} = 0.75\mathbf{H}\left(\frac{\mathbf{M}}{\mathbf{M} + \mathbf{D}}\right)^2 \quad \text{......(60)}$$

$$\mathbf{h} = 0.75H(\cos\theta - 0.1\sin\theta) \times \left(\frac{\mathbf{M}}{\mathbf{M} + \mathbf{D}}\right)^2 \quad \text{...........(61)}$$

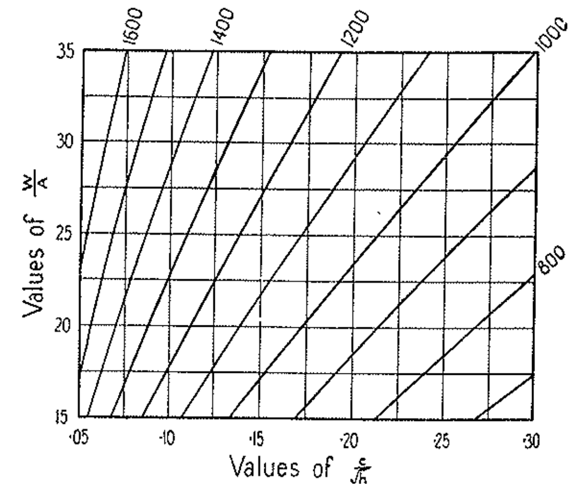


Fig. 18.—*Graph giving* $\dfrac{\textit{Compressive Stress in Pile Head during Driving}}{\sqrt{\mathbf{h}}}$

(Sections II (*b*) and III (*b*) xix) $\mathbf{h} = \text{Actual fall} \times \dfrac{1}{\mathbf{M}}\{\mathbf{M} + \text{Mean net total steam pressure on piston} - \text{Total friction}\} \times \times \left(\dfrac{\mathbf{M}}{\mathbf{M} + \mathbf{D}}\right)^2$ ....(62)

for D.A. steam hammers.

(Sections II (*b*) and and III (*b*) xix) $\mathbf{h} = \{\text{Actual fall} - 2 \text{ in.}\} \times \times \left(\dfrac{\mathbf{M}}{\mathbf{M} + \mathbf{D}}\right)^2$ ....(63)

for S.A. steam hammers.

(Section III (*b*) xix) $\mathbf{L} = L + \lambda$ ....(50)

(Section III (*b*) xx) $\mathbf{N} = \left(1 - 0.3\dfrac{\mathbf{d}}{L}\right)$ ....(53)

where the supporting strata are fairly uniform for the full depth of penetration.

(Section III (*b*) xx) $\mathbf{N} = \left(1 - 0.2\dfrac{\mathbf{d}}{L}\right)$ ....(54)

where the main supporting strata are in the lower half of the full depth of penetration.

(Section III (*c*) vi) $\dfrac{\mathbf{U}}{\mathbf{R}} = \dfrac{3}{1 + \dfrac{\mathbf{R}}{50\sigma\mathbf{d}}}$ ....(55)

for drop hammers.

(Section III (*c*) vi) $\dfrac{\mathbf{U}}{\mathbf{R}} = \dfrac{3}{1 + \dfrac{\mathbf{R}}{150\sigma\mathbf{d}}}$ ....(56)

for S.A. steam hammers.

(Section III (*c*) vi) $\dfrac{\mathbf{U}}{\mathbf{R}} = \dfrac{3}{1 + \dfrac{\mathbf{R}}{250\sigma\mathbf{d}}}$ ....(57)

for D.A. steam hammers.

(Section III (*c*) vii) $\mathbf{c}$ = Measured compression of dolly and cushion + 0.1 in..........(58)

$\mathbf{A}$ = Area of pile (sq. in.)
$\mathbf{D}$ = Weight of dolly (lb.)
$\mathbf{d}$ = Depth of pile toe below ground (ft.)
$\mathbf{H}$ = Restrained drop of monkey (ft.)
H = Restrained oblique drop of monkey in raking guides (ft.)
L = Length of pile (ft.)
λ = Length of timber striking piece (ft.)
$\mathbf{M}$ = Weight of monkey (lb.)
$\mathbf{R}$ = Driving resistance (lb.)
$\mathbf{s}$ = Set per blow (in.)
σ = Perimeter of pile (in.)
$\mathbf{U}$ = Bearing resistance (lb.)
θ = Inclination of raking guides to vertical.

The following problem illustrates the application of the above formulae.

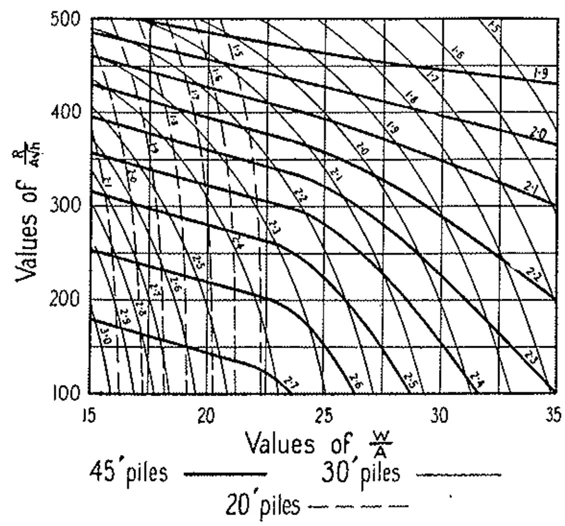


Fig. 19.—*Giving Percentage of Tensile Reinforcement* (*p*) *for Piles of* 3,000 *lb. per sq. in. Concrete.*
(*For other Strengths of Concrete the Percentages are proportional to the Strengths. For Piles longer than* 45 *ft., use Percentages as for* 45 *ft. Piles.*)

A 15 in. × 15 in. concrete pile, 40 ft. long, for a light highway bridge over a river is driven 34 ft. into ground known to be of rather soft soil for 20 ft. depth, with a thick layer of stiff clay beneath. A 5,400 lb. drop hammer worked direct from a friction winch with $7\frac{1}{2}$ ft. fall, gives a total set for 5 blows, of $2\frac{3}{4}$ inches. No record is kept of the compression of cushion and dolly, but the concrete in the pile head shows signs of weakness for the last blows, and is thought to be of a compressive strength of 3,000 lb. per sq. in. at the time of driving. The pile helmet weighs 450 lb., and has a timber striking piece 12 in. long.

From (59) $\mathbf{W} = 5{,}400 + 450 = 5{,}850$ (lb.)

$$\text{therefore } \frac{\mathbf{W}}{\mathbf{A}} = \frac{5{,}850}{225} = 26.0$$

From (60) $\mathbf{h} = \frac{3}{4} \times 7\frac{1}{2} \times \left(\dfrac{5{,}400}{5{,}850}\right)^2 = 4.80$ (ft.)

$$\text{and } \sqrt{\mathbf{h}} = 2.19$$

Assume maximum pile head stress equals 2,400 lb. per sq. in. (See section III (*c*) iii). Then

$$\frac{2{,}400}{\sqrt{\mathbf{h}}} = \frac{2{,}400}{2.19} = 1{,}100 \text{ (lb. per sq. in.)}$$

From Fig. 18, for 1,100 lb. per sq. in. and $\dfrac{W}{A} = 26.0$,

$$\frac{c}{\sqrt{h}} = 0.18.$$

The value of $\dfrac{s}{\sqrt{h}} = \dfrac{2\frac{3}{4}}{5} \times \dfrac{1}{2.19} = 0.25$

From (60), $L = 40 + 1 = 41$ (feet)

From Fig. 17, with $\dfrac{c}{\sqrt{h}} = 0.18$ and $\dfrac{s}{\sqrt{h}} = 0.25$

$$\text{for } \frac{W}{A} = 24, \text{ and } L = 30, \quad \frac{R}{A\sqrt{h}} \times \frac{1}{N} = 453$$

$$\text{for } \frac{W}{A} = 24, \text{ and } L = 45, \quad \text{,,} \quad = 387$$

$$\text{for } \frac{W}{A} = 35, \text{ and } L = 30, \quad \text{,,} \quad = 745$$

$$\text{for } \frac{W}{A} = 35, \text{ and } L = 45, \quad \text{,,} \quad = 645$$

From these values (and preferably others for $\dfrac{W}{A} = 17$, and $L = 60$) an interpolated value of 485 is found which is the value of $\dfrac{R}{A\sqrt{h}} \times \dfrac{1}{N}$ for $L = 41$, $\dfrac{W}{A} = 26.0$, $\dfrac{s}{\sqrt{h}} = 0.25$, and $\dfrac{c}{\sqrt{h}} = 0.18$.

For the conditions of driving $d = 34$ and formula (54) gives

$$N = \left(1 - 0.2 \times \frac{34}{40}\right) = 0.83$$

The value of the driving resistance is thus

$$R = 225 \times 2.19 \times 485 \times 0.83$$
$$= 198{,}000 \text{ lb.}$$

The bulk of the support given to the pile comes from the clay and hence formula (55) is inapplicable.

The following factors of ignorance are used
(1) for method of calculation $1\frac{1}{4}$
(2) for type of ground and loading $1\frac{1}{2}$
and a margin of safety of 2 is added.

Thus the factor of safety to be used is

$$1\tfrac{1}{4} \times 1\tfrac{1}{2} \times 2 = 3\tfrac{3}{4}$$

The safe bearing load of the pile is thus

$$\frac{198{,}000}{3\frac{3}{4}} = 52{,}800 \text{ lb.}$$
$$= 23\tfrac{1}{2} \text{ tons.}$$

(*c*) *Comparison with other formulae.*—A method similar to that given above was used to find results for Table V. *The author's modification of Hiley's formula in Table V consists of putting the compression of the pile capping (see sub-heading iv of section II (f)) as equal to the value found from Fig.* 18 *and calculating the energy lost from cushion compression independently. The remaining energy is then put equal to* Mh *in formula* (20), *and* C *of that formula is taken as due to Hiley's value of the pile compression alone (i.e.,* $1\frac{1}{2}$ *times the pile compression). The modification thus really consists of modifying Hiley's value of* C.

For the larger sets, the author's values agree roughly with the average values given by other formulae, and fairly closely with the results given by the modified Hiley formula. *For large sets outside the range of the graphs of Fig.* 17 *it is suggested that the modified Hiley formula be used, as examination of the wave forms for large sets inclines the writer to the opinion that the theory of this formula is then approximately correct.*

It is noted from the following results that most of the formulae are conservative for the smaller sets.

TABLE V.

16 in. × *16 in. Concrete Piles. Loads in tons for Factor of Safety,* 4, N *taken* 0.85, *and* D 400 *lb.*

Method or formula	2 ton monkey 7 ft. free fall				4 ton monkey 4 ft. free fall			
	Length 30 ft.		Length 60 ft.		Length 30 ft.		Length 60 ft.	
	Set ⅜ in.	Set 1 in.	Set ¼ in.	Set ¾ in.	Set ⅜ in.	Set 1 in.	Set ⅜ in.	Set 1 in.
Author's (3,000 lb. concrete)	33.4	10.8	36.6	8.5	48.9	17.8	36.3	9.8
Author's (2,500 lb. concrete)	28.5	9.7	30.5	7.6	39.5	16.1	30.5	9.2
Author's modification of Hiley (3,000 lb. concrete)		9.3		7.4		17.6		11.8
Author's modification of Hiley (2,500 lb. concrete)		8.7		6.9		15.9		10.7
Hiley	29½	14	22	10½	43½	22½	29	15
"Hiley ½"	24½	13½	17½	10	35	21	23½	14
Brix	25½	9½	28½	9½	32	12	29	11
Eytelwein	39	15	36	12	66	25	45	17
Author's applying formula (55) (3,000 lb. concrete)	33.4	13.8	36.6	16.7	48.9	17.8	36.3	18.3
Author's applying formula (55) (2,500 lb. concrete)	28.5	12.6	31.5	15.5	39.5	16.1	31.5	17.5
Wellington	30½	22	33½	24	35	24	35	24
Gow	19	15	11	9½	36½	25	22	17

Space will not permit a discussion of the apparent correctness of the new theory when applied to timber piles, but it can be shown that an approximation to the Goodrich formula may be obtained. Goodrich's experimental results for the exact movement of the pile head and hammer base under a blow can be explained by the theory. A certain periodic movement noted in these results can be seen to coincide with the time taken for a wave to complete a cycle in the pile, and the total observed time during which the pile moves (averaging about $\frac{1}{30}$th second) appears in agreement with the theory.

It may be objected that the author's method cannot be applied as rapidly as, for instance, the Wellington formula; but days may be spent in obtaining results by a static loading test. *The few minutes extra which may be required to apply the author's method to piles is well spent when it is remembered that many variables are allowed for, and any of these separately may largely influence the final result.*

The new method cannot yet be taken as definitely giving all that is desired. Many of the points demonstrated give food for thought, and much investigation is still required. Particularly is this so in regard to the relationship between driving resistance and bearing resistance for various classes of ground, and the correlation of load tests with pile formulae. Tests of long piles in various classes of ground will do much towards defining the losses due to propagation and clinging earth; and an endeavour should be made to determine whether there is any factor which reduces the tensile stress in driving below the values already calculated. Further research may indicate an empirical solution. Present knowledge, however, if intelligently applied can be very useful, and in certain classes of ground may serve nearly as well as a static load test.

The author wishes to thank Mr. W. D. Chapman, M.C.E., M.I.E.Aust., for information which led him to make the investigations given above, and also for most willingly reading through the manuscript of the paper.

Discussions & Communications.

HIGH EARTHEN DAMS.

With Particular Relationship to the Silvan Dam for Melbourne Water Supply.

By E. G. Ritchie.*

Reply by the Author to discussion.—Interesting contributions by way of discussion of the above paper having been received and published in The Journal,† the author desires to answer questions and criticisms.

Mr. W. A. Robertson has made a valuable contribution by way of discussion. The author agrees with him that an articulated concrete slab paving may in many cases be used with advantage on the upstream slope of an embankment.

If the embankment be composed entirely of clean hard rocky material not readily disturbed or disintegrated by water percolations, a pavement on slope would be distinctly preferable to a central core wall.

Mr. Robertson has, however, voiced one great objection to this form of construction, viz., insufficient flexibility to insure against cracking under settlement conditions. In the author's judgment, this objection has strong application where the embankment is of clay or earth formation, as at Silvan. The paving is liable to be cracked not only by settlement, but by thermal movements. Caving is likely to occur partly as a result of subsidences beneath the concrete layer, and also as a result of wave action. If such a paving were not perfectly watertight, or if it became dislodged, the consequences would not be serious in the case of a properly designed rock fill dam of hard material, owing to the free draining nature of the material supporting the paving.

The author would not care to rely on such a paving for watertightness, however carefully articulated. Where watertightness is important, he considers the central core wall has marked superiority for use in an embankment of earth or clay. Not only is the core wall removed from the influences referred to, but it has the advantage of dense material on the waterside to impede any flow of water which may penetrate cracks or other imperfections in the concrete wall.

No matter how carefully and thoroughly an embankment is consolidated, there is always vertical settlement. This is much larger than horizontal movement in any well-designed work. The inclined paving would be subjected to all this vertical movement, while the conventional core wall if founded, as it ought to be, on an unyielding foundation would be subject only to horizontal movement.

The dangers of slipping of upstream embankment, as referred to by Mr. Robertson, are not often found in the case of properly consolidated earthen embankments, where suitable materials and conservative slopes are used on a reasonably good foundation. The lowering of water level in a large reservoir, such as that at Silvan, is a very gradual process, not amounting to more than an inch or two a day and there is ample time for the saturated material to drain out under these conditions.

One most necessary objective in the use of a central concrete core wall is dense consolidation of the downstream filling to support the pressure from the upstream side. This is where loose rock fill fails in comparison with earth. Not only is it incapable of proper compaction, particularly in the upper parts of the embankment, but, on account of voids, its average weight is only some 70% of that of consolidated earth. The liability of rock fill to yield by sliding on the base is therefore greater than that of earth, where dimensions are similar.

Information being asked re foundation conditions and difficulties, it may be stated that the core wall for its whole length was founded on solid stone into which excavations were carried at least 5 feet in depth and in places deeper. Where the bottom of the trench showed seamy rock, which it was thought advisable to grout, holes were drilled in the foundations and piped. Afterwards cement grout was pumped in to refusal at 100 lb. per sq. inch.

Water was usually met with and dealt with as follows :—

A 4 ft. × 3 ft. sump was excavated on the downstream side of the core wall about 4 feet below bottom level. Leaks from the rock were piped into the sump and water pumped from the sump during concreting, the water level in the sump being kept below concrete level until the concrete had set.

This sump was brought up together with the concrete pourings until the level of ground water was reached. As the level of the concrete was brought up, the sump was filled with packed stone. Leak pipes were carried up through sump and concrete wall to surface level for purposes of grouting. The packed stone was finally grouted.

In reply to Mr. East's question regarding the regulator valve, a recent examination has disclosed that, after three years of continuous use, there was not the slightest sign of wear or cavitation on either body of valve or sector gate. During the whole of the three year period, the gate was operated at partial opening, discharging freely into air and under 90 to 98 feet head.

Mr. Sanders has advocated the use of dams without core walls. The author does not desire to add to the observations in his paper dealing with this form of construction and points out that conventional practice is, except perhaps in India, overwhelmingly on the side of using a core wall.

The author desires to acknowledge the valuable contribution to the discussion by Mr. A. E. Kelso, who has shown the importance of consolidation and the desirability of regular tests to determine the density and percentage of moisture of compacted filling. He has also emphasised the great variability of borrow pit materials. This was one of the considerations mentioned by the author in his paper, when expressing some opposition to the practice of dispensing with a core wall.

Mr. E. D. Shaw has, like Mr. Robertson, espoused the use of concrete paving on the upstream slope in preference to the use of a core wall. The author's reply to Mr. Robertson covers the points raised. He desires to emphasise the fact that the conventional central core wall is protected from variations in temperature, whereas concrete paving on the upstream face would be subject to the full range of temperature between freezing and hot sunshine. The fact of being alternately wet and dry would assist the various other causes bringing about movement, with liability to cracking and displacement of paving. On low dams, such as those quoted by Mr. Shaw, there would be less danger than in the case of a high dam, such as Silvan.

Mr. Shaw states that very extensive tests have been carried out at Hume Dam with the object of measuring the yield of a concrete core wall supported on the points of a layer of stone, and that no yield took place under full working load.

If no yield took place, the simplest explanation is probably that the horizontal pressure did not act on the core wall. It is difficult to see how it could, since the dam has so far had only a minor depth of impounded water opposed to it. It would add to the value of Mr. Shaw's statement if he would supply details of how the "full working load" on the core wall was measured.

The concrete drainage slabs now being employed at Hume, details of which are given by Mr. Shaw, are in the author's judgment distinctly superior to the stone and gravel drainage layer formerly used. They are really an application of the cellular form of construction as used at Silvan.

The author, in conclusion, again thanks all those who have contributed to the discussion.

*See The Journal, Vol. 2, No. 12, December, 1930, p. 455, for text of paper.

†See The Journal, Vol. 3, No. 4, April, 1931, p. 133, and Vol. 3, No. 7, July, 1931, p. 245, for discussion.

CONTRIBUTED ARTICLES.

Members are reminded that the section of The Journal headed *Other Matters of Technical Interest* is included with the object of providing facilities for the publication of articles of general engineering interest, i.e., other than those activities described in the papers which may be presented before the Institution.

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Contributed articles of the nature referred to above may be forwarded direct to Headquarters, or transmitted through Division Secretaries or Division Correspondents for submission to the Editor.