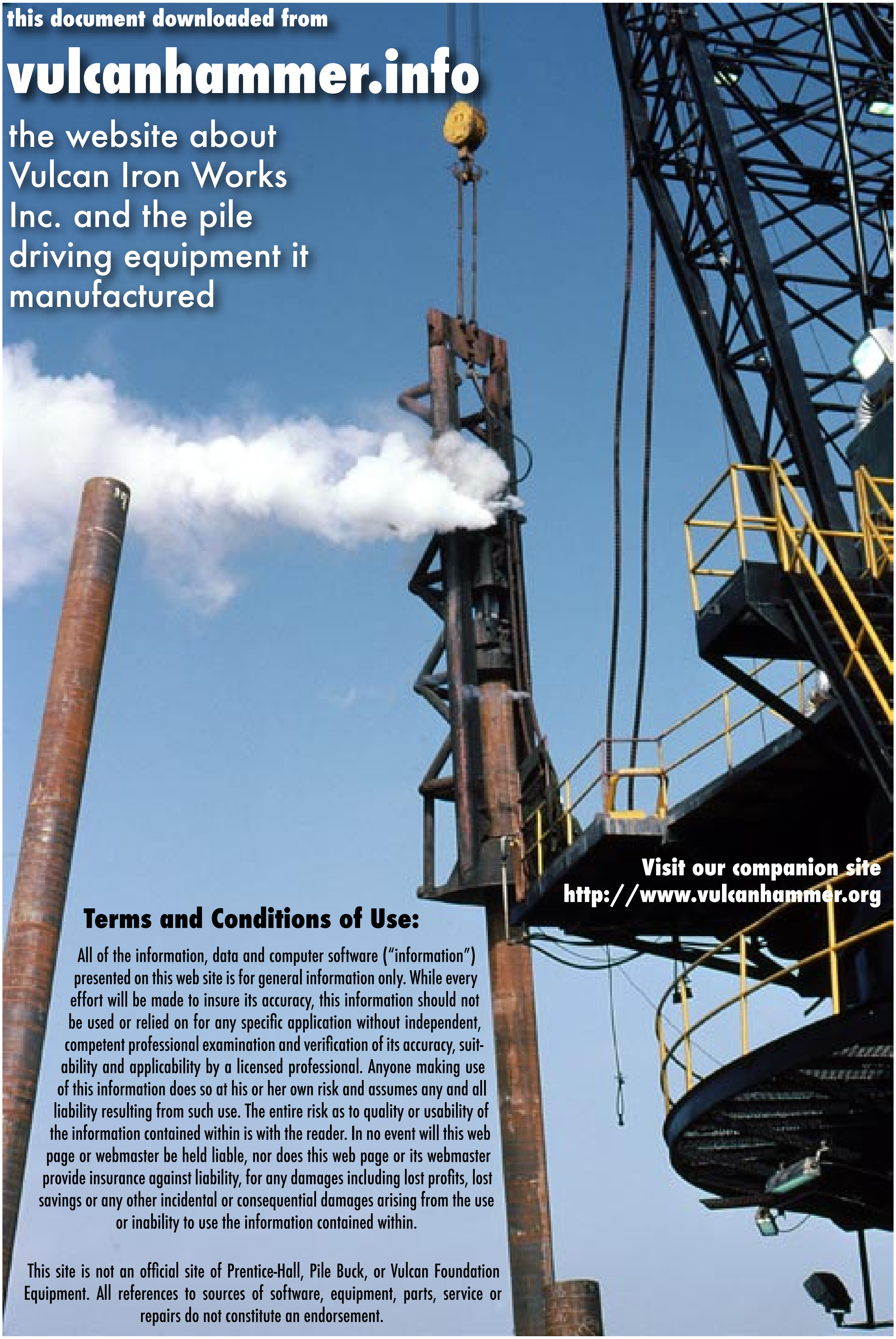

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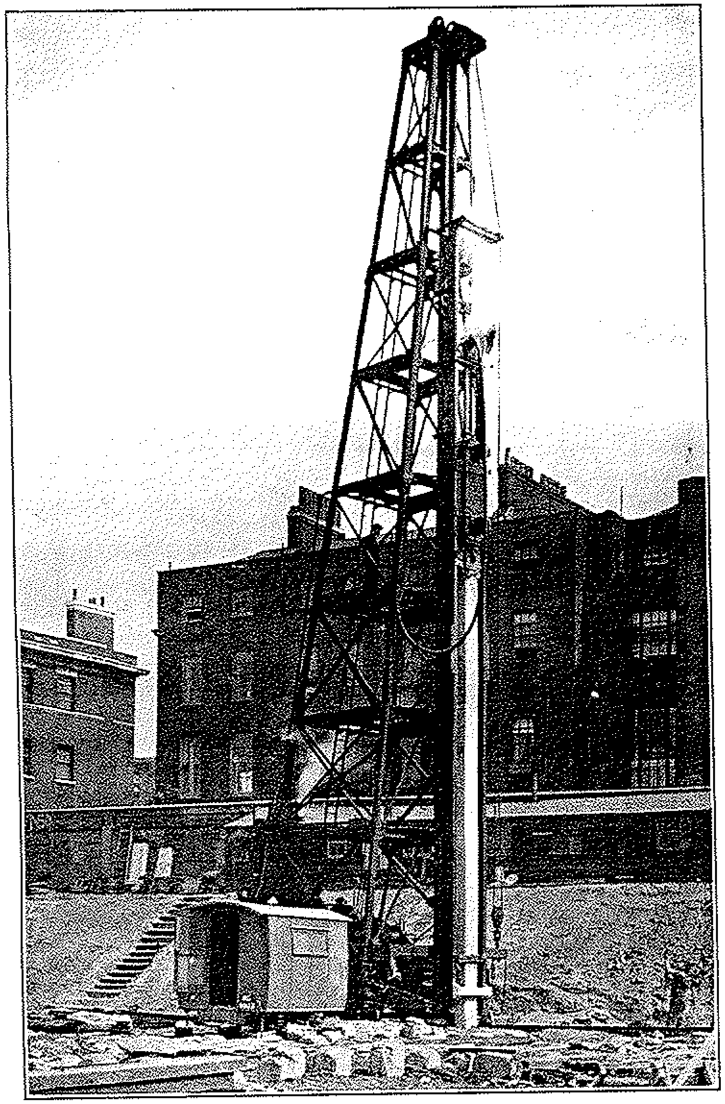

Pile driving test in progress.

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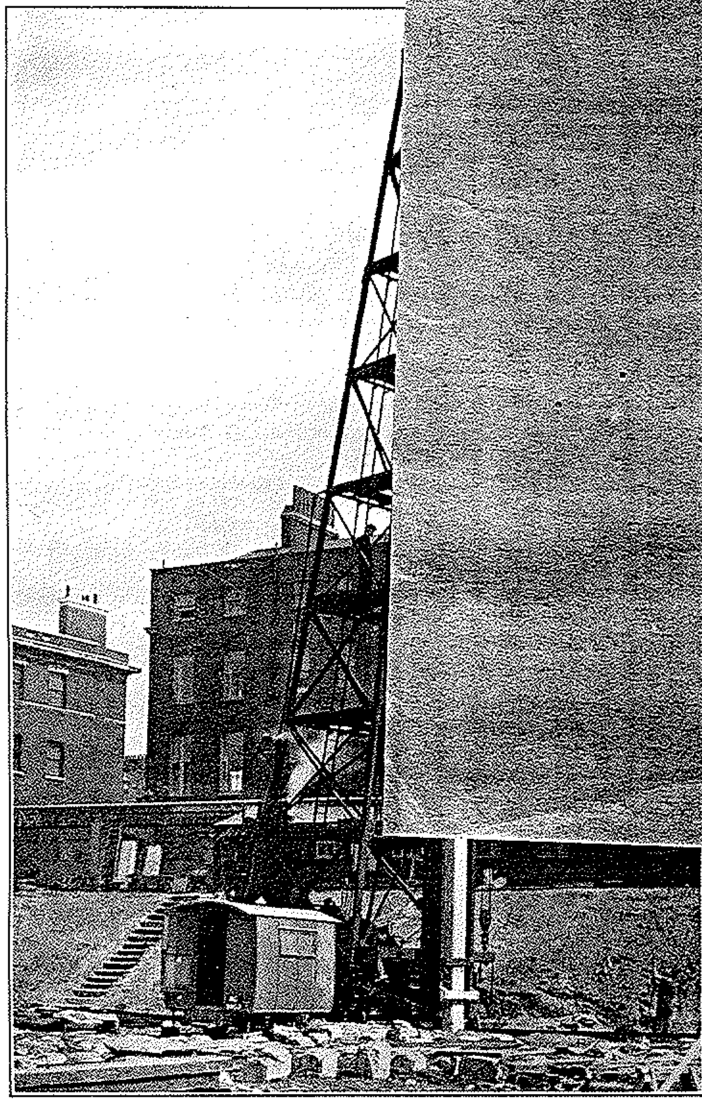

Pile driving test in progress.

DEPARTMENT OF SCIENTIFIC AND INDUSTRIAL RESEARCH

BUILDING RESEARCH

TECHNICAL PAPER No. 20

AN INVESTIGATION OF THE STRESSES IN REINFORCED CONCRETE PILES DURING DRIVING

BY

W. H. GLANVILLE, D.Sc., Ph.D., M.Inst.C.E., G. GRIME, M.Sc.,
E. N. FOX, M.A., B.Sc., Ph.D., and W. W. DAVIES, B.Sc.,
Assoc.M.Inst.C.E.

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PREFATORY NOTE

FOR many years engineers concerned with pile driving have recognised the unsatisfactory state of our knowledge of the stresses set up in reinforced concrete piles while they are being driven. Serious difficulties arose under certain conditions of hard driving, where it was found impossible to comply with certain specifications and at the same time to avoid serious damage or the possibility of serious damage to the piles. Instances had occurred where failure took place not only at the head, but at the foot of the pile and without any indication above ground.

The problem was of such urgency that the Federation of Civil Engineering Contractors asked the Building Research Station to collaborate in an investigation of the subject. Research on the problem was begun at the Station in 1932 and is now being continued with the co-operation of the Institution of Civil Engineers. A brief account of the work has already appeared in the Journal of the Institution.

The outcome of these investigations has been the formulation of a theory that explains the phenomena observed and which has made it possible to construct charts for the guidance of engineers in safeguarding the pile from excessive driving stresses. A general understanding of this theory will be invaluable in diagnosing the causes of failure when such occurs. Furthermore, the work has led to definite recommendations concerning the manufacture, handling and driving of reinforced concrete piles.

The investigations still in hand relate in the main to the production of an improved head cushion, and on simple methods of measuring the maximum head stress.

The progress of the research has been greatly assisted by the readiness of engineers and contractors to grant facilities for tests and by their helpful advice and criticism; in particular, thanks are due to Mr. G. M. Burt, Assoc.Inst.C.E., Mr. A. Hiley, Assoc.M.Inst.C.E., Mr. W. L. Scott, M.Inst.C.E., Mr. H. E. Steinberg, M.Inst.C.E., Mr. R. Travers Morgan, M.Eng., M.Inst.C.E., Messrs. Concrete Piling Ltd., Messrs. Samuel Williams & Sons, and the British Steel Piling Co.

R. E. STRADLING,
Director of Building Research.

Building Research Station,
Garston, Herts.

December, 1937.

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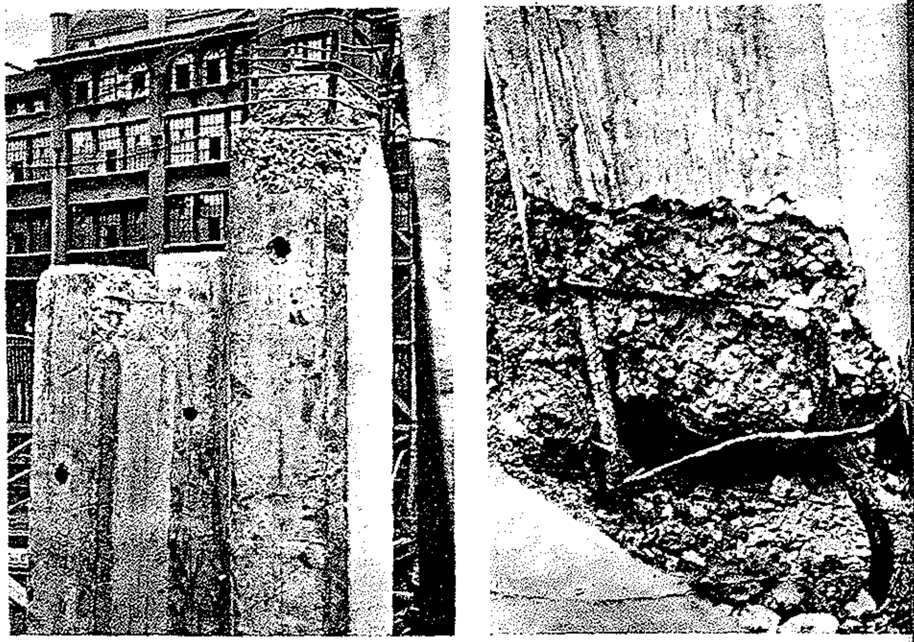


FIGURE 1.—*Examples of Failure in Reinforced Concrete Piles.*

AN INVESTIGATION OF THE STRESSES IN REINFORCED CONCRETE PILES DURING DRIVING

INTRODUCTION

THE immediate purpose of the investigation into the behaviour of reinforced concrete piles during driving, which is described in the present paper, was to ascertain the causes of driving failure and to discover methods for preventing damage. The work was carried out at the Building Research Station with the collaboration of the Federation of Civil Engineering Contractors, and was initiated as a direct result of difficulties experienced while driving piles through hard strata.

In and around London there are many building sites where the ground consists of alluvial or made-up soil of very low bearing power for perhaps 10 feet to 30 feet from the surface. Below this a stratum of hard, compact gravel is to be found varying in thickness from perhaps a foot or two to 20 feet, a variation of this order occurring over any one site. Below the gravel a stratum of comparatively soft earth of low bearing power is again found and at a greater depth still a hard compact clay is reached. In designing structures for such sites, the engineers, owing to the uncertain thickness of the gravel belt, have, in many cases, thought it advisable to found below the gravel on the hard clay. In penetrating the gravel very hard driving conditions are experienced, and it had been found difficult to construct pre-cast piles of sufficient strength to withstand the very severe conditions. Typical examples of failures which had occurred are shown in Figs. 1, 2 and 3. Information as to the effect of driving conditions on the behaviour of a pile was scanty, and what was available was not correlated on the basis of a rational theory. For instance, no recognised standards existed for determining the correct weight of hammer, height of drop or amount of head packing for a given pile, and there was no method of estimating the effect of the conditions at the foot on the stresses in the pile.

Empirical rules based upon experience provided a rough guide for practical use, but could not be relied upon to indicate, for any known set of conditions, whether troubles would occur or not, and the only method of deciding whether piles would stand up to specified conditions was by actual driving.

Consequently, the main problems of the investigation were to devise means of estimating the impact stresses, due to specified driving conditions, and the impact-resistance of the concrete. This involved first an examination, both analytical and experimental, of the nature and magnitude of the stresses induced in piles by the impact of the hammer, and of the extent to which these stresses were modified by changes in driving conditions; second an examination of the effect upon impact-resistance of the different factors controlling the design and manufacture of the pile itself and third the development of methods of estimating and measuring the stresses during driving.

The examination of driving stresses and their dependence on driving conditions was carried out by the use of a piezo-electric strain recorder, yielding photographic records on a time basis of the strain in the concrete. The mathematical theory, aided by the strain records, enabled charts to be produced from which the driving stresses effective in producing damage, may be estimated. A simple instrument, the peak stress indicator, has been devised, which, when attached to the hammer, enables the maximum value of the compressive stress at the head to be measured. The effect upon

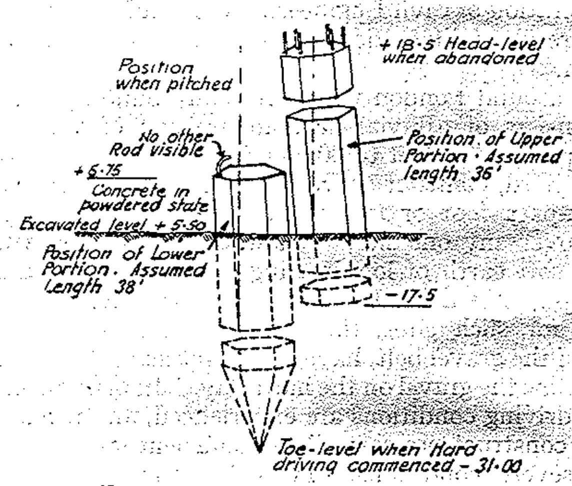


FIGURE 2.—*Failure at centre of Pile.*

impact-resistance of the factors of design and manufacture were investigated by testing short reinforced concrete piles to destruction and by impact tests on small specimens of plain concrete.

By combining the findings of the tests on impact-resistance with the charts for driving stresses, a simple method was evolved for determining the maximum safe height of drop of the hammer for concrete of known cube strength, and for estimating when the lower portion of the pile is endangered.

OUTLINE OF MATHEMATICAL THEORY

Early mathematical work on pile driving, being chiefly concerned with the quest for a universally applicable pile bearing formula was based on the assumption of the instantaneous propagation of stress throughout the pile. It is, of course, well known that a stress

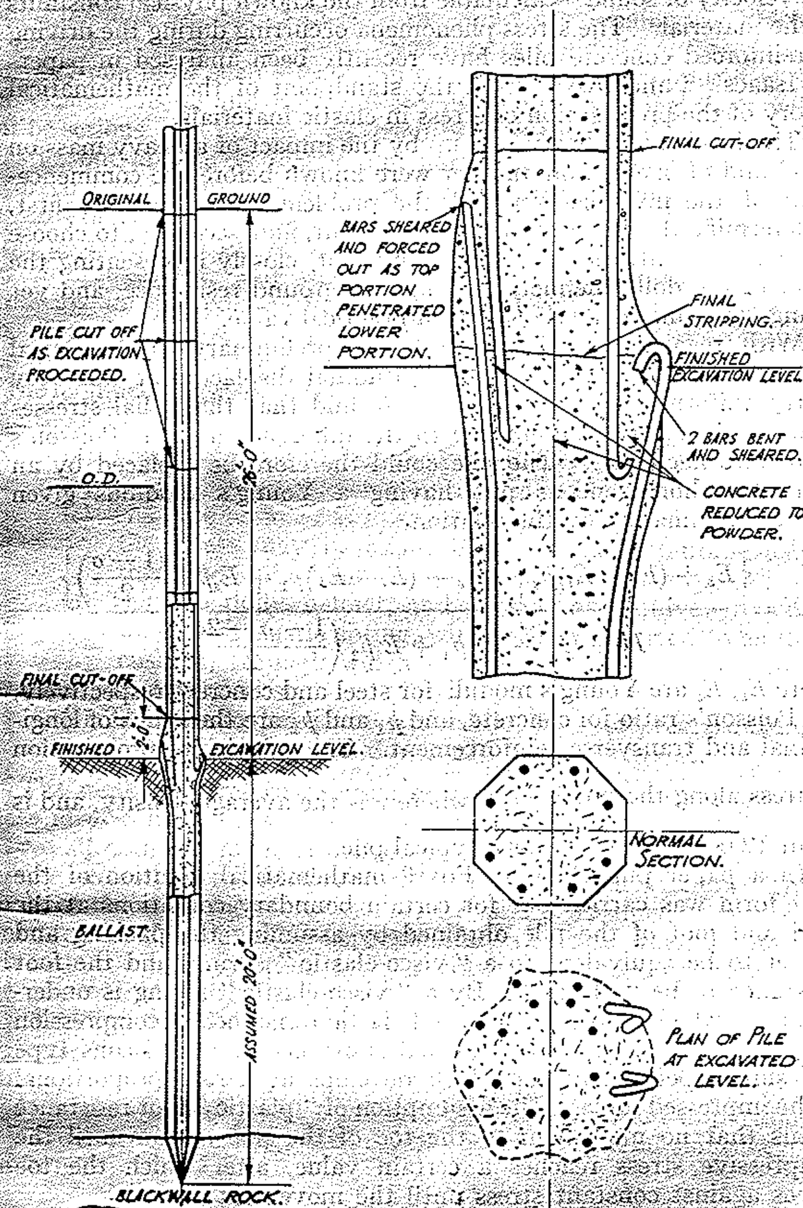


FIGURE 3.—*Failure of the lower portion of the pile.*

wave travels through an elastic medium at a definite velocity—the velocity of sound—calculable from the known physical constants of the material. The stress phenomena occurring during the driving of reinforced concrete piles have recently been analysed in papers by Isaacs[(1)]* and Fox[(2)] from the standpoint of the mathematical theory of the propagation of stress in elastic materials.

The forms of the waves set up by the impact of a heavy mass on a rod and of one rod on another were known before the commencement of the investigation and the problem therefore was, first, to determine the effect of the reinforcement, and, secondly, to choose boundary conditions at the head and foot, closely representing the effect of the dolly, helmet, packing and ground resistance, and yet giving mathematical expressions of practical value.

With regard to the reinforcement, preliminary mathematical investigation showed that the longitudinal displacement was very nearly uniform over the cross-section, and that the radial stresses between steel and concrete due to the difference in their Poisson's ratio were negligible. The pile could therefore be replaced by an equivalent homogeneous pile having a Young's modulus given very approximately by the relation

$$E = \frac{\left\{E_c + (E_s - E_c)p_1\right\}\left\{E_c + (E_s - E_c)p_1 + E_s p_2\left(\frac{(1-\sigma)}{2}\right)\right\}}{E_c + (E_s - E_c)p_1 + E_s p_2\left(\frac{1-\sigma-2\sigma^2}{2}\right)}$$

where E_s, E_c are Young's moduli for steel and concrete respectively, σ is Poisson's ratio for concrete, and p_1 and p_2 are the ratios of longitudinal and transverse reinforcement. The velocity of propagation of stress along the pile is $\sqrt{\frac{E}{\rho}}$, where ρ is the average density, and is about 12,000 ft. per sec. for a typical pile.

In a paper published by Fox[(3)] mathematical solution of the wave-form was carried out for certain boundary conditions at the head and foot of the pile obtained by assuming the packing and helmet to be equivalent to a "visco-elastic" spring, and the foot resistance to be "plastic." By a "visco-elastic" spring is understood one which possesses the usual elastic resistance to compression and, in addition, exerts a frictional resistance of the viscous type and suffers permanent set which develops at a rate proportional to the impressed force. The assumption of "plastic" foot resistance means that no movement of the toe of the pile occurs until the compressive stress reaches a certain value, after which the toe moves against constant stress until the movement ceases.

When the results of preliminary experimental work had been examined it was found desirable to simplify the assumptions concerning the conditions at the head and foot, in order to express the solution in terms of quantities measurable in practice. It was not

* The numbers in brackets relate to the list of references on page 87.

found possible to assign values to the constants defining the effects of permanent set and frictional resistance in the dolly and packing. For any one blow such effects will usually be small compared with the elastic effects, and the error involved by neglecting them will not be serious. In practice, the conditions at the foot are not purely plastic, but are partly plastic and partly elastic. For conditions of driving where the foot stress is likely to become important, i.e., in hard driving, the resistance is concentrated at the foot, and a large proportion of the set is elastic. An elastic foot condition was therefore assumed, and when applying the theory to practical problems an approximate formula was used equating the observed plastic and elastic sets to an equivalent purely elastic set to which the theory could be applied direct. The following is a brief outline of the modified theory. A full treatment will be found in Appendix I. The theory is based on the following assumptions :—

(a) That the pile is undamaged when driven.
(b) That the pile behaves as an elastic rod.
(c) That the hammer is so short compared with the pile that the stress may be assumed to spread instantaneously throughout the hammer.
(d) That the dolly, helmet, and packing are equivalent to a weightless elastic spring, which will be referred to as the cushion, through which the compression is propagated instantaneously.
(e) That the foot resistance is elastic, the foot pressure being proportional to the downward foot movement.

The method of relating actual cushions and foot resistances to these ideal conditions will be given later.

No allowance is made either for propagation loss, that is, loss of energy due to internal friction in the pile, or for skin friction. The whole of the resistance to motion of the pile is assumed to be concentrated at the toe, as is approximately the case when a pile is driven into a hard stratum, such as gravel, close to the surface of the ground.

The frictional resistance in the hammer guides has also been neglected in the theory; a correction for this factor is introduced later.

It is shown in physical textbooks that the equation applicable to wave motion in a long, thin elastic rod has a general solution of the form

$$\xi = f\left(t - \frac{x}{a}\right) + F\left(t + \frac{x}{a}\right)$$

where ξ = displacement of cross-section from initial position,
t = time (after beginning of impact in this case),
x = Co-ordinate of any cross-section measured from one end (the head of the pile),
a = velocity of longitudinal waves in the rod.

The equation states that the displacement of any cross-secti is obtained by adding together two functions, the first of whi $f\left(t-\frac{x}{a}\right)$, represents a wave travelling down and the second, $F\left(t+\right.$ a wave travelling up the pile.

It is obvious that there will be no wave in the upward directi until a time $\frac{l}{a}$, where l is the length of the pile, has elapsed, that until the reflection has taken place from the foot, and that this wa will not reach the head until an interval of time equal to $\frac{2l}{a}$ has passe

The equation which holds for the motion of the hammer duri the initial period, $0 \leqslant t \leqslant \frac{2l}{a}$ before the reflected wave arrives at t head, has a simple solution, from which, with the aid of the assum head conditions, the displacement function f, that is, the dow travelling wave, can be determined for that interval. From t assumption as to the character of the foot resistance, the function representing the reflected wave travelling up the pile, can then expressed in terms of the function f for a time $\frac{2l}{a}$ earlier. Henc since f is known for the interval $0 \leqslant t \leqslant \frac{2l}{a}$, F will be known for t interval $\frac{2l}{a} \leqslant t \leqslant \frac{4l}{a}$. This procedure, applied to successive interva $\frac{2l}{a}$, determines all waves travelling up and down the pile at a selected moment. By combining waves at any point the displac ment, and therefore the stress, may be found at any desired time

Figure 4 has been drawn to illustrate the way in which t characteristics of the stress-time curves may be determined with t aid of the theory. The head, middle, and foot pressures in t concrete have been plotted against time for foot conditions decreasi in hardness from a fixed foot (elastic set = 0) to a free foot (elast set = ∞). Figure 4 is applicable to a pile 14 in. by 14 in. by 40 driven with a 3-ton hammer and a soft head cushion, or to any p and hammer for which $M/M_p = 0{\cdot}8$, and $kl/EA = 1$. (The symb are explained later, p. 88).

The curves have several interesting features which are n restricted to the particular head condition chosen and are comm to a wide range of practical head conditions.

It will be noted that the time scales have different zeros for t head, middle and foot of the pile, due to the finite velocity of prop gation, a, of the stress wave from the head, and that a point dista x from the head is unaffected by the impact until a time $\frac{x}{a}$ has elapse

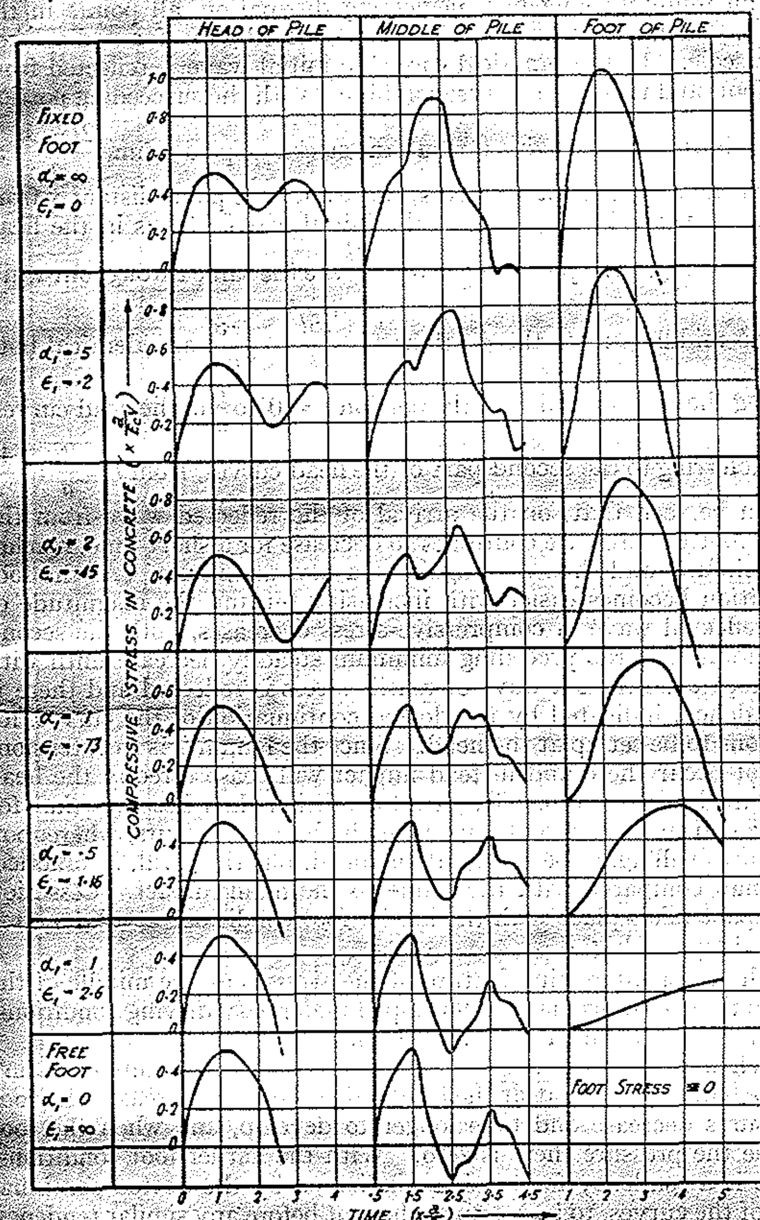


FIGURE 4.—*Compressive Stresses in Concrete* $\left(\times \frac{a}{E_cV}\right)$ *v. Time* $\left(\times \frac{a}{l}\right)$ *when* $\frac{M}{M_p} = \cdot 8$, $\frac{kl}{EA} = 1$.

(For the Symbols see p. 88)

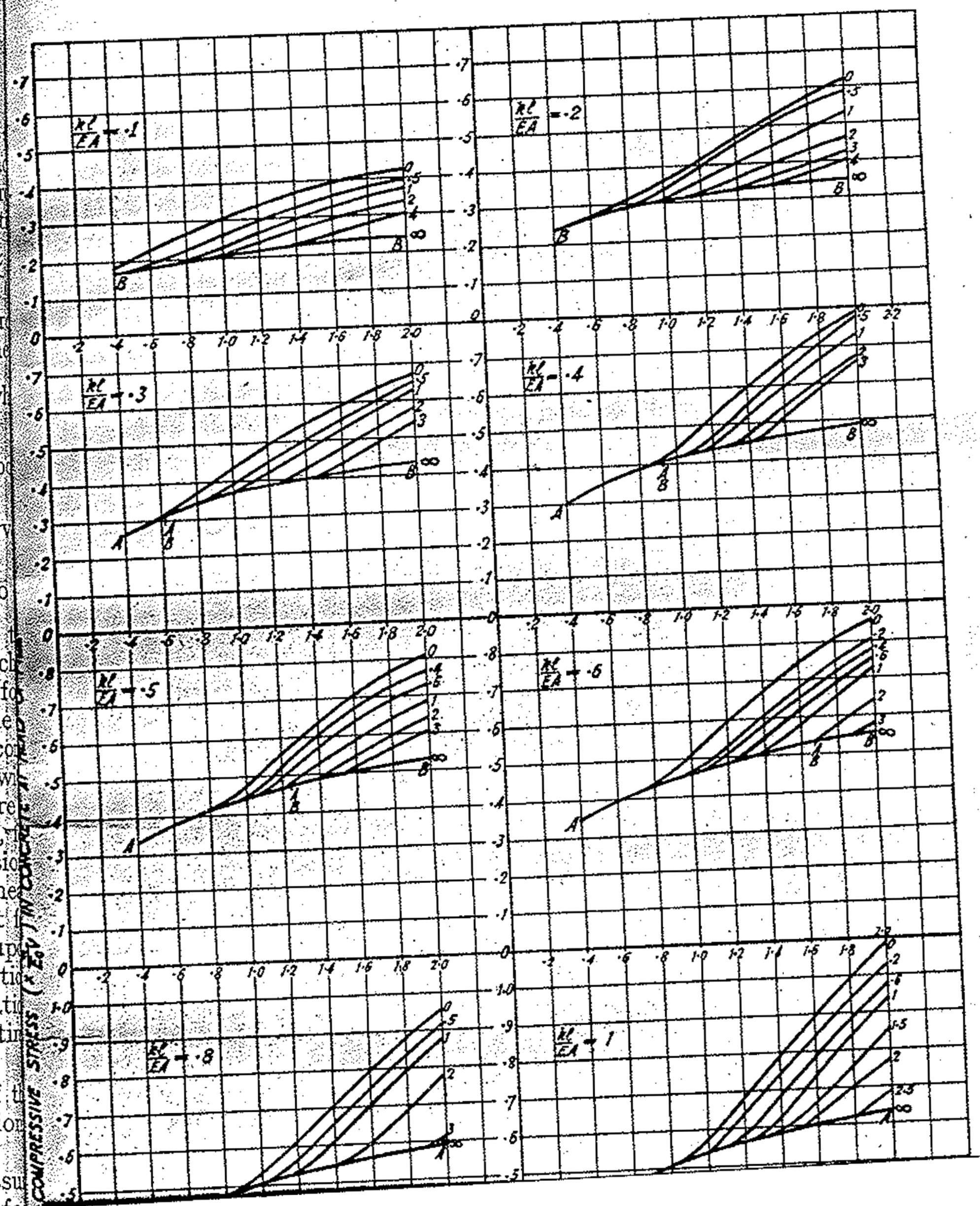
For an infinitely long pile, neglecting dissipation, all points in
pile would have the same stress-time record save for a displa
time scale. In the practical case, the initial wave is reflected fr
the foot, and travels upwards, combining with the succeeding port
of the initial wave to give a resultant stress. Up to a time $\frac{2l-x}{a}$
which the reflected wave arrives, the stress at a point distant x fr
the head will still be given by the initial wave. Thus in the h
curves of Fig. 4 the portions $t=0$ to $\frac{2l}{a}$ are the same throughout wh
in the middle curves the portions $t=\frac{.5l}{a}$ to $\frac{1\cdot5l}{a}$ are identical b
among themselves and with the portion $t=0$ to $\frac{l}{a}$ of the head curv

Referring to the second part of the head curves, from $t=\frac{2l}{a}$ to
it will be seen that, on the arrival of the reflected wave from t
fixed foot the pressure continues to decrease for a short time, reach
a minimum, and then rises again to a new maximum. As the fo
condition becomes easier with increasing set and the magnitude
the reflected wave of compressive stress decreases, both the seco
maximum and the preceding minimum steadily decrease until w
soft foot conditions a wave of tensile stress is reflected and there
a tendency, indicated by the dotted continuation of the curves, f
tensions to be set up at the head. Since the hammer is free, tensio
cannot occur there and instead impact will cease directly the he
pressure becomes zero. The head condition then becomes that
a free end until a second impact of the hammer occurs, whereup
pressures will again be set up at the head, but they will, in practic
be small compared with those due to the main impact. Cessati
of impact takes place with the harder foot conditions at a later tir
than shown in Fig. 4.

The most interesting feature of the stresses at the middle of th
pile is that tensions may be developed under easy driving conditio
i.e., large sets as in the two lowest curves.

When no movement of the foot takes place the maximum pressu
at the foot is twice that at the head but as the set increases the fo
pressures decrease and take longer to develop, and when the fo
is free the pressure there is zero. With the harder foot condition
there is a tendency for tensions, indicated by the dotted continu
tion of the curves, to be set up at the foot before any similar tendenc
is exhibited at the head, which means physically that the pile wi
rebound from the ground before the hammer rebounds from the pil

Finally, it will be seen that the maximum pressure occurs at th
head under easy driving conditions but that as the resistance at th
foot becomes greater the foot pressure increases, and for very har
driving the maximum pressure occurs at the foot.



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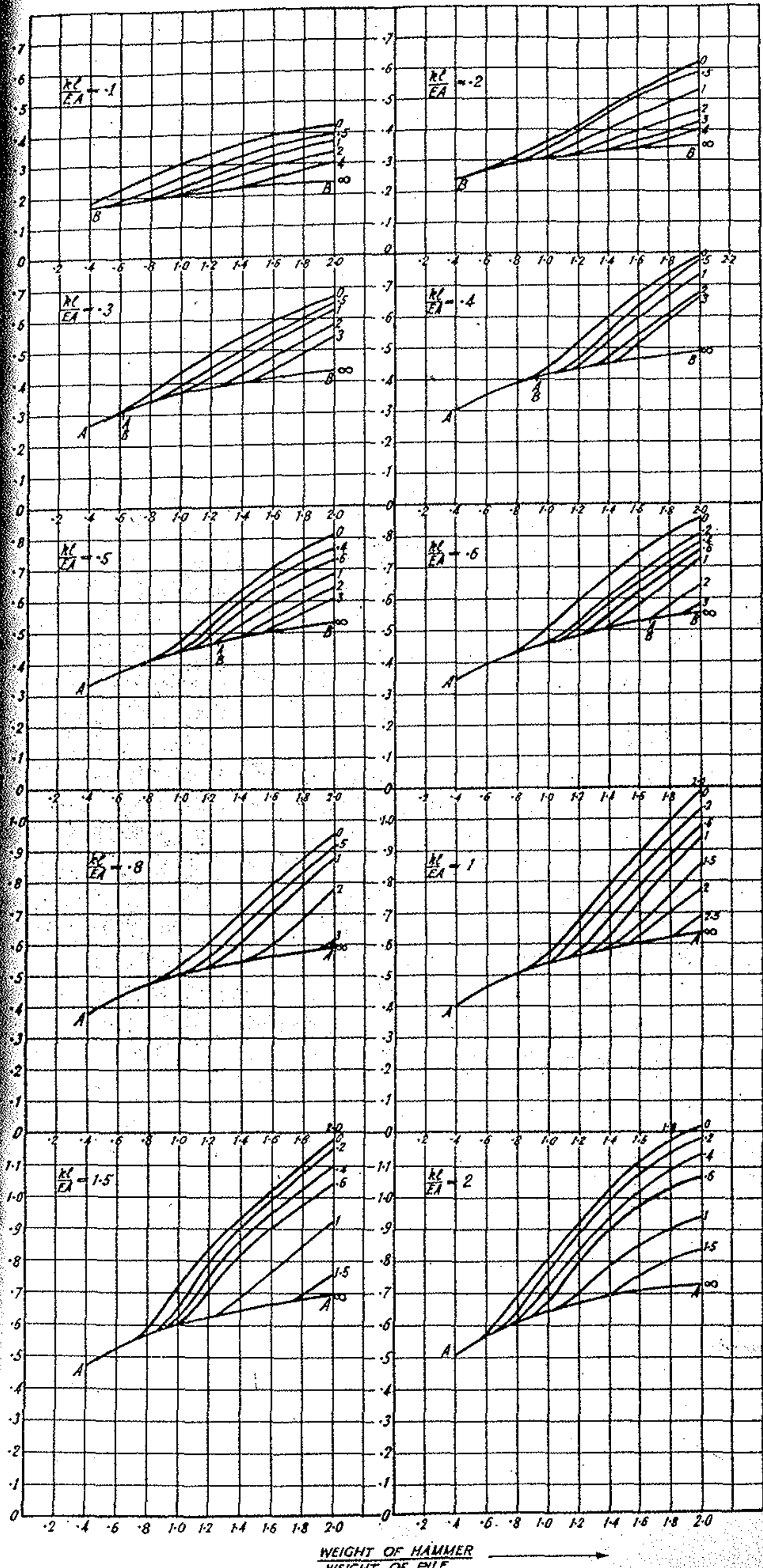

FIG. 5. MAXIMUM COMPRESSIVE STRESS $(\times \frac{a}{E_c V})$ IN CONCRETE AT HEAD OF PILE.
(Numbers to Curves give values of ϵ, constant along any one curve).

rts for the Determination of the Maximum Compressive Stress at the Head and Foot

The mathematical expressions for the stresses are too complicated practical use and the results have therefore been expressed in form of charts giving the maximum compressive stresses at the d and foot for different weights of hammer, stiffnesses of cushion, . The importance of the maximum compressive stresses at these itions in causing failure will be discussed later (p. 41).

The symbols used on the charts are as follows :—

M = the mass of the hammer
h = the height of fall of the hammer
V = the striking velocity of the hammer
M_p = the mass of the pile
A = the area of the pile head
l = the length of the pile
E_c = Young's modulus of the concrete
E = Young's modulus of the equivalent homogeneous pile
a = velocity of stress along the pile
k = force to produce unit compression of the assumed head cushion
ϵ_1 = equivalent elastic set $\div VL/a$

The assumptions on which the theory is based indicate that the iables of importance in determining the stresses in the pile are the ght and height of free fall of the hammer, the stiffness of the head hion and the resistance at the foot of the pile. To understand the rts it is necessary to know how the stiffness of the head cushion the resistance at the foot are defined.

The stiffness of the head cushion is denoted by k/A. For an umed head cushion in which strain is proportional to stress, k is usual stiffness constant and is equal to the force required to duce unit compression, that is, $k = E'A'/l'$, where E' denotes ng's modulus of the dolly, A' its cross-sectional area, and l' its gth. If A denotes the area of the pile head, k/A is, therefore, the ss on the pile head to produce unit compression, and is equal to ng's modulus divided by the length or thickness of the material, n the cushion is of the same cross-sectional area as the pile. his case, k/A is independent of the stress and varies inversely as thickness of the cushion. The stiffness constant for a hard-wood y is obtained with sufficient accuracy from this expression. met packings, however, exhibit a non-linear relationship between ss and strain, and the appropriate value of k/A depends upon magnitude of the imposed stress. It is therefore necessary to ify k/A as "at . . . pounds per square inch." *

* A fuller explanation of k/A is given in Appendix II.

If the dolly and packing have effective constants k_1/A and k
then k/A for the combination is given by

$$\frac{A}{k} = \frac{A}{k_1} + \frac{A}{k_2}$$

and similarly for any number n of packings on top of one anothe

$$\frac{A}{k} = \frac{A}{k_1} + \frac{A}{k_2} + \ldots + \frac{A}{k_n}$$

The foot resistance has been assumed in the theory to be w
elastic and the charts are drawn for various values of ϵ_1, the m
mum theoretical elastic set at the foot, multiplied by a/Vl. Th
rather than the stiffness has been used because information on r
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the use of the expression

Equivalent elastic set = actual elastic set + twice actual plasti

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the same maximum stress, up to the point at which moveme
reversed, is independent of the relative amounts of plastic
elastic set. The assumption was tested theoretically by evalu
particular cases for a purely plastic foot and comparing the r
with those obtained for a purely elastic foot giving a set equ
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in good agreement.

In Figs. 5 and 6 the maximum stresses $(x\,a/E_c v)$ in the concre
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stress may be determined from the simpler diagram of Fig. 7,
one curve replaces the several curves of Fig. 5.

The stress at the foot only becomes important when it is
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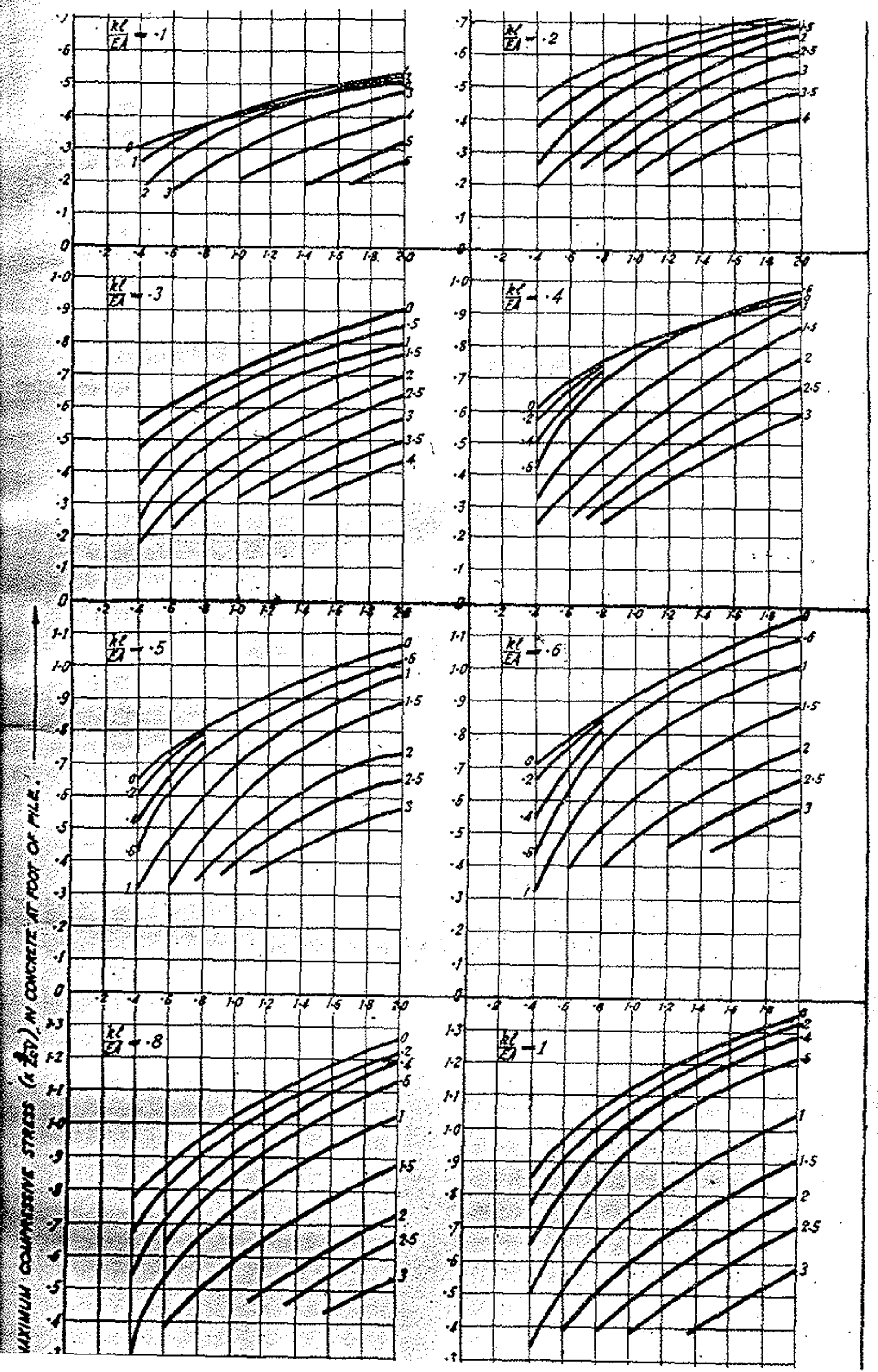
refer only to sets sufficiently small to ensure that the foot st
comparable with or greater than the head stress.



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one curve replaces the several curves of Fig. 5.

The stress at the foot only becomes important when it is
parable in magnitude with the head stress. The curves of F
refer only to sets sufficiently small to ensure that the foot str
comparable with or greater than the head stress.

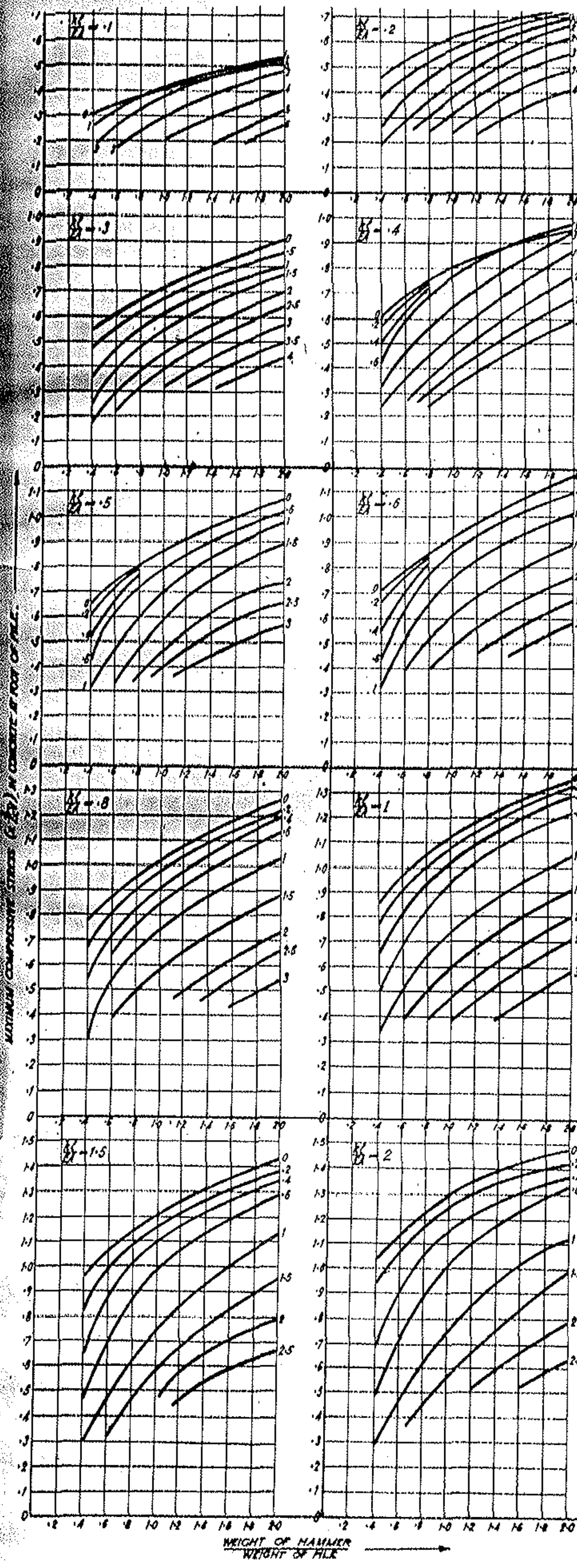

FIG. 6. MAXIMUM COMPRESSIVE STRESS $(\times \frac{a}{E_c v})$ IN CONCRETE AT FOOT OF PILE.
(Numbers to Curves give values of ϵ_1 constant along any one curve).

It can be shown from Fig. 6 that the maximum foot stress increases with increasing M/M_p and for small sets with increasing kl/EA also, but for the larger sets is almost independent of kl/EA. This implies that, for a given hammer, for small sets, *i.e.* in hard driving, the set increases with increased stiffness of head cushion, but for large sets the set is almost independent of the stiffness of the head cushion.

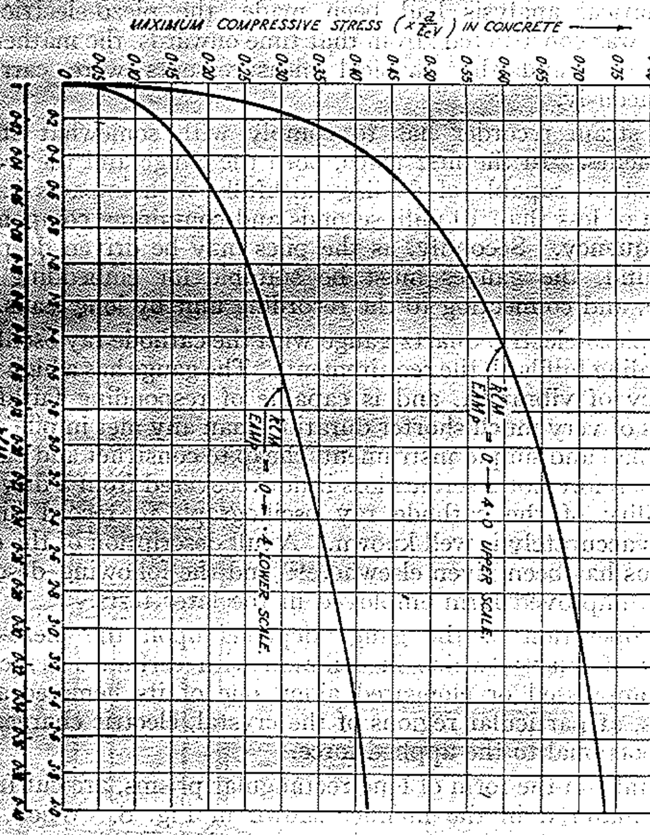

Figure 7.—*Maximum Compressive Stress* $\left(x \frac{a}{E_cV}\right)$ *in Concrete due to Initial Wave.*

The charts given in Figs. 5, 6 and 7, are sufficient for the determination of the maximum compressive stresses at the head and foot of any reinforced concrete pile driven with a normal hammer and head cushion. For rapid practical use graphs less comprehensive in character but of greater simplicity have been constructed; they will be considered in a later section as they are partly based on the results of practical measurements.

APPARATUS FOR THE MEASUREMENT OF STRESS AND SET

Piezo-electric Strain Recorder

To test the validity of the mathematical theory, it was necessary to devise a method of recording the variation of strain with time at selected points in a pile. For this purpose, as soon as the first mathematical analysis had been made, the piezo-electric strain recorder was constructed; from that time onwards, the mathematical investigation and the physical measurements were carried on simultaneously.

The strain recorder has to comply with somewhat stringent requirements. In the first place, both the gauge and the recording unit must be capable of dealing faithfully with impacts having a duration of less than 10 milli-seconds and containing components of high frequency. Secondly, as the piles may be driven deeply into the ground, the gauges must be suitable for embedding in the concrete, and connecting to the recording unit by long leads.

The piezo-electric quartz gauge with the cathode ray oscillograph for recording fulfils all the requirements. The gauge has a high natural frequency of vibration, and is capable of responding faithfully to impulses of very much shorter duration than any met in pile driving. It is a small and simple instrument, cheap to construct and calibrate, and is therefore not a serious loss if not recovered from a driven pile. The ability of the cathode ray oscillograph to record high frequencies accurately is well known. A full description of the original apparatus has been given elsewhere(3) and the following description is of the improved form employed in the later tests.

The operation of the gauge depends upon the piezo-electric property of quartz, which in common with certain other crystals, when compressed or elongated along one of its hemihedral axes, develops, at particular regions of the crystal, electric charges which are proportional to the applied force.

Crystals, in the form of long rectangular prisms, are cut from the natural material in the manner shown in Fig. 8. When, as the sensitive elements of gauges, they are subjected to pressure along the "third" axis, electric charges, proportional to the applied pressure, are liberated on electrodes attached to the faces perpendicular to the electric axis. Such crystal prisms, mounted in small water-tight chambers, described later, are cast in the pile at selected points, and their electrodes are connected to the recording system by highly insulated leads.

The charge liberated on the electrodes sets up, across a known capacity, a voltage which is applied to the grid of the input valve of an amplifier, which serves the double purpose of magnifying the voltage and of isolating the crystal from the comparatively low resistance of the deflector plates of the cathode ray oscillograph.

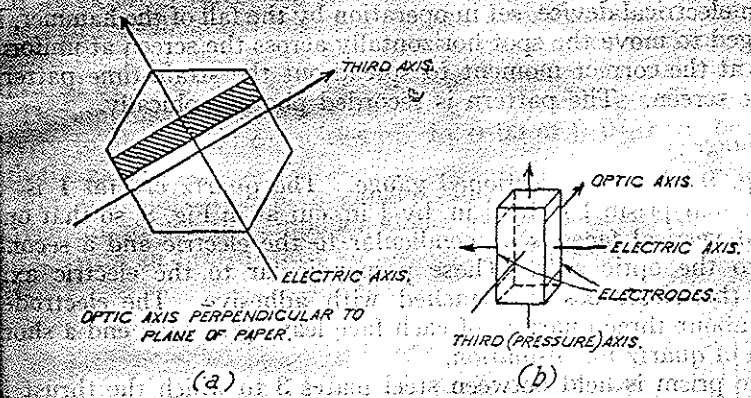

Figure 8.—*Position of Axes in Cut Quartz Crystals.*

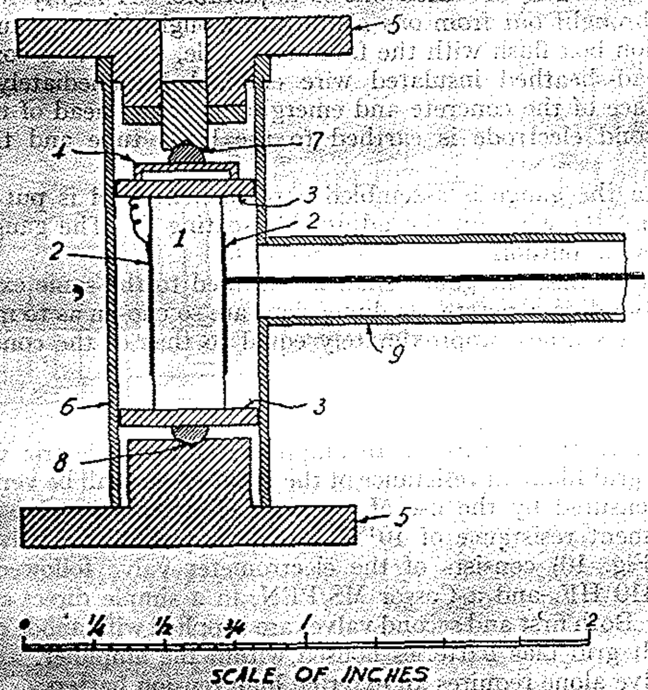

Figure 9.—*Section of a Gauge.*

The output from the amplifier controls the vertical deflection of th fluorescent spot of the cathode ray oscillograph.

An electrical device, set in operation by the fall of the hammer, arranged to move the spot horizontally across the screen at unifor speed at the correct moment to spread out the strain-time patte on the screen. The pattern is recorded photographically.

The Gauge

Fig. 9 shows a sectioned gauge. The quartz crystal 1 is rectangular prism 1 in by ½ in. by ½ in. cut as in Fig. 8, so that o pair of vertical faces is perpendicular to the electric and a seco pair to the optic axis. Those perpendicular to the electric ax carry the electrodes 2, attached with adhesive. The electrod cover about three-quarters of each face leaving at each end a sho length of quartz for insulation.

The prism is held between steel plates 3 to which the thrust applied through steel balls. At one end a stiff spring is insert between ball and plate to reduce the stress in the quartz.

Two heavy circular steel end-plates with projecting flanges connected to a thin brass cylinder 6, form a water-tight chamb enclosing the crystal assembly. The crystal is held between conic seatings 7 and 8, of which one is adjustable. A highly insulat lead is brought out from one electrode through the brass tube 9 a junction box flush with the face of the pile, where it is connect to a lead-sheathed insulated wire embedded immediately belo the surface of the concrete and emerging near the head of the pil The second electrode is earthed to the brass tube and the le sheath.

When the gauge is assembled a permanent load is put on th crystal by screwing up the adjustable seating 7. The gauge th responds to tension.

In order that the gauge shall be strained to the same extent the surrounding concrete, its dimensions are so chosen as to make i equivalent stiffness approximately equal to that of the concrete replaces.

The Amplifier

The essential feature of an amplifier for piezo-electric work that the grid-filament resistance of the input stage shall be very hig This is ensured by the use of a special electrometer valve with grid-filament resistance of $10^{15}\Omega$. The complete three valve a plifier (Fig. 10) consists of the electrometer valve followed by Cossor 410 HF. and a Cossor MS/PEN, in a simple direct-coupl circuit. Both first and second valves are supplied with anode curr by 9 volt grid bias batteries housed within the amplifier, and t third valve alone requires an external high tension battery.

Highly insulated condensers, variable in steps up to a to capacity of 0·015μF, are mounted within the amplifier and are co nected in parallel with the gauge in use.

Potentiometer control of grid bias on the first valve in conjunction ith an external voltmeter provides means of calibrating the am- lifier and oscillograph together or the amplifier alone, and also nables the grid bias to be adjusted to the correct working value ndicated by a milliammeter in the plate circuit of the last valve.

The amplification is about 240, a figure providing sufficient sen- itivity to allow total capacities up to 0·03 or 0·04μF to be used ith the gauges.

For the calibration of a gauge, which is done statically, it is onvenient to use the amplifier alone, and to measure the output oltage with an electro-static voltmeter. The procedure is described ater.

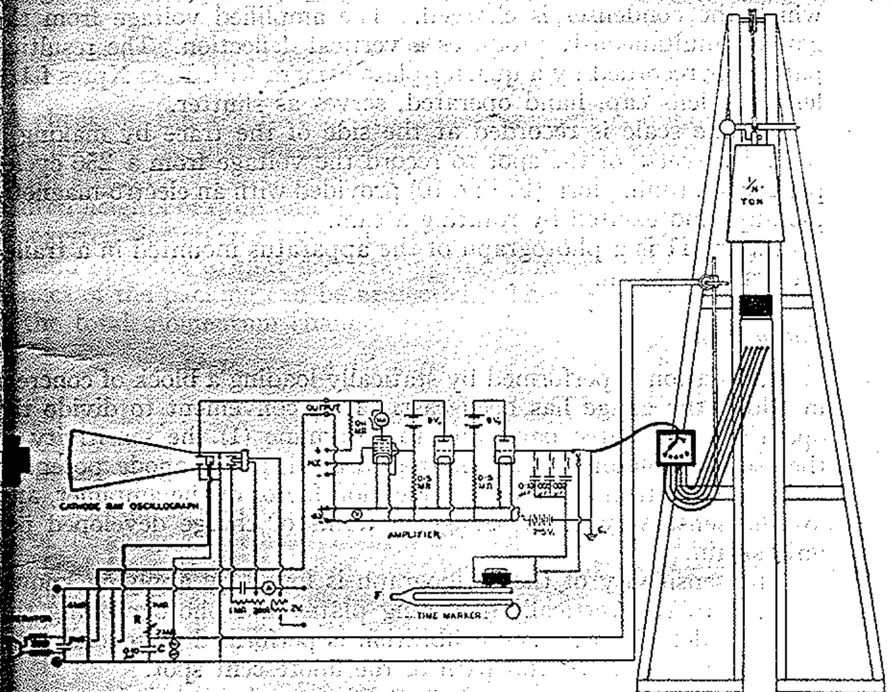

FIGURE 10.—*Electrical Connections of the Strain-recorder.*

Recording-equipment

The oscillograph, manufactured by Messrs. Cossor, Ltd., is perated at 700 to 1,000 volts provided by a direct-current generator riven by 12 volt motor. Current for the motor is provided by a 12 olt accumulator. A variable resistance in the motor circuit allows or manual control of the output voltage of the generator, and has een found quite satisfactory.

The method of registering the strain-time curve has already been riefly described. It involves the use of a single-traverse time-base nd a stationary plate camera.

The time base circuit (Fig. 10) consists of a high resistanc through which a condenser C is charged from the high tension su of the oscillograph, a bias potentiometer by which the spot ma shifted horizontally across the screen, two small neon lamps parallel with the condenser, and a switch which, when closed, s circuits the condenser. When a record is to be taken the swit closed and the spot brought to the left hand side of the scree adjusting the bias potentiometer. The hammer, in falling, o the switch, allowing the condenser to be charged through R. voltage across the condenser, impressed on the horizontally flecting plates of the oscillograph, causes the spot to traverse screen from left to right, the neon lamps limiting the voltag which the condenser is charged. The amplified voltage from gauge simultaneously produces a vertical deflection. The resul pattern is recorded by a quarter plate camera with Ross Xpres lens. A lens cap, hand operated, serves as shutter.

A time scale is recorded at the side of the trace by makin second traverse of the spot to record the voltage from a 256 cy per second tuning fork (F. Fig. 10) provided with an electro-magn pick-up, and excited by rotating a cam.

Figure 11 is a photograph of the apparatus mounted in a tr for transportation.

Calibration

Calibration is performed by statically loading a block of conc in which the gauge has been cast. It is convenient to divide operation into three parts and to determine (1) the sensitivity the oscillograph and camera in mm. per volt at the anode poten to be used in tests, (2) the amplification factor of the amplifier, (3) the sensitivity of the gauge in terms of charge developed unit strain.

The sensitivity of the oscillograph is found by applying kn voltages to the vertically deflecting plates and photographing resulting deflections. The calibration is performed at several p tions along the horizontal path of the fluorescent spot.

To measure the gain of the amplifier a voltmeter is inserted the grid bias circuit and the output leads to the oscillograph disconnected and joined to an electrostatic voltmeter. A curve output against input volts is thus obtained.

The third part of the calibration entails the determination of voltage set up across a standard condenser by the gauge when i subjected to strain. The gauge is cast at the mid-point of length of a block of concrete 16 in. by 4 in. by 4 in. of the sa composition and having the same Young's modulus as the conc in which stresses are to be measured. The block is stressed steps up to 3,000 lb./sq. in. in a testing machine, measurements strain being taken with a two-inch roller extensometer with telesc and scale, over the two-inch central portion in which the gauge

ated. The charge developed by the gauge when strained, duces, across a standard condenser, a voltage which is measured h the aid of the amplifier and the electrostatic voltmeter.

The procedure is to measure strain and charge developed for a nber of increments of stress, and to plot the results as a graph. all three parts of the calibration provide approximate linear tionships, it is possible to express the combined results as a ar relationship between photographic deflection and strain. sensitivity in tension is obtained by extrapolation. In the inter- tation of records, strains are converted to stresses by multiplying he statically determined Young's modulus.

The accuracy of calibration in terms of strain is estimated to be ect to an error of ± 5 per cent. It is, however, possible to be rror by ± 10 per cent. in the stresses calculated from the sured strains, due probably to the difficulty of assigning the ect Young's Modulus to the concrete in which the gauge is cast.

A check on the accuracy of stress-measurement is obtained by paring two calculations of the momentum of the hammer, one the stress record at the head of the pile, and the other from the sured heights of drop and rebound. At the head of the pile the exerted downwards over the cross-section is, at any instant, l and opposite to that decelerating the hammer, provided that effect of the packing can be neglected. Then it is easily shown the total momentum change

$$M(V-v) = A\int_0^T p\,dt$$

e M is the mass of the hammer, V and v its velocities imme- ely before and after striking, A the area of the pile-head, p the s at the head of the pile, and T the duration of the blow.

The integral is proportional to the area under the stress-time e of the gauge at the head, and the left hand term is immediately rmined from the heights of drop and rebound. Calculations made for piles driven without a helmet, the head protected by layers of packing, show agreement within 10 per cent.

Set Recorder

A simple method of recording sets was found to be very satis- ry. Fig. 12 gives details of the apparatus. A board carrying a of paper was firmly fixed to the front face of the pile by heavy ps. A straight edge, along which a pencil was drawn to trace he set record on the paper, was nailed to a heavy baulk of timber, d from the ground at either end by wooden blocks. The timber ed an effectively fixed base for measurement since its vibration ot become apparent on the record until after the set had been ded, and was even then of small amplitude. From the record lastic and elastic sets can be obtained.

The Peak Stress Indicator

The peak stress indicator was designed to provide a simple
tical instrument for measuring the maximum head stress durin
driving of a pile.

The three possible situations for such an instrument are o
pile, helmet or hammer. Any device attached to the pile or h
would have to be of the nature of a strain gauge and for remo
dication would have to be electrically operated. The difficult
fixing a gauge of this type securely, and of simplifying the indic
circuit to such an extent as to render it of practical value, app
the present moment to be insuperable.

The peak stress indicator measures the deceleration of the ha
from which the maximum stress on the pile head may be ded

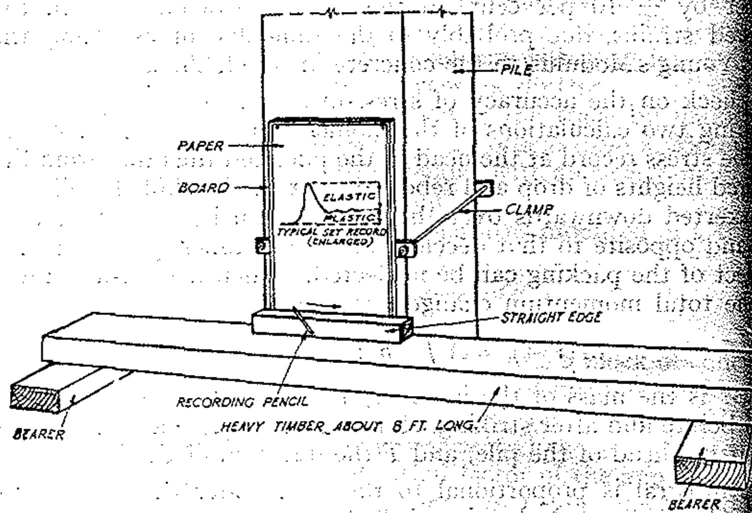

FIGURE 12.—*Set-Recording Apparatus.*

For, if the assumption that the mass of the helmet may be neg
is approximately correct, then the maximum force on the head
pile

$$P_{max} = Mf_{max}$$

where M is the mass of the hammer and f_{max} the maximu
celeration of the hammer. Maximum acceleration indicators
previously been used to indicate peak values of the accelerat
road surfaces[4], subjected to traffic vibrations. The prese
strument embodies several improvements and alterations a
employed with a new method of visual indication.

Fig. 13 is a diagrammatic representation of the instrument
simplest form and of the indicating circuit. A mass m is held a
an insulated stud I by a spring S, whose compression can be
by a screw, calibrated in terms of spring load. Flat spri

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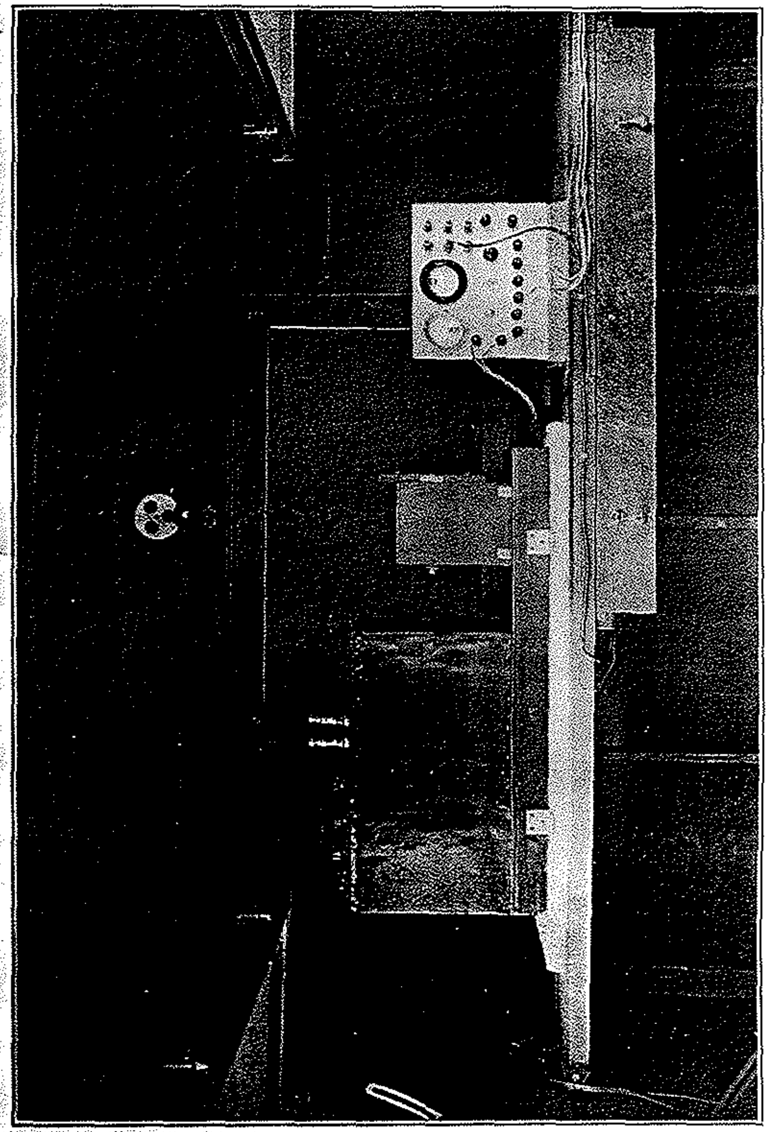

FIGURE 11.—*Recording Apparatus in the Trailer.*

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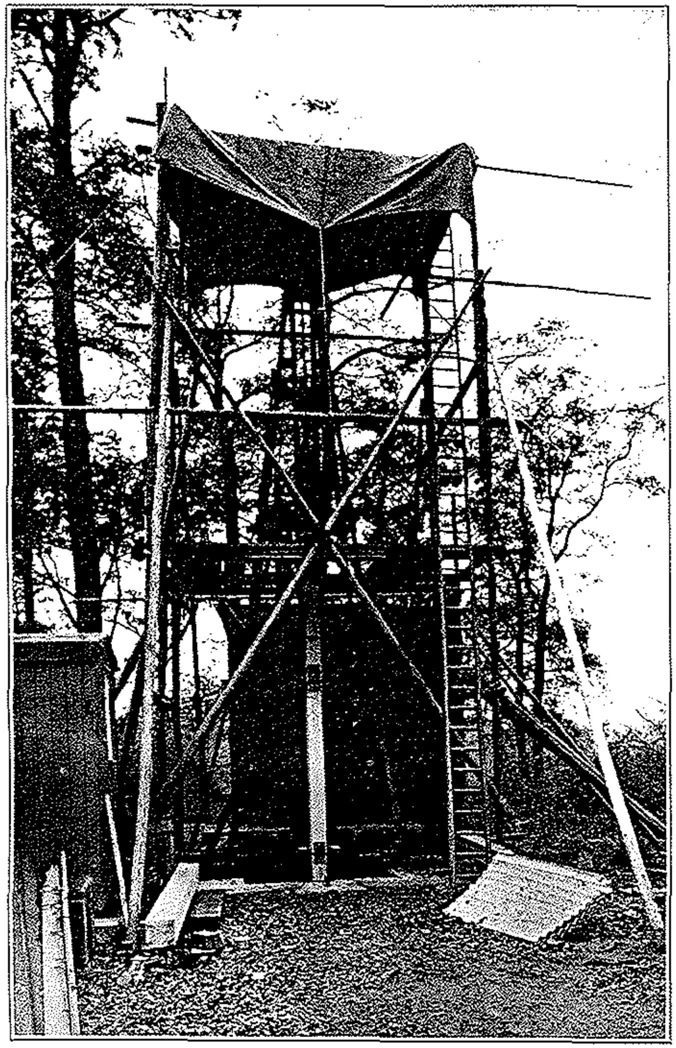

FIGURE 15.—*The Small Pile-Driving Frame.*

sure that the motion of the mass m is parallel with the pillars P. e assembly is screwed to the top of the hammer with the axis of spring vertical. An electric circuit is completed by the contact the insulated stud I and the mass m.

The mass m is subjected to the same deceleration as the hammer, d therefore exerts a maximum force mf_{max} on the spring tending open the contact. If the force mf_{max} is greater than that erted by the spring the contact opens and the neon indicator mp lights. By means of the calibrated screw the compression of spring can be increased until the contact no longer opens. Then $_{max} = u \cdot k_s$ where k_s is the spring stiffness and u the compression the spring. By adjusting u the two sides of this expression may made as nearly equal as desired, and the required value of the ximum deceleration found.

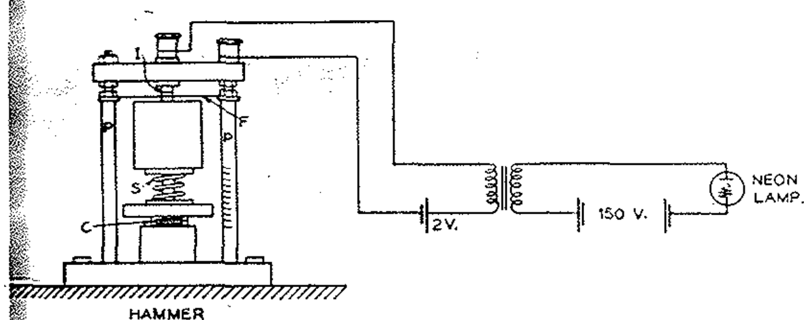

FIGURE 13.—*Peak Stress Indicator.*

The indicating circuit is also shown in Fig. 13. A two-volt cell pplies current to the primary of a suitable transformer, the circuit ing completed by the contact between the mass and the insulated ud. Across the secondary of the transformer a small neon lamp d a dry battery of about 150 volts are connected in series, the ltage of the battery being adjusted to a value lying between the shing and extinction voltages of the lamp. When the contact is oken the voltage induced across the secondary by the break in e primary current raises the voltage across the neon lamp above at necessary for flashing. The lamp lights and remains lit since the ttery voltage is above the extinction voltage.

The simple deceleration indicator of Fig. 13 gives accurate easurements of maximum deceleration for impacts cushioned by cking only without a helmet, or by a light helmet with a reasonably ft dolly above, but may be in error with hard dollies and heavy lmets.

To eliminate these inaccuracies it is necessary to render the peak ress indicator insensitive to the decelerations of short duration e to the initial impact of the hammer on the dolly. A mathe- atical discussion and an experimental investigation of the effect

of the helmet are given in Appendix IV, with fuller particul
the methods adopted to improve the accuracy of the indicato

Fig. 14 shows a peak stress indicator incorporating these im
ments, with which satisfactory results have been obtained in
tests at the Building Research Station; further tests are now
carried out under practical conditions.

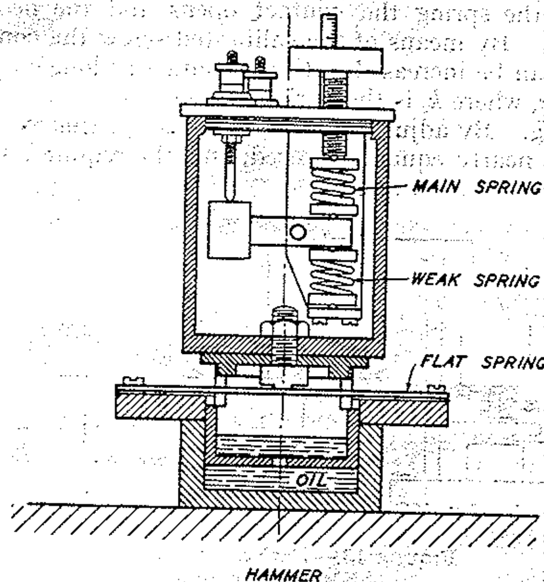

FIGURE 14.—*Improved peak stress indicator.*

THE RECORDING OF STRAINS IN TEST PILES

The experimental tests with the piezo-electric strain recorde
be divided into two sections (1) preliminary tests in a speciall
structed driving pit, and (2) tests on piles driven into ground.

Preliminary Tests

The objects of the preliminary tests were to make a trial
piezo-electric recorder under conditions approximating to
obtaining in practice and to test the validity of the theory.

The piling plant consisted of a small experimental frame l
by the British Steel Piling Company, and three drop ham
weighing 2,000 lb., 980 lb. and 480 lb. The hammers were
by a hand winch fitted with a trigger release. A cast-iron h
fitted with an elm dolly and weighing approximately 3 cwt.
be used at the head of the pile.

In carrying out the tests, the piling frame shown in Fig. 15
mounted over a concrete-lined pit, 4 ft. deep and 6 ft. square

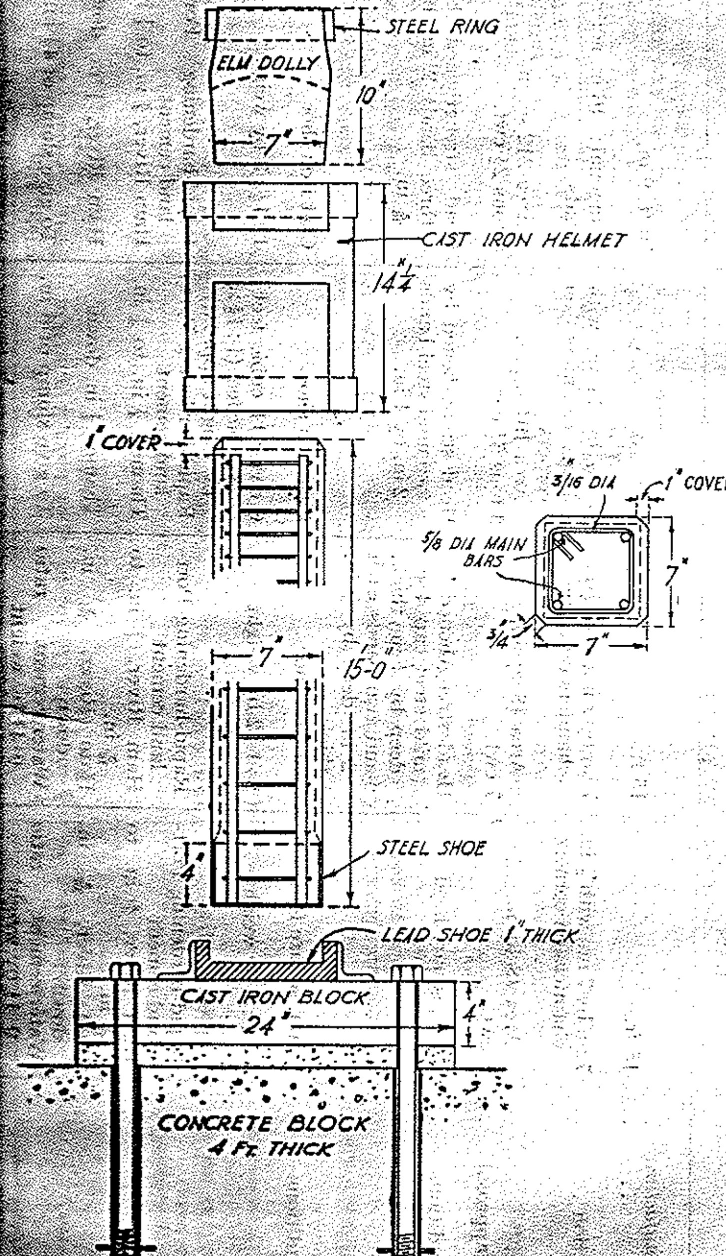

FIGURE 16.—*Driving Conditions for Piles tested in the Concrete Pit.*

TABLE I.—*Particulars of Test Piles.*

Conditions	Pile 49L driven at Building Research Station for preliminary tests	Piles A5 and A6 driven at Building Research Station	Test pile at London University site, Bloomsbury	Test piles driven at Lots Road Power Station
Manufacture				
Length	15 ft.	15 ft.	35 ft.	50 ft. 9 in.
Cross section	7 in. by 7 in., with ½-in. chamfers on corners Area 48 sq. in.	7 in. by 7 in., with ½-in. chamfers on corners Area 48 sq. in.	14 in. by 14 in. with 1-in. chamfers on corners Area 194 sq. in.	Octagonal; 16 in. across flats Area 212 sq. in.
Weight (approx.) ..	7 cwt.	7 cwt.	3 tons 5½ cwt.	5 tons 2½ cwt.
Head conditions ..	Plain, square head	Head strengthened by mild steel cap 4 in. deep by ¼ in. thick, and two bands each 2 in. deep by ⅛ in. thick	Head strengthened by two mild steel bands 2 in. by ¼ in.	Head strengthened by three bands 2 in. by $\frac{3}{16}$ in.
Foot conditions ..	Plain, square foot	Cast iron shoe	Cast iron shoe	Cast iron shoe
Mix	1 : 1 : 2 by weight	1 : 1 : 2 by weight	6 cwt. : 11 cubic feet : 23 cubic feet (1 : 1½ : 3 by weight approx.)	1 : 1½ : 3 by weight
Cement	Rapid-hardening Portland cement	Rapid-hardening Portland cement	Rapid-hardening Portland cement	Rapid-hardening Portland cement
Aggregate	Ham River, ¾ in. to $\frac{3}{16}$ in.	Ham River, ¾ in. to $\frac{3}{16}$ in.	Thames, ¾ in. to ¼ in.	Ham River, ¾ in. to ¼ in.
Sand	Ham River, $\frac{3}{16}$ in. down.	Ham River, $\frac{3}{16}$ in. down	Thames, ¼ in. down	Ham River, ¼ in. down
Water	Water/cement ratio [illegible]	Water/cement ratio [illegible]	To give slump of 6 in.	To give slump of 4½ in.
Reinforcement :—				
Main	Four [illegible]-in. bars	Four [illegible]-in. bars	Four 1-in. bars	Eight [illegible]-in. bars
Transverse	[illegible]-in. links, 1½ in. to 3 in. centres	[illegible]-in. links, 1½ in. to 3 in. centres	¼-in. links, 2 in. to 6 in. centres	⅜-in. helical binding, 2-in. pitch.
Strength of Control Specimens				
Crushing: /lb. sq. in.	6,500 (at 7 days)	(A5) 8,254	6,699	6,953
Young's modulus: /lb. sq. in. × 10^6	5·0	(A6) 5·0	4·5	4·9
Driving				
Hammer	Cast iron drop hammer (980 lb.) released by trip gear	Cast iron drop hammers (weighing 480 lb., 980 lb. and 2,000 lb. respectively) released by trip gear	Single acting steam hammer Weight of ram 3 tons	Cast iron drop hammer weighing 3·3 tons, released by clutch action on winding drum
Head packing ..	10 in. by 7 in. by 7 in. elm dolly, 3-cwt. cast iron helmet and various packings	Cushion consisting of four, twelve, or twenty-four layers of felt (nominal thickness, ¼ in.)	10 in. by 14 in. by 14 in. pinkadou dolly, 10-cwt. cast iron helmet and packing of 3½ in. of deal in four layers, on top of four layers of sacking	9-in. cylindrical hickory dolly, 15 in. average diameter; 8-cwt. cast iron helmet, and packing of two criss-crossed layers of 2-in. manila rope and eight layers of sacking
Ground	Driven in concrete pit with various thicknesses of felt under foot	*See* Fig. 21	*See* Fig. 21	*See* Fig. 21

bottom of which was formed by a block of concrete 4 ft. thick cast-iron block, bolted down in the centre of the pit, formed a base on which to drive the piles. The arrangement is shown grammatically in Fig. 16, together with details of the lead shoe which the foot of the pile was placed, and of the dolly and helme

The piles tested were 15 ft. long by 7 in. square. Details of reinforcement, mix and curing treatment of typical piles are give Table I.

Three gauges were cast in along the axis of each pile, one a distance of ten inches from either end and one at the mid The highly-insulated leads from the gauges to the amplifier were first, merely bare copper wires on porcelain insulators, but th were later replaced by lead-sheathed rubber-insulated leads. recording apparatus was set up in a hut close to the frame and The registration of the record was synchronized with the momen impact by the switch operating the time-base circuit, which, secu to the frame by an adjustable mount, was arranged to be tripped the correct moment by the falling hammer.

The registration of a strain-time record involved the follow operations :—(1) switching on the amplifier and oscillograph, raising the hammer to the required height; (3) setting the time-b switch at the correct position; (4) opening the dark slide in the p camera and removing the lens cap, and (5) giving the signal for hammer to be dropped.

The gauges and recording apparatus were found to func satisfactorily and records were obtained showing the effect of chan in the following variables on the form and magnitude of the stresses

(*a*) The weight of the hammer.
(*b*) The height of fall of the hammer.
(*c*) The weight of the helmet.
(*d*) The amount and type of packing in the helmet.
(*e*) The resistance at the foot of the pile.

As most of the features of the records of the preliminary t were reproduced later in the records of the piles driven into grou the results of the preliminary tests will not be taken alone but be considered in the general discussion.

Piles Driven into the Ground

The preliminary tests in the concrete pit were followed by driving of six piles into the ground; two, 15 ft. by 7 in. by 7 were driven at the Building Research Station, one, 35 ft. by 14 in 14 in. at the London University site, two, 50 ft. long, octagonal cross-section and 16 in. between flat surfaces, at the Lots Ro Power Station, and one, 50 ft. by 14 in. by 14 in. at Messrs. Mowle Yard, Royal Albert Dock. Three or more piezo-electric gaug equally spaced along the length, were cast in each pile.

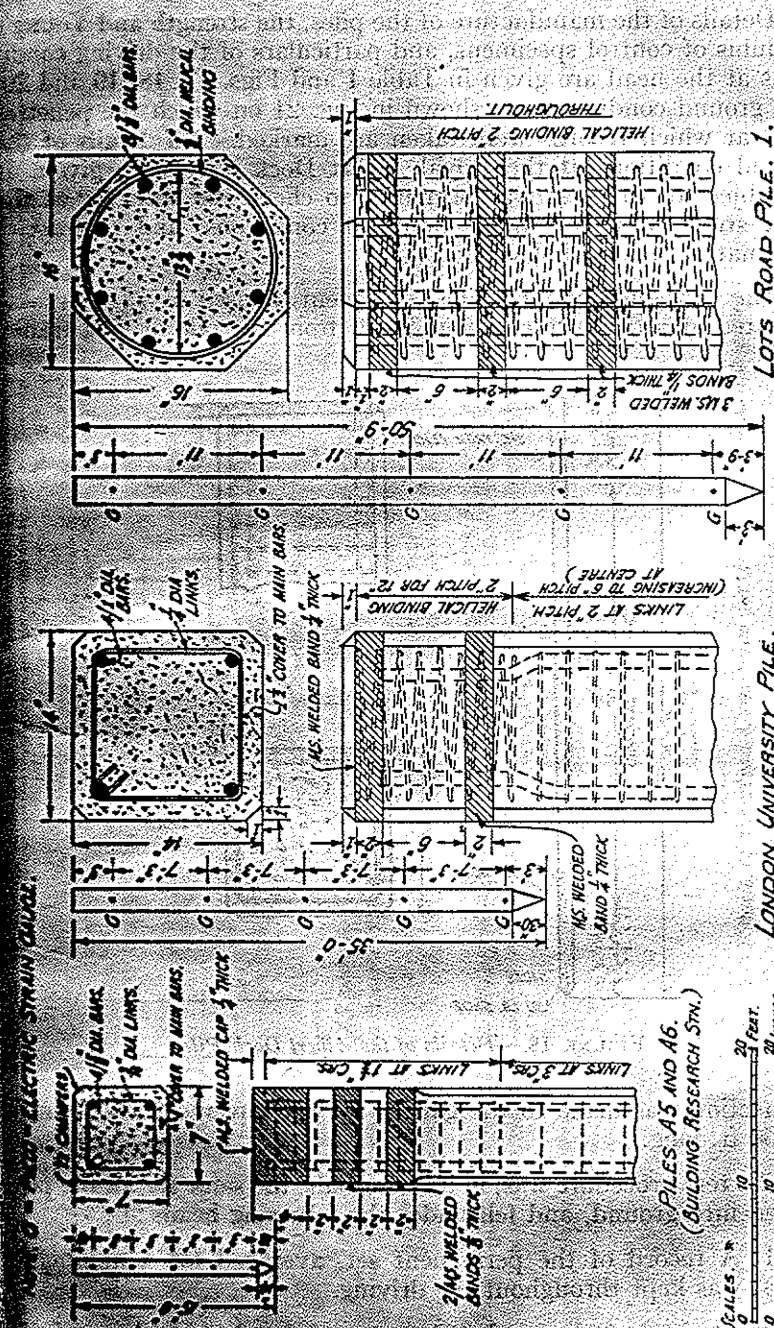

FIGURE 17.—Pile Details

Details of the manufacture of the piles, the strength and Young['s] modulus of control specimens, and particulars of the driving con[di]tions at the head are given in Table I and Figs. 17, 18, 19 and 2[0]. The ground conditions are shown in Fig. 21 on which the penetr[a]tions at which records were taken are marked. No details of t[he] ground conditions at the Royal Albert Dock have been given, [as] the object of this test was primarily to check the accuracy of t[he] peak stress indicator under practical conditions, and no prec[ise] information was available.

The piezo-electric recording equipment, housed in a trailer, w[as] operated at a distance of about 20 ft. from the pile driving fra[me].

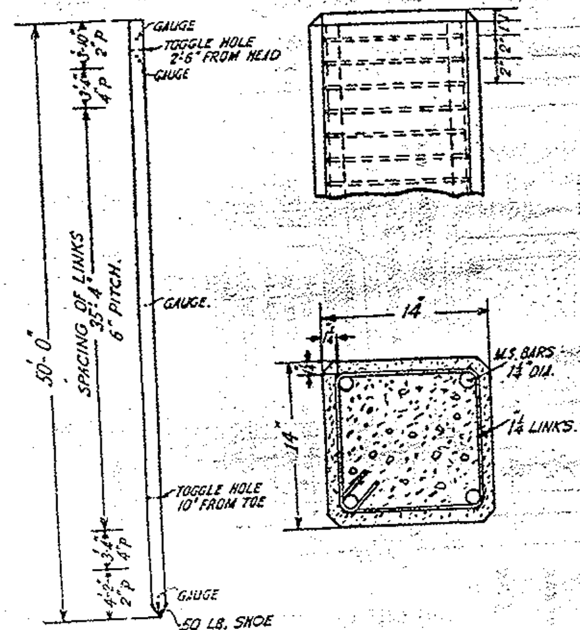

FIGURE 18.—*Details of the Albert Dock Pile.*

Connection was made to the gauges by lead-covered leads fro[m] to 100 ft. long.

The test procedure was, in its essentials, the same for all the [piles] driven into ground, and fell under the following heads :—

(1) A record of the permanent set, averaged for a numbe[r of] blows, was kept throughout the driving.

(2) Strains were recorded for several heights of drop o[f] hammer at four or five stages of penetration, the positions of w[hich] were decided by the ground conditions. (See Fig. 21.)

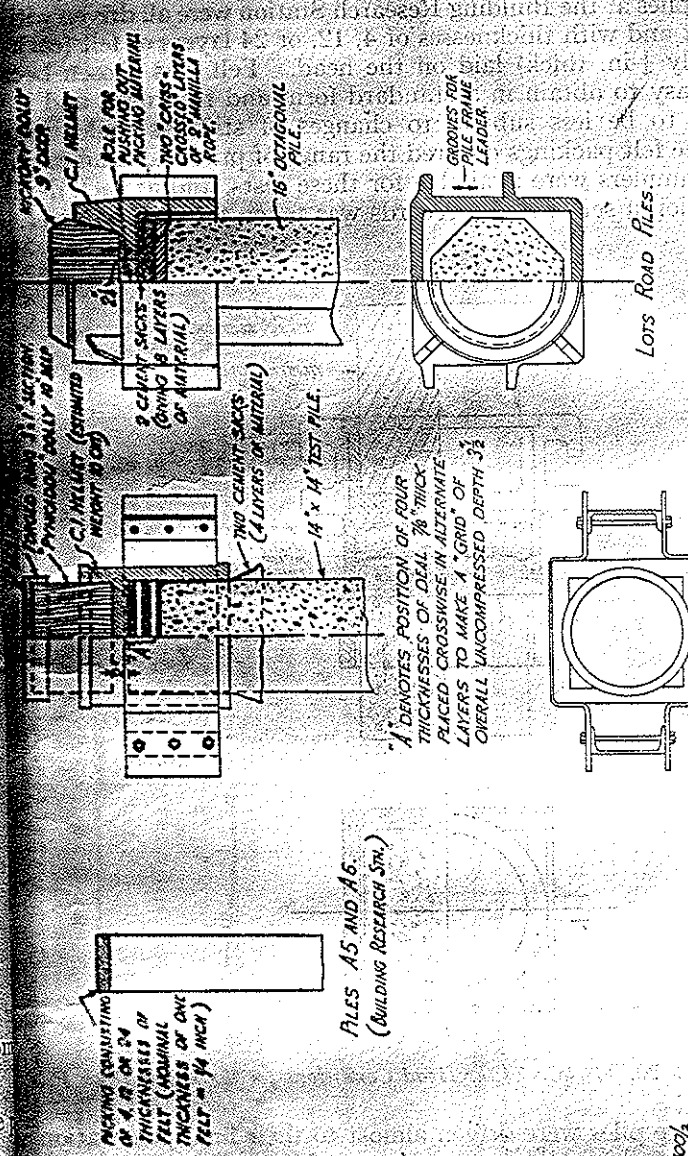

FIGURE 19.—*Details of the Head Conditions for Piles driven into the ground.*

(3) Each set of strain measurements was accompanied corresponding records of plastic and elastic set.

The piles at the Building Research Station were all driven a helmet and with thicknesses of 4, 12, or 24 layers of felt (ea nominally ¼-in. thick) laid on the head. Felt was chosen it was easy to obtain in a standard form and thickness, and thought to be less subject to changes of stiffness under The three felt packings covered the range of practical head cu Three hammers were available for these tests, and at each de penetration a set of strain records was taken for each hamme

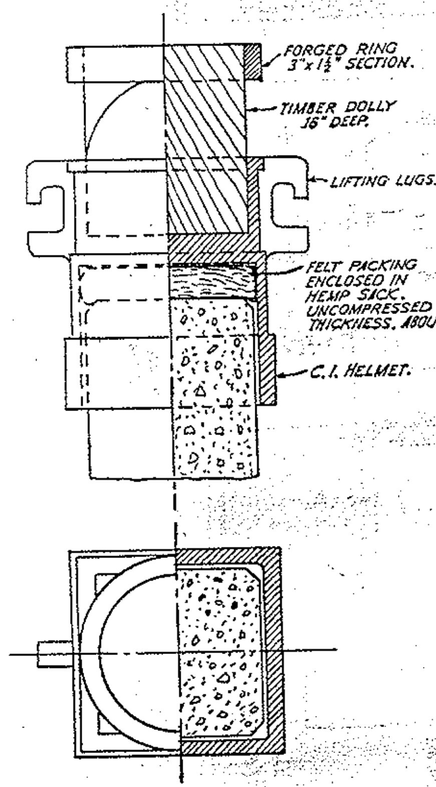

FIGURE 20.—*Details of the Head Conditions for the Albert Dock*

All large piles were driven almost to their final penetra the helmet and using the contractors packing. For purposes calculation with known packing conditions a set of records and 24 thicknesses of ¼-in. felt as head packing and no helm then taken.

Various alterations and readjustments of packing mate made during driving, as necessity arose.

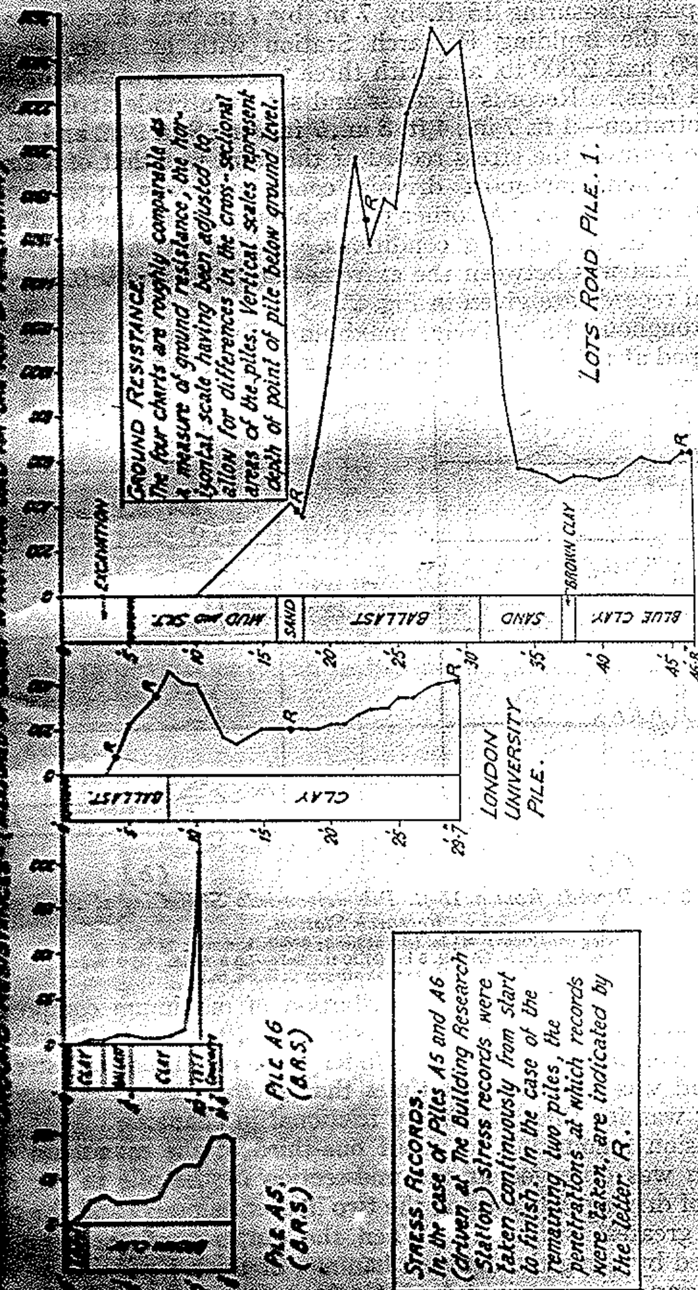

FIGURE 21.—*Comparative Charts showing character and approximate resistance of ground.*

(1) 15-*ft. Pile driven into Stiff Clay at the Building Research St*

A pile, measuring 15 ft. by 7 in. by 7 in. was driven into clay at the Building Research Station with hammers wei 480, 980, and 2,000 lb. and with three thicknesses of packing and 24 felts). Records of stress and set were made at five de of penetration—1 ft. 7 in., 4 ft. 3 in., 7 ft. 9 in., 9 ft. 9 in. and 12

The form of the stress record at the head was that of a sm curve, which, for most driving conditions, was approxim triangular in shape. At other points along the pile the form v greatly with the driving conditions. The duration of the r at the head was between the extremes of 0·004 and 0·020 se Typical records are given in Fig. 22.

Throughout the driving, maximum compressive stresses developed at the head, falling off along the pile to a minimum a

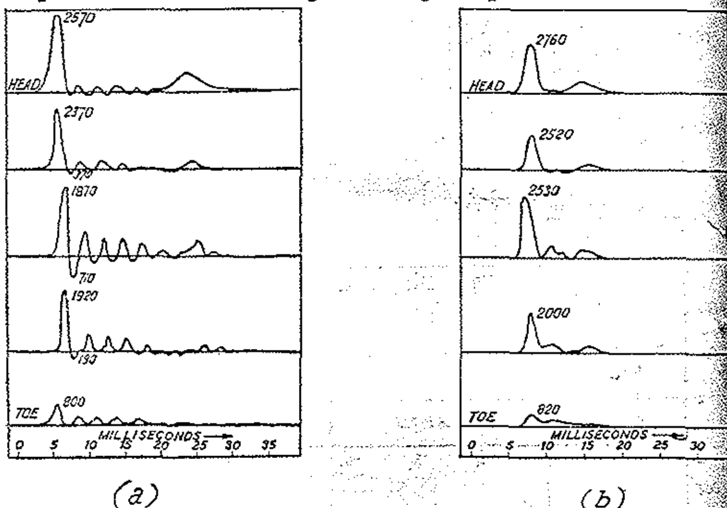

FIGURE 22.—*Records from a* 15-*ft. Pile driven into Stiff Clay at the Bu Research Station.*

Driving conditions:—4½-in. felts at head; 980-lb. hammer; 24-in. drop.
Penetration of point:—(*a*) 4-ft. 3 in. (*b*) 10-ft Set:—(*a*) 0·55-in. (*b*) 0·08-in.
(Figures indicate peak stresses in lb./sq. in.)

foot. Figure 23 shows the distribution of maximum compr stress along the pile when approximately 10 ft. of its length been driven into the ground. Under these conditions the str the head was three to four times that at the foot. Resistan driving was for this pile chiefly frictional in character.

Tension amounting to about one-third of the maximum pression was observed at the mid-section of the pile in the stages of driving, when using the two lighter hammers and the ing of greatest stiffness. When the duration of the blow increased by using the heaviest hammer or thicker packing or the driving resistance increased, tension was not recorded. Re showing tension are reproduced in (*a*) Fig. 22; the suppressing of increased ground resistance is shown in (*b*).

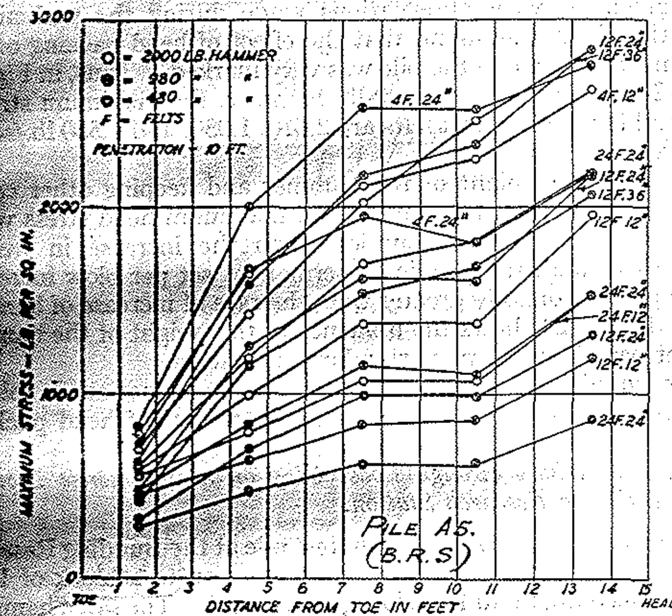

FIGURE 23.—*Pile A.*5.

...bution of Maximum Compressive Stress along the pile at a penetration ... feet, for various packing conditions, hammer weights, and heights of drop.

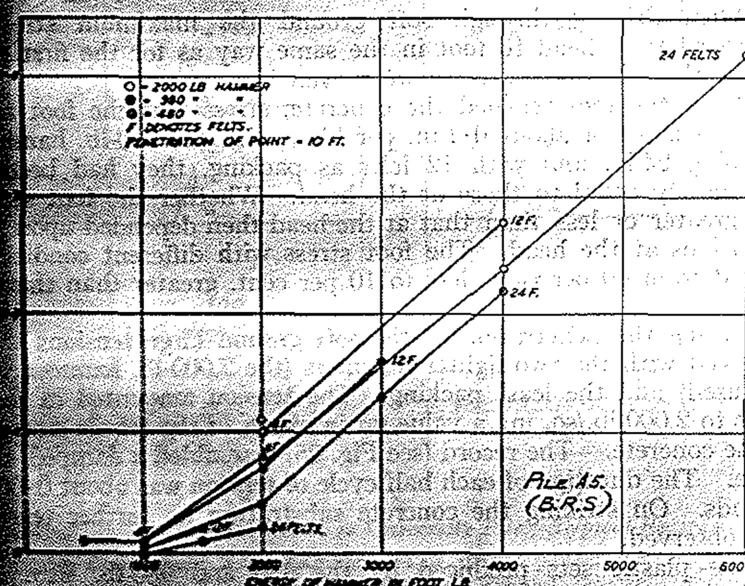

FIGURE 24.—*Pile A.*5.

...between the energy of the Hammer Blow and the Permanent Set ... blow.

In agreement with observations made during the prelim tests in the pit, it was found that the effect of head packing o maximum stresses along the pile was very marked. Thus, in Fi with a 12-in. drop using a 2,000-lb. hammer, the maximum st at the head for 4, 12 and 24 felts are 2,610, 1,950, and 1,530 lb./s respectively.

Increasing the weight of the hammer and keeping other c tions constant produced an increase of maximum head stress w was, however, proportionately less than the increase in we On the other hand, the plastic or permanent set per blow incr at a rate proportionately greater than the rate of increase of ha weight. Figure 24 shows the influence of the weight of the ha on the set per blow for one particular penetration.

Other conditions remaining the same, a reduction in p set was brought about by increasing the amount of head pac

(2) 15-*ft. Pile driven through Loose Clay on to a Concrete Bl the Building Research Station.*

The second 15 ft. by 7 in. by 7 in. test pile at the Building Res Station was driven through fairly loose clay on to a block of concrete placed about 10 ft. below ground level. The same ham and packings were used as for the first pile. Two objects w view—first, to obtain records driving through soft ground second, to examine the condition of moderate side support and resistance at the foot.

When driving through soft ground the maximum st decreased from head to foot in the same way as for the firs except that the foot stresses were even less.

When the foot reached the concrete, stresses at the foot until, at a set of about 0·1 in. per blow with the 980-lb. ha dropping 24 in. and with 12 felts as packing, they had be very nearly equal to those at the head. Whether the foot was greater or less than that at the head then depended upo conditions at the head. The foot stress with different cond ranged from 40 per cent. less to 10 per cent. greater than th the head.

During the penetration of the soft ground large tensions observed with the two lighter hammers (the 2,000-lb. hamme not used) and the least packing. The tension amounted in cases to 2,000 lb./sq. in., a value far in excess of the tensile st of the concrete. The record (see Fig. 25) resembled a damped wave. The duration of each half cycle of tension was about 0 seconds. On striking the concrete substratum no further t was observed.

The plastic sets produced by the 480 and 980-lb. ham when driving through the soft clay, were, for a particular hei drop, approximately proportional to their respective weights the energy efficiency of the two hammers was the same. It wa

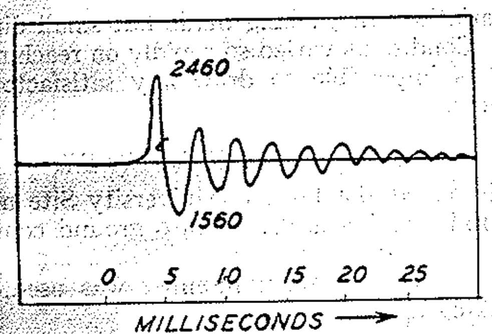

FIGURE 25.—*Record showing tension at the mid-point of the length of the pile. (Figures indicate peak stresses in lb./sq. in.)*

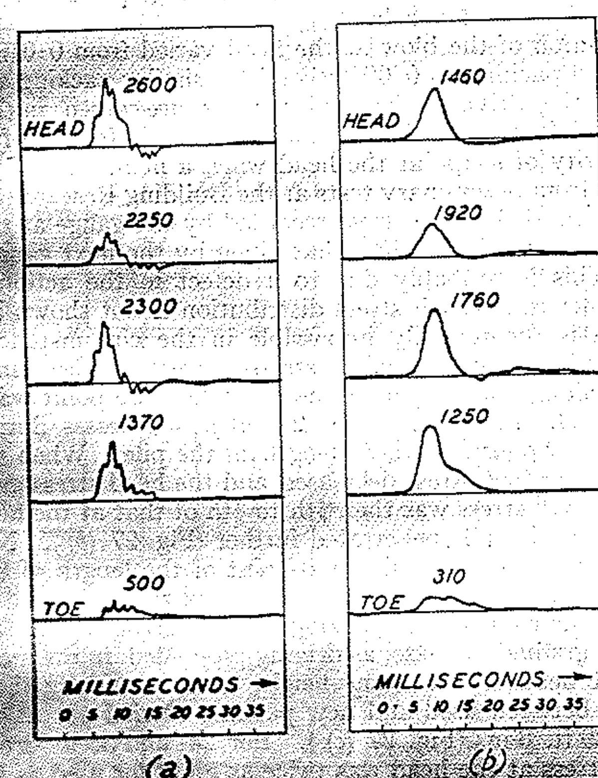

FIGURE 26.—*London University Pile; Typical Records.*

***Driving Conditions*: 3 ton hammer, 24 in. drops, penetration of point 29 ft. 6 in.; (*a*) contractor's packing with 10 cwt. helmet, (*b*) twelve felts without helmet. Set: (*a*) 0·07 in., (*b*) 0·04 in.**

(**Figures indicate peak stresses in lb./sq. in.** The low value at the head in (*b*) **is due to back-pressure in the hammer.**)

found that variations in packing made but small difference to set per blow. Conditions varied so rapidly on reaching the conc block that it is impossible to draw any satisfactory conclus from the records.

(3) 35-*ft. Pile driven at the London University Site*

The pile driven at the London University Site measured 35 by 14 in. by 14 in. and was driven into ground consisting of 7 of ballast overlying clay.

A 3-ton single acting steam hammer was used, with helm dolly and packing as in Fig. 19.

Strain and set records were taken at the depths marked in Fig with 12-in., 24-in. and 36-in., drops. At the greatest depth penetration the packing was removed and further records taken with new packing of the same type. Tests were then m with two different thicknesses of felt packing, placed directly the pile head without a helmet.

The duration of the blow at the head varied from 0·016 sec for new wood packing to 0·008 second for similar packing after throughout the driving. Typical records are given in Fig Their general form is similar to that of records from 15-ft. p the irregularity of shape at the head when a helmet is used ag with results from preliminary tests at the Building Research Sta It will be noticed that the stress recorded by the gauge at the h with felt packing is lower than that given by the next gauge a the pile. This is probably due to a defect in the action of steam hammer, since such stress distribution is not shown in o records and is theoretically impossible in the circumstances.

The maximum compressive stresses were almost unifo distributed along the length of the pile when the point was b driven through the ballast (Fig. 27 (*a*)). Stresses during period were low over the whole length of the pile. When the was reached the foot stress decreased and the head stress grad rose. The foot stress was then ⅛th to ⅓th of that at the head rose slightly as the pile penetrated further (Fig. 27 (*b*)). No tensions were recorded at the mid-point of the length of the

A matter of considerable importance was the continued ch observed in the contractors' packing during driving. This is sh both by the gradual increase, as driving proceeded, in the maxi stress at the head for the same height of drop and by the d comparison between used and new packing carried out at a p tration of 29 ft. 6 in. (Fig. 27 (*c*)). For 24- and 36-in. drops, maximum stress at the head was twice as great for used as for packing.

(4) 50-*ft. Piles driven at Lot's Road Power Station*

Both piles driven at Lot's Road Power Station were 50 ft. and of octagonal cross section 16 in. across the flats.

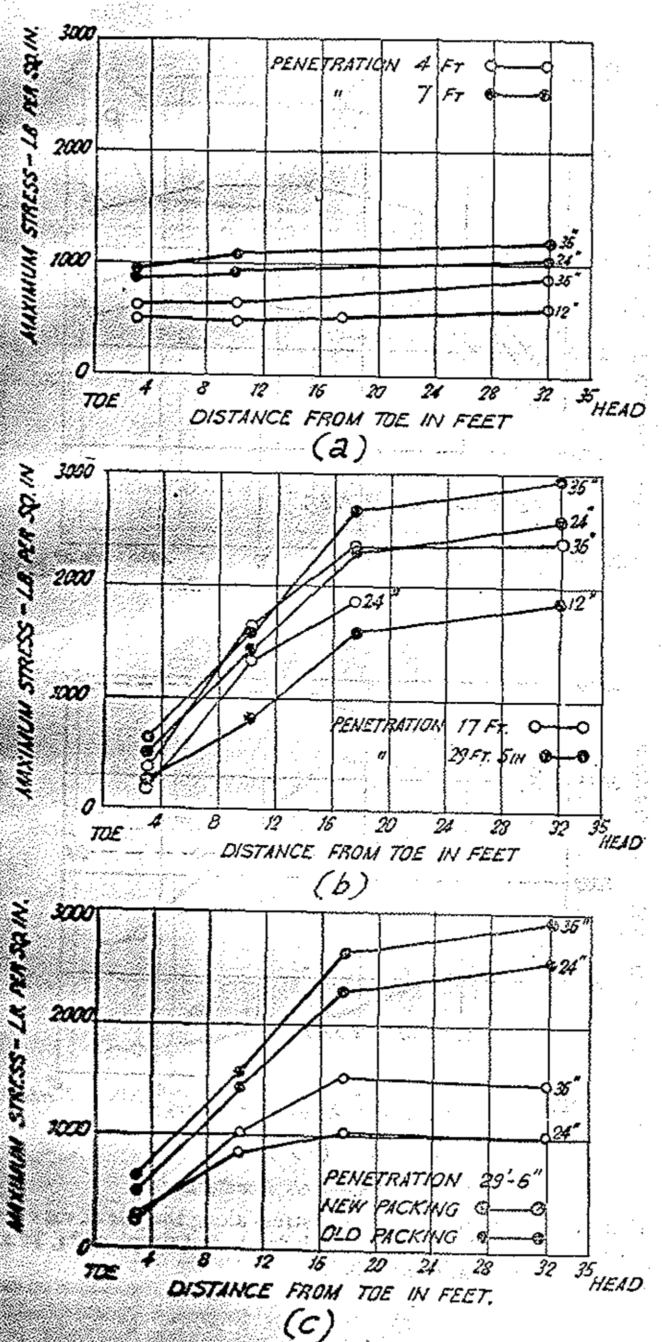

FIGURE 27.—*London University Pile.*
and (b) Distribution of maximum compressive stress along the pile at penetrations up to 29 ft. 5 in., with contractor's packing and helmet, and drops of 12, 24 and 36 in. (*c*) Comparison of new and used packing for in. and 36 in. drops.

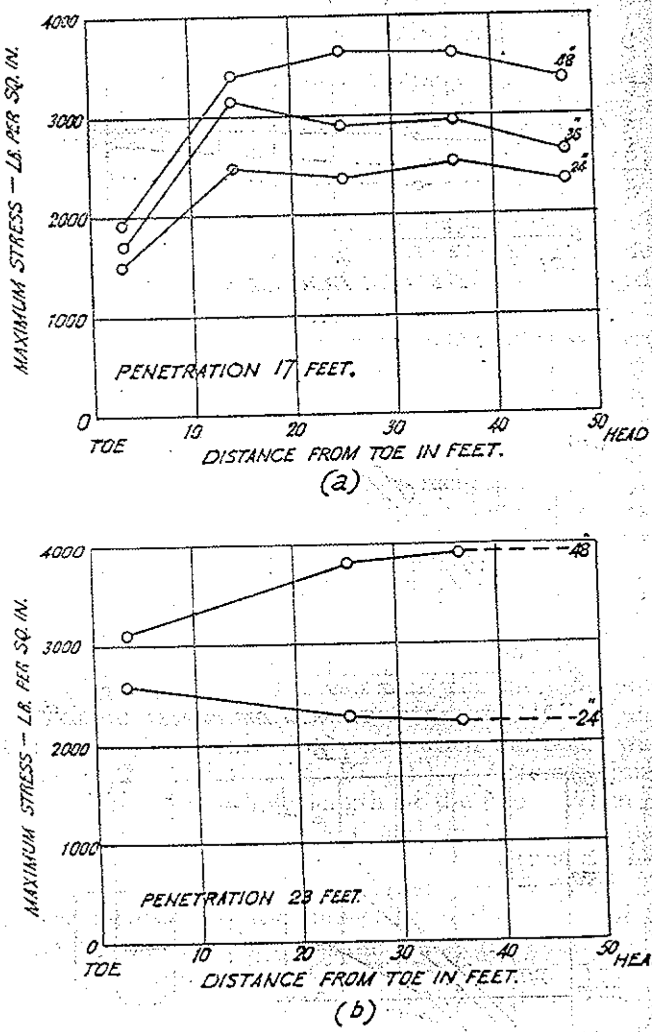

FIGURE 28.—*Lots Road Piles.*

Distribution of maximum compressive stress along the pile at various pe[netra]tions, with contractor's packing and helmet, and drops of 24, 36 and [48 in.] (*a*) and (*b*), pile 1; (*c*) and (*d*), pile 2.

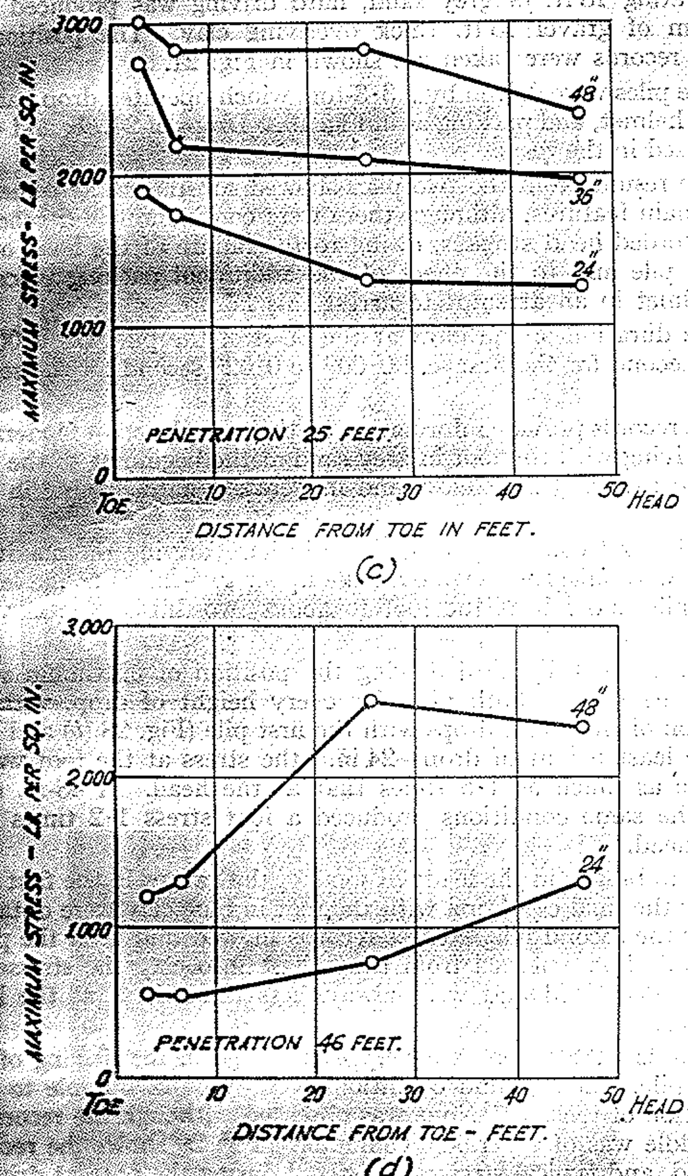

FIGURE 28.—*Lots Road Piles—continued.*

[Distri]bution of maximum compressive stress along the pile at various penetra[tio]ns, with contractor's packing and helmet, and drops of 24, 36 and 48 in. [(a)] and (b), pile 1; (c) and (d), pile 2.

The ground conditions were similar in each case (Fig. 21); penetrating 18 ft. of grey sand, hard driving was produced stratum of gravel 13 ft. thick overlying clay. The position which records were taken are shown in Fig. 21.

The piles were driven by a 3·3-ton, winch-operated drop ha with a helmet, and packing as in Fig. 19. Drops of 24, 36, and were used *in the tests*.

The results from the two piles showed substantial agreeme *their main features*, although there were considerable differen the recorded head stresses, owing to the failure of the head o second pile and to the effect of the additional packing plac the helmet in an attempt to arrest this failure.

The duration of the blow at the head of the pile was 0·0 0·012 second for the first and 0·009 to 0·020 second for the se pile.

The records provide information on *three conditions*—moder hard driving with the foot in grey sand at the beginning of the very hard driving in ballast, and moderately hard driving in clay.

In the first condition of moderately hard driving the maxi stresses were fairly uniform over the upper two-thirds of the l of the pile and fell at the foot to about two-thirds of that (Fig. 28 (*a*)).

During the very hard driving the position of maximum was at the foot of both piles for every height of drop, wit exception of the 48-in. drops with the first pile (Fig. 28 (*b*) and For the least height of drop—24 in.—the stress at the foot w one case as much as 1·5 times that at the head. A 48-in. under the same conditions produced a foot stress 1·2 times at the head.

Due to breaks in the leads to some of the gauges in the first data for the final condition with the foot in blue clay were *ob* only for the second pile. These records showed that after the *of the pile had emerged* from the ballast into clay the stres the foot decreased and were always less than those at the (Fig. 28 (*d*)).

Records were taken at a very early stage in the driving o second pile, when ground conditions were most favourable to production of tension. None was observed, although the rec the middle was of a periodic character. Fig. 29 shows the r obtained, and typical stresses registered during the hardest dr

It is of interest to note that the increase in the k/A value head cushion was not at all pronounced. For 48-in. drops wi first pile, where the same packing was used throughout, the st the head increased only 16 per cent. during the driving.

The failure of the head and the additional packing placed helmet of the second pile had a marked effect on the set per blow

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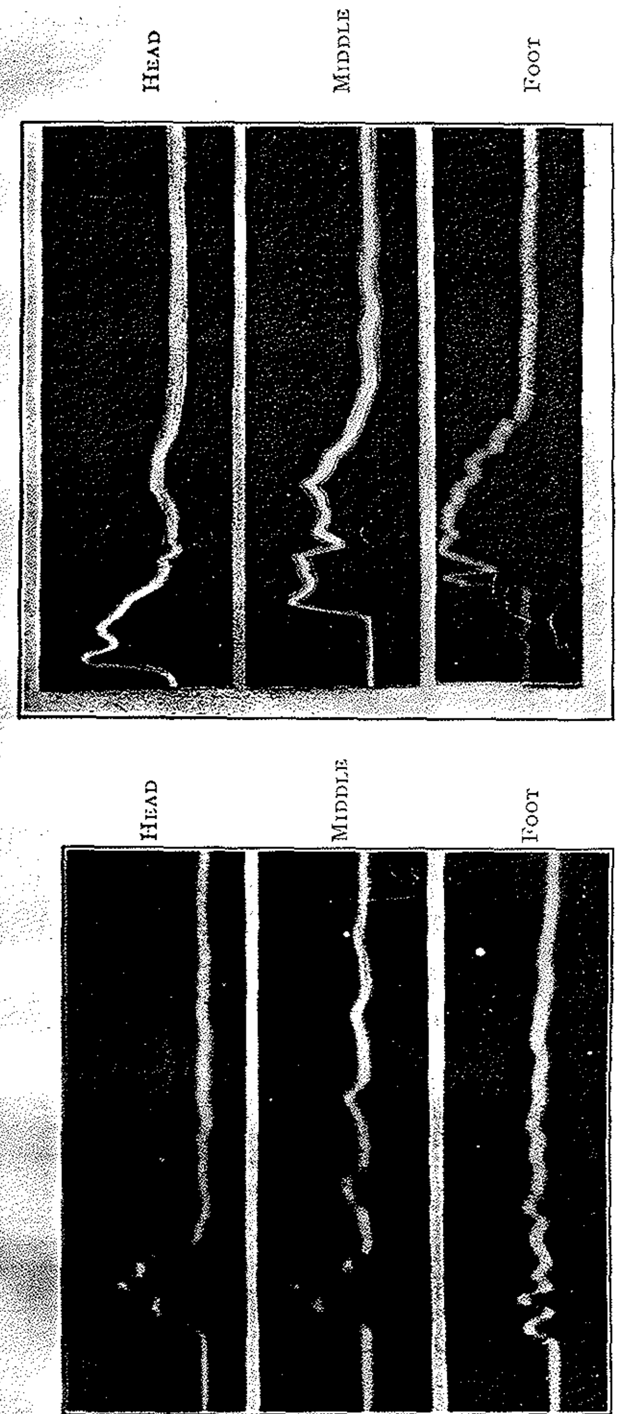

FIGURE 29.—*Lots Road Pile 2. Typical Records for (a) Easy and (b) Hard Driving.*

Driving conditions: contractor's packing in 8-cwt. helmet; 3·3-ton hammer; drops (*a*) 24 in.; (*b*) 36 in.; penetration of point (*a*) 14 ft.; (*b*) 25 ft.

Maximum compressive stresses: (*a*) head—1,590, middle—1,400, foot—520 lb. per sq. in.; (*b*) head—1,930, middle—2,170, foot—2,760 lb. per sq. in.

Sets: (*a*) 0·94 in.; (*b*) 0·06 in.

Duration of blow at head: (*a*) 0·010 second; (*b*) 0·009 second.

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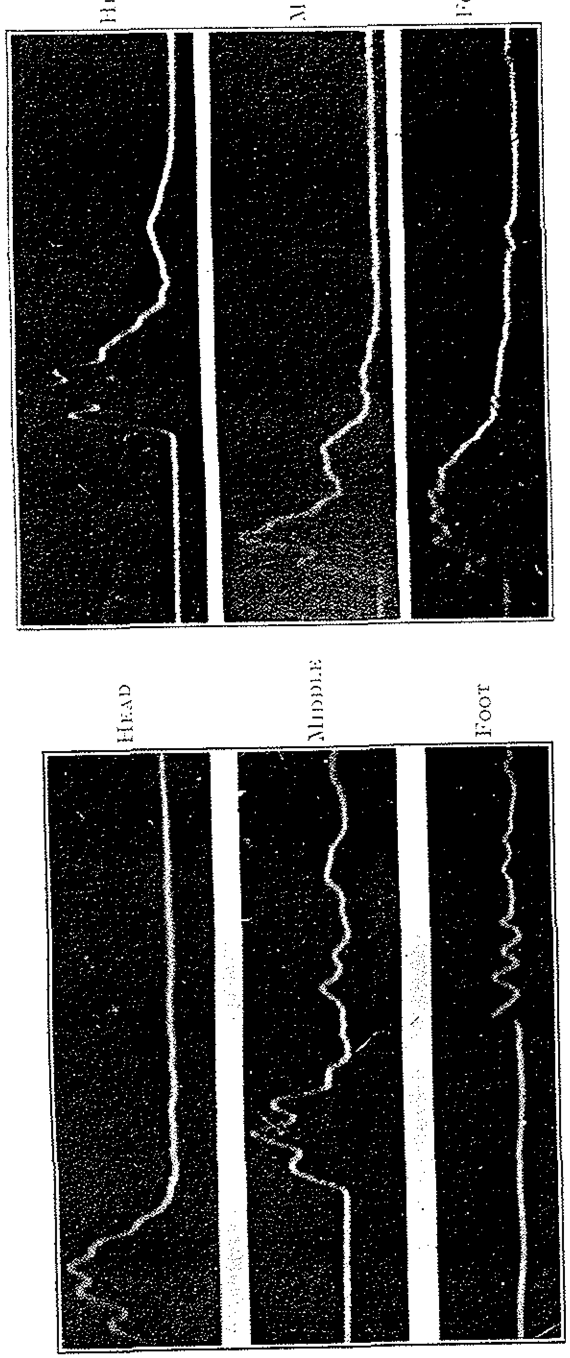

FIGURE 32.—ALBERT DOCK PILE.—*Typical Records for (a) Easy and (b) Moderate Driving.*
Driving conditions: contractor's packing in helmet; 3-ton hammer; 24-in. drops; penetration of foot—(a) 19 ft. (b) 40 ft. 9 in.
Maximum compressive stresses: (a) head—1,300, middle—1,070, foot—220 lb. per sq. in.; (b) head—1,850, middle—1,550, foot—700 lb. per sq. in.

driving was hardest. A decrease in maximum head stress was accompanied by decreased set.

The necessity to secure a uniform thickness of packing was demonstrated by the failure of the head of the first pile after the insertion of fresh packing under the helmet. Up to this time the head, specially strengthened with steel hoops, had withstood very heavy driving without a sign of fracture. As soon as the new packing was inserted, however, progressive disintegration of the head began. Upon removal of the helmet it was found to be due to the fact that the rope and canvas mattress had slipped towards the back of the pile, probably at the time the helmet was lowered over it, resulting in serious failure as shown in Fig. 30 over the uncovered area

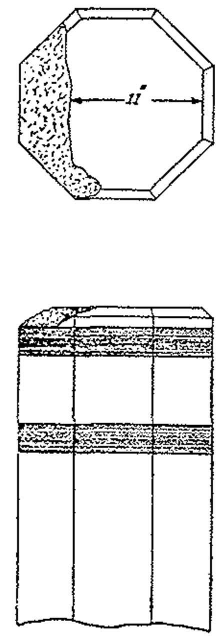

FIGURE 30.—*Lots Road Pile* 1—*Damage to Pile Head after* 13 *blows from a height of* 2 *feet, using new packing.*

(5) 50-*ft. Pile Driven at Messrs. Mowlem's Yard, Albert Dock*

A pile, measuring 50 ft. by 14 in. by 14 in., was driven at Messrs. Mowlem's Yard, Albert Dock, as a test of the peak stress indicator. The test necessitated only one gauge at the head of the pile, but in order to take advantage of the opportunity of acquiring further information thus afforded, gauges were also embedded at the foot and middle of the pile.

The ground conditions were not accurately known. It was anticipated that ballast would be reached at 40-45 ft. below ground level, underlying Thames mud and silt.

A 3-ton winch operated hammer was used with a helmet packing as in Fig. 20. As the pile head was damaged immedi after the commencement of driving, it was only possible to few test blows at heights of drop greater than 24 in., and the ma of driving was done at this height of drop.

Throughout the driving, the peak stresses were greatest upper portion of the pile, falling off to a minimum at the foot, they were a little over one-third of the head stresses (Fig Driving had to be discontinued at a penetration of 40 ft. ow rapid disintegration of the head. The stress records show no c indicating the presence of a hard layer, and it is unlikely tha foot of the pile had reached the ballast when driving was aband

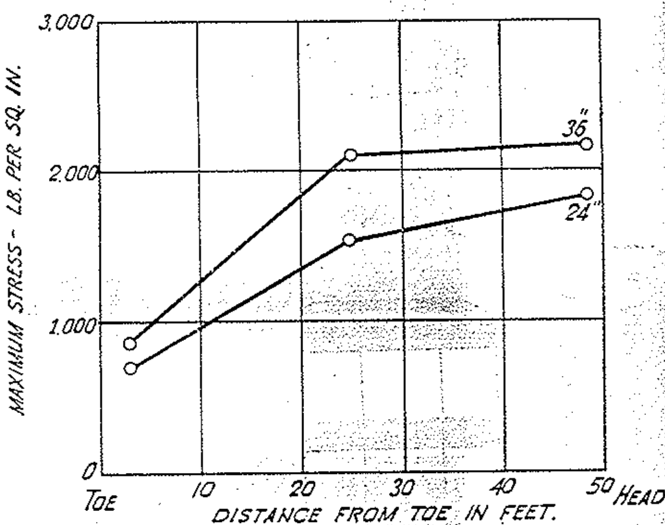

FIGURE 31.—*Albert Dock Pile.—Distribution of maximum compressiv along pile at penetration of* 40 *feet, with contractor's packing and drops of* 36 *inches.*

Under favourable conditions of easy driving, an attemp again made to detect tension in the early stages of penetration. records were similar to those of the second Lots' Road pile in periodic character, but the stresses recorded were purely compr

Fig. 32 shows typical records, at two stages of penetratio

Discussion of Results

The Effect of Driving Conditions on the Stress in the Pil

When the results of the strain measurements are examined the most general point of view, the first fact to emerge is for the conditions of driving most productive of damage, the st developed were purely compressive and might attain a ma

lue as great as 4,000 lb./sq. in. Under certain conditions of driving ferred to again later, tensile stresses of short duration and high lue were set up, but as they did not result in serious failure, it is cluded that compressive stresses are the main cause of the lure of reinforced concrete piles. It is not known, at present, how e destructive effect of repeated impact depends on the number of petitions of stress, and the form of the stress-time curve, but tests destruction on small piles and specimens of plain concrete, desbed later, have shown that there is a relationship between the ximum compressive stress in the concrete and the resistance to ure. The relation is not exact, and more extensive research might eal the effect of the other factors, but as at present there is no dence that the differences of form and duration between the stresses different points along a pile have any serious effect upon impact sistance, the maximum compressive stress must be regarded as the ly useful criterion of the probability of failure.

The theory and the experimental measurements are based on the sumption that the strains are uniform over the cross-section of e pile. Failure, however, is never general and commences at aces subjected to local concentrations of stress which it is not acticable to record. As a consequence, the average stress over e cross-section where failure occurs is most probably somewhat s than it would be in the ideal case of perfectly even stress tribution.

In the majority of cases the highest compressive stress induced ing driving occurs at the head of the pile. In every test where, s generally the case in practice, the resistance to motion of the e was mainly due to skin friction, the peak value of the compressive ess decreased from the head to the toe of the pile (Figs. 23, 27, 28 d 31), although in certain circumstances values only slightly er than those at the head occurred at other positions. At the ddle of a pile, stresses have been recorded equal to, or even asionally greater than, those at the head (Fig. 28 (*d*)), but these e within the experimental error of the stress measurements. The ximum value of the stress at the head is in most cases independent the ground conditions. When the conditions at the head, and length of the pile are such that the maximum value of the stress the head is attained soon enough to be unaffected by the reflected ve from the foot, the value of the maximum stress is determined y by the conditions at the head, that is, by the weight of the mmer, the height of drop, the area of the pile head, the physical stants of the pile, and the stiffness of the cushion. The records w that, for normal head conditions, the stress maximum is ched in about 0·005 second. During that time the stress ve travels about 60 ft. For a pile 30 ft. long, therefore, the ected wave will arrive at the head at the moment when the ximum value of the stress occurs there, but unless the reflected ve is one of compressive stress of considerable magnitude, such as one from a rigidly restrained foot, the maximum at the head will be unaffected thereby. A reflected wav tensile stress in these circumstances has the effect of making stress fall off sharply after its arrival at the head, but does affect the stress maximum. A reflected compression wav considerable magnitude is unlikely to occur even in the ha driving (see page 48), and as a general rule it is safe to state for piles over 30 ft. long the head maximum is independent of conditions. It is difficult to quote figures from the strain meas ments to illustrate the independence of head stresses from gro conditions, owing to the difficulty of keeping the conditions cons at the head. The following figures for the maximum head s of Pile A.5, the 15-ft. pile driven into stiff clay at the Buil Research Station, in which the head conditions were approxim constant, show only a small change of head stress with penetra The maximum head stresses for a 24-in. drop of a 480-lb. ham on a head cushion of *four thicknesses* of ¼-in. felt were 2,000, 2 2,180 and 2,400 lb./sq. in. at penetrations below ground lev 4 ft. 3 in., 7 ft. 9 in., 9 ft. 9 in. and 12 ft. respectively, the corresp ing sets per blow being 0·18, 0·04, 0·01 and 0·00 in.

A deduction of some importance from the foregoing is that packing has initially a high stiffness constant, the head stress the early or easy stages of driving may be nearly as high as at stages when the driving is hard. This state of affairs is like obtain when a pile is driven with a head packing which has consolidated by previous use. In such circumstances it is u to increase the height of drop of the hammer during the early stages of driving.

The maximum stresses throughout the pile increase with ratio $\frac{\text{weight of hammer}}{\text{weight of 1 ft. of pile}}$. This is a more general and acc way of stating the obvious fact that the stresses in a given pil with a given height of drop increase with the weight of the ham At the head, where a comparison is most easily made, the var of maximum stress with the value of the ratio is found to co with theory: for example, in Table 2 (p. 47) for the 15-ft. pile the theoretical values of the maximum head stress for a pack four felts with hammers weighing 480, 980 and 2,000 lb. and drop of 12 in. were 1,360, 1,780 and 2,180 lb./sq. in. respectively the corresponding recorded values 1,250, 1,720 and 2,560 lb./sq. reasonably good agreement. In practical pile driving with a h helmet an effective value of the ratio corrected for the weig the helmet, has to be used.

As the height of drop of the hammer is increased, other fa remaining constant, the stresses increase not in proportion to square root of the height of drop, as would be the case if the helmet and packing were equivalent to the assumed linear sp but in such a way that at higher drops the stresses are greater

those given by the square root relationship (Table 2). This is due to the non-linear stress-strain characteristics of practical head cushions, and in particular to the characteristics of the packing material in the helmet. The static stress-compression curve of felt given in Fig. 33, is characteristic of helmet packing materials. The determination of the dynamic, that is, the impact stress-compression curves for helmet packings is described later (p. 70), and in Appendix I a method is explained for determining for any packing material an equivalent linear spring, to which the theory is directly applicable. The equivalent linear spring is such that the work done in compressing it up to a given maximum stress is the same as that done in compressing the packing material to the same stress. The stiffness of the packing material at the value of the

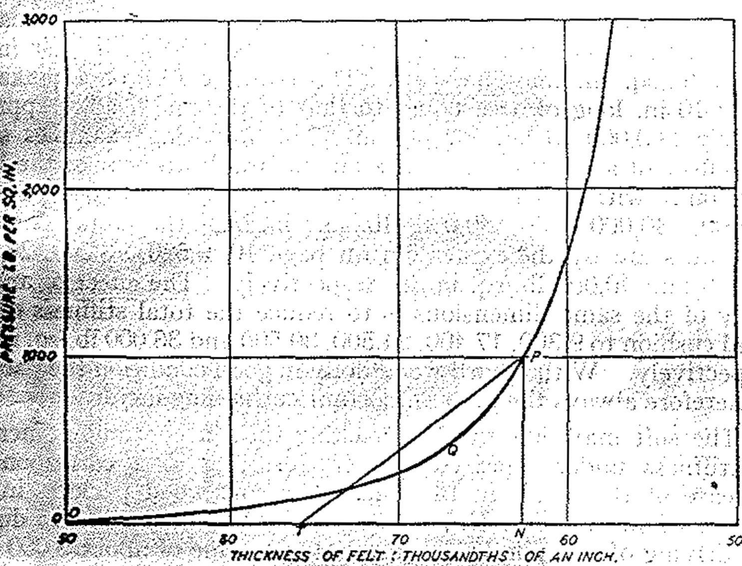

FIGURE 33.—*Compression Curve for one Felt initially ¼ in. thick.*

maximum head stress considered, is measured by k/A, the constant denoting the stiffness of the equivalent linear spring, and is dependent on the magnitude of the maximum stress at the head. For many helmet packings the k/A value is approximately proportional to the maximum stress.

The stiffness of the head cushion is a factor of the greatest importance in determining the stresses in the pile. If no cushion were present between the hammer and the pile head the stress at the head would rise almost instantaneously to a maximum. The head cushion decreases both the rate of increase of stress and its maximum value. Stresses throughout the pile are similarly affected. The lower the value of the stiffness constant, k/A, of the head cushion,

the smaller is the rate of increase of stress at the head and the lo
are the stresses in the pile. Numerous instances of the effect of t
stiffness of the head cushion have been observed during the investig
tion; an example from the preliminary tests is given in Fig. 34.

The cushioning effect of the head covering is chiefly due to t
packing beneath the helmet. The effect of the dolly is relativ
small except in cases where the packing has been consolidated
such an extent that its stiffness has become very high, or where t
dolly has become soft by brooming. The stiffness constants
of several types of head packing have been deduced from the stres
recorded during the driving of test piles, and the results show t
the value of k/A for the packings used in practice may lie anywh
between 1,000 and 50,000 lb./sq. in. per in. at 3,000 lb./sq.
Values as low as 1,000 lb./sq. in. per in., however, only apply
the first blows with new packing, and for practical stress estimat
upper and lower limits of 10,000 and 50,000 lb./sq. in. per in
3,000 lb./sq. in. may be used. The stiffness k/A of a pinka
dolly 10 in. long of area equal to that of the pile head is appr
mately 200,000 lb./sq. in. per in. The following examples s
the effect of a dolly of this type on the total stiffness of the h
cushion: with packing having constants $k/A = 10{,}000$, 20,0
30,000, 40,000 and 50,000 lb./sq. in./in., the total stiff
constants are by the expression on page 10, 9,520, 18,200, 26,0
33,500 and 40,000 lb./sq. in./in. respectively. The effect of an
dolly of the same dimensions is to reduce the total stiffness of
head cushion to 9,300, 17,400, 24,500, 30,500 and 36,000 lb./sq. in
respectively. With a hard wood dolly in good condition the pack
is therefore always the more important cushioning factor.

The soft materials used as packing show a continuous incr
of stiffness under repeated impact, resulting in a correspon
increase of the stress in the pile. The soft wood packing in
helmet of the London University pile increased in stiffness du
the driving of one pile from an initial k/A value of about 3,000
final value of nearly 50,000 lb./sq. in./in. at 3,000 lb./sq. in. an
maximum head stress at the end of the test that is, after a
1,300 blows, was 100 per cent. greater than at the beginning o
driving. The above figures refer to the helmet packing alone
were deduced from the total stiffness of the head cushion, by allo
for the effect of the pinkadou dolly (see p. 10). It was not pos
to separate the effect of the helmet and dolly for the Lot's R
piles as the dolly showed signs of deterioration at the end o
driving and therefore could not be regarded as having the
value of k/A at the end as at the beginning. The increase in
total stiffness of the head cushion for the first Lot's Road pile
much less than for the London University pile, and caused a s
increase of only 16 per cent. at the head, but the explanatio
thought to be that the increased stiffness of the helmet packing
largely compensated by the decreased stiffness of the deterio

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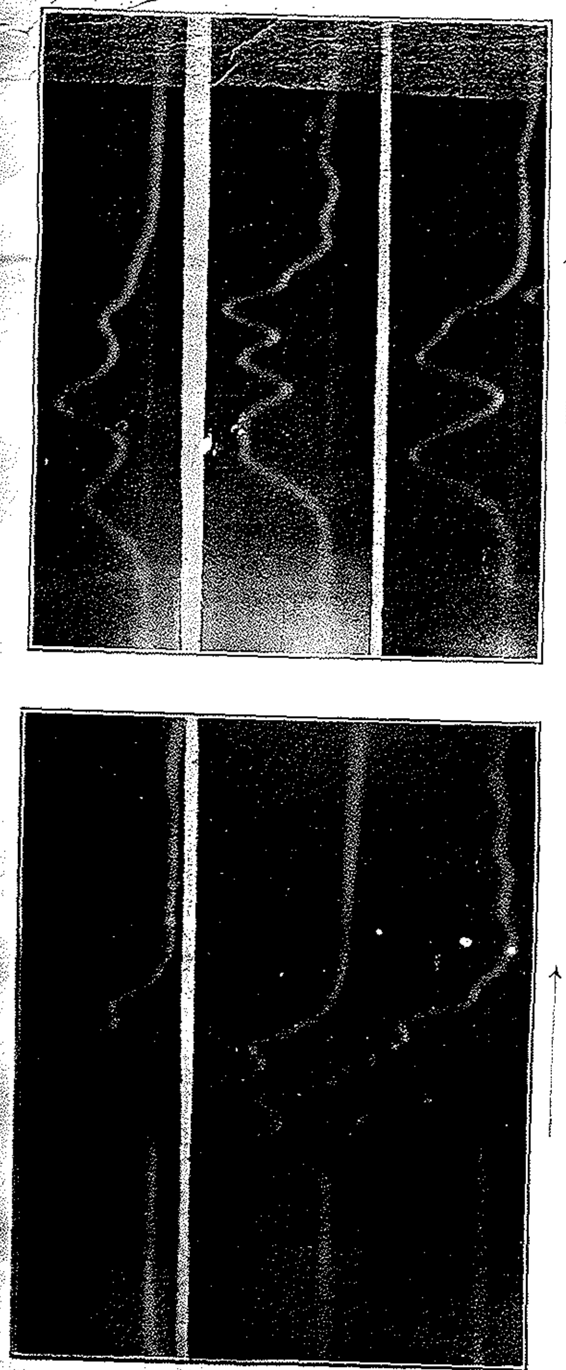


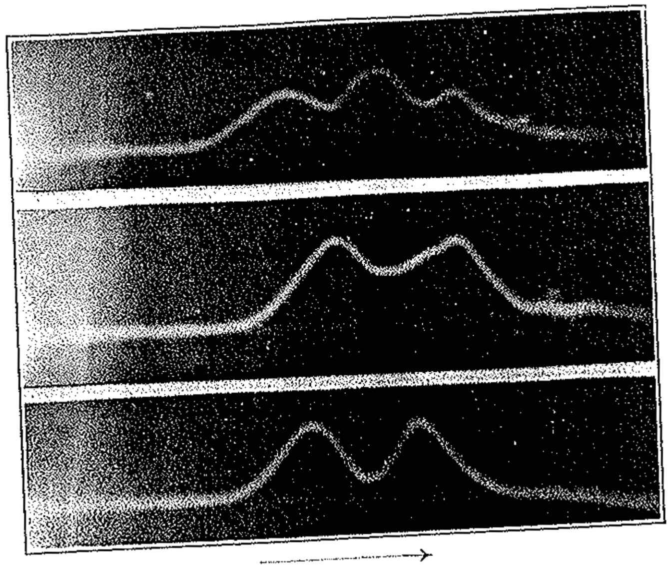
(a) (b)

FIGURE 34.—*The Effect of Packing on the Stresses in a 15-ft. Pile.*

Driving conditions: (*a*) two ¼-in. felts; (*b*) four ¼-in. felts; 980-lb. hammer; 12-in. drops; foot clamped down on ¼ in. of felt.

Maximum compressive stresses: (*a*) head—2,000, middle—2,500, foot—3,600 lb. per sq. in.; (*b*) head—1,500, middle—1,650, foot—2,500 lb. per sq. in.

Duration of blow at head: (*a*) 0·0065 second; (*b*) 0·011 second.



(c)

FIGURE 34 (*Contd.*)—*The Effect of Packing on the Stresses in a 15-ft. Pile.*

Driving conditions : (c) 2 in. sawdust in the helmet.

Maximum compressive stresses : (c) head—1,200, middle—1,350, foot—1,900 lb. per sq. in.

Duration of blow at head : (c) 0·0135 second.

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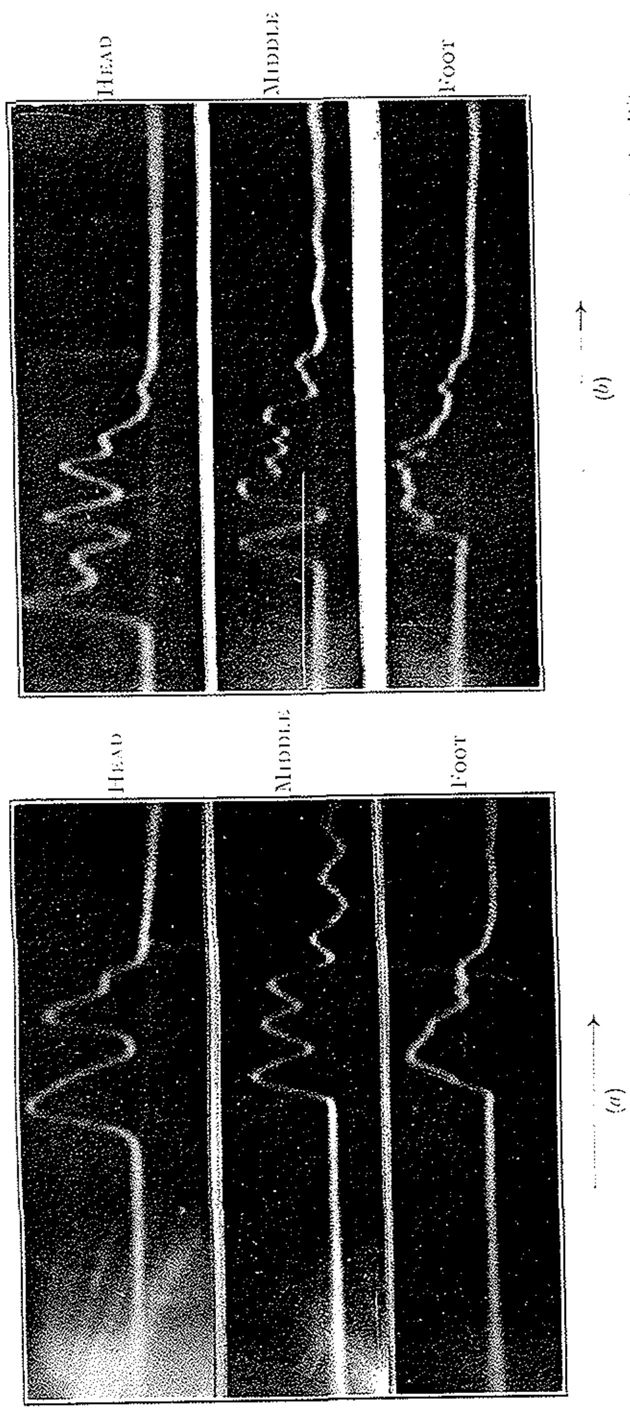

FIGURE 35.—*The Effect of the Helmet on the Form of the Stress-Time Curves—15-ft. Pile Driven in the Pit.*

Driving conditions: (*a*) two ¼-in. felts on head without helmet; (*b*) two ¼-in. felts with helmet; 980-lb. hammer; 12-in. drops; foot resting on hydraulic ram.

Maximum compressive stresses: (*a*) head—1,750, middle—1,300, foot—1,300 lb. per sq. in.; (*b*) head—1,950, middle—1,250, foot—1,050 lb. per sq. in.

Duration of blow at head: (*a*) 0·01 second; (*b*) 0·0075 second.

dolly, a conclusion supported by the results of the test giving the impact characteristics of coiled rope packing, described on page 70.

Dangerous local concentrations of stress may result from unevenness in placing the packing material on the pile head. An instance of this was met during the driving of the first Lot's Road pile, where the head, which had withstood thousands of blows without sustaining any damage, failed immediately after the insertion of fresh packing. The failure was definitely attributable to the uneven distribution of the packing which had slipped towards the back of the pile. Similar head failures have been observed due to the use of hard packing materials, such as hard wood blocks, placed directly on the pile head. A layer of soft material to take up slight irregularities prevents failure from this cause.

The effect of the helmet is to superimpose a vibration of comparatively high frequency on the stress-time curve, which in the absence of the helmet, would have the form of a smooth curve. The effect is shown in Figs. 26 and 35. The helmet is a mass supported between two springs, the dolly above and the helmet packing below, and is set into vibration by the impact of the hammer; the frequency of vibration is determined by the mass of the helmet and the stiffness of the springs. Apart from producing high frequency vibrations, the effect of the helmet is to increase the effective weight of the hammer, and to decrease the effective height of drop. When impact commences the hammer has first to accelerate the helmet. An approximate idea of what occurs may be obtained by neglecting the effect of the high frequency vibrations and assuming that, on impact, the hammer and helmet move on together. Then, by conservation of momentum, the velocity of the hammer and helmet is

$$v = \frac{M}{M + M_h} V$$

where M denotes the mass of the hammer
 ,, M_h ,, ,, ,, ,, helmet
 ,, V ,, ,, velocity of the hammer before the impact or in terms of the height of fall

$$h' = \left(\frac{W}{W + W_h}\right)^2 h$$

where h' denotes the effective height of fall, i.e. the height from which the hammer and helmet together would have to be dropped to acquire the velocity v.

W denotes the weight of the hammer.
W_h ,, ,, ,, ,, ,, helmet.
h ,, ,, height of free fall of the hammer.

The effective weight of the hammer is $W + W_h$.

The neglect of the effect of the helmet in the theory was justified by the fact that the weight of the helmet is usually a small fraction, of the order of 1/10th, of the weight of the hammer, and therefore has only a small influence on the stresses in the pile. This has b confirmed by the experimental tests, but where the weight of helmet is greater than 1/10th of the hammer weight, its ef becomes appreciable.

The main function of the dolly is to protect the upper surf of the helmet. By virtue of the compressibility of the dolly acceleration imparted to the helmet by the hammer on striki kept within limits such that the stress in every part of the hel is below the yield point. Due to the oscillation of the helmet, stress in the dolly may reach a value considerably higher than in the packing, and the difference will increase with the weigh the helmet. Although no observations have been made to the statement, it appears likely that heavy helmets are more destr tive of dollies than light ones.

Having considered separately all the chief factors influen the form and magnitude of the stress-time curves at the head comparison of the calculated and recorded stresses can be m Table 2 contains theoretical and recorded values of the maxim head stress for the piles driven into ground. The agreement i the whole as close as may be expected bearing in mind the possibility of knowing the exact condition of the packing fo particular blow. The results for the London University pile s greater discrepancies than any of the others. Some difficulty experienced in this test with the valve controlling the steam ham and it is probable that the low values recorded are accounted by a loss of energy due to back-pressure in the hammer.

A considerable length of the upper portion of a pile may subjected to a maximum stress almost as great as that at the h if the reflected wave from the toe does not reach that part of pile in time to affect the stress maximum. The only decrease in part of the pile is that due to dissipation of energy by internal skin friction. The effect is shown best by the 50-ft. piles drive Lot's Road, where the maximum stress in the upper half of the was constant within experimental error (Figs. 28 (*a*) and though the maxima do not, of course, occur simultaneously the maximum is attained in about 0·005 second after the begin of impact and the velocity of stress in the pile is about 12,00 per sec., the upper 20 ft. of the pile might be expected to be affected by the reflected wave as was found experimentally.

The maximum stresses in the lower portion of the pile may as great or greater than those at the head only if resistance to mo of the pile is largely concentrated at the foot. When no movem of the foot is possible and skin friction is absent, it is theoretic possible for the foot stress to attain a maximum value, neglec propagation losses, equal to twice that at the head, and in some the preliminary tests at the Building Research Station stress this order were recorded at the foot. Fig. 36 shows the str

* Discussed mathematically in Appendix III.

TABLE 2. *Comparison of Calculated and Recorded Head Stresses*

Pile	Weight of hammer (lb.)	Height of drop (in.)	Head cushion	Maximum head stress (lb./sq. in.)	
				Calculated*	Recorded
15 ft. × 7 in. × 7 in. Driven at the Building Research Station	480	12	4 felts	1360	1250
		24	4 ,,	2220	2360
			12 ,,	1260	1270
		36	12 ,,	1760	1790
	980	12	4 ,,	1780	1720
		24	4 ,,	2810	2920
			12 ,,	1920	1940
		36	12 ,,	2540	2620
	2000	12	4 ,,	2180	2560
35 ft. × 14 in. × 14 in. Driven at London University	6720	24	12 ,,	1950	1920
			24 ,,	1450	1260
		36	12 ,,	2680	2680
			24 ,,	2000	1600
		54	24 ,,	2700	1850
0 ft. × 16 in. octagonal Driven at Lot's Road	7400	24	12 ,,	1920	1850
			24 ,,	1370	1300
		48	12 ,,	3100	3160
			24 ,,	2380	2740
0 ft. × 14 in. × 14 in. Driven at Royal Albert Dock	6720	36	24 ,,	1800	1880

* Using h/A values derived from the impact tests described on p. 70

ime curves at the head, middle, and foot of a 15-ft. pile driven gainst a rigid base. The maximum stress at the foot was 1·84 imes that at the head. Under practical conditions, foot stresses xceeding those at the head were recorded only on the Lot's Road ite, where, as shown in Fig. 21, the pile was driven through a layer f ballast 13 ft. thick, at a rate of penetration of 200 to 400 blows o the foot. The driving resistance was, therefore, very great and as concentrated at the foot. The measured maximum stresses or the two test piles at Lot's Road are given in Fig. 28, and at a enetration of 23 to 25 ft. the foot stresses are seen to range from ·5 times to 0·8 times the value at the head. Further consideration eveals that the ratio of the maximum foot stress to the maximum ead stress decreases as the stresses in the pile increase, that is, he foot stress does not increase at the same rate as the head stress. here is, in fact, a tendency towards a constant foot resistance. he normal height of drop of the hammer on the Lot's Road contract was 48 in., and for that drop the foot stress under the harde driving conditions for the first pile was 0·8 and for the second pi was 1·2 times that at the head, the difference being mainly due t the lower head stress recorded for the second pile. It is clear advantageous, under such driving conditions, to use high streng concrete, permitting high maximum head stresses, resulting in reduction of the ratio $\frac{\text{maximum foot stress}}{\text{maximum head stress}}$, and causing failure, any, to commence at the head rather than at the foot.

It is unlikely that the theoretical upper limit at the foot is ev realised in practical pile-driving, as, apart from the propagati loss, there is always a certain amount of elastic or recoverab deformation of the ground, and the foot is, therefore, never i movable as the theory requires. Foot maxima approaching t limit may be anticipated when driving to refusal against roc particularly for short piles where propagation losses are small.

A small set per blow is, in itself, no certain indication of hi foot stresses; when the smallness of the set is due to high sk friction, the stress at the foot is low compared with that at the hea since the effect of skin friction is always to reduce the stresses the embedded part of the pile. The 15-ft. pile A.5 driven into s clay at the Building Research Station showed a permanent set 0·28 in. at a penetration of 12 ft. for a 36-in. drop of the 2,000- hammer with 24 felts as head cushion; the similar pile A.6 driv through loose clay against a concrete block showed a set of 0·32 i under the same conditions; the foot stress for the first pile was 87 and for the second was 2,150 lb./sq. in. The results of the other te piles driven into ground show that when the foot penetrates cl or similar material the maximum stress is always at the head.

The ground conditions under which high foot stresses are set are, in the limiting case, represented by the theoretical assumpti that resistance to motion of the pile is concentrated at the toe. It w therefore anticipated that the theoretical analysis on this assumpti would furnish a reasonable estimate of the actual stresses in drivi against a hard stratum, when frictional resistance is likely to be r latively low. The maximum foot stresses have been estima theoretically for the second 15-ft. pile driven into ground at t Building Research Station and for the piles tested at Lot's Roa The estimates were made with the aid of the charts described in preceding section (pp. 9-11), and in Table 3 the results are compar with the recorded values. The theoretical values are 20 to 30 p cent. higher than those recorded. It is probable, therefore, th even in extreme cases of high foot resistance, skin friction a propagation loss effect a considerable reduction in the magnitu of the stress wave travelling down the pile.

Tensile stresses of short duration but considerable magnitud occurring in the middle portion of the pile, have been recorded f short piles driven into ground but not in any test on a practical pi

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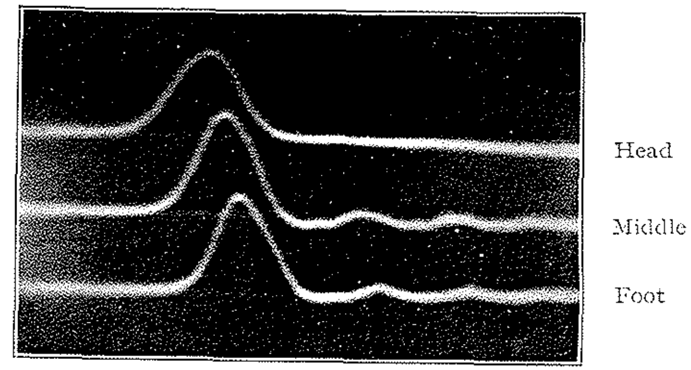

FIGURE 36.—*Records Showing High Foot Stresses Registered when Driving a 15-ft. Pile Against a Rigid Base.*

Driving conditions : 1½ in. sawdust packing on head; no helmet; 980-lb. hammer; 12-in. drops; foot clamped down on ¼ in. of felt.

Maximum compressive stresses : head—1,350, middle—1,650, foot—2,480 lb. per sq. in.

Duration of blow at head : 0·01 second.

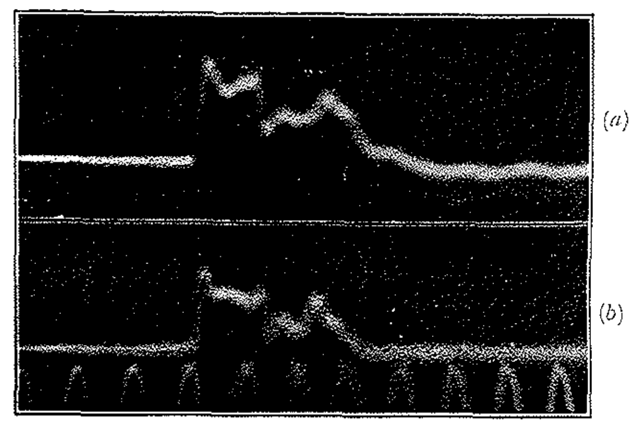

FIGURE 37.—*Lots Road Pile 2.*

Records from gauges placed (*a*) on the pile axis at the mid-point of the length, and (*b*) 4½ in. from the pile axis and 3 ft. above the mid-point, 36 in. drops in each case. The time scale (bottom) marks intervals of $\frac{1}{256}$ second.

TABLE 3.—*Comparison of Calculated and Recorded Foot Stresses*

Pile	Weight of Hammer (lb.)	Height of drop (in.)	Head Cushion	Corrected equivalent elastic set (in.)	Maximum foot stress (lb./sq. in.)	
					Calculated*	Recorded
15 ft. × 7 in. × 7 in. Driven at the Building Research Station	980	12	12 felts	0·16	1750	1300
		24		0·36	2100	1670
		36		0·37	3050	2000
	2000	12		0·18	2500	1780
		24		0·48	2850	2230
50 ft. × 16 in. octagonal	7400	24	Contractors standard	0·20	3200	2600
		48		0·40	3800	3100
Driven at Lot's		24		0·21	2300	1900
Rd. (No. 1)		36		0·30	3150	2760
(No. 2)		48		0·39	3600	3040

* The k/A values for the pile driven at the Building Research Station were derived from impact tests; those for the Lot's Road piles were deduced from the maximum head stresses.

driving contract. The results show that to set up high tensile stresses the pile must be free to vibrate at its fundamental longitudinal mode, with antinodes, that is, places of maximum motion and minimum stress, at its ends. To fulfil these conditions the ground resistance must be low and the head conditions such that the hammer rebounds early and leaves the head free, *i.e.*, a hard packing and a light hammer must be used. Such conditions were realised in the driving of 15-ft. piles through clay at the Building Research Station. Fig. 22 shows the records obtained under conditions favourable to this phenomenon, and the suppressing effect of increased ground resistance. It was anticipated that similar conditions might exist in the early stages of driving piles at Lot's Road and the Albert Dock. The recorded stresses (Fig. 29 and 32) were, however, purely compressive, although in their periodic character those at the mid-point of the length are similar to the record (*b*) of Fig. 22. From theoretical consideration it is seen that the ground resistance was too high to allow tensions to be set up. For large and small piles to yield records of similar form, it is necessary that the ratios M/M_p, kl/EA and the quantity ϵ_1 equal to the equivalent elastic set divided by Vl/A should remain constant. The first two ratios are approximately the same for the large and small piles but for the quantity ϵ_1 to remain constant the equivalent elastic set must be increased in the ratio of the pile lengths, that is, for tensions of the same order at the middle of the 50-ft. pile as were recorded for the 15-ft. pile, the equivalent elastic set would have had to be 50/15ths, or more than three times that for the 15-ft. pile. It is interesting to note that no sign of failure in tension was observed duri the driving of the 15-ft. test pile at the Building Research Statio referred to above, although tensile stresses greater than the tens strength of concrete were recorded.

The tensile stresses due to flexural vibration caused by a sing impact delivered eccentrically have not been proved to reach a hi value and mathematical investigation* shows that such tens stresses should be small. Fig. 37 shows a record from a gau embedded 22 ft. from the head and 4½ in. from the axis of the seco pile driven at Lot's Road. The penetration of the pile at the ti was 25 ft. with a set of 0·1 in. per blow. There is no trace of flexu vibration. There is a possibility of resonance, however, where t number of impacts per minute is large, as with double acting stea hammers. For the occurrence of resonance, the interval betwe the blows of the hammer must be a small multiple of the period flexural vibration of the pile, a relationship unlikely to occur wi drop hammers and single acting steam hammers, for, assuming t pile is supported at both ends, a pile 70 ft. by 18 in. by 18 in. wou vibrate at about 105 per minute, one 40 ft. by 14 in. by 14 in. 250 per minute, and one 20 ft. by 7 in. by 7 in. at 500 per minute

The Effect of Driving Conditions on the Set of the Pile

Though primarily concerned with the stresses set up during t driving of reinforced concrete piles, the investigation has resulted the acquisition of a considerable amount of information on the eff of driving conditions on set.

It is widely recognised that the energy efficiency of driving creases with the weight of the hammer used. Fig. 24 showing variation of plastic set with weight of hammer for a 15-ft. pile (driven into stiff clay demonstrates that the set for a given ene increases with the weight of the hammer. Thus 2,000 ft.-lb energy produced sets of 0·02, 0·04 and 0·07 in. per blow with 480, 980 and 2,000-lb. hammers respectively and a head cushion sisting of 24 thicknesses of ¼-in. felt. The increase of energy efficie is less marked for easy than for moderate and hard driving, shown by the results of the 15-ft. test pile A.6, driven into soft The energy efficiencies of the 480, and 980-lb. hammer (the 2,00 hammer was not used) were nearly equal.

For a given maximum stress at the head, obtained by suita adjusting the height of drop of the hammer, the set increases w the weight of the hammer. For easy and moderately hard driv this is shown by the results of tests on the 15-ft. piles driven ground, one of which provided the data for Fig. 38. The pla sets are plotted against the maximum stresses at the head, the l connecting points obtained by varying the heights of drop of hammers. For example, with a packing of 12 felts and a h stress of 2,000 lb/ sq. in. the sets are 0·02, 0·06, 0·11 in. per b

* Appendix IV.

with the 480, 980 and 2,000-lb. hammers respectively. Similar results were obtained from the test driving of the pile A.6 into soft clay. Experimental evidence is lacking for hard driving, but theoretically, the same effect should be obtained. It is an advantage, therefore, to use a heavy hammer even when the driving is so easy that the gain in energy efficiency is negligible, since the same driving efficiency is obtained with lower stresses in the pile.

The stiffness of the head cushion has been shown to be an important factor in determining the stresses in the pile; its effect on set is also considerable, but is largely dependent on ground conditions.

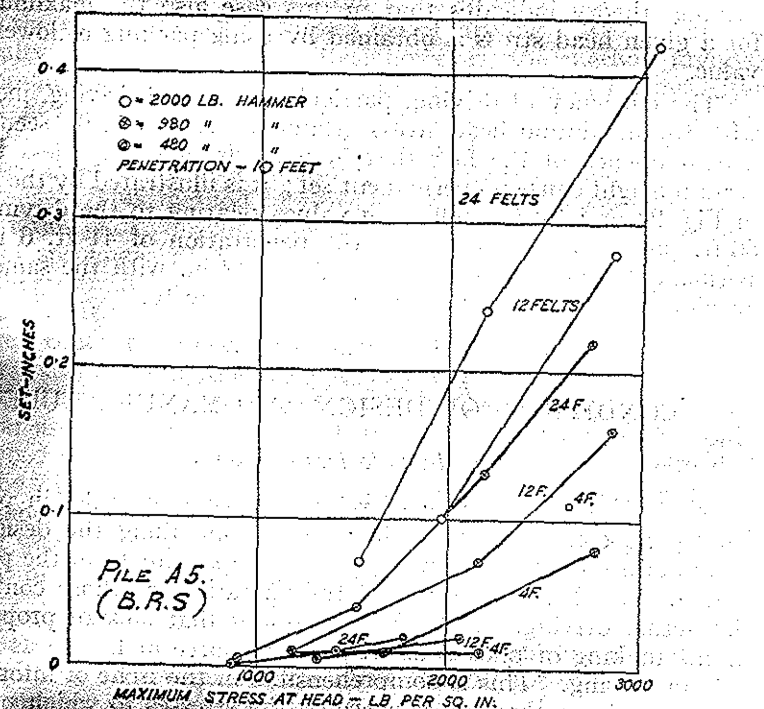

FIGURE 38.—*Pile A.5. Variation of Permanent Set with maximum Head Stress.*

Within the range investigated, the energy efficiency of a hammer was greatest when the stiffest packing was used, the gain of efficiency increasing with hardness of driving. During the last stages of the driving of the 50-ft. Lot's Road piles, test blows were delivered with thicknesses of 12 and 24 ¼-in. felts as packing, and it was observed that the sets for a particular height of drop of the hammer were, in every instance, greater with 12 than with 24 felts. The set with the contractor's packing and drops of 48 in. had an average value of 0·10 per blow; with 12 and 24 felts the sets were 0·17 and 0·12 in. respectively. For moderate driving Fig. 24 shows that an appreciable increase of energy efficiency accompanies a reduction in the stiffness constant, but for driving as easy as that experienced when testi[ng] the 15-ft. pile A.6 in soft clay the set is practically unaffected by th[e] change in the stiffness constant due to replacing a packing of 4 by o[ne] of 24 felts.

Greatest set for a given maximum head stress is produced b[y] the use of the packing of lowest stiffness. It is clear that this m[ust] be so for easy driving, since the energy efficiency is practica[lly] unaffected by packing stiffness while the maximum head stress [is] dependent on it. Fig. 38 demonstrates the effect for moder[ate] driving, and although there is no experimental evidence for ha[rd] driving, theory indicates that in this case also the maximum s[et] for a given head stress is obtained by using packing of lowest k[/l] value.

The efficiency of driving, particularly in hard driving, increas[es] with the maximum head stress attained. This is easily seen to b[e] a consequence of the fact that in hard driving drops less tha[n a] certain height cause no permanent set; it is illustrated by the cur[ves] of Fig. 24 and by the following values relating to the driving o[f a] 50-ft. pile at Lot's Road: at the penetration of 41 ft. 6 in. t[he] permanent sets for drops of 24, 36 and 48 in. with the same he[ad] cushion were 0·04, 0·10 and 0·19 in. respectively.

CONDITIONS OF DESIGN AND MANUFACTURE

Tests to Destruction

A study of the failure of reinforced concrete piles in prac[tice] led to the conclusion that the factors controlling the design [and] manufacture of the piles were quite as important from the poin[t of] view of the ultimate success of the operation, as those control[ling] the actual driving. There is little doubt that lack of proper c[are] in the making of piles is as frequent a source of failure as lack [of] care in driving. This is comprehensible in the light of informat[ion] which has been obtained concerning the stresses to which piles [are] subjected during driving, and which indicates that under "norma[l"] contract conditions stresses are imposed which approach, and in [the] case of very hard driving exceed, the impact strength of the concr[ete]. Under these conditions a small reduction in strength arising f[rom] faulty manufacture may make all the difference between succ[ess] and failure.

In order to study the effect of manufacturing conditions u[pon] driving resistance it was apparent that tests to destruction wo[uld] be necessary. Within the means available two courses were op[en]: to test a small number of 15-ft. piles under standardised condit[ions] representing as nearly as possible those met in practice, or to [test] a large number of small specimens under conditions which could [be] controlled within close limits but which might fail in certain resp[ects]

to represent practical conditions, particularly those governing the head and foot of an actual pile.

Since neither method would provide the whole of the necessary information it was decided to explore the field in a general manner by means of tests upon 15-ft. piles, and to use impact tests on small specimens as a means of obtaining supplementary information and following up promising lines of investigation.

The question of standardising conditions of test for 15-ft. piles was next considered. Obviously the ideal method would be to drive the piles into ground representing a range of practical conditions from the easiest to the hardest type of driving. Apart from the difficulty of finding suitable sites, however, the method involved the prohibitive expense of digging out the test piles for inspection after they had been driven. It was decided, therefore, to drive the test piles in an open pit on to a rigid base, using packing of a suitable type at the foot to simulate the desired conditions of toe resistance. This method of testing permitted the standardisation or variation of driving conditions as required, facilitated the examination of specimens at all stages of testing and enabled the corresponding stress measurements under similar conditions to be obtained. It also enabled an approximation to be made to the severest conditions likely to be met in practice, since these usually occur when the toe resistance is very high and the skin support small. The driving equipment used for these tests was the same as that used on earlier tests with the piezo-electric recorder and is described on page 20. In Series D and E, however, an elm dolly 7 in. by 7 in. by 18 in. long was substituted for the cast iron helmet and short timber dolly. In these tests also an electrically driven winch was used instead of a hand operated unit. In all cases the weight of hammer used was 980 lb.

Before proceeding to deal with individual factors it was considered necessary to carry out preliminary tests to determine what range of heights of drop would be the most suitable for subsequent tests, what number of blows would be required at each height and what form of failure would be likely to occur. For this purpose a series of 10 piles 15 ft. long and 7 in. square was made, using rapid-hardening Portland cement, Ham River sand $\frac{3}{16}$ in. down, Ham River gravel $\frac{3}{4}$ in.–$\frac{3}{16}$ in., and water in the proportions 1 : 2 : 4: 0·55 by weight. The reinforcement consisted of four $\frac{5}{8}$-in. diameter M.S. bars at $4\frac{3}{8}$-in. centres and $\frac{3}{16}$-in. diameter lateral ties spaced uniformly at 3-in. centres. Strongly braced wooden moulds were used for casting and the piles were finished with square, unchamfered ends. The main reinforcement was extended to the extreme ends of the pile, the first lateral tie at each end being placed $1\frac{1}{2}$ in. from the extremity. The piles were tested at 7 days old after having been cured for 2 days under damp sacks and for the remainder of the period in conditioned air. The atmosphere in which this and subsequent groups of piles were hardened was maintained at a temperature of 64° F. and a relative humidity of 64 per cent.

At the time of casting each pile, control specimens were made for use in determining those properties of the concrete which it was thought might be of value in elucidating the results of the tests. For static compression tests 4-in. cubes were made, for Young's modulus tests 8 in. by 4 in. cylinders and for transverse tests 16 in. by 4 in. by 4 in. beams. These were all cured under conditions identical with those of the pile to which they referred, and were tested at the same age.

During driving, the piles were held in a vertical position against the frame with two wooden clamps secured to the leaders 4 ft. 6 in. from each end of the pile. The foot was supported upon two layers of standard felt ¾ in. thick placed in the lead shoe shown in Fig. 16. Two layers of felt were also placed at the head of the pile under the helmet. These conditions were reproduced when testing subsequent groups of piles.

Each pile was tested with a fixed height of drop of the 980-lb. hammer, the range of drops for different piles being from 3 in. to 39 in.

In these and all subsequent tests, failure was taken as sufficient local disintegration of the concrete to expose the main reinforcement. Each pile was damaged as far as possible to the same extent, the possible variation in the number of blows required to do this being of the order of 10 per cent. A descriptive record was kept of the behaviour of each pile from the first sign of failure onwards, but it was not found possible to define accurately a point at which failure might be said to have just commenced. The difficulty was mainly due to the fact that failure began in different ways—sometimes in the form of cracks and sometimes as spalling—and that it sometimes ceased altogether at the point at which it began. In general, however, it was found that the results obtained from defining failure as complete local disintegration were not materially different from those which would have been obtained from considering only the first signs of failure.

The results of the group of tests under discussion are given in the form of a graph in Fig. 39. They show that the higher the drop the smaller was the proportion of the total number of blows causing final failure which was required to give signs of appreciable damage. Thus, with an 8-in. drop no real damage had occurred after 75 per cent. of the total number of blows, whereas 50 per cent. of the blows delivered with a 29-in. drop caused considerable spalling.

Observations were made on all the piles to detect any cracks which might form, but the central part of the piles gave no indication of deterioration except where a drop of 3 in. was used, when, after 400 blows, small cracks began to appear over the whole face of the pile; these had not widened appreciably after 1,300 blows. It is of interest to note that oscillograph records taken under similar

driving conditions and using a 6-in. drop showed maximum head and foot stresses of 1,300 and 2,300 lb./sq. in. respectively. The pile tested with a 6-in. drop in this series failed at the foot after 1,300 blows, while the cube compressive strength of the control specimens was approximately 5,000 lb./sq. in. This indication of a critical impact stress in the neighbourhood of 50 per cent. of the cube strength is in agreement with the results of impact tests on small specimens.

From the point of view of the conduct of future tests to destruction the present group indicated the probability of failure at one or other of the ends of the pile, and the immunity of the central portion. It also showed that the conditions chosen would enable destruction to be effected within a reasonable number of blows and at reasonable heights of drop. The existence of a critical height of drop, below which failure would not occur except after a large number of blows, suggested that use could be made of this phenomenon in devising

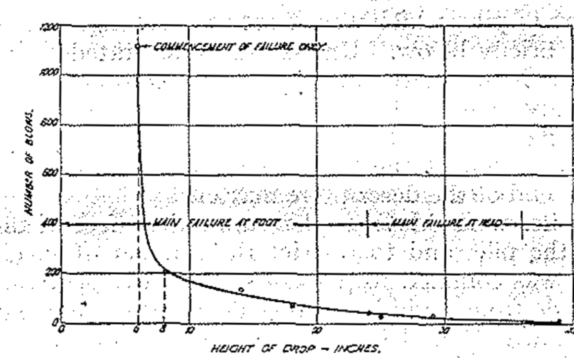

FIGURE 39.—*The effect of the Height of Drop of the Hammer on the Impact Resistance of a reinforced Concrete Pile.*

a measure of impact strength. It will be seen from the curve that this critical height is revealed after 200–300 blows; if, therefore, a scale of increasing heights of drop could be adopted, beginning at a sufficiently low value and applying about 200 blows at each height, the impact strength of the specimen could be measured by the numerical value of the height of drop at which failure ultimately occurred. The difficulty of putting this method into practice was entirely one of equipment. With the equipment then available it was felt that to adopt this method would involve too cumbersome and lengthy a procedure and make it extremely difficult to test the requisite number of specimens within a reasonable period. For this reason it was decided in subsequent tests to adopt a somewhat simpler procedure beginning at a greater height of drop and reducing the number of blows delivered at each height. A description of this procedure is given in the appropriate place, but may be anticipated to the extent of stating that, since comparative and not absolute results were required, no great difficulty was experienced in their interpretation. Instead of impact resistance being measured directly by height of drop at failure it was assessed by carefully considering this value in conjunction with the total number of blows delivered. For the last two series of tests (D and E), when improved equipment was available, the former method was adopted.

A great many factors are involved in the making of reinforced concrete piles, but in view of the limited number of specimens which could be tested it was obviously essential to concentrate attention upon those factors which appeared likely to be the most important. These were considered to be :—

(1) the form of the head of the pile;
(2) the amount and disposition of reinforcement;
(3) the type of cement, and the age at test;
(4) the cement content;
(5) the curing conditions;
(6) the type of aggregate.

The groups of piles which were tested to investigate these factors are described briefly below. Unless otherwise stated the piles were 15 ft. long and 7 in. square, and the driving equipment was similar to that already described. It was found convenient in certain cases to investigate more than one factor in a single group; for this reason the discussion of the results has been placed in a separate section at the end of the descriptive matter.

(A) Ten piles were driven to determine the effect of the form of the head of the pile, and to restrict the number of specimens the investigation was confined to piles of square cross-section. All the piles were made from rapid-hardening Portland cement, Ham River sand $\frac{3}{16}$ in. down, Ham River gravel $\frac{3}{4}$ in.—$\frac{3}{16}$ in. and water, the proportions by weight being 1 : 2 : 4 : 0·55 respectively. The longitudinal reinforcement consisted of four $\frac{5}{8}$ in. diameter mild steel bars at $4\frac{3}{8}$-in. centres; the lateral ties were $\frac{3}{16}$ in. diameter, placed 3 in. apart in the central portion of the pile and at the required pitch in the head. To ensure that failure should occur at the head the foot of each pile was cast in a square-ended welded steel shoe 4 in. deep and $\frac{1}{4}$ in. thick. Some of the shoes were made with open and some with closed ends; the two types appeared to be equally satisfactory. The design of the head was varied according to which of the following factors it was desired to investigate :—

(1) spacing of lateral ties.
(2) end cover to main reinforcement.
(3) chamfering the horizontal arises $\frac{3}{4}$ in.
(4) the relative effect of separate and continuous ties.
(5) the effect of casting the head of the pile in different forms of steel ring.

All the piles were cured for 2 days under damp sacks and subsequently left to mature in an atmosphere of controlled humidity and temperature. The age at the time of test was in each case 7 days

Control specimens for determining compressive strength, transverse strength and Young's modulus were made at the same time and cured under similar conditions. Driving was begun at a height of drop of 18 in. and increased by 6-in. steps, 75 blows being given at each height. Failure, as previously stated was taken to be complete local disintegration of the concrete.

The results are given in Table 4.

(B) Seventeen piles were driven to investigate the effect of type of cement, curing conditions and age at time of test. All were made with plain heads having $\frac{3}{4}$-in. chamfer on the top edges and 1-in. end cover to the main reinforcement. The reinforcement was similar to that used in the last series except that for a length of 18 in. at the head and foot of the pile the lateral ties were spaced at $1\frac{1}{2}$-in. centres instead of 3-in. The foot of each pile was strengthened with a welded steel shoe as before. The concrete varied with the factor it was desired to investigate. Three types of cement were used: normal Portland, rapid-hardening Portland and high alumina. The piles were tested at different ages up to 28 days after having been cured for varying periods under damp sacks and subsequently in conditioned air. Ham River sand and aggregate of similar grading to the last series was employed and the same mix (1 : 2 : 4 : 0·55) was used for every pile.

The tests were conducted on identical lines with those of the previous series. The results are given in Table 5.

(C) Four piles were tested to investigate the effect of cement content. The form of the piles, the reinforcement and the methods of making and driving were identical with the last series except that normal Portland cement was used throughout and each pile was cured under damp sacks for 7 days and subsequently in conditioned air until being tested at 28 days. One pile, No. 17 M, was common to both this and the previous series. The results are given in Table 6.

(D) To supplement the data obtained from the last series regarding the effect of cement content upon impact strength, and to investigate the influence of longitudinal reinforcement, a group of eight piles was tested. Four piles were reinforced with the standard reinforcement consisting of four $\frac{5}{8}$-in. diameter main bars and $\frac{3}{16}$-in. diameter ties, and four with four $\frac{5}{16}$-in. diameter main bars and $\frac{3}{16}$-in. diameter ties. The ties in each case were spaced at 3-in. centres in the central portion of the pile and $1\frac{1}{2}$-in. centres for a length of 18-in. at the foot. Each pile was strengthened at the foot by a welded steel shoe 4-in. deep and $\frac{1}{4}$-in. thick, with a closed end. The horizontal edges at the head of the pile were chamfered $\frac{3}{4}$-in. and 1-in. end cover provided for the main reinforcement. Four different mixes were employed, varying from 1 : 2 : 4 : 0·55 to 1 : 1 : 2 : 0·41, the ingredients being similar to those of previous series. Each pile was tested at 7 days after being cured under damp sacks for 3 days and subsequently in conditioned air.

TABLE 4.—*Tests to Destruction—Series A. Effect of Form of Head on Impact Resistance of Reinforced Concrete Piles*

Description of pile						Results of tests		
						Height of drop		
Pile No.	End-cover over main bars (In.)	Pitch of laterals (In.)	Type of laterals	Edges	Type of end	At first failure (In.)	At final failure (In.)	No. of blows at final failure
83L	1	2½	Continuous binding	Chamfered	Chamfered	18	24	116
80L	1	2½	Separate ties ..	Chamfered	Chamfered	18	24	120
85L	1	3	Separate ties ..	Chamfered	Chamfered	18	24	121
88L	Nil	2½	Continuous binding	Chamfered	Chamfered	18	24	141
92L	Nil	3	Separate ties ..	Square ..	Square	18	24	149
82L	1	2½	Separate ties ..	Chamfered	Ring ⅜ in. by ¾ in.	30	30	209
84L	1	2½	Separate ties ..	Chamfered	Ring ⅜ in. by ¾ in.	24	30	220
87L	1	1½	Separate ties ..	Chamfered	Ring ⅜ in. by ¾ in.	30	36	269
2M	1	1½	Separate ties ..	Chamfered	Ring ⅛ in. by 2½ in.	30	36	248
89L	Nil	1½	Separate ties ..	Chamfered	Ring ⅛ in. by 2½ in.	30	36	257

Average crushing strength (cube) of concrete = 5,360 lb. per sq. in.

TABLE 5.—*Tests to Destruction—Series B. Effect of Type of Cement, Curing and Age upon Impact Resistance of Reinforced Concrete Piles*

Ref. No.	Type of cement	Mix	Time of curing under damp sacks (days)	Age at test (days)	Driving			Control tests		
					No. of blows at:			Cube strength (lb/sq. in.)	Modulus of rupture (lb./sq. in.)	Modulus of elasticity (lb./sq. in. × 10^{-6})
					18-in. drop	24-in. drop	30-in. drop			
16M	Normal Portland	1 : 2 : 4 : 0·55	Nil	7	32(5)*	—	—	2440	436	3·9
20M	,,	,,	Nil	28	42(4)	—	—	4000	510	5·0
17M	,,	,,	7	28	75(4)	—	—	4450	535	5·1
11M	Rapid-hardening Portland.	,,	Nil	3	24(4)	—	—	2085	358	3·8
100L	,,	,,	Nil	7	43(9)	—	—	2870	374	4·3
96L	,,	,,	Nil	28	54(7)	—	—	4070	416	4·5
7M	,,	,,	3	3	47(5)	—	—	2240	464	3·7
99L	,,	,,	7	7	75(70)	19	—	3440	440	4·5
95L	,,	,,	7	28	75(48)	44	—	5728	553	5·9
51M	High Alumina	,,	1	1	53(4)	—	—	8520	620	6·1
50M	,,	,,	2	2	75(67)	16	—	8600	618	5·7
59M	,,	,,	2	2	70(7)	—	—	9120	700	5·9
57M	,,	,,	3	3	63(12)	—	—	9720	691	6·0
54M	,,	,,	3	3	75	75(32)	20	9460	640	5·9
46M	,,	,,	7	7	75(25)	26	—	10060	715	6·5
41M	,,	,,	2	28	75(7)	44	—	11400	591	5·8
55M	,,	,,	Nil	28	38(5)	—	—	10400	591	5·9

* Figures in brackets denote the number of blows at which first signs of failure appeared.

TABLE 6.—*Tests to Destruction—Series C. Effect of Cement Content upon Impact Resistance of Reinforced Concrete Piles*

Normal Portland Cement 28 days old: cured damp for 7 days.

Tests on piles					Control tests		
Ref. No.	Mix	No. of blows at			Cube strength (lb./sq. in.)	Modulus of rupture (lb./sq. in.)	Modulus of elasticity (lb./sq. in. × 10^{-6})
		18-in. drop	24-in. drop	30-in. drop			
22 M	1 : 2½ : 5 : ·70	38	—	—	3220	486	4·3
17 M	1 : 2 : 4 : ·55	75	—	—	4450	537	5·1
24 M	1 : 1½ : 3 : ·45	75	22	—	6230	573	5·8
31 M	1 : 1 : 2 : ·41	75	75	32	6370	508	5·4

TABLE 7.—*Tests to Destruction—Series D. The Effect of Reinforcement and Mixing Proportions upon Impact Resistance of Reinforced Concrete Piles*

Ref. No.	Mix (using rapid-hardening Portland cement)	Reinforcement		Test on pile					Control test		
		Main	Transverse	No. of blows at					Cube strength (lb/sq.in.)	Modulus of rupture (lb./sq.in.)	Modulus of elasticity (lb./sq. in. × 10^{-6})
				6-in. drop	8-in. drop	10·67-in. drop	14·23-in. drop	18·97-in. drop			
A3	1 : 1 : 2	Four ⅝-in. dia. bars (2·5 per cent.)	$\frac{3}{16}$-in. ties at 3-in. centres	—	—	200(25)*	200	25	4,650	436	4·23
B1	1 : 1·2 : 2·4			—	—	200	200(116)	100	4,750	468	—
C1	1 : 1½ : 3			200	200	200(53)	200	85	4,780	434	—
D1	1 : 2 : 4			—	—	140(70)	—	—	3,400	426	5·06
A2	1 : 1 : 2	Four $\frac{5}{16}$-in. dia. bars (0·62 per cent.)	$\frac{3}{16}$-in. ties at 3-in. centres.	200	200	200	200	80(10)	4,670	412	4·75
B2	1 : 1·2 : 2·4			—	200	200	200	76(40)	4,630	438	4·65
C2	1 : 1½ : 3			—	200	200	200(150)	105	4,730	429	4·82
D2	1 : 2 : 4			—	200(200)	200	40	—	3,430	426	4·18

* The figures in brackets denote the stage at which first signs of failure were observed.

C 2

Driving conditions were similar to those of previous tests, except that improved equipment enabled a modified system of height of drop increments to be introduced, the reasons for which have already been explained. Except where experience of previous tests suggested that a higher minimum could be adopted driving was begun at a drop of 6 in. and increased every 200 blows by one-third. The full scale of drops was thus 6, 8, 10·67, 14·23, 18·97 and 25·29 in. The adoption of a logarithmic scale restricted possible error in determining critical drop to 33⅓ per cent., and probably to 15 per cent. or less. A second modification was that an elm dolly 7 in. square and 18 in. long, with hooped ends, was used in place of the cast-iron helmet and short dolly employed in previous tests. This modification was introduced to facilitate the changing of specimens and to enable the heads of the specimens to be kept more easily under observation.

The results of this series are given in Table 7.

(E) Four piles 5 ft. long and 7 in. square were tested to compare the effect of a crushed stock brick aggregate with that of gravel. Except in respect of the aggregate the piles were made and cured as in the previous series. The brick aggregate consisted of good quality common stocks crushed to the same grading as the gravel and used in combination with Ham River sand. Immediately before mixing this aggregate was well saturated with water, and sufficient water was added to the mix to give the same slump as for the concrete made with gravel aggregate. For both types of aggregate a 1 : 1½ : 3 mix was employed, using rapid-hardening Portland cement. The proportion of water with the gravel concrete was 0·45 by weight. An electric vibrator was used on the sides of the moulds during placing. Without this aid it would have been impossible to compact the brick concrete to a satisfactory degree of consolidation. It was originally intended to test at 7 days; but as at this date the piles made with brick aggregate still had a very "green" appearance, and were obviously not sufficiently matured, testing was postponed for a further 3 weeks.

The results are given in Table 8.

Discussion of Results

Form of Head

Close spacing of lateral ties, as was to be expected, was found t give a marked increase of resistance to failure, and the only limit t the advantage which can be obtained in practice would appear t be one arising from the need to allow sufficient room between ties f the placing of the concrete. The tests furnished no evidence tha the use of continuous helical binding in place of separate ties i advantageous. The effect of end cover was not well-defined, althoug the piles in which the reinforcement was finished flush with the to surface of the pile appeared to be slightly stronger than comparab

TABLE 8.—*Tests to Destruction—Series E. Effects of Type of Aggregate upon Impact Resistance of Reinforced Concrete Piles*

Ref. No.	Coarse aggregate	Fine aggregate	Age at test	Number of blows: 6-in.	8-in.	10·67-in.	14·23-in.	18·97-in.	25·29-in.	Control tests: Cube strength (lb./sq.in.)	Modulus of rupture (lb./sq.in.)	Modulus of elasticity (lb./sq.in. × 10^{-6})
C.10	Brick	Ham River Sand	28 days	200	200	200(200)*	200	30	—	4,220	468	1·72
C.11	Gravel	,,	,,	200	200	200	200	150(40)	—	7,440	519	5·40
C.12	Brick	,,	,,	200	200	200	200	120(70)	—	3,800	440	1·70
C.13	Gravel	,,	,,	200	200	200	200	200	60(10)	7,130	512	5·18

* Figures in brackets indicate first signs of failure.

piles having 1-in. of end cover. In spite of the reduction of area no loss of impact resistance resulted from chamfering the head of the pile.

The use of a welded steel band, the upper edge being placed nearly flush with the pile-end, was found to give a greatly increased resistance. Two types of band were tested and were nearly equally effective. Each was 7 in. square overall, the first $\frac{3}{8}$ in. thick and $\frac{3}{4}$ in. deep, the second $\frac{1}{8}$ in. thick and $2\frac{1}{2}$ in. deep. Under test the top surface of the concrete remained intact throughout whilst the reinforcement was exposed by spalling for about 14 in. below the ring.

Amount and Disposition of Reinforcement

The effect of varying the percentage of main reinforcement was investigated in Series D, in which four piles reinforced with [illegible] diameter bars were compared with similar piles having [illegible] diameter bars, equal to 2·5 and 0·6 per cent. respectively. The results show plainly that under the test conditions no advantage was obtained by using the heavier reinforcement. The development of transverse cracks during driving, on the other hand, was considerably more pronounced in the piles with lighter reinforcement, though occurring in both. They began to open out at a fairly early stage in driving, but in spite of the high impact stresses to which the piles were subjected, the cracks at no time reached a condition at which they could be said to endanger the stability of the piles. Previous tests have shown, moreover, that hair cracks occurring in a structure under conditions favourable to corrosion of the reinforcement tend to seal up in the course of time through further hydration of the cement, and prevent such corrosion taking place. No information is available to suggest at what width transverse cracks are likely to become a source of danger through corrosion or by reducing impact resistance.

Type of Cement and Age at Test

Information on the effect of type of cement and age at test is contained in the results given in Table 5. The results do not show a marked superiority for any particular type of cement, except in respect of the time required for hardening. Considering the possible sources of error in tests of this character the results for both normal and rapid-hardening Portland cement are reasonably consistent. Those for high alumina cement are not so consistent, and indicate that there may be factors of some importance to impact strength of which nothing is at present known. It is interesting to note also, that although consistently high compressive strengths were developed in all the piles made with high alumina cement, compared with those made from the other two types, this trend is not uniformly reflected in the impact strengths as indicated by driving tests.

From the point of view of the impact strength developed at certain ages the tests show that approximately equal strength is obtained by normal Portland cement at 28 days, rapid-hardening Portland cement at 7 days and high alumina cement at about two days. Because of the inconsistencies already referred to, however, the last figure should be taken with some reserve.

Cement Content

The effect of cement content is illustrated in Table 7, both sections of which demonstrate that for rapid-hardening Portland cement no great increase of impact strength is obtained by using a mix richer than 1 : $1\frac{1}{2}$: 3. The same tendency is revealed in the compressive strength of the control specimens and has, in fact, been well-established by previous investigations. The fact that a similar feature is not revealed in Table 6 referring to normal Portland cement concretes, is not held to disprove the application of this characteristic to Portland cements in general. Apart from the fact that to do so it would be necessary to accept the evidence of a single pile, it is known from the previous tests referred to above that the static strength of concrete made with normal Portland cement does follow the same rule.

Curing

The effect of curing (Table 5) is more marked than that of any other factor. The results demonstrate conclusively that, whatever type of cement is used, a substantial increase of strength can be obtained by curing the concrete under damp sacks at least during the initial stages of hardening. Piles so cured withstood approximately twice as much driving as those which were allowed to harden under atmospheric conditions, and the increase was in every case more marked than the corresponding increase in the static compressive strength.

Type of Aggregate

The results given in Table 8 show that concrete made with a crushed stock brick aggregate gave a slightly lower impact strength than piles made with gravel. No information has yet been obtained about the effect of granite and other stone aggregates upon impact strength. It is expected that this matter, and others connected with the effect of aggregates in general, will be more fully investigated in the near future.

IMPACT TESTING MACHINE

Earlier in this paper it was stated that two forms of test to destruction appeared to be necessary: the first a series of tests on large specimens under conditions approximating to those encountered in practice; the second a series of tests on small specimens under conditions in which a greater number of specimens could be tested and more accurate control obtained over the driving variables. The tests on 15-ft. piles which have been described above, while providing useful indications of the effect of manufacturing conditions upon impact strength, demonstrated that final conclusions could only be reached by a more extended series of tests on small specimens under controlled conditions. Some of the directions in which useful work could be done have been indicated in the discussion on 15-ft. piles, and there are others of which mention has not yet been made. In the course of investigations on outside contracts, for example, the possibility was mooted of a critical period in the hardening stage of a pile during which its impact strength reaches a value lower than that preceding and following. This is a point which can only conveniently be investigated by an extended series of tests on small specimens. Other factors are the effect of grading and type of aggregate, the effect of varying the water/cement ratio, the best cement content, the effect of vibration-compaction on impact strength and a number of others.

Two types of impact machine for small scale tests have been used during the course of the investigation. The first was designed for impact tests on 4-in. cubes and was used in connection with Series B of the tests to destruction. The cubes were mounted on a machined steel base (Fig. 40) and impact applied by dropping a 5-lb. tup on to a $1\frac{1}{2}$-in. diameter steel dolly with a hemispherical lower end resting on the centre of the top face of the specimen. The height of drop of the first blow was $\frac{1}{2}$ in. and the height of succeeding blows increased by $\frac{1}{2}$ in. per blow. When the point of failure was reached the specimen usually split into two nearly equal portions. The height of drop at failure was taken as a measure of its impact strength. Subsequently two variations of this method were tried. In the first a steel plate $\frac{3}{4}$ in. thick and 4 in. square was placed between the dolly and the specimen. In the second 6-in. cubes were used and the impact of the dolly applied successively at 8 points evenly spaced on a circle of 3 in. diameter. In both cases the weight of the tup was increased to 10 lb. Although the results exhibited a fair degree of consistency over a range of similar cubes it was not found possible, with any of the three methods tried, to establish a clear relationship between the strength obtained in this way and the resistance to driving of the 15-ft. piles of Series B.

A machine of this type is theoretically unsatisfactory for two reasons: first, the mode of failure is different from that occurring under practical pile-driving conditions, and, second, the duration of impact is not of the same order. As stated above, failure with the original apparatus generally occurred by the specimen splitting into two pieces, whereas in practice failure is generally by crushing. An attempt to produce failure by crushing was made with the steel plate, but failure then occurred first at the corners and its commencement was ill-defined. Assuming a maximum compressive stress at failure of 3,000 lb./sq. in. and a rebound of 8 in.

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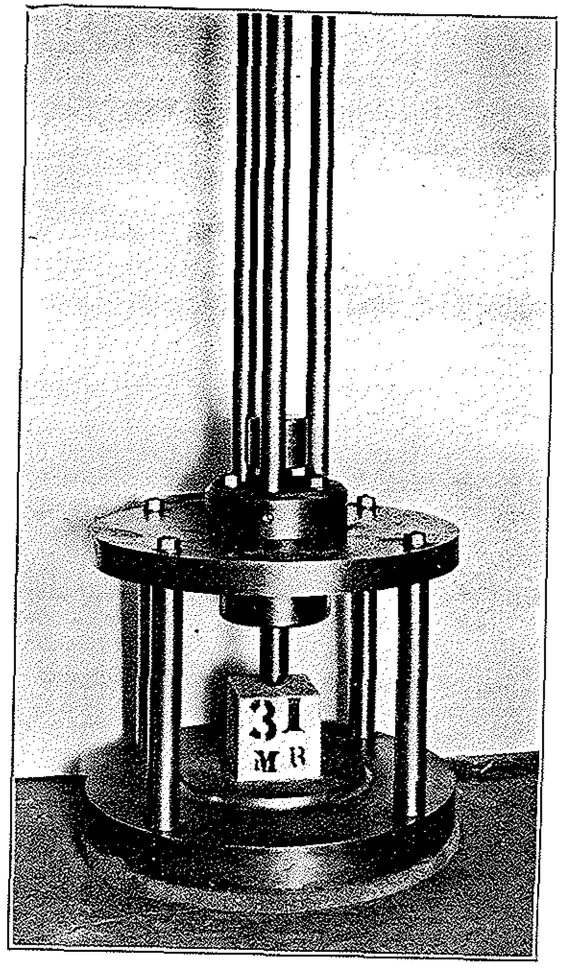

FIGURE 40.—*Apparatus used for Impact Tests on Small Specimens.*

for a drop of 2 ft. with a 10-lb. tup, calculations show that, even if uniform conditions of stress distribution could be achieved, the duration of impact would be of the order of 0·0002 second, that is, one-fiftieth of the normal practical duration.

The second machine was designed to reproduce practical conditions more closely. In its final form, by the insertion of suitable buffers between the specimen and the hammer, it is possible to vary the duration of impact on a 4-in. square specimen from 0·001 to 0·010 second, and at the greatest duration to apply a maximum

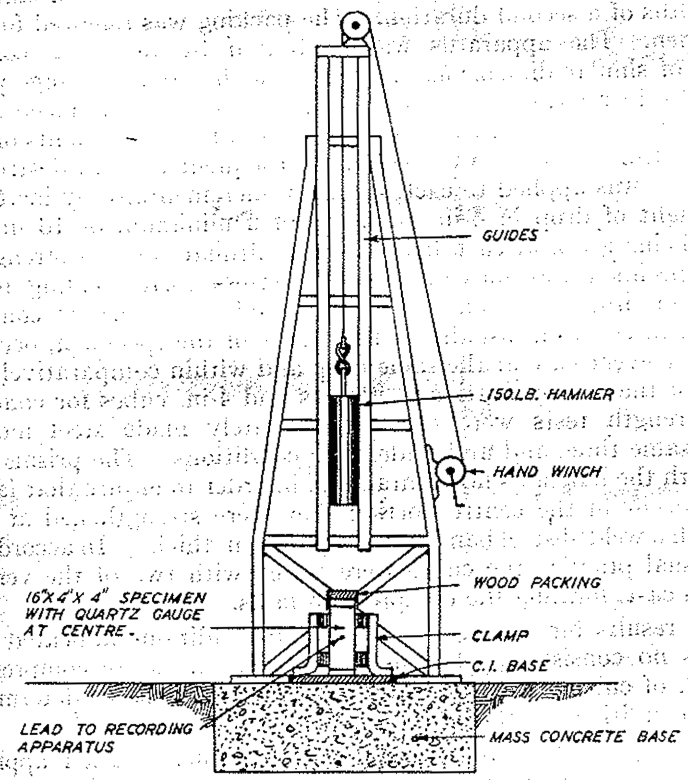

FIGURE 41.—*Impact machine for 16 in.* × *4 in.* × *4 in. specimens.*

compressive stress of 4,000 lb./sq. in. The form of the stress-time curve in the specimen can also be varied by alteration of the material and form of the buffer. It is therefore possible to apply to the specimen stresses of the type most productive of failure under practical conditions. A small machine was first built to obtain data for design purposes and with it a series of tests was carried out on concrete specimens. The machine, shown diagrammatically in Fig. 41, consists essentially of a steel frame supporting a cast iron cylindrical hammer weighing 150 lb. and operated by a hand winch. The specimens, in the form of prisms 16 in. long by 4 in. square, were placed on a heavy cast iron base bolted to a mass concrete foundation, an adjustable clamp being provided to hold them rigidly in a vertical position.

As a result of preliminary tests the packing for the whole series was standardised using, below the specimen one thickness of three-ply wood, and above the specimen a piece of elm 4 in. square and 1 in. thick laid with the grain horizontal. It was found that this form of packing ensured a uniform transverse distribution of stress, maintained reasonably constant properties for the full extent of each test, and gave a smooth stress-time curve of approximately 3/1000ths of a second duration. The packing was renewed for each specimen. The apparatus was calibrated by testing a concrete prism of similar dimensions, but fitted with a quartz gauge placed centrally in relation to all axes. The impact stress induced at the centre of the specimen was determined for a range of heights of drop varying from zero to 36 in. In the subsequent tests to destruction the stress was applied to each specimen incrementally by increasing the height of drop in 4-in. stages from a minimum of 16 in., 20 blows being given at each height. The ultimate impact strength of each specimen was measured by the stress corresponding to the height of drop at which first signs of failure occurred; complete local disintegration, usually at the centre of the specimen, occurred in nearly every case at the same drop and within comparatively few blows of the first indications. Prisms and 4-in. cubes for compressive strength tests were cast in accurately made steel moulds at the same time, and under identical conditions. The prisms were cast with the long axis horizontal, and in order to ensure that failure would occur in the centre portion they were strengthened at each end with a welded steel band 1 in. deep by ⅛ in. thick. In accordance with usual practice the cubes were tested with two of the vertical sides as cast, forming the compression faces.

The results for a range of mixes and conditions indicated that there is no consistent relationship between the static compressive strength of cubes and the impact strength of prisms as determined by the conditions of the test.

The impact strength of individual specimens varied approximately between 50 and 80 per cent. of the cube compressive strength. It is probable, therefore, that impact stresses less than the lower limit will not cause failure within the number of blows likely to be given in practice. This fact is of considerable importance and is referred to again in the conclusions and recommendations on page 7

As a result of the experience gained with the machine used in these tests, a new impact machine, capable of automatic operation (Fig. 42), was constructed. It consists essentially of two rigid built-up steel columns 21 ft. long mounted on a cast iron base 6 in. thick and weighing approximately 2 tons, the base itself being bolted down to a mass concrete foundation approximately 7 tons in weight. Ample precautions have thus been taken to ensure th

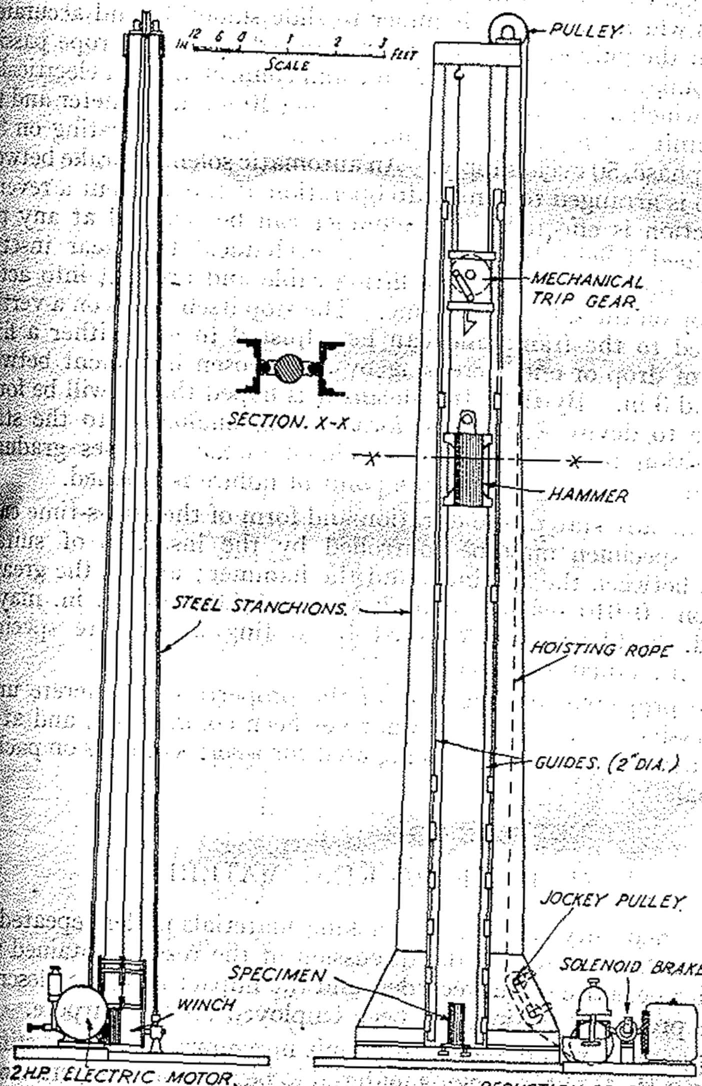

FIGURE 42.—*Large Impact Machine.*

external movements in the apparatus itself shall be reduced to minimum. Bolted to the columns are two circular steel guides 2 in diameter and 10 in. apart, between which the hammer runs at the base of which the specimen is placed. The hammer is cylindrical cast-steel unit with a machined bottom face and proje ing legs which enable the hammer to slide smoothly and accura between the guides. The hammer is raised by a steel rope pass over a pulley at the top of the frame and connected to an electrica driven winch at the base, the winch being 10 in. in diameter and power unit a 2 h.p., A.C. " squirrel-cage " motor operating on volt, 3-phase, 50 cycle supply. An automatic solenoid brake betw the two is arranged to come into operation at the moment a reve of direction is effected. The hammer can be released at any p determined height by means of a mechanical trip gear inser between the hammer and the lifting cable and brought into act by a stop on the side of the frame. The stop itself slides on a vert bar fixed to the frame and can be adjusted to give either a fi height of drop or one increasing by any chosen increment betw $\frac{1}{4}$ in. and 3 in. By the latter means it is hoped that it will be fou possible to devise a standard form of test analogous to the st compression test, giving to a specimen impact stresses gradu increasing in intensity until the point of failure is reached.

As already stated, the duration and form of the stress-time cu for the specimen may be controlled by the insertion of suita buffers between the specimen and the hammer; and at the grea duration—0·010 second—a peak stress of 4,000 lb./sq. in. may applied. Calibration is effected by testing a concrete speci fitted with a quartz gauge.

The projected investigation of the properties of concrete u compressive impact stress has not yet been commenced, and at present time the machine is being used for a series of tests on pa materials.

TESTS OF PACKING MATERIALS

The necessity for tests of packing materials under repeated pact has been shown in the discussion of the results obtained the piezo-electric strain recorder, and the impact machine, desc in the preceding section, has been employed for the purpose.

Apparatus was designed to permit measurements of the sti constant (k/A) of the packing material to be made at any period test. The specimen of packing is placed between a steel anvil steel cap which fits closely over it. The anvil is bolted firmly to the cast-iron base of the machine, and contains a centrally p piezo-electric quartz gauge for recording stresses (Fig. 43) stress-recording apparatus, used in conjunction with the gau assembled in a special cabinet forming an integral part of the e

ment. The specimen is subjected to repeated blows, each giving a peak stress which is maintained as nearly as possible at a constant value of 3,000 lb./sq. in. by gradually reducing the height of drop as the packing consolidates. At suitable intervals during the test records are taken of stress and strain for selected heights of drop, the former by means of the piezo-electric gauge and the latter by the trace of a stylus fixed to the hammer and recording on metallised paper mounted rigidly on a suitable frame. By this method the

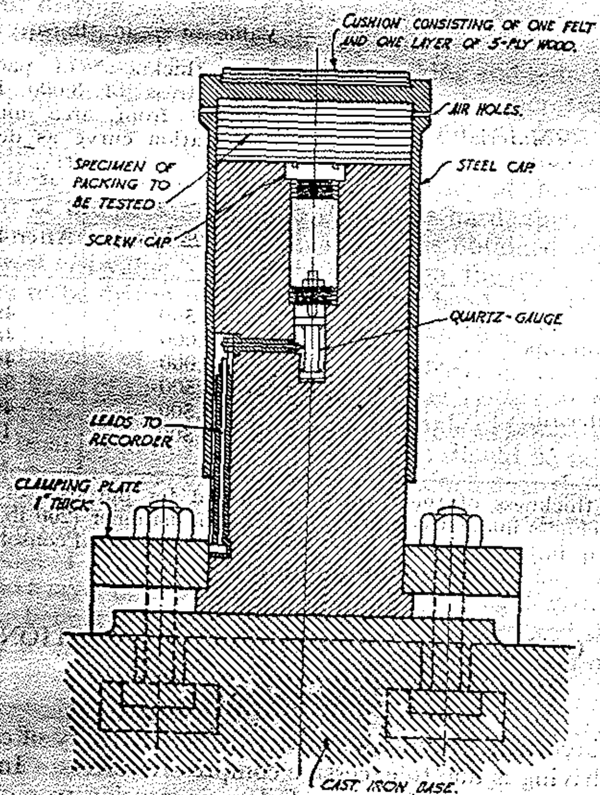

FIGURE 43.—*Anvil for testing head packings.* (Scale: Quarter size.)

progressive deterioration of the packing and the change which is occurring in the value of the stiffness factor k/A are recorded.

The results of tests of a number of materials used as helmet packings are given in Table 9. They suggest that the materials in common use are far from ideal. There are considerable differences in the value of the stiffness constant k/A as between one packing and another, and in most cases a very considerable increase in the constant as a result of repeated impact. It will be noticed, however, that a packing consisting of asbestos fibre exhibited not only a reasonably low stiffness constant at the beginning but maintained this condition almost unchanged. The suitability of this material for use under practical conditions has not yet been confirmed.

TABLE 9.—*Helmet Packings. Variation of Stiffness Factor* $\frac{k}{A}$ *with Number of Blows*

Material	Value of $\frac{k}{A}$ in lb./sq. in. for a 1 in. thickness* of packing and at a stress of 3,000 lb./sq. in. (derived from area under stress deformation curve as described in text)	
	After 25 blows	After 4,000 blows
Felt	18,500	49,000
Coiled hemp rope	30,000	48,000
Sack-cloth	23,000	40,500
Asbestos fibre	28,500	30,500
Hard wallboard	35,500	49,000
Rubber	13,500	13,500

* The thickness of the packing is defined for purposes of comparison as the thickness under the static weight of the hammer (200 lb., equivalent to 16 lb./sq. in.) after 25 blows have been given for initial consolidation.

CONCLUSIONS AND RECOMMENDATIONS

Driving Failures

The investigation has shown that in the majority of cases failure during driving is due to excessive compressive stress. In the course of the tests no case of serious failure in tension was observed, in spite of the fact that certain of the 15-ft. piles were subjected to tensile stresses many times greater than the static tensile strength. There is, however, a considerable amount of evidence from other sources to show that tensile failure does occur, though rarely, in the driving of reinforced concrete piles. The reasons for such failures are discussed later.

Failure in compression may occur at any position along the length of the pile, but most commonly takes place at the head (Fig. 1). Since the maximum head stress is, in most cases, independent of ground conditions, head failure may occur when driving into any type of ground. The most common causes of head failure are

consolidation of the helmet packing and unevenness in placing the packing on the head of the pile. The effect of the gradual consolidation of packing materials under repeated impact in causing a progressive increase in the maximum stress at the head has been discussed on pp. 34 and 44. The effect is naturally most pronounced on contracts where the driving is hard and each pile requires a large number of blows to attain its final penetration, but in easy driving the same phenomenon may be anticipated, if the same helmet packing is used for a number of piles. Where head failure has occurred, it is advisable in driving subsequent piles to add more or substitute fresh packing in the helmet after about 1,000 blows. In cases where there is a choice between retaining the same packing and reducing the height of drop of the hammer on the one hand and increasing the amount of packing and retaining the same height of drop of the hammer on the other, the latter is always the more efficient procedure: for, if driving is easy, the energy-efficiency of the hammer is independent of the packing, and, if driving is hard, then for the same maximum stress at the head, greater set is obtained with the packing of lowest k/A value. For the same reason, the packing, for a pile which has to be driven on with a damaged head, should be increased so as to allow as large a drop as possible to be used.

The second cause of head failure, unevenness in placing the packing on the head of the pile, may be due either to carelessness or to the nature of the packing material. An example of failure due to this cause has been given in Fig. 30. Great care should be taken to see that a uniform thickness of packing is maintained over the entire area of the head, and also that the layer in immediate contact with the head has no sharp discontinuities in its surface. Wooden slats, for instance, should not be placed directly on the head, but should have interposed a layer of softer material, such as sacking. Excessive side-play in the helmet allows the packing to be displaced and should be avoided by wrapping material over and around the head of the pile.

Uneven stress distribution at the head may be due to inaccurate delivery of the blow, that is, to the path of the hammer not being coincident with an extension of the long axis of the pile.

Failure in the lower part of the pile can only occur in exceptionally hard driving through a dense stratum, when the maximum compressive stress may reach a higher value at the foot than elsewhere in the pile. The actual failure does not necessarily occur at the foot itself for in such cases a considerable length of the pile may be subjected to almost the same maximum stress, with the result that failure may occur in a region of weakness further up the pile, due perhaps to damage in handling, inadequate transverse reinforcement, or poor concrete. Again, the foot may derive considerable lateral support from the highly compressed strata around it, tending to prevent failure at that point, and counteracting the effect of the reduction in area of the pile at the toe. An example of failure in the lower part of the pile has been given in Fig. 3. Failure occurred not at the toe itself but at a distance from the toe equal to about one quarter of the pile length. The stresses in the lower region of the pile were undoubtedly higher than elsewhere, and it is probable that the pile below the region of failure was safeguarded by the lateral support from the highly compressed strata.

The tests have shown that, as the maximum head stress is in creased, in hard driving the maximum foot stress increases at a slower rate. High strength concrete has the advantage, therefore, not only of permitting higher driving stresses throughout the pile, with consequent increase of driving efficiency, but also of enabling the ratio $\frac{\text{maximum foot stress,}}{\text{maximum head stress}}$ for a particular ground condition, to be reduced.

It has been stated that compression failure may take place at any position along the pile and it has been shown how it may occur at the head and in the lower part of the pile. But for a long pile, the maximum stress for a considerable distance below the head may be nearly equal to the maximum at the head, and compression failure may therefore commence at some distance below the head if a region of weakness exists.

Though no serious tensile failures were encountered during the course of the tests, there is a considerable body of evidence to show that such failures do occasionally occur in the form of transvers cracks sometimes spaced about 2 ft. apart along the length of th pile. The driving conditions under which the cracks are develope appear to be well defined, and are those of hard driving against relatively impenetrable medium close to the surface of the ground the major part of the pile being above ground and unsupported The appearance of the cracks definitely proves the occurrence of ten sile stresses, but it does not appear possible, at the present moment to show conclusively how the tensile stresses originate. The fir possibility is that the conditions may be such as to give rise to stres time curves as in Fig. 44, where a 15-ft. pile driven against a har base vibrated longitudinally, giving rise to alternating tensile an compressive stresses in the middle part of the pile. The pile wa free at both ends, owing to the rebound of the hammer from th head and the pile from the ground, and damping of the vibration was therefore reduced to a minimum. The rarity of these failures an argument against this explanation, and points rather to a chan combination of unfavourable conditions. An alternative expla tion is that the cracks are due to excessive transverse vibration. has been stated that the tensile stresses due to transverse vibratio produced by a single impact on a sound pile were found mathema cally to be insufficient to cause failure, but that it was possible for t vibration to be augmented by resonance. The driving conditions a such as to minimise the damping of transverse as well as longitudi vibrations, and if the interval between the blows of the hamm

To face page 74

FIGURE 44.—*Record of the longitudinal vibration of a 15-ft. pile. Driving conditions; two ½-in. felts in helmet; 980-lb. hammer; 6-in. drop; foot resting on two ½-in. felts. Frequency of longitudinal vibration—455 per second.*

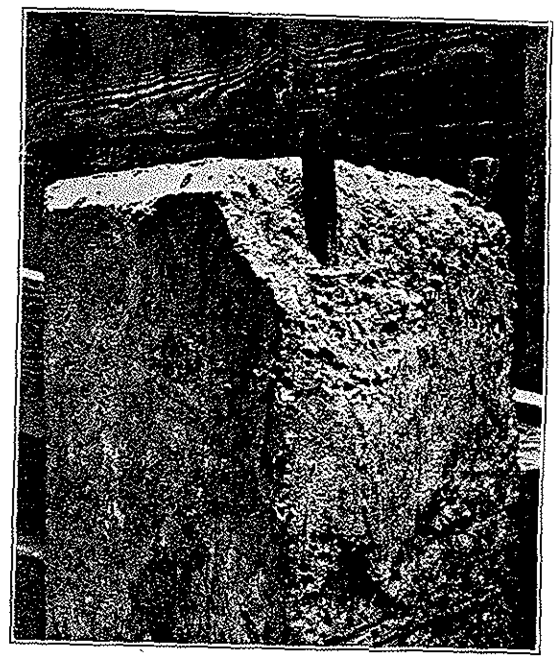

FIGURE 45.—*Movement of the longitudinal reinforcement.*

is a small multiple of the period of transverse vibration of the pile the amplitude of the vibrations may be sufficiently augmented by slight resonance to cause tension failure. The required combination of conditions to produce this state of affairs is likely to occur rarely. If the first explanation is correct the only cure is to use thicker packing and/or a heavier hammer; if the second explanation is adopted the preventive measures are (1) to ensure that the blow is struck axially and that the helmet and dolly fit evenly on the pile-head and (2) to restrain the pile at several points along its length and to prevent swaying of the frame.

Several cases of vertical movement of the longitudinal reinforcement have been reported in conjunction with transverse cracking (see Fig. 45). The movement of the bars may be as great as four inches. Evidently bond failure has taken place. It is possible to explain the upward movement of the bars by supposing that the cracks are first opened symmetrically by transverse vibration and are then closed under the compressive impact-stress by the concrete above the crack slipping down the bars. Of greater importance than the actual movement of the bars is the state of affairs indicated below ground. It is difficult to explain how a protruding bar can be forced upwards by as much as 4 in. unless failure has taken place below ground level.

The Design and Manufacture of Reinforced Concrete Piles

The tests have shown that the amount of longitudinal reinforcement does not greatly affect the ultimate driving resistance of a pile provided the latter is initially free from harmful transverse cracks. If sufficient longitudinal reinforcement is used to enable the pile to be transported and pitched without fracture it is unlikely that failure due to insufficient reinforcement will occur during subsequent driving.

The amount of lateral reinforcement, on the other hand, profoundly affects the impact resistance of a pile, particularly at the head and toe. For piles subjected to hard driving it is recommended that for a length from the extremities of $2\frac{1}{2}$-3 times the external diameter of the pile the volume of lateral reinforcement should not be less than 1 per cent. of that of the gross volume of the corresponding length of pile. The diameter of the ties should conform with the usual practice for reinforced concrete and be not less than $\frac{3}{16}$ in. or one-fourth the diameter of the main bars, whichever is greater. The minimum spacing of the ties at head and foot should be such as to provide ample facility for placing the concrete. It was observed on an outside contract that the performance of piles reinforced with heavy spirals ($2\frac{1}{4}$ per cent.) was definitely good; and although patches of surface spalling occurred, they did not materially affect the resistance of the pile to further driving.

In the mid-section of the pile the amount of lateral reinforcement may with safety be somewhat reduced. At first sight this statement may appear to be in contradiction with the experimental evidence which has shown that impact stresses in the middle of the pile may be nearly as great as those at the head. It should be remembered, however, that at the extremities of the pile the lateral support is not as great as in the middle; at the ends of the pile, also, the stress in the main reinforcement is not fully developed. Although little actual evidence is available, it appears reasonable to reduce the percentage of lateral reinforcement in the middle portion to not less than 0·4 per cent. of the gross pile volume, the spacing being increased gradually over a length of 2-4 ft. In any case the spacing should not be greater than half the overall width of the pile.

For sites where driving is easy and resistance is chiefly due to skin friction it is still advisable to strengthen the head in accordance with the above recommendations.

The cover of concrete over the reinforcement should be decided solely from consideration of the protection required by the steel and should therefore be as in ordinary reinforced concrete practice. There is no evidence to show that end cover at the head influences impact strength; it is therefore suggested that a reasonable procedure would be for the main reinforcement to be finished, without hooks, 1 in. from the top surface of the pile.

Reinforcement by means of external bands placed at the head of the pile was found to increase driving resistance very considerably. In the experimental work at the Building Research Station two types of welded steel band were tested, one $\frac{3}{4}$ in by $\frac{3}{8}$ in., and the other 2 in. by $\frac{1}{8}$ in., and both were found to give good results. It is probable that the latter type would be more suitable for practical conditions. The following range of sizes is suggested :—

For square piles up to 12 in. overall diameter	2 in. by $\frac{1}{8}$ in.
,, ,, ,, 12 to 15 in. ,, ,,	$2\frac{1}{2}$ in. by $\frac{3}{16}$ in.
,, ,, ,, over 15 in. ,, ,,	3 in. by $\frac{1}{4}$ in.

For octagonal or round piles the cross-section of the bands could be somewhat reduced. Two or three bands should be used, according to the strength required, the first being placed 1 in. clear of the head and the remainder at centres equal to about half the overall diameter of the pile. To ensure that the top ring shall not work loose it should be either anchored into the concrete or roughened on the inside to provide good bond. The necessity for good welded joints is obvious. Since the heads of piles are practically always removed after driving these bands can be used repeatedly.

A similar form of strengthening at the toe will also be useful in cases where very hard driving is to be encountered.

It is considered very important to ensure that the head of the pile should be carefully formed by arranging the shuttering forming the head at right angles to the length of the pile and by taking care that it is smooth and free from wind.

Of the types of cement investigated, normal Portland, rapid-hardening Portland, and high alumina, there is no evidence from the tests at the Building Research Station, to show, considerations of hardening period apart, that any of these types is definitely superior to the others. It was observed, however, that high alumina cement piles gave more erratic results than piles made with Portland cement. The experience of practical engineers appears to be in favour of normal rather than rapid-hardening Portland cement. The reason for the preference may be that greater care is necessary in the manufacture of rapid-hardening cement concrete.

Within the range of mixes investigated no great increase in impact strength is obtained by increasing the cement content beyond the proportions by weight of 1 part of cement, $1\frac{1}{2}$ parts of sand and 3 parts of aggregate. A 1 : 2 : 4 mix, on the other hand, is considerably weaker than 1 : $1\frac{1}{2}$: 3. This trend is similar to that already known to exist in reference to static compressive strength and appears to apply generally to all Portland cements. It is not at present known whether high alumina cement concrete also exhibits this tendency, or to what extent, in the case of all types of cement, it is dependent upon other factors, such as the grading of the aggregate.

No evidence is available at present to show that any particular type of aggregate possesses marked superiority. Tests with a crushed stock brick aggregate gave a slightly lower impact strength than gravel and a static strength little more than half. It is possible that by using a crushed brick aggregate of high compressive strength impact strength greater than that of gravel concrete might be obtained. This view is supported by experiments conducted elsewhere[5] which have indicated that increased impact strength can be obtained by using aggregate the individual stones of which have a pronounced angularity. A disadvantage of angular aggregates in practice, however, is that the operation of compacting is made considerably more difficult, and may result in a loss of strength due to insufficient compacting or to the necessity of using a higher water/cement ratio. It was found in making the piles with brick aggregate at the Building Research Station that satisfactory compacting could only be obtained by the use of an electric vibrator on the sides of the moulds.

The water/cement ratio should be kept as low as possible consistent with proper compaction of the concrete. The use of a sloppy mix in one of the piles tested was undoubtedly a major cause of its early failure. The effect of water/cement ratio on static strength is well established and it is certain that its effect on impact strength is no less marked. The use of vibrators in placing the concrete should enable a low water content to be used.

The influence of wet curing upon impact strength is well defined. It is recommended that wet curing should be applied for as long as is practically possible; unless conditions of driving are easy, for Portland cement concrete this period should be not less than 14 days. Provided the concrete is kept continuously and uniformly moist the method of curing is immaterial; but it is recommended that piles should be covered with canvas, cement sacks or other suitable material, rather than left exposed and dependent upon frequent sprinkling.

The age at which the piles are driven will depend upon the strength which it is desired they should attain. It has been suggested that in the case of certain cements there is a critical period during which the impact strength of the concrete is temporarily reduced and subsequent to which the strength again increases. Tests at the Building Research Station have not shown any signs of such a period, but indicate a progressive increase of impact strength.

The impact tests have shown that the impact strength of the concrete may be as low as 50 per cent. of the cube crushing strength. Since, in hard driving, maximum compressive stresses exceeding 3,000 lb./sq. in. can be set up, a concrete of crushing strength exceeding 6,000 lb./sq. in. is necessary in these circumstances. For easy driving conditions the impact strength should be not less than 2,000 lb./sq. in., that is, the concrete should have a crushing strength of not less than 4,000 lb./sq. in.

To obtain strengths greater than 6,000 lb./sq. in., proportions not leaner than 1 : $1\frac{1}{2}$: 3 (nominal), that is, 1 cwt. cement, $1\frac{7}{8}$ cu. ft. sand, $3\frac{3}{4}$ cu. ft. coarse aggregate, should be used, and the greatest care exercised in the selection of aggregates, the control of water content, and curing. (It is of interest to note that a crushing strength of only 3,300 lb./sq. in. is required for 1 : $1\frac{1}{2}$: 3 high grade concrete under the Code of Practice.[61]). For easy driving conditions, where the lower crushing strength is adequate, the strength might be obtained with careful control with a 1 : 2 : 4 mix.

Handling Stresses

Great importance is attached to the avoidance of serious transverse cracks during handling. This is not because cracks of this nature are likely to impair the strength of the piles through corrosion over a period of years. It is known that under most conditions the cracks are satisfactorily sealed by further hydration of the cement during the later stages of hardening. The danger lies in the fact that under severe driving conditions a wide transverse crack may prove to be the starting point from which the ultimate failure of the pile begins. Very fine hair cracks of the order of a few thousandths of an inch, only visible under very close inspection, are inherent in the nature of the material and will occur at steel stresses of the order of 18,000 lb./sq. in. or less. These in general will only be visible during lifting and will close when the pile has been erected in position for driving.

The pile should be designed and the operations of transportation and pitching carried out with a view to keeping the stresses sufficiently low to prevent the formation of cracks of a larger size which

will not close when the pile is erected. It is often found that job methods of handling result in stresses far in excess of those for which the pile has been calculated. It is obviously harmful, for example, to pitch a pile which has been designed for two-point suspension by lifting it at the head and dragging the toe along the ground; and a pile should not be lifted at the mid-point unless it has actually been designed for this condition. An opportunity of serious damage also occurs during transportation when a considerable length of the pile overhangs the tail-board of a lorry. Impact stresses set up by irregularities in the surface of the road may be many times greater than the static stresses due to the dead weight of the pile, and in many cases not even the latter are fully provided for. There is scope in this direction not only for greater care on the part of contractors, but for greater watchfulness on the part of engineers to make certain that the piles receive no harmful treatment previous to their erection in the frame.

Driving Conditions

It is important to realise that normal factors of safety, as employed in other branches of engineering, do not generally apply to the driving of reinforced-concrete piles by present-day methods; the margin of safety is in many cases small, and is sometimes non-existent. It has been stated previously that the impact-strength of concrete varies between 50 and 80 per cent. of the cube compressive strength. Since it is impossible at present to define the relationship within closer limits, a maximum stress of 50 per cent. of the cube compressive strength may be regarded as representing what may be termed a driving factor of safety of unity. The calculations given later in this section are made on this basis, but it must be emphasised that it is not to be regarded as adequate.

As the margin of safety is frequently very small, it is important that reinforced concrete piles should be driven by the best possible method. The most favourable driving conditions may be defined in two different ways: they may be either those conditions which produce the greatest set for the least expenditure of driving energy, irrespective of the stresses induced in the pile; or those conditions which produce the greatest set for the lowest stresses in the pile, irrespective of the energy expended. Since in most cases protection of the pile from failure is the first consideration and the amount of energy expended, within reasonable limits, of minor importance, the recommendations which follow are concerned mainly with the reduction of driving stresses to a minimum.

Within the range of conditions investigated it was found that the most favourable conditions of driving, represented by the value of the factor: $\frac{\text{set}}{\text{maximum head stress}}$, occurred without exception when the heaviest hammer was used in conjunction with the head-cushion of lowest stiffness (lowest k/A value), the height of drop being adjusted to give a safe stress. There is reason to suppose that this rule can be applied generally and is virtually independent of the type of ground into which the pile is driven.

Under certain conditions of moderate and hard driving the use of a head cushion of low stiffness involves a loss of energy-efficiency. In few cases, however, is the loss likely to be of sufficient magnitude to justify the employment of stiffer packing.

Pile driving equipment should be chosen so that the piles are driven under the conditions productive of least damage. Since in all cases the heaviest possible hammer is required the choice of hammer weight depends only upon considerations of plant available. It is suggested that a reasonable minimum for the weight of the hammer is obtained by making the value of the ratio $\frac{\text{weight of hammer}}{\text{weight of 1 ft. of pile}}$ not less than 30. This gives for 12 in., 14 in., 16 in., and 18 in. square piles hammers of $2\frac{1}{4}$, 3, $3\frac{1}{2}$ and $4\frac{3}{4}$ tons respectively. Of the types of hammer in common use—drop hammers and single-acting steam hammers—there appears to be no type possessing outstanding advantages. In all cases there is a certain loss of kinetic energy due to frictional and other causes, and it is usual to make allowance for this factor when calculating the striking energy of the hammer. A head cushion should always be used. The best way of using the materials at present available would be simply to place packing on the head of the pile, but owing to the difficulty of retaining the packing in position it is generally necessary to use a helmet. A light helmet is to be preferred to a heavy one, as a heavy helmet is likely to cause more rapid deterioration of the dolly. The interior surfaces of the helmet and more particularly the surfaces parallel with the head of the pile, should be truly plane. The helmet should be well-fitting and should be prevented from abrading the sides of the pile by layers of material such as sacking placed over and around the head. No definite pronouncement can be made as to the best material for the construction of the dolly, but it was observed that the pinkadou dollies used for certain of the tests showed no signs of deterioration. The problem of packing is complex, for, during the course of the investigation no form of packing was encountered which satisfied completely the theoretical requirements, viz.:—

(1) Low stiffness as represented by the factor k/A.
(2) No increase of stiffness during driving.
(3) Low cost in relation to durability.

The packing tests have shown two materials which satisfy the first two requirements. The first is rubber, which, however, has the disadvantage that room must be allowed for lateral expansion; it cannot therefore be used in ordinary helmets, but might be used with a helmet of suitable design. The second material is asbestos fibre, which also fulfils the first two requirements. Its first cost is rather high, however, and it remains to be proved whether the material is of practical value. Of the packings in common use, tested in the impact machine, sack-cloth exhibited the best characteristics. Whatever material is used it is advisable that the layer in contact with the head of the pile should be of soft material (low k/A value) so that any small irregularities in the pile-head may not be the cause of failure.

The estimation of driving stresses in practice will now be considered. It has been shown that the most important stresses in causing failure are the maximum compressive stresses at the head and foot of the pile. Simplified charts, derived from those given on pp. 83 and 84 have been prepared with the aid of which, for a pile of normal constitution, the conditions at the head and foot may be examined.

From Fig. 46 the maximum stress at the head of a pile can be estimated when the driving conditions at the head are known. The value is derived from Fig. 7, by inserting the values of the physical constants for a pile of normal constitution, i.e. $2\frac{1}{2}$ per cent. longitudinal reinforcement and concrete of Young's modulus $4{\cdot}5 \times 10^6$ lb./sq. in. The figure therefore is applicable to piles where the maximum head stress is unaffected by the reflected wave from the foot, that is, under normal conditions, to piles over 30 ft. long. The head stress in lb./sq. in. is plotted against the ratio $\frac{\text{weight of hammer}}{\text{weight of 1 ft. of pile}}$ for different values of the stiffness, k/A, of the head cushion. Since the effective k/A of the head cushion is a function of the maximum stress in the packing (i.e. at the head of the pile), which is unknown, it follows that the maximum head pressure cannot be determined immediately from the curves of Fig. 46. One can, however, proceed by successive approximation as follows. Assume a value P for the maximum head pressure, then from the curve showing the variation of k/A with pressure for the head cushion (Fig. 53)* derive the corresponding k/A. It is sufficiently accurate for this calculation to take the stress in the head cushion as equal to that in the concrete at the head. Using this value of k/A in Fig. 46 obtain a first approximation P_1 to the maximum head pressure. The correction for the weight of the helmet given on p. 45 should be applied, that is, the actual weight of the hammer and height of free fall should be replaced by the effective weight of the hammer $(W+W_h)$ and the effective height of fall $h' = \left(\frac{W}{W+W_h}\right)^2 h$ (notation as on p. 45). Starting with the first approximation P_1 the process is repeated to obtain a second approximation P_2 and is continued until two successive approximations to P differ by a negligible amount.

* The derivation of this curve is explained in Appendix II.

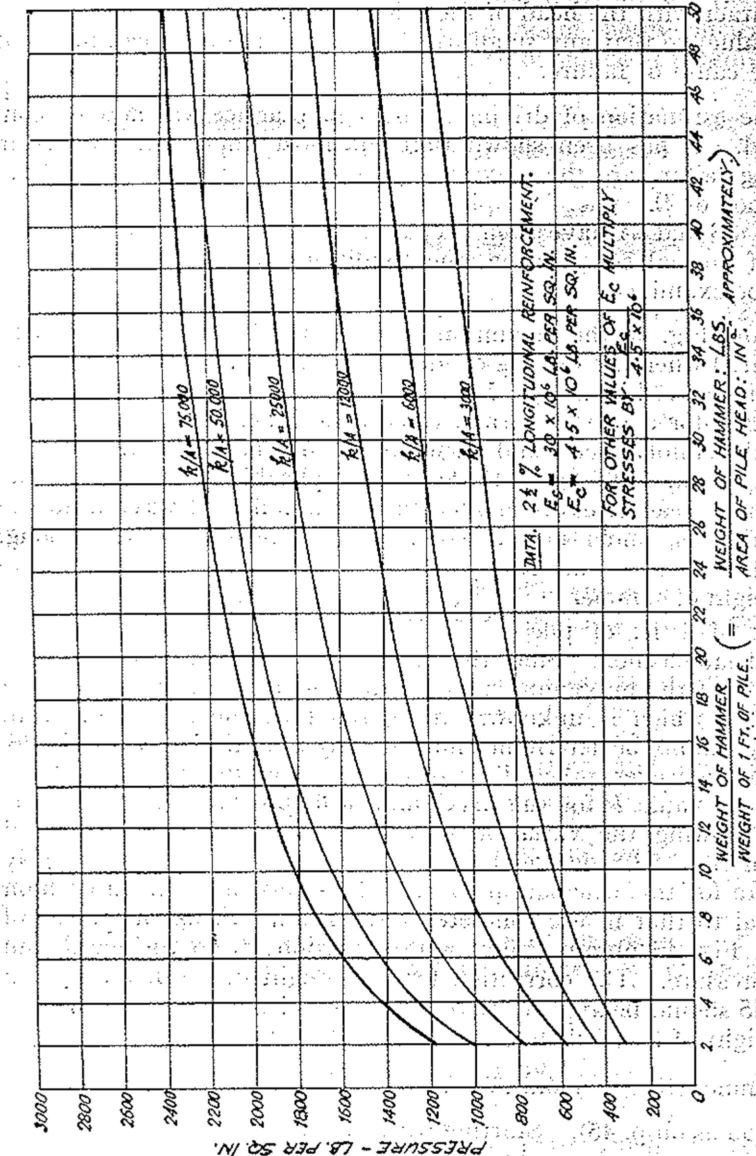

FIGURE 46.—The maximum Head Stress for a Drop of One Foot.

It should be noted that in this and other estimations made in this section, unless otherwise stated, the height of drop to be used or determined is the equivalent height of free fall. The actual height of fall of the hammer is related to the equivalent height of free fall by the use of the customary rules.

Figure 46 is of greatest assistance when the characteristics of the packing and dolly are known with some accuracy, and it is desired to determine the maximum stress at the head

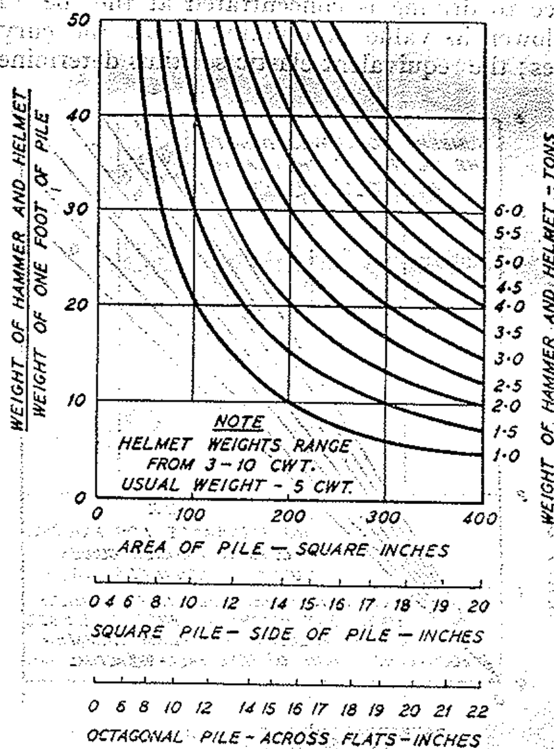

FIGURE 47.—*Diagram giving the ratio* $\frac{\textit{weight of hammer and helmet}}{\textit{weight of 1 foot of pile}}$ *for a range of pile sizes and hammer weights.* (*Weight of reinforced concrete taken as* 160 *lb./cu. ft.*)

for a particular height of drop of the hammer. A series of charts of greater simplicity has been prepared for more general use, when the characteristics of the packing are not known with any certainty. They include curves for the examination of both head and foot conditions. The first figure (Fig. 47) is for the quick estimation of the ratio $\frac{\text{weight of hammer and helmet.}}{\text{weight of 1 ft. of pile}}$ The second, Figure 48, consists of two sets of three graphs, one set applicable to conditions giving rise to a stress of 2,000 lb./sq. in. in the concrete and the other to a stress of 3,000 lb./sq. in. The three graphs of each set are for soft, medium and hard head cushions, i.e. for k/A values of 10,000, 20,000 and 40,000 lb./sq. in./in. at a stress of 3,000 lb. per sq. in. Each graph consists of two curves, derived from Fig. 7, the first giving the effective height of drop to produce the specified stress (2,000 or 3,000 lb./sq. in.) at the head, and the second the equivalent elastic set at the foot to produce a similar stress at the foot when the pile is being driven through a hard stratum and resistance to driving is concentrated at the toe. Equivalent elastic sets lower in value and falling below the curve produce higher stresses; the equivalent elastic set thus determined is there-

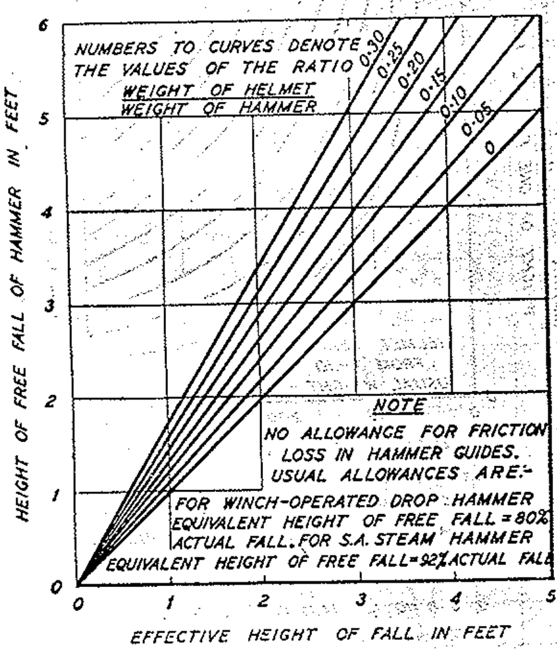

FIGURE 49.—*Conversion of effective height of fall to height of free fall of hamm[er] for various ratios* $\frac{\text{weight of helmet}}{\text{weight of hammer}}$.

fore the minimum permissible, assuming the head and foot to be equal strength.

The third figure (Fig. 49) is used to convert the effective heig[ht] of drop to produce the specified head stress to the height of free f[all] of the hammer and is drawn for a range of values of the ra[tio] $\frac{\text{weight of helmet}}{\text{weight of hammer}}$. The resulting height of free fall is the maxim[um] permissible for the concrete considered.

The curves for the determination of the minimum permissi[ble] equivalent elastic set were obtained by combining the results

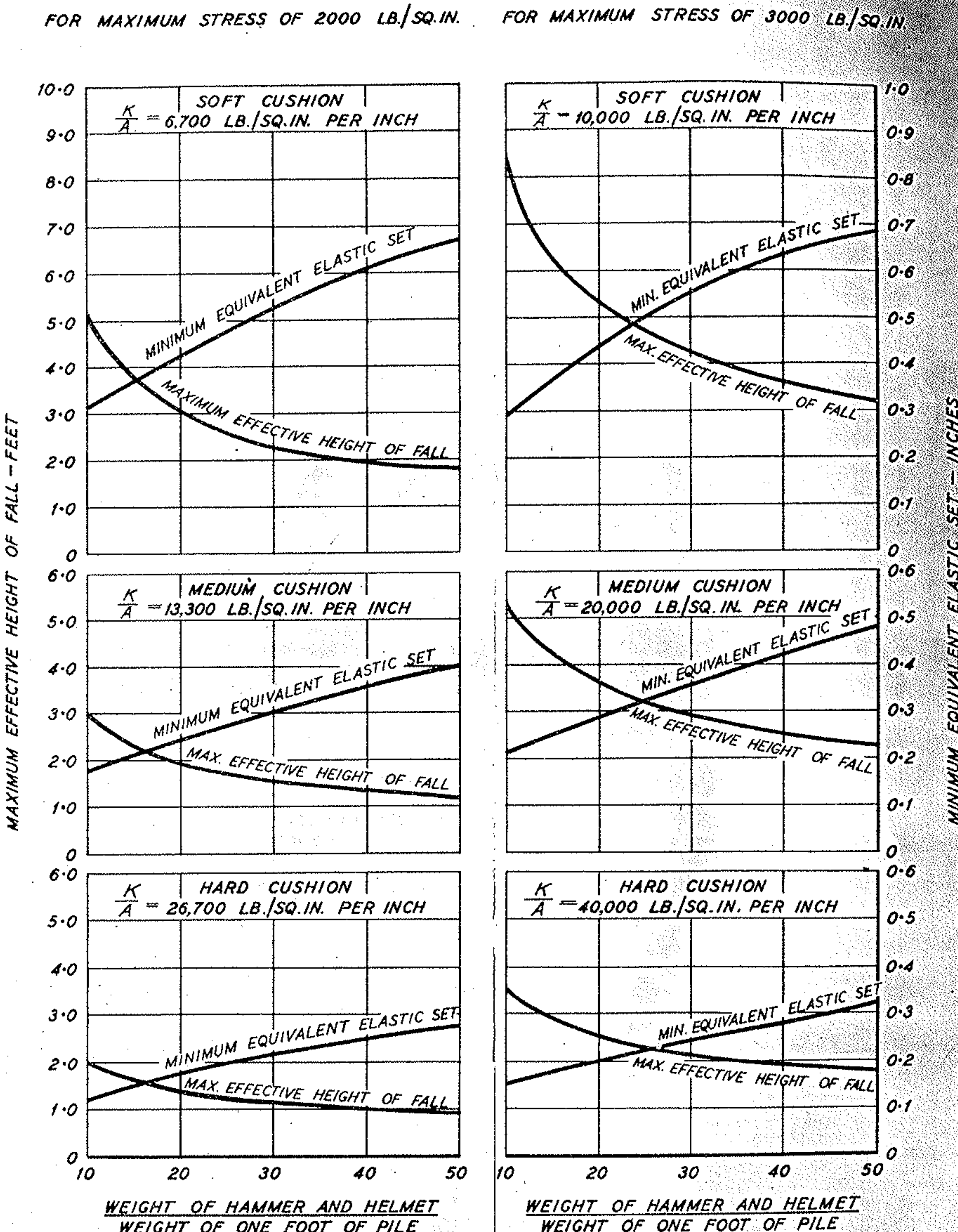

FIG. 48. RELATION BETWEEN THE RATIO $\frac{\text{WEIGHT OF HAMMER AND HELMET}}{\text{WEIGHT OF ONE FOOT OF PILE}}$ AND THE EFFECTIVE HEIGHT OF FALL AND THE EQUIVALENT SET FOR A GIVEN MAXIMUM STRESS IN THE PILE. ($2\frac{1}{2}$ PER CENT. LONGITUDINAL REINFORCEMENT IN PILE. YOUNG'S MODULUS FOR CONCRETE $4\cdot5 \times 10^6$ LB. PER. SQUARE INCH.)

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the experimental tests with deductions from the charts for the foot stress (Fig. 6). The equivalent elastic set to produce the specified stress (2,000 or 3,000 lb./sq. in.) in the concrete at the foot of the pile, when the height of drop of the hammer is such as to produce the same maximum stress at the head, was estimated for a range of values of the ratio $\frac{\text{weight of hammer and helmet}}{\text{weight of 1 ft. of pile}}$, and for various pile-lengths and stiffness constants, making allowance for the difference between the theoretical and actual foot stresses by *calculating for a maximum stress at the foot 30 per cent. higher than that at the head.* It was found that the set was almost independent of the pile-length, so that a single curve for each value of the packing constant is accurate enough for practical purposes.

In making use of the curves, it may be assumed that for most types of head cushion the hard condition applies after about 1,000 blows.

The value of the equivalent elastic set has to be determined in order to assess the foot conditions. The set-recording apparatus described in a preceding section (p. 18), provides a simple method. The recorded set has to be corrected for the elastic compression of the embedded length of the pile; this is done by subtracting from *the measured set* 0·004 *in. per ft. of pile embedded* where the maximum head stress is 3,000 lb./sq. in. or 0·003 in. where the stress is 2,000 lb./sq. in. The equivalent elastic set is given by the expression—

Equivalent elastic set = twice measured plastic set + corrected elastic set.

It should be noted that Figs. 47 to 49 are only applicable to piles over 30 ft. long and *that dangerous foot stresses can only be set up when resistance to motion of the pile is concentrated at the toe.* For piles less than 30 ft. long when driving into soft ground, the height of drop of the hammer derived from the above figures would give a slightly lower stress *than that specified and a slightly higher* stress when driving is very hard against a dense layer. In such cases Figs. 5 and 6 should be used.

The following example illustrates the use of the charts. A pile, 55 ft. *long by* 14 *in.* square *is driven with a* 2¼*-ton winch* operated drop hammer and a 5-cwt. helmet. The cube strength of the concrete is at least 6,000 lb./sq. in. Hard driving is encountered in a dense layer of ballast. What are the maximum permissible height of drop of the hammer and *the minimum equivalent elastic set?*

From Fig. 47 we find that the ratio $\frac{\text{weight of hammer and helmet}}{\text{weight of 1 ft. of pile}}$ is 26.

Hard driving is to be encountered and with a pile of this length the number of blows to drive one pile is likely to exceed 1,000. The curves for $k/A = 40{,}000$ must be used. The impact strength is at least 3,000 lb./sq. in. From Fig. 48 (*c*) the effective height of fall is 2·3 ft.

The ratio $\frac{\text{weight of helmet}}{\text{weight of hammer}}$ is 0·11 and from Fig. 49 the height of free fall of the hammer is 2·85 ft. The height of free fall of the hammer for a winch operated drop hammer is usually taken as 80 per cent. of the actual height of fall. The actual height of fall of the hammer should, therefore, be 3·6 ft.

The minimum equivalent elastic set is read off from Fig. 48 (*c*) using the same value of the weight ratio as before and is found to be 0·23 in.

Future Work

Further research might profitably be pursued in several directions. It will be obvious from what has gone before that one of the chief sources of uncertainty and trouble is the unsatisfactory nature of existing methods of cushioning the blow. A head cushion of constant properties and capable of ensuring a uniform distribution of stress over the pile head would not only prevent the majority of head failures but would enable data to be collected on bearing capacity under controlled and definite head conditions. Two methods of constructing an improved head cushion are at present under consideration: the first is based on the idea of using a light helmet, in which the major portion of the packing material is placed above the diaphragm; the second method contemplates the substitution of a mechanical device for the packing and dolly.

The investigation has shown that the impact-strength of concrete lies between 50 and 80 per cent. of the cube compressive strength. It is very necessary that a more detailed investigation on the effect of manufacturing variables on the impact strength of concrete should be carried out with the new impact machine. Only as a result of such work can the knowledge be obtained which is necessary to predict the impact strength of concrete with accuracy.

Information is required on the magnitude of the actual elastic set at the foot of the pile in hard driving, that is, on the temporary compression of the ground beneath the pile. The importance of the actual elastic set becomes evident when it is required to estimate whether a specified final set (that is, permanent set) is likely to be accompanied by dangerously high foot stresses. Evidence from the tests carried out during the course of the present investigation indicates that the actual elastic set for hard driving varies over a comparatively narrow range around a value of 0·15 in., but owing to the restricted number of observations, the figure must be accepted with reserve.

Finally, several points in connection with the stress phenomena in the pile require further investigation. No attempt has been made, in the work described, to measure the stress distribution over the cross-section at the head of the pile, but if the considerable experimental difficulties could be overcome, the knowledge gained would probably be of value in designing the head of the pile.

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NOTATION

The symbols used in the Appendices have the following meaning :—

M denotes the mass of the hammer
h „ „ height of free fall of the hammer
V „ „ velocity of the hammer on impact
M_h „ „ mass of the helmet
M_i „ „ total mass of the peak stress indicator
M_p „ „ mass of the pile
ρ „ „ mass density of the pile as a whole
A „ „ total area of cross-section of the pile
l „ „ length of the pile
p_1 „ „ ratio of longitudinal reinforcement
p_2 „ „ ratio of transverse reinforcement
E_c „ Young's modulus for concrete
E_s „ Young's modulus for steel
E „ the effective Young's modulus for the material of the pile
σ „ Poisson's ratio for concrete
K „ the radius of gyration of a cross-section of the pile about it intersection with the neutral surface
a „ „ velocity of longitudinal waves in the pile
g „ „ acceleration of gravity
t „ „ time after the beginning of impact
x „ „ distance measured along the length of the pile from th head of the pile as the origin
y „ „ displacement in the direction at right angles to the lengt
ξ „ „ displacement of a cross-section along the x axis
ξ_o „ „ displacement of the head of the pile along the x axis
ξ_l „ „ displacement of the foot of the pile along the x axis
η „ „ displacement of the hammer along the x axis, measur from its position at $t = O$
γ „ „ displacement of the helmet along the x axis, measur from its position at $t = O$
z „ „ displacement of the peak stress indicator along the x a from its position at $t = O$
B „ „ amplitude of flexural vibration
P_o „ „ stress at the head of the pile
P_m „ „ maximum stress at the head of the pile
P_l „ „ stress at the foot of the pile
k „ „ force to produce unit compression of the assumed he cushion
k_l „ „ force to produce unit compression of the assumed fo resistance
k_d „ „ force to produce unit compression in the dolly
k_p „ „ force to produce unit compression in the packing, assum linear elasticity
k_i „ „ force to produce unit compression of the spring on wh the peak stress indicator is mounted
μ „ „ frictional force for unit velocity exerted by the damp device on the peak stress indicator
ϵ_1 „ „ equivalent purely elastic set at the foot $\div \frac{Vl}{a}$

Other symbols are defined as they occur.

APPENDIX I

THE DEVELOPMENT OF THE MATHEMATICAL THEORY

(1) *Wave Solution*

It has already been stated that the stress-wave travels along the pile with a mean speed dependent on the ratios of longitudinal and transverse reinforcement and on Young's modulus for the concrete. The problem of a reinforced concrete pile can therefore be reduced to that of an equivalent homogeneous pile, which means that the assumption of plane cross sections remaining plane during impact is true for a reinforced concrete pile in the same way as for a homogeneous pile. Before using this assumption, its validity was tested by analysing mathematically the simple case of a circular pile with one circular reinforcing bar at its centre. In the analysis, the effects of radial variations of displacement, stress, etc., were taken into account and it was then found that, save for certain terms containing the factor $\left(\frac{\text{radius}}{\text{length}}\right)^2$ and therefore negligible in practice, the longitudinal displacement was uniform over the cross section, *i.e.* plane sections remained plane. Furthermore, it was found that the radial stresses between the steel and the concrete, due to a difference in their Poisson's ratios, were also negligible. The longitudinal displacement ξ, and therefore the strain $\frac{\partial \xi}{\partial x}$, being uniform over the cross section, it follows that

$$\text{compressive stress in concrete} = -E_c\frac{\partial \xi}{\partial x}$$

$$\text{compressive stress in steel} = -E_s\frac{\partial \xi}{\partial x}$$

$$\text{and mean compressive stress in cross section} = -E\frac{\partial \xi}{\partial x}.$$

The equivalent Young's modulus for the pile as a whole is given by the expression

$$E = \frac{\left\{E_c + (E_s - E_c)\,p_1\right\}\left\{E_c + (E_s - E_c)\,p_1 + E_s p_2\left(\frac{1-\sigma}{2}\right)\right\}}{E_c + (E_s - E_c)\,p_1 + E_s p_2\left(\frac{1-\sigma-2\sigma^2}{2}\right)}$$

In practice the effect of the transverse reinforcement may be neglected; its ratio is so small that it has a negligible effect on the longitudinal waves, as will be seen by inserting numerical values for p_2 in the formula for E given above. Hence

$$E = E_c + (E_s - E_c)\,p_1$$

The original theoretical treatment included constants defining the effects of permanent set and frictional resistance in the head cushion. Experimental evidence has shown that such effects may be neglected.

The foot resistance is, in practice, partly elastic and partly plastic. Any differences between the assumed and actual foot conditions will cause the calculated and observed values to differ most widely at the foot, and it is therefore best to assume a foot condition which corresponds most closely to that in which the foot-stress may become important, namely, severe driving against a hard stratum. Under such conditions the greater proportion of the set is elastic.

It was therefore decided to assume a purely elastic foot condition, and when applying the theory in practice to use an approximate formula equating the observed plastic plus elastic set to an equivalent purely elastic set, to which the theory would be applied directly.

The effect of dissipation of energy due to propagation losses in the pile has not been included in the theory, since very little information concerning it exists. Dissipation will tend to reduce the stresses. The effect of skin friction will be of the same nature from the practical point of view, since it will tend to decrease the amplitude of the stress-waves as they travel along the pile. The effect of neglecting both propagation losses and skin friction is therefore to make the theoretical stresses higher than the actual stresses; that is, the error is on the safe side.

The theory as developed for practical use is based on the following assumptions :—

(1) That the pile is undamaged when driven.
(2) That the pile behaves as a linearly elastic rod.
(3) That stress waves in the hammer may be neglected.
(4) That the conditions at the head are equivalent to a linearly elastic spring interposed between the pile and the hammer.
(5) That the foot resistance is elastic, the foot pressure being proportional to the downward foot movement.

By considering the equation of motion of a small length dx of the pile, the usual wave equation

$$\frac{\partial^2 \xi}{\partial t^2} = a^2 \frac{\partial^2 \xi}{\partial x^2} \quad \ldots \quad (1)$$

follows immediately, where $a^2 = \frac{E}{\rho}$

This has the general solution $\xi = f\left(t - \frac{x}{a}\right) + F\left(t + \frac{x}{a}\right) \quad \ldots \quad (2)$

representing waves travelling down and up the pile. The total pressure across any cross section

$$= - AE\frac{\partial \xi}{\partial x}$$

and the pressure at the head of the pile = the elastic pressure in the packing = the pressure retarding the hammer.

$$\text{Thus } - AE\left(\frac{\partial \xi}{\partial x}\right)_{x=0} = k\,(\eta - \xi_o) = - M\,\eta'', \quad \ldots \quad (3)$$

where the dashes denote differentiation with regard to t.

$$\text{Hence } \eta' - \xi_o' = - \frac{M\eta'''}{k} \quad \ldots \quad (4)$$

$$\left.\begin{aligned} &\text{Now from (2) } \xi_o' = f'(t) + F'(t) \\ &\left(\frac{\partial \xi}{\partial x}\right)_{x=o} = - \frac{1}{a}f'(t) + \frac{1}{a}F'(t) \end{aligned}\right\} \quad \ldots \quad (5)$$

$$\begin{aligned} \text{From (3) and (5), } M\eta'' &= - \frac{AE}{a}f'(t) + \frac{AE}{a}F'(t) \\ &= - \frac{AE}{a}\xi'_o + \frac{2AE}{a}F'(t) \quad \ldots \quad (6) \end{aligned}$$

Hence, eliminating ξ_o from (4) and (6).

$$M\eta''' + \frac{Mka}{AE}\eta'' + k\eta' = 2kF'(t) \quad \ldots \quad (7)$$

It is obvious that no reflected wave reaches the head of the pile until a time $\frac{2l}{a}$ has elapsed, so that, when $O \leqslant t < \frac{2l}{a}$ $F(t) = O \quad \ldots \quad (8)$

For this period, equation (7) therefore has the simplified form

$$\eta''' + 2n\eta'' + p^2\eta' = O \quad \ldots \quad (9)$$

where

$$\left.\begin{aligned} 2n &= \frac{ka}{AE} \\ p^2 &= \frac{k}{M} \end{aligned}\right\} \quad \ldots \quad (10)$$

The initial conditions at $t = O$ are :—

$$\left.\begin{aligned} \eta &= O \\ \eta' &= V = \sqrt{2gh} \end{aligned}\right\} \quad \ldots \quad (11)$$

$$\left.\begin{aligned} \xi &= O \\ \xi' &= O \end{aligned}\right\} O < x \leqslant l \quad \ldots \quad (12)$$

If $p^2 > n^2$, which covers the range of practical conditions, the solution of (9) which satisfies the initial conditions is

$$\eta = \frac{V}{qp^2}\left\{\left[q^2 - n^2\right] e^{-nt}\,\sin qt + 2qn\left(1 - e^{-nt}\cos qt\right)\right\}$$

where $q^2 = p^2 - n^2 \quad \ldots \quad (13)$

η is thus given by (13) for the interval of time $O \leqslant t < \frac{2l}{a}$. Hence from equation (3), $\xi_o = f(t)$ is given, since $F(t) = O$ from equation (8).

The assumption of a linearly elastic foot condition is expressed by the equation

$$- EA\left(\frac{\partial \xi}{\partial x}\right)_{x=l} = k_1\,\xi_l.$$

$$\text{or } \frac{EA}{a}\left\{f'\left(t - \frac{l}{a}\right) - F'\left(t + \frac{l}{a}\right)\right\} = k_1\left\{f\left(t - \frac{l}{a}\right) + F\left(t + \frac{l}{a}\right)\right\} \quad (14)$$

By virtue of equations (8) and (12) equation (14) may be solved for F to obtain the equation

$$F\left(t + \frac{l}{a}\right) = f\left(t - \frac{l}{a}\right) - 2\frac{k_1 a}{AE}\int_{l/a}^{t} e^{-k_1 a(t-\nu)/AE} f\left(\nu - \frac{l}{a}\right)d\nu \quad \ldots \quad (15)$$

which gives F for any one interval in terms of f for the preceding intervals.

The determination of f and F from equations (4), (6), (8) and (15) then proceeds by successive intervals $\frac{2l}{a}$ of t, and as it involves no new mathematical features the results alone will be given here.

It has been shown in the discussion of the results of experimental tests that over the range of durations of impact likely in practice the damaging effect is dependent mainly on the maximum compressive stress. It is therefore sufficient to carry the calculation of stresses as far as the maximum stress since the subsequent smaller stresses will be of little practical interest. In most cases the maximum at any point x in the pile occurs within a time $\frac{4l}{a}$ from the commencement of stress at the point, *i.e.*, within the time interval $t = \frac{x}{a}$ to $t = \frac{x + 4l}{a}$ and thus from (2) it is necessary to know $f(t)$ from $t = O$ to $\frac{4l}{a}$ and $F(t)$ from $t = \frac{2x}{a}$ to $t = \frac{2x + 4l}{a}$. Hence since x may vary from O to l it is necessary to know $f(t)$ from $t = O$ to $\frac{4l}{a}$ and $F(t)$ from $t = O$ to $\frac{6l}{a}$ in order to evaluate the maximum stress at any point.

Since numerical evaluation has to be performed, it is desirable to express all formulæ, as far as possible, in non-dimensional form and this will be done by using the following substitutions :

$$\left.\begin{aligned} t &= \frac{l}{a}.\ \ \tau. \\ x &= l\ z. \\ f\ (t) &= \frac{Vl}{a}.\ \ f_1(\tau). \\ F\ (t) &= \frac{Vl}{a}.\ \ F_1(\tau). \\ \frac{n}{n_1} = \frac{q}{q_1} &= \frac{p}{p_1} = \frac{a}{l} \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (16)$$

The mathematical analysis then gives for the non-dimensional wave functions $f_1\ (\tau)$ and $F_1\ (\tau)$ the following results

$\underline{0 \leqslant \tau \leqslant 2}$

$$f_1\ (\tau) = \frac{2n_1}{p_1^2}\left(1 - e^{-n_1\tau}\cos q_1\tau\right) - \frac{2n_1\cot\theta}{p_1^2}\,e^{-n_1\tau}\sin q_1\tau \quad \ldots \quad (17)$$

$$F_1\ (\tau) = 0 \quad \ldots\ldots\ldots \quad (18)$$

$\underline{2 \leqslant \tau \leqslant 4}$

$$\begin{aligned} f_1\ (\tau) = {} & \frac{2n_1}{p_1^2}\left(1 - e^{-n_1\tau}\cos q_1\tau\right) - \frac{2n_1\cot\theta}{p_1^2}\,e^{-n_1\tau}\sin q_1\tau \\ & + \left\{\frac{2r_3\cot\theta.\sin(\theta_2+\theta)}{p_1} - \frac{4r_2\cot^2\theta.\cos(\theta_1+\theta)}{p_1}\right\}\left\{1-e^{-n_1(\tau-2)}\cos\ q_1(\tau-2)\right\} \\ & - \left\{\frac{2r_3\cot\theta.\cos(\theta_2+\theta)}{p_1} + \frac{4r_2\cot^2\theta.\sin(\theta_1+\theta)}{p_1}\right\}e^{-n_1(\tau-2)}\sin\ q_1(\tau-2) \\ & + 4r_2\,(\cot^2\theta)\,(\tau-2)\,e^{-n_1(\tau-2)}.\cos\left\{q_1\,(\tau-2)+\theta_1\right\} \\ & - \frac{4n_1r_2^2}{r_1^2}\left\{1 - e^{-a_1\,(\tau-2)}\right\} \quad \ldots\ldots\ldots \quad (19) \end{aligned}$$

$$\begin{aligned} F_1\ (\tau) = {} & \frac{2r_2\cot\theta.\sin\,(\theta_1+\theta)}{p_1}\left\{1 - e^{-n_1\,(\tau-2)}\cos q_1\,(\tau-2)\right\} \\ & - \frac{2r_2.\cot\theta.\cos\,(\theta_1+\theta)}{p_1}.e^{-n_1\,(\tau-2)}\sin q_1\,(\tau-2) \\ & - \frac{4n_1}{r_1^2}\left\{1 - e^{-a_1\,(\tau-2)}\right\} \quad \ldots\ldots\ldots \quad (20) \end{aligned}$$

$\underline{4 \leqslant \tau \leqslant 6}$

$$\begin{aligned} F_1\ (\tau) = {} & \frac{2r_2\cot\theta.\sin\,(\theta_1+\theta)}{p_1}\left\{1 - e^{-n_1\,(\tau-2)}\cos q_1\,(\tau-2)\right\} \\ & - \frac{2r_2\cot\theta.\cos\,(\theta_1+\theta)}{p_1}.e^{-n_1\,(\tau-2)}.\sin q_1\,(\tau-2) \\ & - \frac{4n_1}{r_1^2}.\left\{1 - e^{-a_1\,(\tau-2)}\right\} \\ & + \frac{1}{p_1}\left\{r_4\sin\,(\theta_1+\theta_3+\theta) - 4r_2^2\cot^2\theta.\cos\,(2\theta_1+\theta)\right\} \\ & \qquad \left\{1 - e^{-n_1\,(\tau-4)}\cos q_1\,(\tau-4)\right\} \\ & - \frac{1}{p_1}\left\{r_4\cos\,(\theta_1+\theta_3+\theta) + 4r_2^2\cot^2\theta.\sin\,(2\theta_1+\theta)\right\}\times \\ & \qquad e^{-n_1\,(\tau-4)}\sin.\ q_1\,(\tau-4) \\ & + 4r_2^2\,(\cot^2\theta)(\tau-4)\,e^{-n_1\,(\tau-4)}\cos\left\{q_1\,(\tau-4)+2\theta_1\right\} \\ & + \left\{\frac{8n_1r_2^2}{r_1^2} - \frac{r_4\sin\,(\theta_1+\theta_3)}{a_1}\right\}\left\{1 - e^{-a_1\,(\tau-4)}\right\} \\ & - \frac{8a_1n_1r_2^2}{r_1^2}(\tau-4).e^{-a_1\,(\tau-4)} \quad \ldots\ldots\ldots \quad (21) \end{aligned}$$

The constants n_1, q_1, etc. are given in terms of the fundamental constants by the following relations :

$$\left.\begin{aligned} n_1 &= \frac{kl}{2EA} \\ p_1^2 &= \frac{klM_p}{EAM} \end{aligned}\right\} \left.\begin{aligned} q_1^2 &= p_1^2 - n_1^2 \\ \tan\theta &= \frac{q_1}{n_1} \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (22)$$

$$\left.\begin{aligned} a_1 &= \frac{k_1l}{EA} \\ r_1^2 &= (n_1 - q_1)^2 + q_1^2 \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (23)$$

$$\left.\begin{aligned} r_2\cos\theta_1 &= \frac{p_1^2 - a_1^2}{r_1^2} \\ r_2\sin\theta_1 &= \frac{2a_1q_1}{r_1^2} \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (24)$$

$$\left.\begin{aligned} r_3\cos\theta_2 &= \left\{r_2^2 + \frac{2n_1^2}{q_1^2}\right\}r_2\cos\theta_1 \\ r_3\sin\theta_2 &= \quad r_2^3\sin\theta_1 \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (25)$$

$$\left.\begin{aligned} r_4\cos\theta_3 &= 2r_2^2\left[r_2^2 + \frac{4n_1a_1}{r_1^2} + \frac{2n_1^2}{q_1^2}\right]\cot\theta.\cos\theta_1 \\ r_4\sin\theta_3 &= 2r_2^2\left[r_2^2 + \frac{4n_1a_1}{r_1^2} - \frac{2n_1^2}{q_1^2}\right]\cot\theta.\sin\theta_1 \end{aligned}\right\} . \quad (26)$$

In the above the case $p^2 > n^2$ has alone been given since the other case $p^2 < n^2$ corresponds to excessively hard packings which are not, and should not, be used in practice. The solution for $p^2 < n^2$, if required, can easily be obtained from that given above by replacing q by iq and transforming the circular to hyperbolic functions.

In terms of these non-dimensional functions and quantities the displacement at any cross-section of the pile is given by

$$\xi = \frac{Vl}{a}\left\{f_1\,(\tau - z) + F_1\,(\tau + z)\right\} \quad \ldots\ldots\ldots \quad (27)$$

while the compressive stress in the concrete is given by

$$-E_c\frac{\partial\xi}{\delta x} = \frac{E_cV}{a}\left\{f_1'(\tau - z) - F_1'(\tau + z)\right\} \quad \ldots\ldots \quad (28)$$

where the accents to f_1 and F_1 denote differentiation with respect to τ and not with respect to t as in the case of f and F. The expressions for $f_1'\ (\tau)$ and

$F_1'(\tau)$ may be obtained by differentiating (17) to (21) to obtain the following results:

$0 \leqslant \tau \leqslant 2$

$$f_1'(\tau) = 2\cot\theta . e^{-n_1\tau}\sin q_1\tau \qquad (29)$$

$$F_1'(\tau) = 0$$

$2 \leqslant \tau \leqslant 4$

$$\begin{aligned} f_1'(\tau) = {} & 2\cot\theta . e^{-n_1\tau}\sin q_1\tau \\ & + 2r_3\cot\theta . e^{-n_1(\tau-2)}\sin\{q_1(\tau-2)+\theta_2\} \\ & - 4p_1r_2(\cot_2\theta)(\tau-2)e^{-n_1(\tau-2)}\cos\{q_1(\tau-2)+\theta_1-\theta\} \\ & - 2r_3\cot\theta . \sin\theta_2 . e^{-\alpha_1(\tau-2)} \end{aligned} \qquad (31)$$

$$\begin{aligned} F_1'(\tau) = {} & 2r_2\cot\theta . e^{-n_1(\tau-2)}\sin\{q_1(\tau-2)+\theta_1\} \\ & - 2r_2\cot\theta . \sin\theta_1 e^{-\alpha_1(\tau-2)} \end{aligned} \qquad (32)$$

$4 \leqslant \tau \leqslant 6$

$$\begin{aligned} F_1'(\tau) = {} & 2r_2\cot\theta . e^{-n_1(\tau-2)}\sin\{q_1(\tau-2)+\theta_1\} \\ & - 2r_2\cot\theta . \sin\theta_1 . e^{-\alpha_1(\tau-2)} \\ & + r_4 e^{-n_1(\tau-4)}\sin\{q_1(\tau-4)+\theta_1+\theta_3\} \\ & - 4p_1r_2^2(\cot^2\theta)(\tau-4)e^{-n_1(\tau-4)}\cos\{q_1(\tau-4)+2\theta_1-\theta\} \\ & - r_4\sin(\theta_1+\theta_3)e^{-\alpha_1(\tau-4)} \\ & + \frac{8\alpha_1^2 n_1 r_2^2}{r_1^2}(\tau-4) . e^{-\alpha_1(\tau-4)} \end{aligned} \qquad (33)$$

The displacement and velocity of the hammer are obtained from the preceding equations by using the relations

$$\eta = \frac{Vl}{a}\left[f_1(\tau) + F_1(\tau) + \frac{1}{2n_1}\left\{f_1'(\tau) - F_1'(\tau)\right\}\right] \qquad (34)$$

$$\frac{d\eta}{dt} = V\left[1 - \frac{p_1^2}{2n_1}\left\{f_1(\tau) - F_1(\tau)\right\}\right] \qquad (35)$$

The wave solution outlined above is the form best suited to most practical problems; however, it is sometimes advantageous to have the solution in an alternative approximate form.

(2) *Approximate solution for soft head-cushions and heavy hammers*

The treatment by the wave solution of extreme cases of very soft packings with heavy hammers and short piles, involves an excessive amount of labour since calculation has to be made over many intervals $\frac{2l}{a}$ of t before the maximum stress is obtained. Fortunately, however, an approximate solution is possible in such circumstances.

Within the pile equation (1) applies, viz :—

$$\frac{\partial^2\xi}{\partial t^2} = a^2\frac{\partial^2\xi}{\partial x^2}$$

while at the head of the pile, *i.e.* at $x = 0$

$$\xi = \xi_o \qquad (36)$$

$$-E\frac{\partial\xi}{\partial x} = P_o \qquad (37)$$

by the definitions of ξ_o and P_o.

At the foot of the pile, *i.e.* at $x = l$

$$-AE\frac{\partial\xi}{\partial x} = k_1\xi \qquad (38)$$

Integrating (1) with regard to x and using (37)

$$\frac{\partial\xi}{\partial x} = -\frac{P_o}{E} + \frac{1}{a^2}\int_o^x\frac{\partial^2\xi}{\partial t^2}dx \qquad (39)$$

whence, again integrating and using (36)

$$\begin{aligned} \xi(x,t) & = \xi_o(t) - \frac{x P_o(t)}{E} + \frac{1}{a^2}\int_o^x dx\int_o^x\frac{\partial^2\xi(\lambda,t)}{\partial t^2}d\lambda \\ & = \xi_o(t) - \frac{x}{E}P_o(t) + \frac{1}{a^2}\int_o^x(x-\lambda)\frac{\partial^2\xi(\lambda,t)}{\partial t^2}d\lambda \end{aligned} \qquad (40)$$

where ξ, ξ_o, P_o are written more fully as functions of x and t.

From (38), (39) and (40) it follows that

$$P_o(t) = \frac{E\alpha_1}{1+\alpha_1} . \frac{\xi_o(t)}{l} + \rho\int_o^l\left(1 - \frac{\alpha_1}{1+\alpha_1} . \frac{\lambda}{l}\right)\frac{\partial^2\xi(\lambda,t)}{\partial t^2} . d\lambda \qquad (41)$$

where $\alpha_1 = \frac{k_1 l}{EA}$ as in (23)

Hence, substituting in (40) for P_o from (41)

$$\xi(x,t) = \left\{1 - \frac{\alpha_1}{(1+\alpha_1)} . \frac{x}{l}\right\}\xi_o(t) - \frac{1}{a^2}\int_o^l K(x,\lambda)\frac{\partial^2\xi(\lambda,t)}{\partial t^2} . d\lambda \qquad (42)$$

$$\left.\begin{aligned} \text{where } K(x,\lambda) & = \lambda\left\{1 - \frac{\alpha_1}{(1+\alpha_1)} . \frac{x}{l}\right\} \quad o \leqslant \lambda \leqslant x \\ & = x\left\{1 - \frac{\alpha_1}{(1+\alpha_1)} . \frac{\lambda}{l}\right\} \quad x \leqslant \lambda \leqslant l \end{aligned}\right\} \qquad (43)$$

The equation of motion of the hammer is

$$M\frac{d^2\eta}{dt^2} = -AP_o = -k(\eta - \xi_o) \qquad (44)$$

whence eliminating η

$$\frac{d^2P_o}{dt^2} + \frac{k}{M}P_o + \frac{k}{A} . \frac{d^2\xi_o}{dt^2} = 0 \qquad (45)$$

So far no approximation has been made and equations (41) to (45) combined with the initial conditions (11) and (12) will give the exact solution. In cases of soft packing, heavy hammers and short piles the inertia terms become relatively small and a first approximation can be obtained by neglecting these terms.

From (41) and (42) it then follows that

$$P_o(t) = \frac{E\alpha_1}{1+\alpha_1} . \frac{\xi_o(t)}{l} \qquad (46)$$

$$\xi(x,t) = \left\{1 - \frac{\alpha_1}{(1+\alpha_1)} . \frac{x}{l}\right\}\xi_o(t) \qquad (47)$$

The same result could be obtained *ab initio* by assuming a uniform compression of the pile.

From (44), (45), (46) and the initial conditions (11) and (12) the pressure everywhere in the pile is given by

$$P_o(t) = \frac{EV}{a}\sqrt{\frac{M}{M_p\left\{\frac{EA}{kl}+1+\frac{1}{\alpha_1}\right\}}}\ \sin q_2\tau \quad (48)$$

$$\text{where } q_2^2 = \frac{M_p}{M\left\{\frac{EA}{kl}+1+\frac{1}{\alpha_1}\right\}} \quad (49)$$

If the approximate value of ξ_o from (46) and (48) is substituted in the second term of (41), which has been neglected, then the ratio of the neglected term to that retained is

$$-\frac{M_p\left(1+\frac{1}{3}\frac{\alpha_1^2}{1+\alpha_1}\right)}{M\left\{1+\alpha_1\left(1+\frac{EA}{kl}\right)\right\}} \quad (50)$$

The smaller the value of the ratio (50) the greater will be the accuracy of the approximation. Sufficient accuracy will, however, only be obtained in certain cases of hard driving against a rigid base in which α_1 is effectively infinite. Then from (48) with $\alpha_1 = \infty$ the maximum pressure is

$$\frac{EV}{a}\sqrt{\frac{M}{M_p\left\{\frac{EA}{kl}\ 1+1\right\}}} \quad (51)$$

and the ratio (50) becomes

$$-\frac{M_p}{3M\left\{1+\frac{EA}{kl}\right\}} \quad (52)$$

which is small for heavy hammers, soft packing, and short piles. For easy driving where α_1 is effectively zero the formula (48) is not accurate since it gives zero pressure in the pile whereas the exact solution shows that in such cases large pressures may be developed at the head.

A more accurate approximation can, however, be obtained by including the effects of inertia to the first order. The value of P_o is obtained from [illegible] by substituting in the small integral term the value of ξ given by (47), whence after integration,

$$P_o = \frac{C_1E}{l}\cdot\xi_o(t) + D_1 pl\frac{\partial^2\xi_o}{\partial t^2} \quad (53)$$

$$\left.\begin{aligned} \text{where } C_1 &= \frac{\alpha_1}{1+\alpha_1} \\ D_1 &= \frac{1}{1+\alpha_1} + \frac{\alpha_1^2}{3(1+\alpha_1)^2} \end{aligned}\right\} \quad (54)$$

The elimination of P_o from (45) and (53) results in a simple fourth order differential equation for ξ_o which is easily solved, the initial conditions being by virtue of (11), (12), (44) and (53)

$$\left.\begin{aligned} \xi_o = \xi_o' = \xi_o'' &= 0 \\ \xi_o''' &= \frac{kV}{D_1M_p} \end{aligned}\right\}\text{when } t = 0 \quad (55)$$

The solution is

$$\xi_o = \frac{Vl}{a}\cdot\frac{kl}{EAD_1}\left\{\frac{\sin S_1\tau}{FS_1-2S_1^3}+\frac{\sin S_2\tau}{FS_2-2S_2^3}\right\} \quad (56)$$

where S_1, S_2, are the positive roots of

$$S^4 - FS^2 + \frac{klM_pC_1}{EAMD_1} = 0 \quad (57)$$

$$\text{and } F = \frac{C_1}{D_1}+\frac{klM_p}{EAM}+\frac{kl}{EAD_1} \quad (58)$$

From (53) it then follows that

$$P_o = \frac{EV}{a}\cdot\frac{kl}{EAD_1}\left\{\frac{C_1-D_1S_1^2}{FS_1-2S_1^3}\sin S_1\tau+\frac{C_1-D_1S_2^2}{FS_2-2S_2^3}\sin S_2\tau\right\} \quad (59)$$

while from (39) the mean compressive stress P_l at the foot is given to the same order of approximation by

$$P_l = \frac{EV}{a}\cdot\frac{kl}{EAD_1}\left\{\frac{C_1+D_2S_1^2}{FS_1-2S_1^3}\sin S_1\tau+\frac{C_1+D_2S_2^2}{FS_2-2S_2^3}\sin S_2\tau\right\} \quad (60)$$

$$\text{where } D_2 = \frac{3\alpha_1+\alpha_1^2}{6(1+\alpha_1)^2} \quad (61)$$

The formulae (59) and (60) give a good approximation in most cases not covered by the exact solution. Analysis shows that the neglected terms are of order

$$\frac{8+5\alpha_1+\alpha_1^2}{4(1+\alpha_1)^2}\left(\frac{klM_p}{EAM}\right)^{3/2} \quad (62)$$

compared with the terms retained, so that the approximation is valid when this expression is small compared with unity. For very easy driving, when α_1 is virtually zero a very simple formula is obtained. The first term of (59) vanishes and the second becomes, after simplification

$$P_o = \frac{EV}{a}\sqrt{\frac{kl/EA}{1+M_p/M}}\ \sin S_2\tau \quad (63)$$

$$\text{where } S_2^2 = \frac{kl}{EA}\left\{1+M_p/M\right\} \quad (64)$$

Upper limits to the pressure at the head and foot of the pile

Upper limits to the pressures at the head and foot of the pile for any driving conditions may be obtained from energy considerations. The elastic energy stored in the head cushion is

$$\tfrac{1}{2}\frac{(P_oA)^2}{k}$$

which is always less than the initial energy of the hammer. Therefore

$$\tfrac{1}{2}\frac{P_o^2A^2}{k} < \tfrac{1}{2}MV^2$$

$$\text{or} \quad P_o < \frac{EV}{a}\sqrt{\frac{Mkl}{M_pEA}} \quad (65)$$

The upper limit may be approached if the movement of the head of the pile is very small relative to the compression of the head cushion, *i.e.* if the resistance at the foot is very high and the head cushion very soft.

Similarly at the foot, the elastic energy

$$\tfrac{1}{2}\frac{(P_l A)^2}{k_1} < \tfrac{1}{2} M V^2$$

whence
$$P_l < \frac{EV}{a}\sqrt{\frac{Mk_1 l}{M_p EA}} \quad \ldots \quad (66)$$

(3) *The results of numerical evaluation*

The expressions (29) to (33) were evaluated as functions of τ for different driving conditions (*i.e.* for different values of n_1, q_1, etc.) and the pressures in the concrete were obtained from the expression

$$-E_c\frac{\partial \xi}{\partial x} = \frac{E_c V}{a}\left\{ f_1'(\tau - z) - F_1'(\tau + z)\right\} \quad \ldots \quad (28)$$

For certain extreme cases of heavy hammers and soft head cushions the approximate solution, (59) and (60), was used.

An examination of (22) to (26) shows that all the constants n_1, q_1, etc. may be regarded as functions of three non-domensional constants only, viz., $\frac{M}{M_p}$, $\frac{kl}{EA}$, and a_1 which is equal to $\frac{k_1 l}{EA}$, and it was thus necessary to evaluate for different sets of values of these constants. Since the pressures in the concrete are directly proportional to $\frac{E_c V}{a}$ this factor is better left in algebraical form for the general evaluation. When a particular problem has to be evaluated the numerical value of $\frac{E_c V}{a}$ pertinent to that problem may be inserted, the only labour involved being that of simple multiplication.

For reasons which have already been given, the numerical work has been carried out with the object of determining the maximum pressures at the head and foot of the pile for different driving conditions. Since the constant a_1 is difficult to assess in practice, the only easily obtained information concerning the ground resistance being furnished implicitly by the set per blow, it is better to use ε_1 rather than a_1 as independent parameter. In the theory the stresses are therefore calculated for different values of a_1 at the same time evaluating the corresponding ε_1.

In Figs. 5 and 6 the maximum head and foot pressures $(\times \frac{a}{E_c V})$ in the concrete are plotted against $\frac{M}{M_p}$ for different sets of values of $\frac{kl}{EA}$ and ε_1. Along any one curve $\frac{M}{M_p}$ is the abscissa and $\frac{kl}{EA}$ and ε_1 are constant but the results could obviously be equally well represented by a series of curves in which either $\frac{kl}{EA}$ or ε_1 was the abscissa. It is well to emphasise here that a constant ε_1 does not imply a constant a_1 and the curves in Figs. 5 and 6 are thus curves of constant set but not necessarily, save for $\varepsilon_1 = O$ and ∞, curves of constant ground resistance.

The curves for $\varepsilon_1 = \infty$ corresponding to easy driving are of considerable practical importance and will be considered in more detail. From (28) and (29) the compressive stress in the concrete at the head in the first interval $\frac{2l}{a}$ is given by

$$-E_c\frac{\partial \xi}{\partial x} = 2\frac{E_c V}{a}.\cot\theta . e.^{-n_1\tau}\sin q_1\tau \quad \ldots \quad (67)$$

Regarded as a function of τ this has a maximum value

$$= 2\frac{E_c V}{a} e^{-\theta/\tan\theta}\cos\theta \quad \ldots \quad (68)$$

occurring at a value τ given by

$$q_1\tau = \theta \quad \ldots \quad (69)$$

Now from (22)

$$\cos^2\theta = \frac{n_1^2}{p_1^2} = \frac{klM}{4EAM_p} \quad \ldots \quad (70)$$

whence the expression (68) does not depend upon $\frac{kl}{EA}$ and $\frac{M}{M_p}$ separately but upon their product. The maximum head pressure in the initial wave may therefore be plotted as a function of $\frac{klM}{EAM_p}$ as in Fig. 7. The curve gives the maximum pressure in the concrete for sufficiently large sets provided the maximum of the initial wave is reached before the reflected wave reaches the

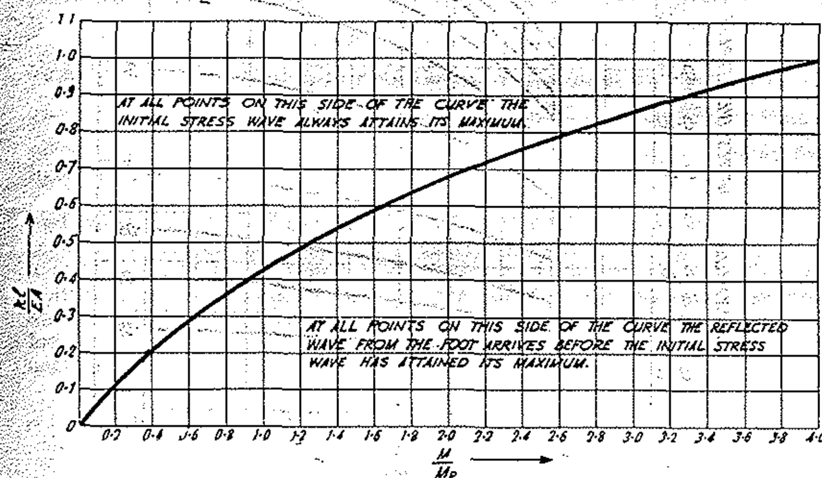

FIGURE 50.—*Curve showing relation between* $\frac{M}{M_p}$ *and* $\frac{kl}{EA}$ *when maximum stress in initial wave is reached at time* $\frac{2l}{a}$ (i.e. *just as reflected wave from the foot reaches the head of the pile*).

head, *i.e.* before $\tau = 2$. In the critical case where the maximum is attained at $\tau = 2$ it follows from (69) that

$$2q_1 = \theta \quad \ldots \quad (71)$$

and since from (22) θ and q_1 may be expressed as functions of $\frac{kl}{EA}$ and $\frac{M}{M_p}$ a limiting curve on which (71) is true may be drawn as in Fig. 50. For a pair of values $\left(\frac{kl}{EA}, \frac{M}{M_p}\right)$ above and on this curve the maximum pressure in the initial wave will be attained in the first interval while for pairs of values below the curve it will not be attained till later. In the former case the maximum head pressure for sufficiently large sets may be obtained from the one curve in Fig. 7 in place of the several curves $\varepsilon_1 = \infty$ in Fig. 5, which require interpolation for values of $\frac{kl}{EA}$ intermediate between those given. The portions of the curves $\varepsilon_1 = \infty$ in Fig. 5 on which the maximum of the initial wave is attained and which are therefore portions of the one curve in Fig. 7 are denoted by the letters *AA*. In the alternative case in which the maximum of the initial wave is not attained, the pressure at the head for large sets will

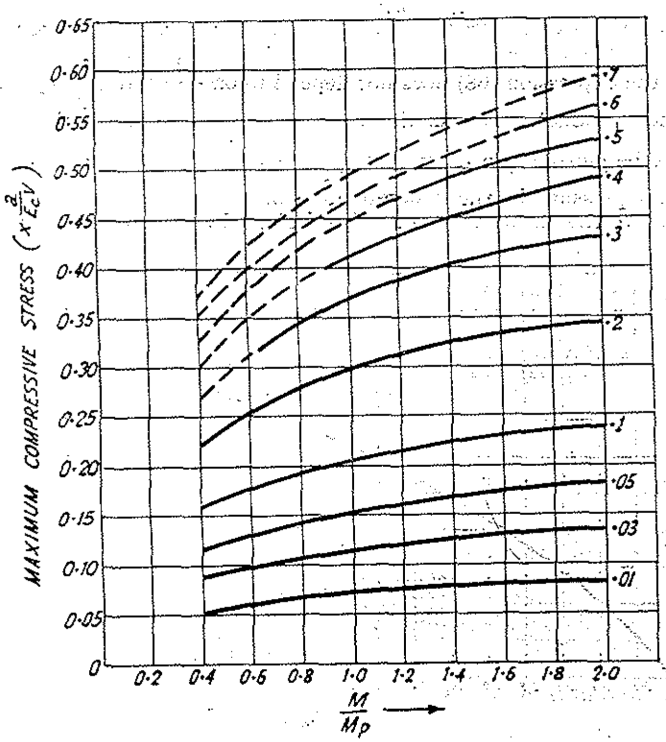

FIGURE 51.—*Maximum Compressive Stress* $\left(\times \frac{a}{E_cV}\right)$ *in Concrete at Head of Pile for Large Sets* ($\varepsilon_1 \to \infty$).

(Numbers to Curves give values of $\frac{kl}{EA}$ constant along any one curve. For larger values of $\frac{kl}{EA}$ and also on dotted portions of curves, the maximum is given by Figure 7, $\frac{M}{M_p} \leq 2$).

commence to drop immediately after $\tau = 2$ and the maximum pressure will thus be the value at $\tau = 2$ in the initial wave and will be less than the maximum of the initial wave. This maximum depends on $\frac{kl}{EA}$ and $\frac{M}{M_p}$ separately and cannot be expressed in one form as in Fig. 7; but only as a series of curves corresponding to the portions *BB* of the curves $\varepsilon_1 = \infty$ in Fig. 5. Such a series is given in Fig. 51 which includes all the portions *BB* of the curves $\varepsilon_1 = \infty$ in Fig. 5. To sum up, the complete curves $\varepsilon_1 = \infty$ for $\frac{M}{M_p}$ from *O* to ∞ are composed of two portions, first, a portion *AA* which gives the maximum of the initial wave, and, second, a portion *BB*, on which the maximum is less than that of the initial wave, since the initial maximum is not attainted in the first interval. The junction of the two parts *AA* and *BB* is given by the curve in Fig. 50 since *it corresponds to the case where* the initial wave just attains its maximum at $\tau = 2$. For the range of values $\frac{M}{M_p} = 0{\cdot}4$ to $2{\cdot}0$ considered in Fig. 5 the curves $\varepsilon_1 = \infty$ range from all *BB* for the softest to all *AA* for the hardest head cushion.

There is no lower limit to the foot pressures for large sets as with the head pressure, but as the set tends to infinity the foot pressure tends to zero. In Fig. 6 the maximum foot pressure is only given for sets sufficiently small to ensure *that it is comparable* with the maximum head pressure and is therefore of practical importance. The maximum foot pressure increases *with increasing* $\frac{M}{M_p}$ and for small sets with increasing $\frac{kl}{EA}$ also but for the larger sets it is interesting to note that the maximum foot pressure for any given set varies but little with varying $\frac{kl}{EA}$. *Physically this implies that for a* given ground resistance and hammer weight the set is to a great extent independent of the head cushion provided the set is not very small.

Finally, a point of theoretical interest is that for soft head cushions and sufficiently heavy hammers, as the set increases from zero, the maximum *foot pressure does not immediately decrease but first* increases until a certain set is reached and then decreases with larger sets. This effect is due to small resonances between the packing, the pile and the ground, and is of little practical importance *since the greatest value of the maximum* foot pressure attained for a finite set exceeds the value for zero set by a negligible amount only.

APPENDIX II

APPLICATION OF THE THEORY TO PRACTICE

(1) *The stiffness constant of the packing*

The stiffness of the head cushion has been defined in the theory by the constant k, but since this is proportional to the area of the head cushion, which is usually equal to A, the area of the pile head, it is better to consider k/A, which is a more intrinsic constant since it depends only on the thickness and nature of the packing.

It has already been stated that for materials of constant Young's modulus, i.e., having a linear stress-strain characteristic over the range of stress considered, and of the same area as that of the pile head,

$$\frac{k}{A} = \frac{E'}{l'} \quad \ldots \quad (72)$$

where E' = Young's modulus of material
l' = length or thickness of the material.

The expression is applicable to hard wood dollies in good condition. If the

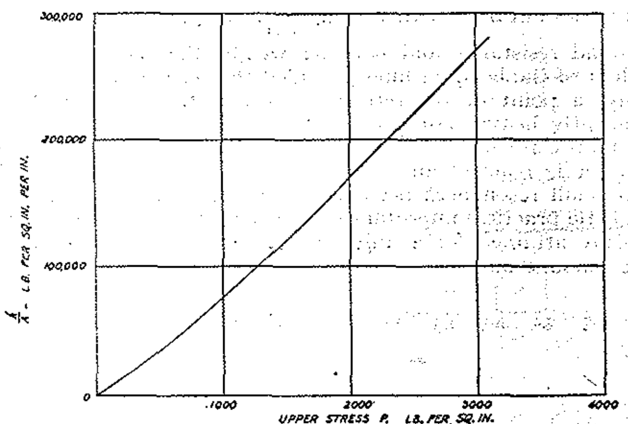

FIGURE 52.—*Variation of the packing-constant* $\frac{k}{A}$ *with the Maximum Stress in the* ½ *in. of Felt.*

dolly has a cross-sectional area A' different from that of the pile head A, the effective value of the stiffness constant is easily seen to be

$$\frac{k}{A} = \frac{E'}{l'} \cdot \frac{A'}{A} \quad \ldots \ldots \quad (73)$$

For soft materials, such as those used for helmet-packings, the relation between stress and compression is non-linear, and is of the form shown in Fig. 33 for felt. The simple relationship given above no longer holds, and the fourth assumption of the theory is no longer exact. If the fourth assumption is replaced by the exact assumption of a spring having a compression curve similar to that of Fig. 33 the equations can no longer be solved save by a numerical method using a step by step integration and involving a prohibitive amount of labour to cover *all possible driving conditions.* However one can obtain a very good approximation by assuming that a

packing of this non-linear type when compressed up to a stress P lb./sq. in. will be equivalent, so far as the stresses in the pile are concerned to a linearly elastic spring of equal area, such that the work done in compressing it to a stress P lb./sq. in. is the same as for the actual packing. Expressed graphically in Fig. 33, if PT defines the equivalent elastic spring, then

$$\text{Area } OQPNO \text{ under curve} = \text{Area } PTN \text{ under line} = \tfrac{1}{2}\frac{(PN)^2}{k/A}$$

$$= \tfrac{1}{2}\frac{P^2}{k/A}$$

$$\text{Hence } k/A = \frac{P^2}{2 \times \text{Area } OQPNO} \quad \ldots \ldots \quad (74)$$

The variation of k/A with P may thus be found and plotted as in Fig. 52.

A similar graph has been plotted in Fig. 53 for a complete head cushion, using the relationship given on p. 10 to obtain the constant k/A for the combination

$$\frac{A}{k} = \frac{A}{k_1} + \frac{A}{k_2}$$

$\frac{k_1}{A}$ and $\frac{k_2}{A}$ here denote the effective packing constants of the dolly and packing.

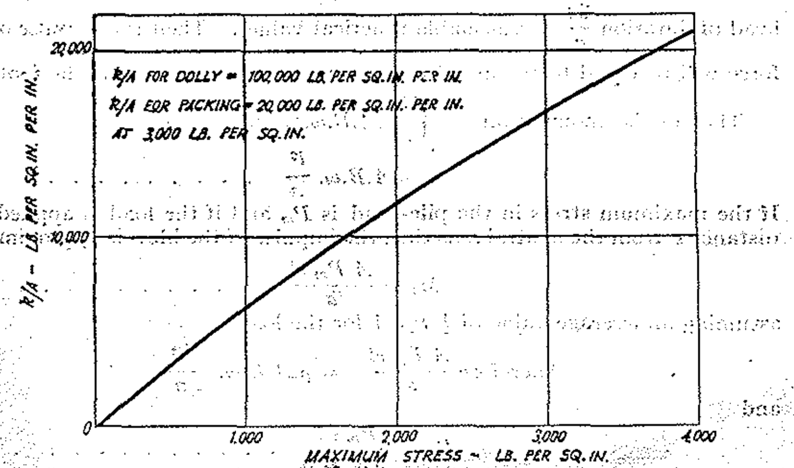

FIGURE 53.—*The Stiffness-constant Curve for a Complete Head Cushion.*

To estimate the accuracy of the approximate method of obtaining an effective value of k/A one particular case was evaluated numerically by the exact method assuming the curve in Fig. 33 and integrating step by step, and the result was compared with that obtained by the approximate method. The approximate method is more accurate the softer the packing, so a reasonably hard packing of four thicknesses of ½-in. felt under a 1,000 lb. hammer on a pile head 49 sq. in. in area was chosen; even in this rather extreme case the error by the approximate method was only 6 per cent. The approximate method, therefore, provides a means of extending the present theory to cover all types of packing with sufficient accuracy for practical purposes.

APPENDIX III

FLEXURAL VIBRATIONS

The general equation for the flexural vibration of the pile, neglecting the effect of the direct compressive stress, and assuming the pile to be homogeneous, is

$$EK^2 \frac{\partial^4 y}{\partial x^4} = -\rho \frac{\partial^2 y}{\partial t^2} \quad \ldots \ldots \quad (75)$$

The end conditions, which most nearly represent practical conditions, are probably those in which both ends of the pile are supported. The solution of equation (75) satisfying these conditions is

$$y = B \sin \frac{n\pi x}{l} \sin \omega t \quad \ldots \ldots \quad (76)$$

$$\text{where } \omega = \frac{Ka\pi^2 n^2}{l^2} \quad \ldots \ldots \quad (77)$$

Only the fundamental mode where $n = 1$ will be considered.

Suppose the pile is set into flexural vibration by an eccentric blow at the head of duration $\frac{2l}{a}$ (a reasonable practical value). Then the impulse of this force will be equal to the angular momentum of the pile about the foot.

$$\text{The angular momentum} = \int_0^l \rho.A.B.\omega.x.\sin\frac{\pi x}{l}.dx$$

$$= \rho.A.B.\omega.\frac{l^2}{\pi} \quad \ldots \ldots \quad (78)$$

If the maximum stress in the pile-head is P_m and if the load is applied at a distance ε from the neutral axis then the impulse of the blow is approximately

$$M_t = \frac{A\,P_m\,\varepsilon l}{a} \quad \ldots \ldots \quad (79)$$

assuming an average value of $\frac{1}{2} P_m A$ for the load.

$$\text{Therefore } \frac{A\,P_m\,\varepsilon l}{a} = \rho.A.B.\omega.\frac{l^2}{\pi}$$

and

$$B = \frac{P_m\,\varepsilon\,l}{\rho\,a^2\,K\,\pi} \quad \ldots \ldots \quad (80)$$

since

$$\omega = \frac{K\,a\,\pi^2}{l^2} \quad \ldots \ldots \quad (77)$$

The tension at a distance y_1 from the axis is $y_1 E \frac{\partial^2 y}{\partial x^2}$

$$\text{Now } \frac{\partial^2 y}{\partial x^2} = -B\frac{\pi^2}{l^2}\sin\frac{\pi x}{l}\sin\omega t \quad \ldots \ldots \quad (81)$$

The maximum value of the tension in the pile, which is set up at $x = \frac{l}{2}$ i.e., at the mid-point of the length, at a distance y_1 from the axis, is therefore

$$y_1 E \frac{\partial^2 y}{\partial x^2} = -\frac{y_1 E B \pi^2}{l^2}$$

$$= \frac{P_m \varepsilon \pi y_1}{lK} \quad \ldots \ldots \quad (82)$$

substituting for B from (80).

The period of vibration $\frac{2\pi}{\omega}$ is $\frac{2l^2}{Ka\pi}$

As a practical example, consider the tensile stress in a pile 50 ft. long by 14 in. square, for a head stress of 3,000 lb./sq. in., assuming the load to be applied at a distance of 5 in. from the neutral axis.

For a square pile we find:

$$\text{Maximum tension} = \frac{P_m \varepsilon \pi}{l} \cdot \sqrt{3} \quad \ldots \ldots \quad (83)$$

In the above example, therefore, the maximum tension, which occurs in the outer fibres at the mid-point of the length, is 136 lb./sq. in.

A few such examples show that the tensile stress due to a single blow is insufficient to cause failure.

APPENDIX IV

THE PEAK STRESS INDICATOR—THE EFFECT OF THE HELMET

The disturbance introduced by the presence of a heavy helmet may be understood from the following approximate analysis.

The following assumptions will be made:

(*a*) that both dolly and packing may be represented by springs of linear elastic characteristics;

(*b*) that stress waves in the systems may be neglected;

(*c*) that the movement of the pile head is negligible compared with movement of the helmet.

By considering the forces acting on the hammer and the pile we have

$$\left.\begin{aligned} M\eta'' &= -k_d(\eta-\gamma) \\ M_h\gamma'' &= -k_p\gamma + k_d(\eta-\gamma) \end{aligned}\right\} \quad \ldots \ldots \quad (84)$$

where the dashes denote differentiation with respect to t.

Hence, by eliminating γ,

$$\eta^{\text{IV}} + \left(\frac{k_d}{M} + \frac{k_d+k_p}{M_h}\right)\eta'' + \frac{k_d\,k_p}{MM_h}\eta = 0 \quad \ldots \ldots \quad (85)$$

Expressing the equation as

$$\eta^{\text{IV}} + 2n_2\eta'' + p_2^2\eta = 0 \quad \ldots \ldots \quad (86)$$

where

$$\left.\begin{aligned} 2n_2 &= \left(\frac{k_d}{M} + \frac{k_d+k_p}{M_h}\right) \\ p_2^2 &= \frac{k_d\,k_p}{MM_h} \end{aligned}\right\} \quad \ldots \ldots \quad (87)$$

the solution of practical interest is

$$\eta = A\cos mt + B\sin mt + C\cos m_1t + D\sin m_1t \quad \ldots \quad (88)$$

where

$$m = \sqrt{\left\{n_2 - \sqrt{(n_2^2-p_2^2)}\right\}}$$

$$m_1 = \sqrt{\left\{n_2 + \sqrt{(n_2^2-p_2^2)}\right\}}$$

The initial conditions at $t = O$ are

$$\left.\begin{aligned} \eta &= O \\ \eta' &= V \\ \gamma &= O \\ \gamma' &= O \end{aligned}\right\} \quad \ldots \ldots \quad (89)$$

The solution satisfying these initial conditions is

$$\eta = \frac{V\beta}{m(\beta-\alpha)}\sin mt + \frac{V\alpha}{m_1(\alpha-\beta)}\sin m_1t \quad \ldots \ldots \quad (90)$$

where

$$\left.\begin{aligned} \alpha &= \frac{k_d - Mm^2}{k_d} \\ \beta &= \frac{k_d - Mm_1^2}{k_d} \end{aligned}\right\} \quad \ldots \ldots \quad (91)$$

The acceleration of the hammer is therefore

$$\eta'' = \frac{mV\beta}{(\alpha-\beta)}\sin mt - \frac{m_1V\alpha}{(\alpha-\beta)}\sin m_1t \quad \ldots \ldots \quad (92)$$

and the force exerted by the hammer is

$$M\eta'' = MV\left\{\frac{m\beta}{(\alpha-\beta)}\sin mt - \frac{m_1\alpha}{(\alpha-\beta)}\sin m_1t\right\} \quad \ldots \ldots \quad (93)$$

The displacement and therefore the acceleration of the hammer are made up of two components, the first of a low frequency equal to $\frac{m}{2\pi}$, the second of a high frequency equal to $\frac{m_1}{2\pi}$.

It is important to calculate the ratio of the maximum value of the acceleration due to the high-frequency term to the maximum value of the acceleration due to the low frequency term. From (92) the ratio is found to be $\frac{m_1\alpha}{m\beta}$ which may be written as

$$\frac{m}{m_1}\cdot\frac{k_d/Mm^2 - 1}{k_d/Mm_1^2 - 1} \quad \ldots \ldots \quad (94)$$

Also from (92) and (84) we find

$$\gamma = V\left\{\frac{\alpha\beta}{m(\beta-\alpha)}\sin mt + \frac{\alpha\beta}{m_1(\alpha-\beta)}\sin m_1t\right\} \quad \ldots \quad (95)$$

and therefore the force on the pile-head is

$$k_p\gamma = k_pV\left\{\frac{\alpha\beta}{m(\beta-\alpha)}\sin mt + \frac{\alpha\beta}{m_1(\alpha-\beta)}\sin m_1t\right\} \quad \ldots \quad (96)$$

The ratio of the maximum forces on the pile head due to the high and low frequency terms is $\frac{m}{m_1}$.

It is obvious that the ratio (94) is always greater in absolute value than $\frac{m}{m_1}$, and by the insertion of practical values of the constants k_d, k_p, M, and M_1 it is found that for an average case $\frac{m}{m_1} = \frac{1}{10}$ and $\frac{m}{m_1}\cdot\frac{k_d/Mm^2-1}{k_d/Mm_1^2-1} = \frac{1}{2}$.

An examination of pile driving records shows that the high frequency vibration is highly damped, dying away very rapidly. The foregoing relations can therefore only apply approximately to the first one or two oscillations.

The analysis shows that on impact, the hammer undergoes a deceleration of short duration equal to approximately one-half the maximum value of the deceleration. The expressions for the stress on the pile head and the deceleration of the hammer are composed of two sine functions, one of high and one of low frequency. The effect of the high frequency term on the deceleration of the hammer is approximately five times as great as its effect on the stress on the pile head.

To provide a check on the analysis an experimental study of the helmet effect was made on the model scale using the impact machine described on p. 68. The pile head was represented by a concrete block 16 in. long by 4 in. square, containing a centrally placed piezo-electric gauge. The hammer weighed 200 pounds. The ratios $\frac{M_h}{M}$, $\frac{k_d}{M}$ and $\frac{k_p}{M}$ were given values corresponding to typical practice. The values of the second and third ratios were only approximately known; the model dolly consisted of two inches of softwood with the grain horizontal and the packing was represented by three layers of one-inch rubber.

Oscillograph records of the deceleration of the hammer and of the stress on the pile head were obtained for different heights of drop; the stress in the pile was recorded in the usual way from the piezo-electric gauge in the concrete block; and the deceleration of the hammer was similarly recorded by the use of a piezo-electric deceleration recorder mounted on the hammer. The instrument consists essentially of a mass m_1 of 50 grams mounted on a quartz crystal Q (see Fig. 54). When the hammer undergoes deceleration the crystal is compressed by a force $m_1 f$, setting up a voltage proportional to this force across the terminals of the amplifier. The variation of voltage with time is recorded with the cathode ray oscillograph; the recorded deflection at any instant is proportional to the deceleration of the hammer. The deceleration recorder is calibrated by loading the crystal statically.

Examples of oscillographic records of the hammer deceleration and the stress in the specimen are reproduced in Fig. 55. The duration and character of the lower record agree well with records from driven piles. The upper record is of the type predicted by the theory outlined in this Appendix, but

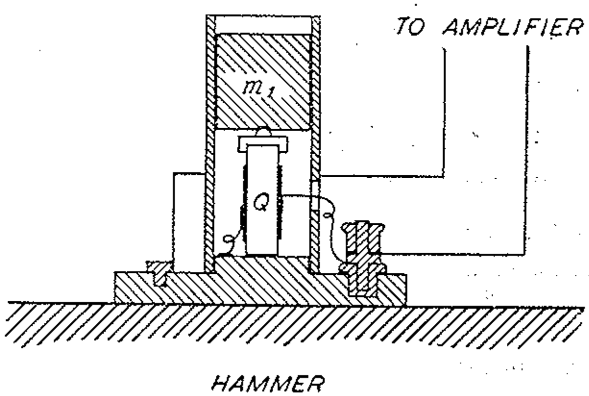

FIGURE 54.—*Piezo-electric Deceleration Recorder.*

shows certain differences that can be explained by the approximate nature of the assumptions made.

It was implied that on striking the helmet the hammer became attached to the dolly, and that tension could be set up in the dolly. The record shows that after the initial impact the helmet and dolly outstrip the hammer, which is therefore not decelerated until impact recommences.

Due to the non-linear load-compression characteristics of the packing and dolly the frequency of the helmet oscillation increases slightly as the pressure increases.

The deceleration due to the high frequency term is of the anticipated order; the initial peak itself is greater than one-half of the maximum deceleration.

Even allowing for the high rate of damping of the high frequency term, its influence may introduce a considerable error in the value of the head stress as deduced from measurements of the deceleration of the hammer. Suppose for example that the high frequency term contributes 3 per cent. to the value of the head stress. The analysis shows that it will contribute 15 per cent. to the deceleration. It is therefore necessary that the peak stress indicator should be insensitive to this component of the deceleration.

For this purpose two alterations have been made: the indicator has been mounted on damped springs; and the arrangement of the weight and the calibrated spring has been changed.

Consider the effect of mounting the peak stress indicator on a spring provided with a damping device. With the notation on p. 88 we find

$$k_i(z - \eta) + \mu(z' - \eta') = -M_i z''.$$

$$z'' + \frac{\mu}{M_i} z' + \frac{k_i}{M_i} z = \frac{\mu}{M_i}\eta' + \frac{k_i}{M_i}\eta \quad \dots \dots \quad (98)$$

From (90) the displacement of the hammer is

$$\eta = C \sin mt + D \sin m_1 t \quad \dots \dots \quad (99)$$

$$\left.\begin{aligned} \text{where } C &= \frac{V\beta}{m\,(\beta - \alpha)} \\ D &= \frac{V\alpha}{m_1\,(\alpha - \beta)} \end{aligned}\right\} \quad \dots \dots \quad (100)$$

Therefore

$$\left.\begin{aligned} z'' + \frac{\mu}{M_i} z' + \frac{ki}{M_i} z &= \frac{\mu}{M_i}(Cm \cos mt + Dm_1 \cos m_1 t) \\ &\quad + \frac{k_i}{M_i}(C \sin mt + D \sin m_1 t) \\ &= E \cos (mt + \delta_1) + F \cos (m_1 t + \delta_2) \end{aligned}\right\} (101)$$

$$\left.\begin{aligned} \text{where } E &= \frac{C\sqrt{k_i^2 + \mu^2 m^2}}{M_i}, \tan \delta_1 = -\frac{k_i}{\mu m} \\ F &= \frac{D\sqrt{k_i^2 + \mu^2 m_1^2}}{M_i}, \tan \delta_2 = -\frac{k_i}{\mu m_1} \end{aligned}\right\} \quad \dots \dots \quad (102)$$

The solution of equation (101) is

$$z = G_e{}^{-\frac{1}{2}\frac{\mu}{M_i}t} \cos (qt + \delta_o) + R \cos (mt + \delta_1 + \varepsilon_1) + R_1 \cos (m_1 t + \delta_2 + \varepsilon_2) \quad . \quad (103)$$

where $q = \sqrt{\frac{k_i}{M_i} - \frac{1}{4}\frac{\mu^2}{M_i^2}}$

$$\left.\begin{aligned} R &= \frac{E}{\sqrt{\left\{\left(\frac{k_i}{M_i} - m^2\right)^2 + \frac{\mu^2 m^2}{M_i^2}\right\}}}, \tan \varepsilon_1 = -\frac{\frac{\mu m}{M_i}}{\frac{k_i}{M_i} - m^2} \\ R_1 &= \frac{F}{\sqrt{\left\{\left(\frac{k_i}{M_i} - m_1^2\right)^2 + \frac{\mu^2 m_1^2}{M_i^2}\right\}}}, \tan \varepsilon_2 = -\frac{\frac{\mu m_1}{M_i}}{\frac{k_i}{M_i} - m_1^2} \end{aligned}\right\} \quad . \quad (104)$$

The initial conditions are:

$$\left.\begin{aligned} z &= O \\ z' &= V \end{aligned}\right\} \text{ when } t = o \quad \dots \dots \quad (105)$$

from which we find

$$\left.\begin{aligned} \tan \delta_o &= \frac{R\left\{\cos (\delta_1 + \varepsilon_1)\right\}\left(q^2 - m^2 - \frac{1}{4}\frac{\mu^2}{M_i^2}\right) + R_1\left\{\cos (\delta_2 + \varepsilon_2)\right\}\left(q^2 - m_1^2 - \frac{1}{4}\frac{\mu^2}{M_i^2}\right)}{\frac{\mu}{M_i}\cdot q \cdot \left\{R \cos (\delta_1 + \varepsilon_1) + R_1 \cos (\delta_2 + \varepsilon_2)\right\}} \\ \text{and } G &= -\frac{R \cos (\delta_1 + \varepsilon_1) + R_1 \cos (\delta_2 + \varepsilon_2)}{\cos \delta_0} \end{aligned}\right\} (106)$$

The acceleration of the peak stress indicator is

$$z'' = G\,e^{-\frac{1}{2}\frac{\mu}{M_i}t}\left\{\frac{\mu}{M_i}q\sin(qt+\delta_0)+\left(\tfrac{1}{4}\frac{\mu^2}{M_i^2}-q^2\right)\cos(qt+\delta_0)\right\}$$
$$- R\,m^2\cos(mt+\delta_1+\varepsilon_1) - R_1\,m_1^2\cos(m_1t+\delta_2+\varepsilon_2) \quad \ldots \quad (107)$$

By increasing the damping sufficiently, the first term, which represents the free oscillation of the peak stress indicator on the damped spring, may be made to die out so rapidly that its effect can be neglected. The other terms represent the acceleration of the peak stress indicator due to the components of low and high frequency respectively in the acceleration of the hammer. In order that the spring mount shall achieve its object of rendering the acceleration indicator insensitive to the high frequency component of the acceleration, while responding accurately to the low frequency component, the ratio $\frac{R}{C}$ must differ from unity by a negligible amount, and the ratio $\frac{R_1}{D}$ must be small compared with unity.

$$\text{Now } \frac{R}{C} = \sqrt{\frac{\frac{k_i^2}{M_i^2}+\frac{\mu^2m^2}{M_i^2}}{\left(\frac{k_i}{M_i}-m^2\right)^2+\frac{\mu^2m^2}{M_i^2}}} \quad \ldots \quad (108)$$

$$\text{and similarly } \frac{R_1}{D} = \sqrt{\frac{\frac{k_i^2}{M_i^2}+\frac{\mu^2m_1^2}{M_i^2}}{\left(\frac{k_i}{M_i}-m_1^2\right)^2+\frac{\mu^2m_1^2}{M_i^2}}} \quad \ldots \quad (109)$$

The ratio of m_1 to m is generally of the order of 10 to 1, and it is therefore possible to choose a value of $\sqrt{\frac{k_i}{M_i}}$, intermediate between m and m_1 such that the requirements are fulfilled. In practical pile driving the duration of the blow is generally of the order of 0·01 second, that is, it is approximately that due to one-half of a 50 cycle sine wave. With a helmet of normal weight the high frequency vibration will be of approximately 500 cycles. Now let the frequency of the peak stress indicator on the spring $\sqrt{\frac{k_i}{M_i}}$, be 2π by 150.

For critical damping

$$\frac{k_i}{M_i} = \frac{1}{4}\frac{\mu^2}{M_i^2}$$

$$\text{or } \frac{\mu}{M_i} = 1886$$

Let the value of μ/M_i be 1500. Then for $m = 2\pi$ by $50 = 314$, and $m_1 = 3140$, and $\sqrt{\frac{k_i}{M_i}} = 943$ we find $\frac{R}{C} = 1{\cdot}09$ and $\frac{R_1}{D} = 0{\cdot}47$. The peak stress indicator will therefore read slightly too high on the main peak, but the acceleration due to the high frequency term has been reduced to less than one-half. A correction may be applied for the departure of the ratio $\frac{R}{C}$ from unity. For the example considered earlier where the maximum deceleration of the hammer due to the high frequency term was 15 per cent. of the main peak maximum, the peak stress indicator would show a stress not greater than 7 per cent. higher than the main peak maximum, that is within 4 per cent. of the required value.

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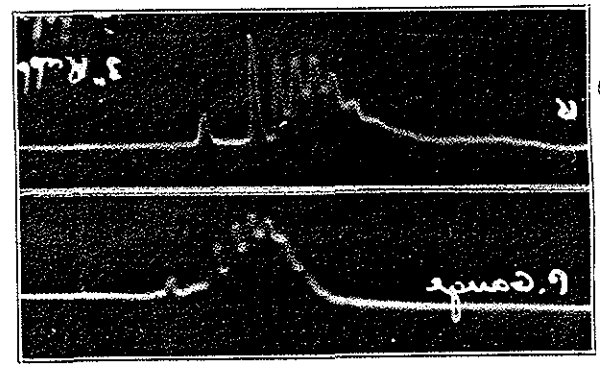

(a)

(b)

FIGURE 55.

(a) The deceleration of the hammer, recorded with the piezo-electric deceleration recorder.

(b) The stress in the specimen, for similar conditions. (The duration of the blow is 0·01 second.)

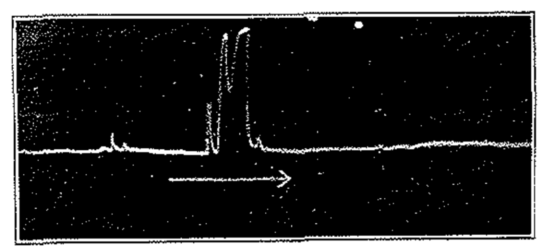

FIGURE 56.—*Record of the breaking of contact.*

The duration of the main break is about 1/2,000 second.

The second artifice employed to reduce the effect of the helmet oscillation depends on the arrangement of the spring supporting the weight against the contact. The simple arrangement of Fig. 12 suffers from the defect that in consequence of the time taken for an impulse to travel along the spring it is progressively more inaccurate as the duration of the impact decreases. With a spring placed below the weight, the spring pressure is reduced during impact, the amount of the reduction becoming greater as the duration of the impact decreases. The indicator therefore reads high and is more sensitive to the high frequency helmet oscillation than to the main peak. It has been found that, in certain circumstances, the initial impact can operate the indicator. An oscillographic investigation of the mechanism of the break with this indicator was made using a simple electric circuit, with the result shown in Fig. 56. The main break is preceded by a number of breaks of exceedingly short duration, due to the spring effect mentioned.

The method of construction now employed makes use of this effect to render the indicator more insensitive to the helmet vibration. A drawing of the indicator is shown in Fig. 13; the spring now acts at one end of a lever arm, the other end of which carries the weight. The spring effect is reversed, and the indicator is less sensitive to high than to low frequencies.

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