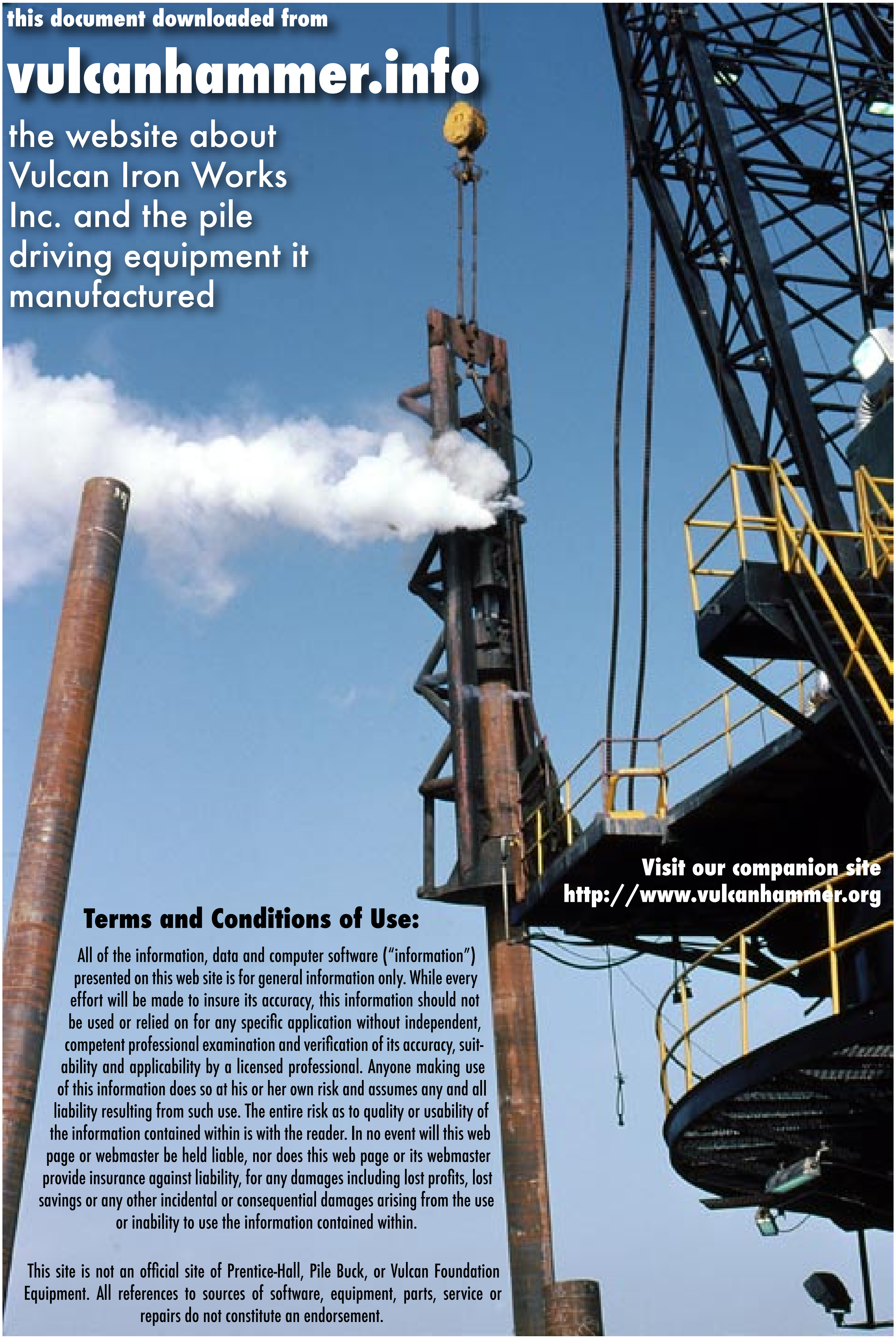

AN OPTIMISATION METHOD FOR PILE DRIVING ANALYSIS

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SYNOPSIS

A numerical method has been developed for analysing field data from piles as they are driven. The basis is a "least squares" technique which is used in conjunction with a non-linear optimisation procedure. It enables the measured stress time curve to be fitted by a wave equation analysis by calculating the values of hammer impact velocity, cushion stiffness, coefficient of restitution, soil quake, damping and skin friction such that the difference between the observed and predicted stress-time curves is a minimum. Good estimates of these parameters are of considerable practical value because they enable a comparison to be made between the data assumed at the design stage and that actually observed during installation.

The acceleration-time curve produced by the optimised parameters is then compared with the measured acceleration curve. This provides a completely independent check on the validity of the calculated parameters. Since acceleration is a very sensitive variable in the computation, this is the most stringent test of the calculated values which can be devised.

Finally, two worked examples are presented, one using a theoretical stresswave produced by the wave equation for which there is a known exact solution and the second analyses the installation of a 1.2 metre O.D. pipe pile driven with a 40 tonne hydraulically operated hammer.

INTRODUCTION

The wave equation is now universally accepted as the best method for carrying out driveability calculations on offshore piles. The size and cost of the piles now being driven is such that extensive studies of the driving performance of hammers, piles and chasers is carried out at an early stage in the design.

Developments in instrumentation over the past decade have now reached a stage where measurement of the stresswave caused by impact of the hammer can be made easily and cheaply, and several commercial organisations now offer this service. The objective of all these systems is to measure the stress-time and acceleration-time curves for the pile wall immediately below the hammer. (Some operators independently measure the hammer impact velocity as well.)

This type of measurement can be used for several purposes, it:

a) enables the hammer performance to be observed and changes in efficiency detected;

b) enables peak driving stresses and accelerations to be monitored and then used by the engineer to guide installation. For underwater driving this application is likely to be very important;

c) enables signals to be recorded for later analysis. When this data is compared with the design assumptions it makes a valuable contribution towards improving the design process;

d) enables bearing capacity estimates to be carried out based on redrive tests. These should be performed when the soil has recovered from disturbance because it is important that the data should not be affected either by pore pressures or any thixotropic regain in strength of the soil. Under these conditions the resistance to driving of the first few blows is numerically equal to the bearing capacity of the pile;

e) may form the future basis of acceptance procedures. This is a comparatively new concept, and no generally accepted principles yet exist.

Whatever the purpose of taking these measurements, there remains the task of analysing the data. There is general agreement that this is best done with the wave equation, but two distinct approaches have evolved.

In the first, the field signals are used to drive the pile using a modified wave equation programme. If this is done then the hammer, cushion and pile cap can be removed from the computation, and it only remains to find, by trial and error, the soil resistance profile which gives best agreement with the observed data.

The second method differs from the first in that the hammer, cushion and pile cap are retained. By trial and error the hammer impact velocity, cushion stiffness, coefficient of restitution and soil characteristics are found which give the best fit with the observed signal.

Trial and error methods are not to be recommended for the novice. They can be very laborious due to the large number of variables and have hidden problems due to nonlinearity and lack of uniqueness in the solution. For dealing with this class of problem, nonlinear optimisation techniques have been developed, and in civil engineering (where perfect solutions do not exist) these often take the form of a least squares criterion. In the present case for example, the criterion is that the best values of the optimised variables are those which produce the smallest possible area between the calculated and observed signals (Fig. 1).

Developments in advanced mathematical and computing techniques over the past few years have led to a situation where most large computing systems offer standard optimisation routines in their software library. The least-sum-of-squares routine available on the ICL 2980 computer used at Queen Mary College is in the NPL Algorithms Library E4/18/F, written by Gill et al (1976).

THEORY

In a basic instrumentation package, strain gauges and accelerometers are mounted on the pile assembly just below the hammer. The package may sometimes allow the hammer impact velocity to be determined but this is not essential in the present treatment.

The least information necessary for analytically describing the behaviour of the pile is either the stress wave or the acceleration wave. Since one can be used to calculate the other, both are not required, and in the present work the stress wave is used because of the greater reliability of the measured signal.

In principle all of the observed data can be used in the optimisation routine i.e. the stress, acceleration, hammer velocity and set per blow. To do so, however, makes the computer routine inefficient, and experience has shown that it is best to use only the stresswave and set per blow. A further advantage in not fitting all the data is that a completely independent check on the quality of the optimised solution can be performed using the extraneous data. This is extremely important since data can be faulty due to instrument malfunction, or there could be incorrect data in the computer model. The probability of detecting such faults is increased by increasing the number of independent checks.

As mentioned previously, the objective of the method is to fit the wave equation to the observed stress signal. However before the wave equation can be used the value of six governing parameters must be known and it is these which are sought by the optimisation routine. It is logical to split the six parameters into two groups:

a) <u>Hammer Parameters</u>

1. Hammer impact velocity, v_H. This depends on the efficiency.
2. Hammer cushion stiffness, k. Depends on wear.
3. Coefficient of restitution, e. Again depends on wear.

b) <u>Soil Parameters</u>

4. Damping, J.
5. Skin friction, τ.
6. Quake, Q.

Where the soil consists of a number of layers, the relative magnitudes of damping, skin friction and quake must be known for each layer. During optimisation these relative magnitudes are maintained. The routine then factors the damping in each layer by the same amount. A similar treatment is used for the skin friction and quake.

THE OPTIMISATION ROUTINE

The wave equation is a finite difference technique and therefore the results from it are a set of stresses at discrete points in time. Referring to Fig. 1, this means that instead of fitting a continuous curve to the data, matching has to be confined to a finite number of points n. Since the observed signal is stored as a finite number of points in the computer this is entirely compatible with the use of a digital computer.

Let t_1, t_2, t_3 ... t_n be the points at which the wave equation is to be required to fit, then the sum of the squares of the residuals at these points is:

$$R = \sum_{123}^{n} [\sigma_1 - w(v_H, k, e, \tau, Q, J, t_1)]^2 \quad .....(1)$$

where $w(v_H, k, e, \tau, Q, J, t_1)$ is a functional representation of the wave equation at time t_1, and σ_1 is the observed signal.

The principle of Least Squares requires eqn (1) to be a minimum with respect to v_H etc. Thus:

$$\frac{\partial R}{\partial v_H} = 0 = \sum_{123}^{n} [\sigma_1 - w(v_H, k, e, \tau, Q, J, t_1)]\left(\frac{\partial w}{\partial v_H}\right)_{t=t_1} \quad ..(2)$$

with similar expressions for k, e, τ, Q, and J. There are thus six equations from which v_H, k, e, τ, Q and J can be found.

The equations are non-linear, and must be solved by a Newton based method. Moreover, w is generated by the wave equation and hence explicit algebraic expressions for $\frac{\partial w}{\partial v_H}$ are not available. These must therefore be generated numerically by running the wave equation with values of v_H, etc slightly less and slightly greater than the current values. To reduce the amount of computation time it is vital to keep the number of runs of the wave equation to a minimum. By using the optimisation package twice, to obtain the two groups of parameters, it is possible for the first and second order derivatives used in the enhanced Newton routine to be calculated by only four runs of the wave equation, compared with ten runs to do six parameters simultaneously.

The difficulties in solving non-linear equations by the Newton Method are well known, viz. converging onto the correct roots, slow oscillatory convergence and divergence. Since the wave equation is a frequently accessed sub-routine, and for large piles involves considerable computation, it is essential that the basic algorithm is used most efficiently. As mentioned above, these considerations necessitated the dividing of the calculations into two parts. The first part, referred to as hammer dominated in Figure 1, has its shape dictated by the hammer characteristics, and is therefore referred to as the "hammer signature", British Petroleum (1978) and Litkouhi (1979). This period is insensitive to the soil conditions because it takes a finite time for the stress wave to travel down the pile and be reflected back, and the motion is therefore essentially that of a hammer striking an infinitely long pile. From this part of the curve, v_H, k and e are found.

Following this stage the signal moves into a region where it becomes strongly influenced by the soil resistance. This is referred to as the soil dominated zone in Fig. 1. The signal becomes attenuated as energy is dissipated overcoming the resistance of the soil, and from this part of the curve, τ , Q and J are found.

In order to speed the convergence of the routine, constraints are provided for the six variables which prevent physically unacceptable solutions from being considered. These limits have been selected to give ranges to each variable which experience shows they cannot exceed:

0	$<$	v_H	$<$	$\sqrt{2gh}$	(drop hammers)
500	$<$	k	$<$	15,000 kN/mm	(small hammers)
10,000	$<$	k	$<$	100,000 kN/mm	(offshore hammers)
0.3	$<$	e	$<$	1.0	
0.0	$<$	J	$<$	5.0 s/m	
0.0	$<$	τ	$<$	C_u	(clays)
0.0	$<$	Q	$<$	5.0 mm	

EFFICIENCY OF THE COMPUTATIONS

The computational effort depends on the number of points, n, chosen to fit the wave equation. This will be different for the two optimisation cycles because of the different nature of the signals being matched. Fig. 2 shows how the quality of fit to a theoretical stresswave (i.e. one generated by the wave equation for which the exact solution is known) depends on the number of data points selected. For convenience, the quality of fit has been described in terms of the average error in the parameters after optimisation.

The number of data points used in the hammer period is relatively small because the duration of the hammer signature is never greater than eight milliseconds, even for an offshore hammer. Furthermore, the shape of the initial peak is a simple, smooth curve, so a small number of points will accurately describe it. From Figure 2 the number of points used to describe the hammer signature has been fixed at 15. This is sufficient for both light and heavy hammers.

The soil domain extends from t_s to $4L/c$ where L is the pile length and c is the velocity of sound in the pile material, Fig. 1. Some sixty or more points are required for this zone. There should be not less than 3 points for each 2 ms of signal.

As a result of dividing the optimisation into two periods it is of major importance that the division be made with some precision. The central problem is thus to ensure that over the hammer dominated zone there has been no significant effects caused by stress waves reflected from the soil. For a long pile with considerable stick-up this presents little difficulty, however, for piles with a stick-up of less than five metres, considerable care is required.

The duration of the hammer signature, t_s, is closely related to the ram weight, and further guidance can be obtained by considering the initial motion of the hammer and anvil, British Petroleum (1978) and Litkouhi (1979). It is logical therefore, to connect t_s with the time to the initial peak of the observed stresswave, t_m. Where stick-up is greater than five metres, the following empirical rule has been found to work:

$$t_s = 1.5 \, . \, t_m$$

For short stick-ups trial and error is used taking

$$t_s = t_m \, , \quad t_s = 1.25 \, t_m \quad \text{and} \quad t_s = 1.5 \, tm$$

in order to find which produces the "best fit".

To summarise therefore, it has been found satisfactory if optimisation is carried out over the first $4L/c$ seconds of the stress wave. This period should then be divided into a hammer dominated and a soil dominated zone depending on the time to peak stress on the measured signal.

Example 1

A theoretical stresswave was produced by the wave equation using the data given in Fig. 3 col. 2. The values of the six optimisation parameters are also given in column 2 of Table 1. The stresswave is shown in Fig. 4 and labelled "measured signal".

The points at which the wave equation is required to fit this signal are given by + for the hammer domain and * for the soil domain.

	Measured Signal	Initial Guess	Optimised Hammer Values	Optimised Soil Values
a) Hammer parameters				
Ram velocity (m/s)	4.50	2.70	4.494	4.494
Cushion stiffness (kN/mm)	4500	3268	4508	4508
Coeff. of restit.	0.90	0.37	0.883	0.883
b) Soil parameters				
Top layer				
Damping (s/m)	0.35	3.50	3.50	0.351
Skin friction (kN/m^2)	150.0	90.0	90.0	149.3
Quake (mm)	1.25	0.20	0.20	1.22
Bottom layer				
Damping (s/m)	0.35	3.50	3.50	0.351
Skin friction (kN/m^2)	100.0	60.0	60.0	99.5
Quake (mm)	2.50	0.40	0.40	2.44

Table 1. Progression of the solution for Example 1

An "initial guess" of the values of the parameters was then supplied to the programme to start the optimisation procedure off. These are given in the third column of Table 1. The corresponding stresswave is shown in Fig. 4 and labelled "pre-optimisation" wave.

The hammer dominated region was calculated to extend over the first 1.55 msec and after 20 cycles of optimisation the hammer velocity, cushion stiffness and coefficient of restitution had the values given in the fourth column of Table 1. The soil parameters have not, of course, changed since these are not optimised until the soil cycle.

The soil domain was calculated to extend from 1.6 to 16.0 ms, and after a further 24 cycles of optimisation the values given in the fifth column of Table 1 were obtained. These final values agree well with the known solution i.e. column 2 of Table 1. As shown in Fig. 4 the two stresswaves differ by less than the thickness of the line.

The acceleration-time curve is used as an independent check on the optimised paramters. The comparison for the final solution is also given in Fig. 4. The error is less than the thickness of the line. The RMS error between these curves was 5.47g for the hammer cycle and 8.71g for the soil cycle.

		PARAMETER						
		Imp. Vel. m/s	Cush. Stiff. kN/mm	Coeff. Rest.	Smith Damp. s/m	Skin Frict. kN/m²	Quake mm	% error
Exact solution		4.50	4500	0.90	0.35	100.0	2.50	0
Case								
1	start.guess	2.70	3268	0.37	3.50	60.0	0.40	
	post-opt.	4.49	4508	0.88	0.35	99.5	2.44	1.35
2	start.guess	7.15	9050	0.72	0.60	220.0	1.00	
	post-opt.	4.49	4509	0.87	0.35	99.3	2.44	2.60
3	start.guess	3.70	6038	0.57	0.05	70.0	4.50	
	post-opt.	4.50	4506	0.91	0.36	99.8	2.47	1.26
4	start.guess	2.20	2159	0.50	1.90	140.0	0.80	
	post-opt.	4.49	4508	0.88	0.35	99.6	2.46	1.63
5	start.guess	5.90	8300	0.95	3.70	55.0	3.70	
	post-opt.	4.50	4506	0.89	0.35	99.7	2.47	1.12
6	start.guess	8.40	2700	0.65	0.20	165.0	3.00	
	post-opt.	4.50	4507	0.90	0.35	100.1	2.48	0.64

Table 2. Solutions of Example 1 for different initial guesses

As a further demonstration of the programmes capability this problem was repeated using six different starting guesses, see Table 2. In all cases the programme successfully found the true solution without difficulty. The average error for each case is shown in column 8.

The soil parameters for layer 1 and at the toe have been omitted for brevity. They are qualitatively as good as those of the second layer, which is shown.

Example 2.

Fig. 5 shows a stress wave obtained during a test on a 40 tonne hydraulic hammer. This was being used to drive a pile 1.22 mm O.D. into a thick bed of chalk. A triangular distribution of skin friction with depth is assumed. Full details of the hammer, pile and soil are given in column 3 of Fig. 3.

The hammer signature was taken over the first 2.7 ms and the + in Fig. 5 show the points at which the wave equation was fitted. The points used for the soil dominated zone are given by * and extend to 32 ms (i.e. 4L/c).

The solution is set out in Table 3 and the stress wave which corresponds to the optimised parameters is shown in Fig. 5. The agreement is reasonable.

The differences between the measured signal and the calculated signal are due to problems associated with bending of the pile during driving and lack of concentricity in the blow. Due to this the measured signal departs from a true wave equation. What the programme has therefore done is to find the wave equation which best fits this set of observations. Since the mechanical difficulties were minor the observed behaviour has not departed significantly from that assumed in the wave equation.

It has been possible to check the cushion parameters using the results of Van Luipen (1979). The cushion was a Bougossi disc 1.2 m diameter and 300 mm thick. The stiffness calculated by the programme is 9202 kN/mm which corresponds to a Young's Modulus of 2.36 kN/mm^2. The coefficient of restitution was 0.767 after 1500 blows. These values agree well with Van Luipen's.

	Initial Guess	Optimised Hammer Values	Optimised Soil Values
a) Hammer parameters			
Ram velocity (m/s)	3.2	4.24	4.24
Cushion stiffness (kN/mm)	11 000	9202	9202
Coeff. of restit.	0.95	0.767	0.767
b) Soil parameters			
Top layer			
Damping (s/m)	0.656	0.656	0.260
Skin friction (kN/m^2)	200	200	99.2
Quake (mm)	2.54	2.54	3.24

Table 3. Progression of the solution for Example 2.

CONCLUSIONS

The paper has shown how a standard nonlinear optimisation routine, which is now commonly available in computer systems, can be used to fit the wave equation to measured stress-time curves. The method therefore provides a useful tool for analysing field data either in real time or from tape recordings. Since the manual analysis of data is very time consuming, there are considerable savings in labour.

Where equipment is poorly maintained, or piles are battered, some care is necessary since measured signals may be influenced by lack of concentricity of blow or bending of the pile. The behaviour of the real system will not then agree with the assumptions of the wave equation. Under these conditions the optimisation procedure finds a best fit wave equation solution for the data.

Further work is required on layered deposites when the relative properties of the soil layers are unknown.

ACKNOWLEDGEMENTS

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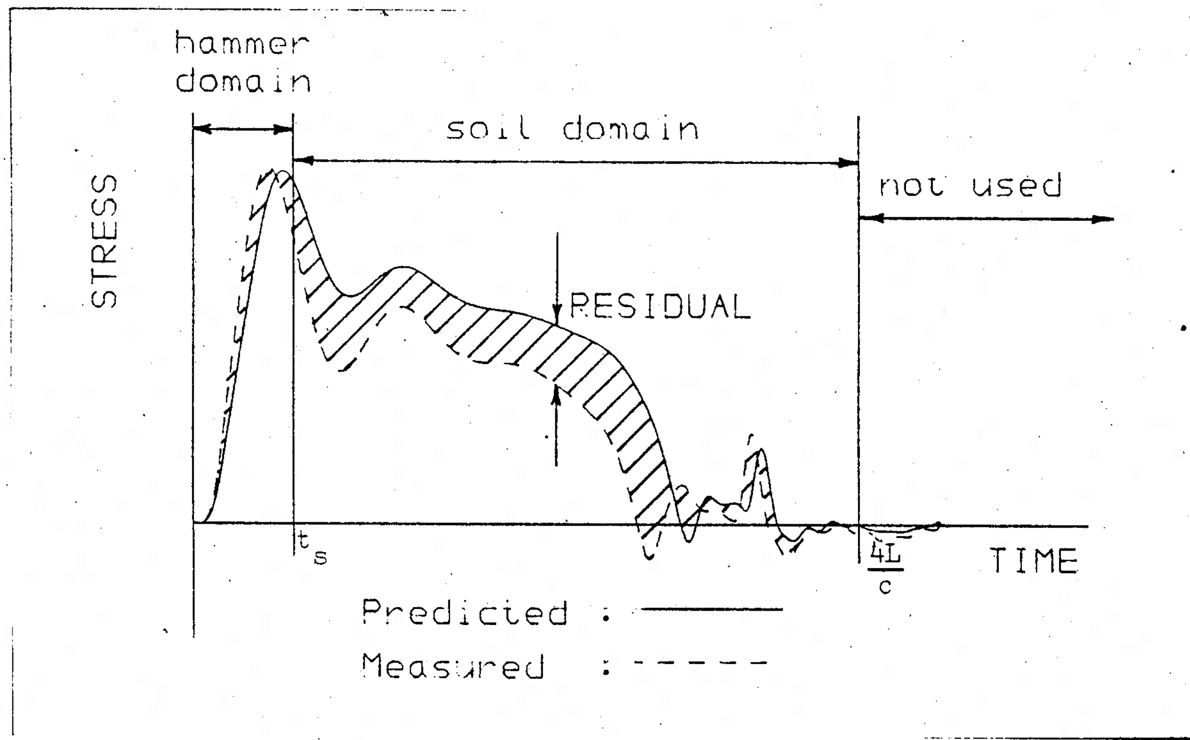


FIGURE 1. Sub-division of a stress-time curve

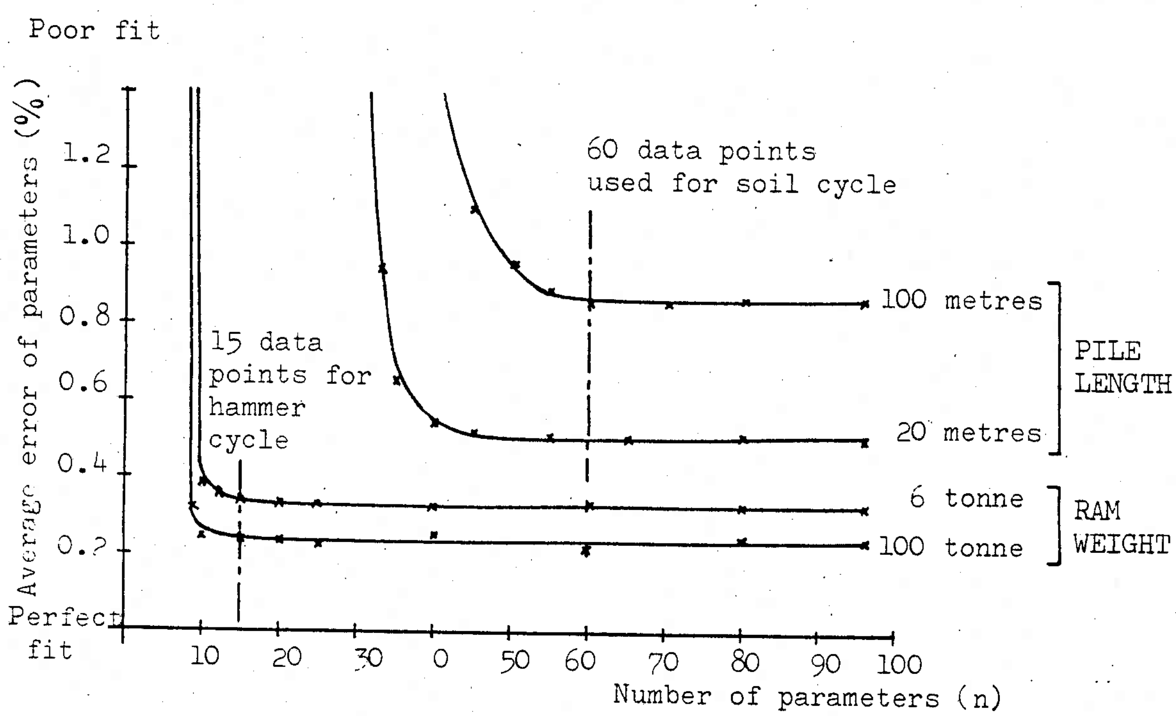
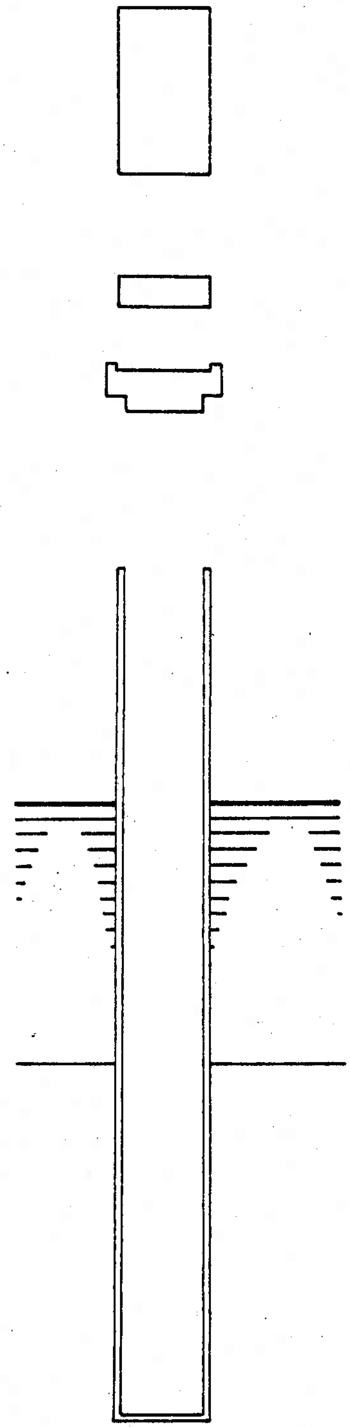


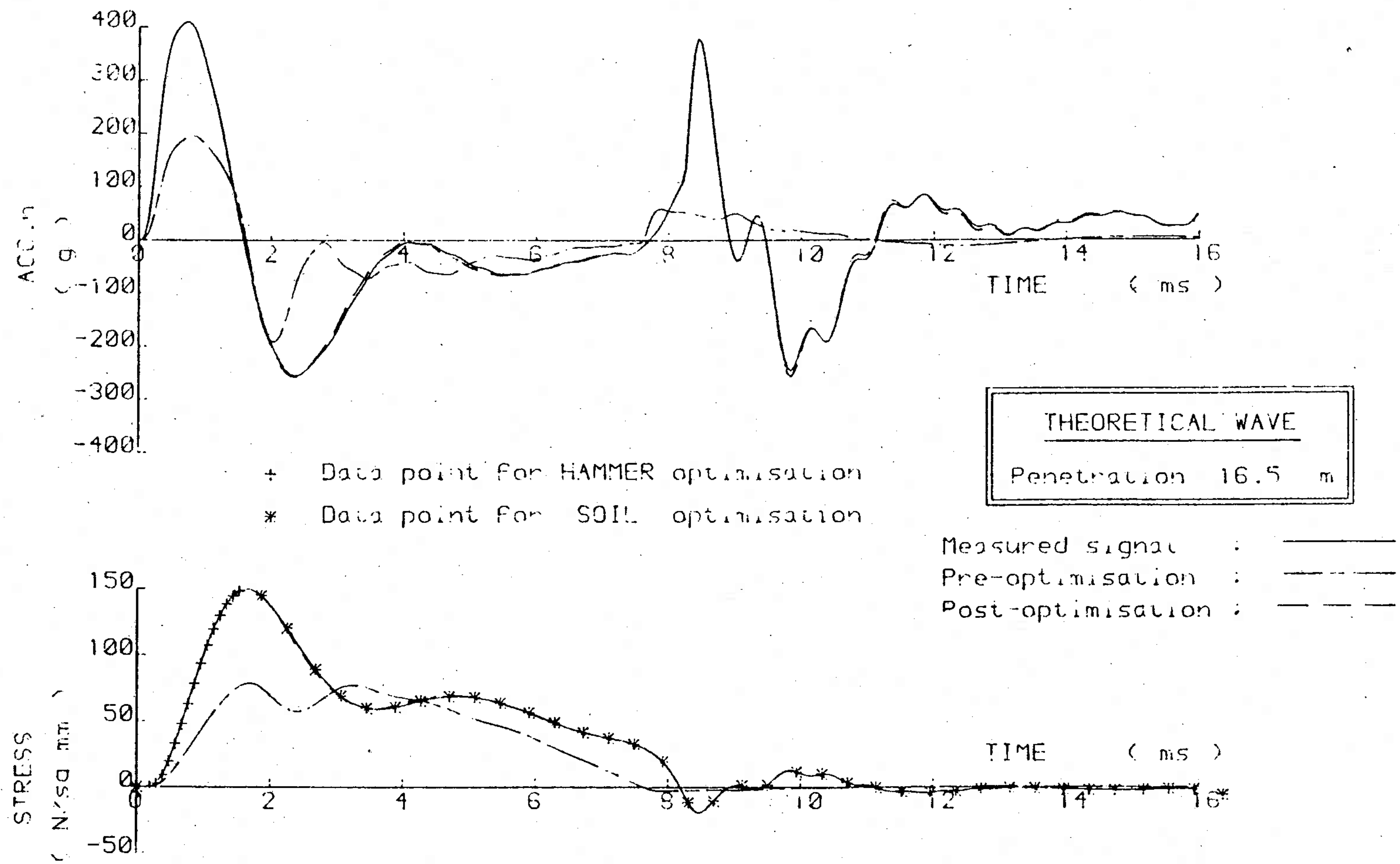
FIGURE 2. Selection of 'n' (the number of data points

	EXAMPLE 1	EXAMPLE 2
HAMMER		
Ram weight (kg)	6 000	40 830
Impact velocity (m/sec)	4.5	*
HAMMER CUSHION		
Stiffness (kN/mm)	4 500	*
Coeff. of restit.	0.9	*
ANVIL		
Anvil weight (kg)	1 000	4 500
PILE CUSHION	none	none
PILE		
Length (m)	20.0	41.5
Embedded depth (m)	18.5	33.4
Outside diameter (m)	0.75	1.21
Wall thickness (mm)	50.0	25.5
E of material (kN/m^2)	207.0×10^6	210.0×10^6
SOIL		
Top layer		
Depth (m)	0-10	0-50
Skin friction (kN/m^2)	150	*
Smith damping (sec/m)	0.35	*
Quake (mm)	1.25	*
Second layer		
Depth (m)	10-30	
Skin friction (kN/m^2)	100	none
Smith damping (sec/m)	0.35	
Quake (mm)	2.50	
End resistance		
Point damping (sec/m)	0.05	0.01
Point quake (mm)	2.50	3.24
Point resistance (kN)	530.0	500.0



* values to be selected by optimisation procedure

FIGURE 3. Data for Examples 1 and 2

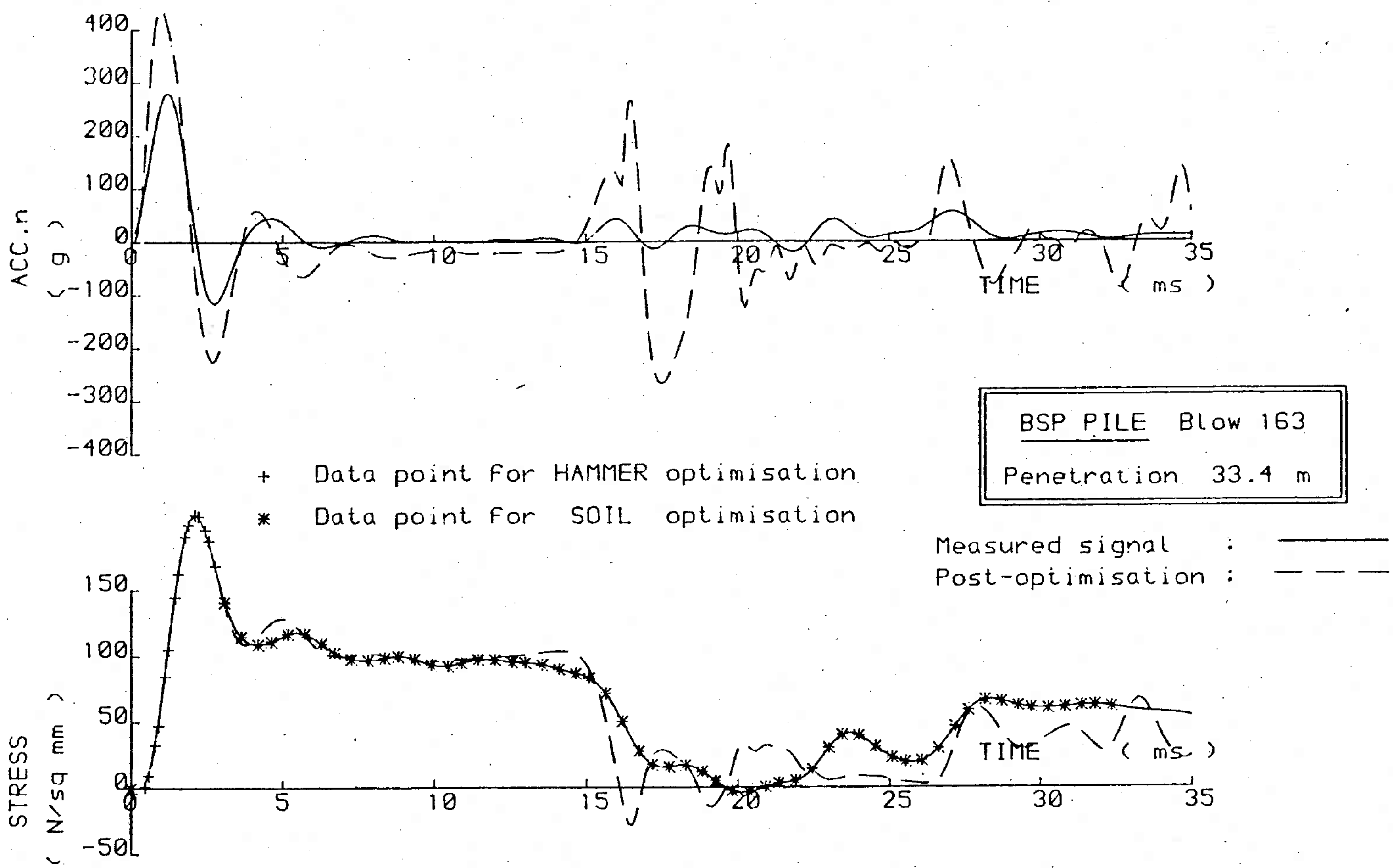


HAMMER PARAMETERS IMPACT VEL. = 4.49 m/sec CUSH. STIFFNESS = 4508. kN/mm COEFF. OF RES. = .98

TOE ELEMENT SOIL VALUES DAMPING = .25 sec/m QUAKE = 2.44. mm SKIN FRICTION = 99.5 kN/sq m

ASSUMED DRIVING RESISTANCE = 5309. kN SET PER BLOW = 4.2 mm

Figure : 4 The solution of EXAMPLE 1



HAMMER PARAMETERS IMPACT VEL. = 4.24 m/sec CUSH. STIFFNESS = 9202.. kN/mm COEFF. OF RES. = .77

TOE ELEMENT SOIL VALUES DAMPING = .26 sec/m QUAKE = 3.24. mm SKIN FRICTION = 99.2 kN/sq m

ASSUMED DRIVING RESISTANCE = 6827. kN SET PER BLOW =31.3 mm.

Figure : 5 The solution of EXAMPLE 2