Survey of Methods for Computing the Power Transmission of Vibratory Hammers[1]

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Abstract

This paper is a survey of analytical methods used to calculate the power consumption and transmission of vibratory hammers used in the installation of piles. The paper discusses the parameters, derivation, and comparative usefulness of various methods of computing the power consumption of these machines. The paper also discusses the importance of torque calculations as well as power calculations. The power consumption of vibratory hammers is important because a) many of the existing methods for estimating the resistance and/or bearing capacity of the piles use power consumption as a parameter, and b) methods being developed to determine the bearing capacity and drivability of piles driven by vibration will probably use these methods. Suggestions for further research in this field, including factors to consider in modeling power transmission and consumption, are set forth.

Introduction

When analytical methods of any kind were first applied to the impact hammer, the most important hammer parameter that was taken into consideration was the energy output of the impact hammer. When the Engineering News formula was first proposed, drop and the new air/steam hammers were in use; the rated striking energy was the product of the height of the drop and the weight of the ram, as given by the equation

$$E_r = g m_r h/1000 \quad (1)$$

where E_r = rated striking energy, kJ
g = acceleration due to gravity = 9.8 m/sec
m_r = mass of the ram, kg
h = height of the ram drop, m

[1]Originally appeared in the Second August Issue 1994 of *Pile Buck*, published by Pile Buck Inc., Jupiter, FL.

Although the Engineering News and the other dynamic formulae are now falling into disuse, the striking energy of the hammer is still the single most important quantity of an impact hammer. This is because the kinetic energy of the ram is transformed during impact into the dynamic force that moves the pile.

On the surface, the analogous quantity for the vibratory hammer is the eccentric moment of the rotating unbalanced weights; this is computed by the formula

$$K = mr \quad (2)$$

where
K = eccentric moment, kg-m
m = mass of eccentric weight(s), kg
r = moment arm for eccentric weight(s), m

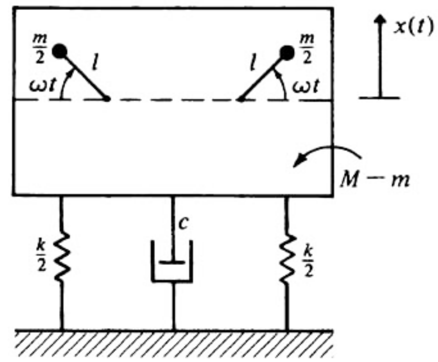
However, the two quantities are not really analogous because the interaction of the pile and the soil is different for impact driven piles and vibrated ones. Although the vibration effects experienced during impact can and do result in soil relaxation (and subsequent freeze) in some soils, vibratory hammers rely on changes in the soil properties during vibration. This allows the static force to push the pile into the soil. The static force is the only net downward force that exists in a vibratory hammer; the force created by the eccentric moment of a vibrator hammer is bidirectional, as opposed to the unidirectional force created by an impact ram. The same problems arise when we consider the ram force of an impact hammer as related to the dyamic force of a vibratory hammer.

At the present time there is no accepted method to determine the bearing capacity of a pile driven solely by vibration. If any solution is to be arrived at to solve this problem, all of the relevant parameters must be properly identified, quantified, and properly included in the solution. One parameter that has frequently appeared in solutions is the energy transfer rate, or power, into the vibratory system, and the derivation of this quantity is the first subject.

Derivation of Power Formulae for Simple Systems

Consider the system shown in Figure 1, where a mass is excited by counter rotating eccentrics similar to those found on a vibratory hammer.

Figure 1. Schematic of Vibratory System (after Meirovitch (1975))



The system is connected to an infinite mass by a spring and dashpot system. According to Meirovitch (1975), the equation of motion of the system is

$$M x''(t) + c x'(t) + k x(t) = m r \omega^2 \sin(\omega t) \quad (3)$$

where M = total mass of system, kg
ω = rotational speed of eccentrics, rad/sec
c = viscous dampening of system, N-sec/m
k = spring constant of system, N/m
t = time, sec
x = displacement, sec

If we first define the resonant frequency of the system as

$$\omega_n = \sqrt{(k/M)} \quad (4)$$

where ω_n = resonant frequency of the system, rad/sec, the viscous damping factor as

$$\zeta = c/(2\ M\ \omega_n) \qquad (5)$$

where ζ = viscous damping factor,

the phase angle as

$$\phi = \arctan(2\ \zeta\ (\omega/\omega_n)/(1-(\omega/\omega_n)^2)) \qquad (6)$$

and the magnification factor as

$$|H(\omega)| = 1/\sqrt{\{[1-(\omega/\omega_n)^2]^2 + [2\ \zeta\ (\omega/\omega_n)]^2\}} \qquad (7)$$

where $|H(\omega)|$ = magnification factor,

we can then solve Equation (3) to

$$x(t) = (K/M)\ (\omega/\omega_n)^2\ |H(\omega)|\ \sin(\omega t-\phi) \qquad (8)$$

Following both Meirovitch (1975) and O'Neill et.al. (1989), the energy used during a cycle of vibration is given by the equation

$$\Delta E_{cyc} = \int_0^{2\pi/\omega} F\ x'\ dt = \int_0^{2\pi/\omega} K\ \omega^2 \sin(\omega t)\ x'\ dt \qquad (9)$$

where F = exciting force, N
ΔE_{cyc} = cycle energy, J

Differentiating Equation (8) for velocity and then substituting and integrating Equation (9), we have

$\Delta E_{cyc} = (K^2\ \omega^2/M)\ (\omega/\omega_n)^2\ |H(\omega)|\ \pi \sin(\phi)$ . . . . (10)

Multiplying this cycle energy by the number of cycles per unit time $\omega/2\pi$, we obtain the power

$N = (K^2\ \omega^3/M)\ (\omega/\omega_n)^2\ |H(\omega)| \sin(\phi)/2$ . . . . (11)

where N = Power, W

If we define the quantity

$N_o = K^2\ \omega^3/M$ . . . . (12)

where N_o = Baseline power, W

and the general equation

$N = \alpha\ N_o$ . . . . (13)

where α = power coefficient

then Equation (11) can be restated as

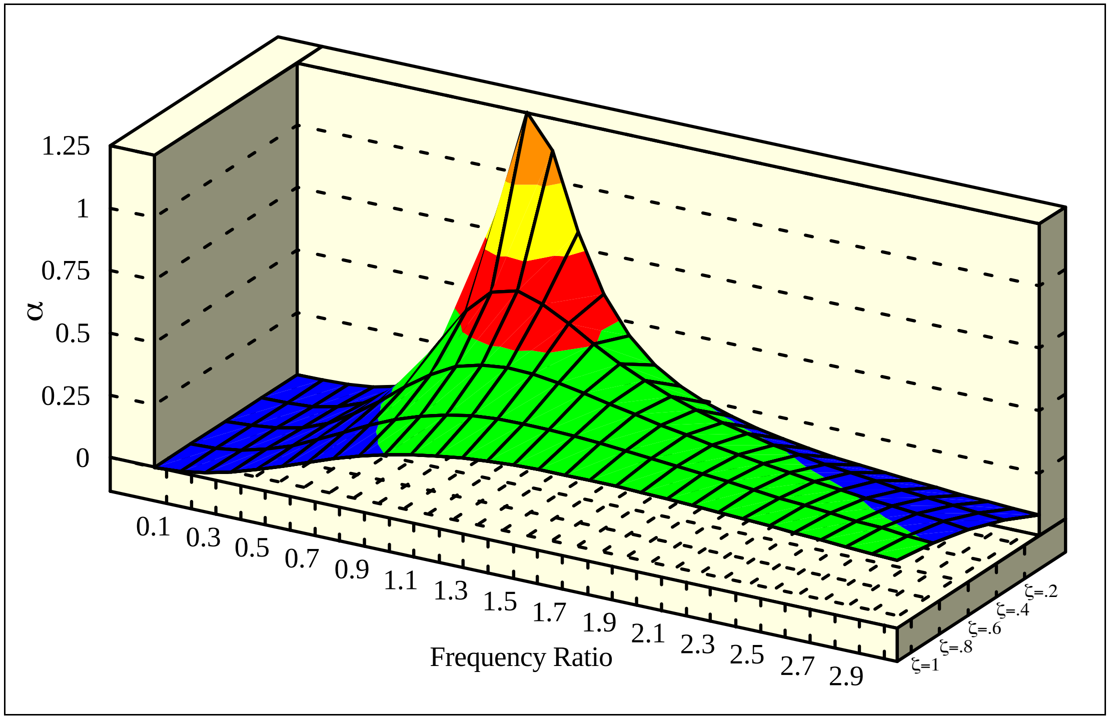
$N = N_o\ (\omega/\omega_n)^2\ |H(\omega)| \sin(\phi)/2$ . . . . (14)

and for this case

$\alpha = (\omega/\omega_n)^2\ |H(\omega)| \sin(\phi)/2$ . . . . (15)

A plot of α for various damping coefficients and frequency ratios is shown in Figure 2. A similar derivation is given in Schmid and Hill (1966).

Figure 2 Plot of α for various damping coefficients and frequency ratios



Equation (13) is a very simple formulation of the power consumption of a vibrating system as a function of eccentric moment, system mass, and rotational speed, all multiplied by a constant. Although the constants presented here are theoretical, the possibility exists for this constant to be formulated experimentally.

The instantaneous torque applied to the eccentrics to maintain constant angular velocity is

$$T_{inst} = (\omega/\omega_n)^2 \, |H(\omega)| \, (\sin(2\omega t-\phi) - \sin(\phi)) \, K^2 \omega^2/(2 M) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

where T_{inst} = Instantaneous torque, N-m

Equation (16) shows that the torque required to maintain the eccentrics at a constant angular velocity varies throughout the rotation of the eccentrics. Under such a condition, the only way to maintain the angular velocity as a constant (a general assumption with vibratory hammers) is for either a) the motor to exactly match the torque changes in its output during an eccentric rotation, or b) the rotational inertia of the eccentrics to be infinite, in which case the hammer could not be started. Since (a) is unlikely (especially for units with hunting tooth pinions) and (b) is impossible, it is reasonable to say that there is variation in the angular velocity of the eccentrics as they rotate. The effects of this variation have not been adequately addressed; however, this variation is potentially important because variations in the angular velocity is the only method by which deficiencies in the power transmission can manifest themselves in the physical system.

Special Case for Simple Systems -- c=0

Although the system described above and the derivation of its equations of motion and energy consumption can be very instructive, they pose two difficulties:

1) Real systems are more complex than described. This is because a) real systems have distributed mass and elasticity, such as is modeled in the wave equation, b) pile/soil interaction is complex, being both nonlinear and variable along the length of the pile and between the pile shaft and toe, and c) losses are inherent in the power transmission of the vibratory machine itself. Item (c) will be addressed below.

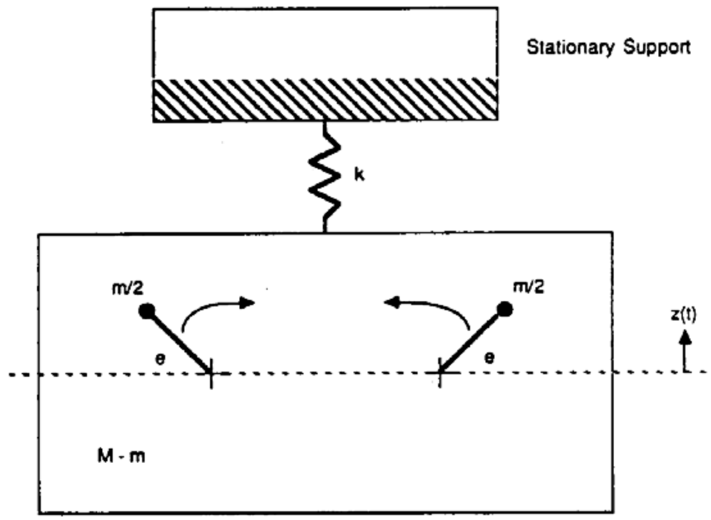
2) It is difficult to make ready generalizations from the derivations given because of the large number of variables. Such a problem defeats the whole purpose of simple system derivations.

To deal with these problems, researchers who have used these systems for one reason or another have either introduced simplifications or empirical factors. The former are achieved by eliminating the dampening or the elasticity. We first need to consider the case where the dampening is eliminated, and c=0.

In the case of these systems, except at resonance if $c=0$, then $\phi=0$ (from Equations (5) and (6)), and thus $\alpha=0$ (from Equation (15).) Thus there is no net power consumption in the system. This makes sense because all of the elements involved are either mass or elastic elements and are thus energy conservative.

Having said this, O'Neill et. al. (1989) derive the theoretical power formulation they use in conjunction with the experimental data using a system without dampening as shown in Figure 3. It is essentially an inversion of the one shown in Figure 1.

Figure 3 Simplified vibrating system (after O'Neill et. al. (1989))



For this system the theoretical power the vibrator generates is computed by the equation

$$N = \theta \cdot (4000 \cdot W_{st} + 2 \cdot K \cdot \omega^2 \cdot (1 + \theta^2/(\theta^2 + \theta_n^2))) \cdot (K \cdot \theta^2/(1000 \cdot M \cdot (\theta^2 + \theta_n^2))) \quad (17)$$

where θ_n = natural frequency of suspension with respect to the vibrator mass, Hz

Rearranging this with the notation used in this paper, this equation is

$$N = (K^2 \omega^3 / M)(|H(\omega)|/\pi)(2W /(K \omega_n^2) + (\omega/\omega_n)^2(1 + (\omega/\omega_n)^2|H(\omega)|)) \quad (18)$$

and this is related to the power actually delivered to the pile top by the formula

$$N' = (0.25+0.063\, n)\, N \quad (19)$$

where N' = power actually delivered to the pile top, kW
n = peak acceleration of the vibrations, g's

The peak acceleration can be computed by the formula

$$n = (3.54-2.186 \cdot D_r) \cdot (8.99+2.76 \cdot d_{10}) \cdot ((39.37 \cdot V_{sys})^{(1.71-s'_h/85.1)}) \quad (20)$$

where s'_h = horizontal effective stress, kPa
D_r = relative density
d_{10} = grain size, mm

Equations (19) and (20) are empirical correlations from laboratory tests.

Special Case for Simple Systems -- k=0

If we now consider the case where k=0, Equation (11) then reduces to

$$N = (\sin(2\phi)/4)\, (K^2\, \omega^3 / M) \quad (21)$$

and for the general Equation (13)

$$\alpha = \sin(2\phi) / 4 \quad (22)$$

A plot of α versus the various phase angles is shown in Figure 4.

Figure 4 Values of α versus phase angle

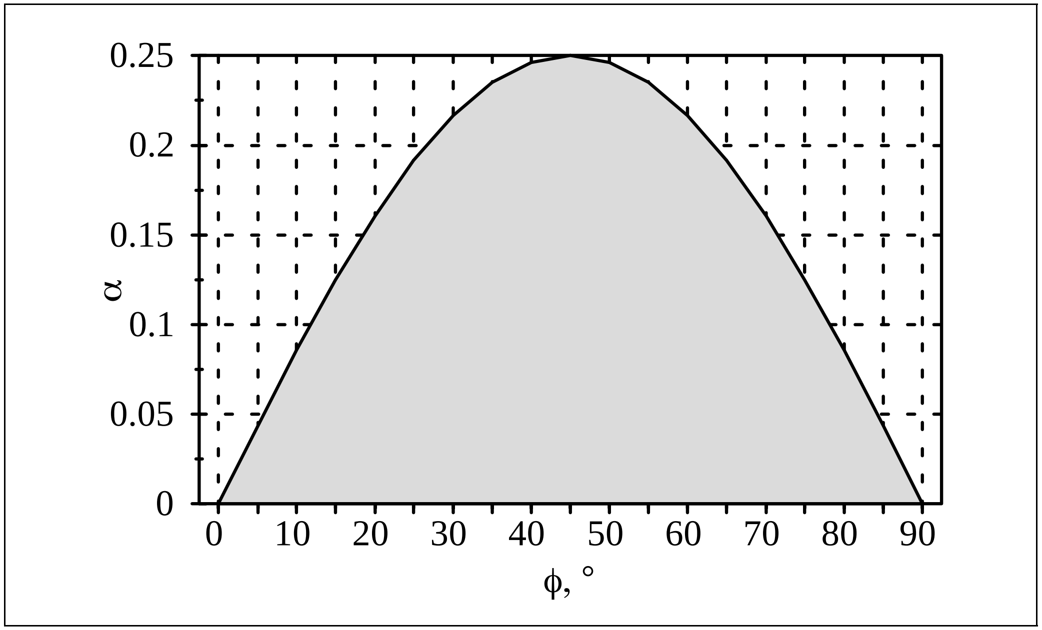


Figure 4 and Equation (22) show that the maximum value of α is 0.25 when φ = 45°. For this model, therefore, the maximum power requirement possible is given by the equation

$N = 0.25\ K^2\ \omega^3 / M$ . . . . (23 -- φ = 45°)

This model has the possibilities of some degree of realism since a) the liquefaction of the soil is an essential part of vibratory driving, and a possible source of a fluid type dampener, and b) the possibility to model energy radiation into the soil as a dashpot, as is done in the wave equation. Equation (23) is also given in Erofeev et. al. (1985). He also recommends that, since this formulation does not take into account energy losses due to ground vibration, the value for this maximum power be increased by 10-20%.

Using this formula and taking into consideration actual operating conditions, Tseitlin et. al. (1987) state that the power necessary for a vibratory hammer is computed by the equation

$N_{дв} = K\theta^3(.079K/M + 3.2\times10^{-6}D)$ . . . . (24)

where $N_{дв}$ = Power Consumed by the Vibrator, kW
D = Diameter of Bearing Race, mm
θ = Frequency of the machine, Hz

Equation (24) was developed with the following assumptions:

a) Efficiency of the power transfer from motor to vibration exciter is 90%.
b) Coefficient of rolling friction in the bearings is 0.1%.
c) Of the power actually sent into the soil, 15% of it is lost in the soil mass.

Rearranging equation to more closely correspond with Equations (11) and (21), we have

$N = 0.318\ K^2\ \omega^3 / M + 1.29\times10^{-5}\ D\ K\ \omega^3$ . . . . (25)

where the second term accounts for power consumption due to bearing losses.

Concerning the torque, Warrington (1989b) derives this formula for this case. If we take this formulation, rearrange the variables, and change the sign convention of the phase angle, the instantaneous torque required to maintain rotation of the eccentrics at a constant velocity is

$T_{inst} = \cos(\phi)\ (\sin(2\omega t-\phi) - \sin(\phi))\ K^2\ \omega^2/(2\ M)$. . . . . (26)

which can likewise be derived from Equation (16).

Energy Methods for Vibratory Hammers

In examining the formulae above, it is important to note that the power consumed by a vibratory hammer is not a fixed quantity, but depends upon a wide variety of variables, such as the static and dynamic weight of the system, the characteristics of the suspension, the frequency and rotational stability of the eccentrics, losses due to operations such as clamping and pile splicing, and the internal dampening of the materials used in the system. Factors of the soil system also include the elastic and plastic properties of the soil and the various types of dampening that take place in soil, such as viscous and radiation dampening. This fact is most succinctly observed by Goncharevich and Frolov (1985), who state the following:

> The power which is required to operate the vibratory machine in the given regime and the power which can be transmitted by a vibrator of a specific type are determined by a whole complex of factors: vibrator parameters, machine characteristics, and the acting loads in the machine. It is not possible to impart additional power to the vibratory machine by simply increasing motor output. Each vibratory machine consumes strictly determined power, whose value is dependent on a whole set of factors acting in the vibratory machine-vibration exciter-load system.

This variability of power consumption has led to the consideration of using the energy used by a vibrating system as a parameter in determining either the resistance of the pile, its bearing capacity, or both. These methods are referred to as energy methods. Warrington (1989a) describes some of these methods. The basic pattern formula for these methods is given by the equation

$R_u V_{sys} = N + (W_{dyn}+W_{st})\ V_{sys}$ . . . . (27)

This equation can be reformulated in two ways; first, to compute penetration velocity,

$V_{sys} = N\ (R_u-W_{dyn}-W_{st})$ . . . . (28)

and for bearing capacity of the pile,

$R_u = W_{dyn}+W_{st}+N\ V_{sys}$ . . . . (29)

where R_u = Soil Resistance, kN
V_{sys} = Pile Penetration Velocity, m/sec
N = Vibratory power, kW
W_{dyn} = Dynamic Weight, kN
W_{st} = Static Weight, kN

Two examples of actual application of this theory are given; the first is that of Davisson (1970). He applied this method to the Guild-Bodine resonant vibratory hammer. The formula he developed is given by the equation

$$R_u = (N+(W_{dyn}+W_{st})\cdot V_{sys})/(V_{sys}+\theta\cdot S_l / 1000) \quad (30)$$

where S_l = Soil Loss Factor, mm/cycle

and for penetration velocity, we can rearrange it to read

$$V_{sys} = (N-R_u\cdot\theta\cdot S_l / 1000)/(R_u-W_{dyn}-W_{st}) \quad (31)$$

Another energy method formula is the Snip (or more properly "Tsnip," named after the Central Research and Testing Station in Ivanteevka, Russia) formula; this is given by O'Neill et.al. (1989) as

$$R_u = (\lambda-30000\cdot V_{sys}/(A_o\,\theta))(245\,N/(A_o\,\theta)+W_{dyn}+W_{st}) / F \quad (32a)$$

and

$$R_u = (245\,\lambda\,N / (A_o\,\theta)+W_{dyn}+W_{st}) / F \quad (32b)$$

where λ = Soil Coefficient
F = factor of safety (generally F=2)

Equation (32a) is for penetration velocities of 0.5-1.67 mm/sec, and (32b) for 0.05-0.5 mm/sec.

Although these methods are based on actual field and/or laboratory testing, their main weakness is their difficulty in taking into account all of the variables of a complex system, especially as they relate to soil interaction. For example, one *a priori* assumption with energy methods is that the power required to insert the pile into the soil increases with resistance. However, with impact hammers we find that, as the resistance increases, the energy the hammer transmits to the pile actually decreases because of rebound effect. This possibility needs to be dealt with as it occasionally appears in the field.

Power Transmission in Vibratory Hammers

Although it is unlikely that a final solution to the problem of the relationship between the performance of vibratory systems and the bearing capacity of the piles will be found in a simple energy formula, the monitoring of the power consumption of vibratory hammers will remain an important part of job site control of pile driving by vibration. It is therefore important to understand the mechanism by which energy is transmitted over time from the prime mover of a system to the pile.

In the U.S. hydraulic power transmission systems predominate in vibratory hammers, so these systems will be considered in this discussions. Energy losses in electrically driven systems, or even those which directly drive the eccentrics (such as the Guild-Bodine resonant driver,) are similar.

In looking at the various components of the system, the following items are worthy of note:

Engine: An engine transmits torque to the pump by generating torque at a certain speed. This torque is not constant with speed; indeed, by definition an internal combustion engine has zero torque at zero speed. This is especially relevant in systems which use fixed displacement pumps. In these systems this can limit the power significantly when the engine speed is reduced, which is necessary to reduce the rotational speed of the eccentrics.

Gearbox: Many hydraulic pumps to not operate either at all or at their optimal utilization at conventional engine speeds, so many but not all systems use a gearbox to both change the rotational speed of the power transmission and to allow the installation of other power transmission items (such as clamp pumps.) Lynwander (1983) states that a three stage industrial gear box can have an efficiency of 94-97%, but this can vary with lubrication and the condition of the gears and bearings.

Pump: The pump converts the energy in the torque of the rotation shaft to pressurized hydraulic fluid. In addition to the usual mechanical losses, there are inevitably volumetric losses due to leakage in the compression of the fluid by the pistons, vanes, or gears. These losses can vary with hydraulic temperature and viscosity. If a variable displacement system is used, the engine can be run at its full speed during any kind of operation, but the power decreases in a linear fashion with the displacement decreasing from full, which means that full power availability to the system is impossible under less than full speed.

Hoses, Valves, Filters and Connections: Once the fluid is pressurized, it flows to the motor on the hammer through the hoses and other in line hydraulic components. Fluid frictional losses in any fluid duct is inevitable, and hoses are no exception, and these can vary with the length of the hose installed. These losses are added to by losses in valves, fluid elbows, filters (whose losses can vary with the amount of dirt in them,) and especially quick disconnects, which are a significant source of losses in mobile hydraulic systems.

Motor: The fluid energy of the hydraulic system must be converted back into torque to turn the eccentrics, and the motor accomplishes this task. Most hydraulic vibratory hammers use fixed displacement hydraulic motors, mostly because variable displacement motors have not been proven to operate for any extended period in this environment. As is the case with fixed displacement pumps, this means that the available energy varies proportionally with the speed because, for a given hydraulic fluid pressure, the torque is essentially constant, as given by the equation

$$T = V \Delta p / 2\pi \quad (33)$$

where V = displacement volume of pump, cm^3
Δp = hydraulic fluid pressure differential between pressure and return ports, MPa

This torque is more or less constant over a rotation of the eccentrics; the interaction of this with the varying torque of Equation (16) is something that has not to date been addressed.

Return Losses: Equation (33) shows that the energy available for the vibratory hammer is defined by the differential between the input and output pressures of the motor; thus, it is important to consider the losses in the oil return system as well, as these set this lower limiting pressure. These losses are similar to the losses experienced in the supply line, and they include losses due to the oil cooler and whatever brake valve is in the return system.

Gears: Once the motor has converted the energy of the hydraulic fluid back into torque, it must be transmitted first to the eccentrics and then the pile. This is done through the power transmission system of the vibratory hammer itself. Some vibratory hammers do not have gear transmission or synchronization systems, instead powering each eccentric individually and relying on the amplitude of vibrations to synchronize the eccentrics. These hammers do not experience gear losses, but losses due to deficiencies in synchronization when the amplitude is minimized. For those systems with gears, as is the case with power pack gearboxes the vibratory hammer gears experiences losses due to meshing, wear, and with splash lubrication systems (common in vibratory hammers) the churning of the oil, which varies widely with the level of oil in the system and the temperature and viscosity of the oil.

Bearings: Anti-friction bearings are virtually universal on vibratory hammers, but they are not free of losses. The most common formula to estimate these losses is the SKF formula, given by the equation

$$T_{bearing} = f_f F (d_b / 2) / 100 \quad (34)$$

where $T_{bearing}$ = bearing torque due to friction, N-m
f_f = coefficient of friction = 0.0011 for cylindrical roller bearings
F = load on bearing, N
d_b = bore diameter of the bearing, m

as reported in Ragulskis and Yurkauskas (1985). In the same source another more general formula is given; it is

$$T_{bearing} = 0.04 D_o + (1.5 A + 1.25 R) \delta D_o / d_b \quad (35)$$

where D_o = diameter of the inner race ring, m
A = axial bearing load, N
R = radial bearing load, N
δ = friction coefficient = $5\text{-}10\times10^{-6}$ m

It is very important to keep in mind that a great deal of variation in these figures has been obtained, and that the same variables that act on gear losses also apply to the bearings, especially relative to the lubricant.

Clamp: The clamps used on vibratory hammers are essentially friction clamps, with assistance from penetration of the pile by the teeth of the clamp jaws. The coefficient of friction can widely vary, and also energy can be lost in the heating of the pile top due to both clamp slippage and hysteresis of the pile material.

Conclusions

Based on the analysis of the information given above, the following conclusions can be drawn:

1) The nature of energy transfer from driving machine to pile is fundamentally different between impact hammers and vibra-

tory ones, and thus requires different treatment both theoretically and in field measurement and evaluation.

2) Although the simple, single degree of freedom model is by no means comprehensive in its consideration of relevant parameters, it can be helpful both in understanding the system and in being used as a baseline for actual correlations between hammer performance and pile capacity or resistance. The general function of $\alpha K^2 \omega^3 / M$ can be especially helpful in this task.

3) Energy methods represent a practical application of power transmission and measurement to the determination of pile resistance or capacity. They may be useful as a job site control method (much as blow count or enthru is with impact hammers;) they are not as useful for the estimation of pile capacity before driving.

4) As is the case with any mechanical device, vibratory hammers have both power generation and transmission characteristics that do not insure a consistent transmission of power from the prime mover to the pile. The use of measuring devices at the power source, such as pressure gauges and engine power measuring devices, cannot be used to measure the power transmitted to the pile because of the intermediate losses; indeed, these devices are part of the diagnostic system of a hammer to indicate trouble.

5) The only way to accurately measure the power transmitted to a pile is to measure it at the pile itself. This is also true with impact hammers, but it is more important with vibratory ones because, while impact hammers only interact with the pile at one part of their stroke (namely the impact itself, and the combustion chamber interaction with diesel hammers,) vibratory hammer eccentrics constantly interact with the soil.

Acknowledgements

The author would like to express his thanks to Mr. Leonard C. Albert of the Church of God Department of Lay Ministries for his help in the preparation of the graphics for this paper and other publications. The author would also like to express his deep appreciation to Messrs. L.V. Erofeev and D.A. Trifonov-Yakovlev of the VNIIstroidormash institute in Moscow, Russia, for their review of this paper, and especially their helpful comments concerning the formulation of power for springless systems. To them, he would like to say, "Друг любит во всякое время и, как брат, явистя во время несчастия" (Prov. 17:17)

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Appendix

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